Analysis of Dual Use of a Trunk Group with Reservation

By

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Abstract

This paper considers the traffic aspects of a multitrunk communication system under a dual use of trunks for both direct and store-and-forward (S/F) traffic, in which one trunk is held in reserve for direct traffic. A method of computation, some results for trunk efficiencies and blocking probabilities of direct traffic and also of S/F traffic are presented. Numerical results are obtained for the various cases. The numerical results show that the trunk efficiency of the system can be twice as high as that of the system with no S/F traffic. This is possible without any significant impairment of the grade of service to direct traffic due to mixing S/F traffic. Moreover, in the system, the grade of service of S/F traffic is not significantly impaired due to the trunk held in reserve for direct traffic.

1. Introduction

Transmission systems with dynamic matching between various information sources and a transmission line have been intensively studied ever since Filipowsky and Scherer (1) indicated the importance of these systems. Hasegawa *et al.* (2) investigated this type of system, aided by various information sources and one transmission trunk. Gimpelson (3) considered a system with wide- and-narrow-band traffic sources and transmission trunks. In his system, the narrow band traffic was under the Lost-Calls-Cleared assumption, and the wide band traffic was allowed to wait in a queue when all trunks were occupied. A system similar to the above was studied in which the wide band traffic had preemptive priority and the narrow band traffic had to wait in a queue on interruption (4).

Otterman (5, 6) studied a system with direct traffic and store-and-forward (S/F)

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traffic sources and transmission trunks. In his system, the direct traffic was under the Lost-Calls-Cleared assumption but a specified number of trunks were reserved for the direct traffic, while the S/F traffic was allowed to wait in a queue when trunks were occupied. However, Otterman's works were based on the assumption that the arrival rate of S/F traffic was so high that there always existed a queue of S/F traffic (5); or that the arrival rate of S/F traffic was so low that the occupancy of trunks by direct traffic was unaffected by the presence of S/F traffic (6). These assumptions appear to be naive in practical applications.

In this paper, omitting the above mentioned assumptions, the authors study the transmission system with dynamic matching between transmission trunks and direct and S/F traffic sources. The direct traffic is under the Lost-Calls-Cleared assumption, but one trunk is reserved for direct traffic whenever possible. S/F traffic waits in a buffer when all trunks (except a trunk in reserve) are occupied. However, S/F traffic is under the Lost-Calls-Cleared assumption when the queue length is over a specified level. A method of computation, some results for trank efficiencies and blocking probabilities of direct traffic and of S/F traffic are presented. Numerical results are obtained for the cases of two to eight trunks and a buffer size of two to ten S/F traffic calls.

In order to clarify the features of the system, the authors discuss another system under the dual use of trunks for both direct and S/F traffic without any reserved trunk. This is on the assumption that S/F traffic is permitted to have access to trunks whenever idle trunks exist. The other assumptions are exactly the same as those in the above system. The numerical values of the blocking probability of direct and S/F traffic in the system are calculated. These results are compared with those of the system with one trunk held in reserve; and several useful conclusions are derived.

2. Model and Symbols

In communication systems, it is of great importance to transmit as much traffic as possible through a given number of trunks. However, in many communication systems it is also required to handle different types of information. To cope with these problems, it is effective to adopt a system under the dual use of trunks for both direct and S/F traffic. That means a system in which not only direct traffic but also S/F traffic can be transmitted through trunks. Direct traffic consists of calls which require immediate transmission to their own destinations. S/F traffic consists of calls which are stored at, or near, a switching center and then transmitted to their own destinations.

In the following, we explain the system in detail. It is assumed that any occupancy of the trunks is always monitored. For direct traffic, one trunk is always reserved whenever possible. When a direct traffic call arrives and has access to the reserved trunk, one of the idle trunks, if one exists, is reserved for direct traffic. If none exist, the first trunk that becomes idle is reserved for direct traffic. Direct traffic is under the Lost-Calls-Cleared assumption. That is, if direct traffic arrives and finds all trunks occupied, the direct traffic is lost and will not reappear in the system. S/F traffic is permitted to have access to one of the idle trunks, with the exception of the reserved trunk for direct traffic. S/F traffic transmitted is not interrupted by the arrival of direct traffic. When S/F traffic is not permitted to have access to trunks, it waits in the buffer. However, if S/F traffic arrives when the buffer is fully occupied, it is under the Lost-Calls-Cleared assumption.

Now we consider the case where the number of subscribers of both traffics is sufficiently large. Then, we can assume that both calls of direct and S/F traffic are independently generated by Poisson processes with arrival rates λ_1 and λ_2, respectively. Holding times for both types of calls are also assumed to be independent, and exponentially distributed with service rates μ_1 and μ_2, respectively. Offered loads for both calls are represented by λ_1/μ_1 and λ_2/μ_2, respectively; and the total offered load is $\lambda_1/\mu_1+\lambda_2/\mu_2$.

The following is a list of all symbols used in this paper.

s; Number of trunks in reserve.

c; Total number of trunks in the system.

M; Buffer size.

λ_1; Arrival rate of direct traffic.

λ_2; Arrival rate of S/F traffic.

μ_1; Service rate of direct traffic.

μ_2; Service rate of S/F traffic.

T_1; Average holding time of direct traffic.

T_2; Average holding time of S/F traffic.

r; Ratio of the average holding time of direct traffic to the average holding time of S/F traffic, T_1/T_2.

a_1; Offered load of direct traffic.

a_2; Offered load of S/F traffic.

i; Number of trunks occupied by direct traffic.

j; Number of trunks occupied by S/F traffic.

k; Number of S/F traffic calls in the queue.

(i, j, k); State of i direct traffic calls being transmitted, j S/F traffic calls being transmitted and k S/F traffic calls in the buffer queue.

$P(i, j, k)$; The probability of state (i, j, k).

3. Mathematical Analysis

To begin with, let us explain the probability of a state transition caused by the arrival or termination of direct or S/F traffic. The probability that one direct (S/F) traffic call arrives in a small interval ΔT is given by $\lambda_1\Delta T$ ($\lambda_2\Delta T$). When n direct traffic calls are transmitted, the probability that one transmission of the direct traffic is over within ΔT is given by $n\mu_1\Delta T$. The same probability for the case of S/F traffic is given by $n\mu_2\Delta T$. According to the list of symbols, (i, j, k) is represented as the state where i is the number of direct traffic calls transmitted, j is the number of S/F traffic calls transmitted and k is the number of S/F traffic calls waiting in the buffer. Therefore, $i+j$ is the number of trunks occupied by direct and S/F traffic. In the following, we explain the transition mechanism of the states by Tables I and II, and derive equations which are satisfied by the probabilities of the states in equilibrium.

3.1 One Trunk in Reserve: s=1

We deal with the transition mechanism of the states for the system with one trunk held in reserve.

Case A: $i+j=c$.

We consider the case where $i+j=c$, that is, all trunks are occupied. When a new direct traffic call arrives, the state is not changed because of the Lost-Calls-Cleared assumption. When a new S/F traffic call arrives and the buffer is not fully occupied, it waits in the buffer and increases the buffer queue length k by one. When a new S/F traffic call arrives and the buffer is fully occupied, the state is not changed because of the Lost-Calls-Cleared assumption. On the other hand, when the transmission service of direct or S/F traffic is over, the new idle trunk is immediately held in reserve for direct traffic; and the S/F traffic in the buffer is not permitted to have access to the idle trunk. Thus, this termination of service decreases $i+j$ by one.

Case B: $i+j=c-1$.

We consider the case where $i+j=c-1$, and therefore one trunk is held in reserve for direct traffic. When a new direct traffic call arrives, it is permitted to have access to the trunk in reserve, and this increases the number of trunks occupied $i+j$ by one. When a new S/F traffic call arrives, it is not permitted to have access to the trunk. Consequently, it waits in the buffer, which thereby increases the buffer queue length k by one, if the buffer is not fully occupied. However, if the buffer is fully occupied, the state is not changed because of the Lost-Calls-Cleared assumption. When the transmission service of direct or S/F traffic is over, and a buffer queue exists, one of S/F traffic calls in the buffer is permitted to have access to the new idle trunk. This termination decreases the buffer queue length k by one and does not change the number of trunks occupied $i+j$. On the other hand, when the transmission service of direct or S/F traffic is over, and a buffer queue does not exist, another trunk (excepting the trunk in reserve) becomes idle and this termination decreases the number of trunks occupied $i+j$ by one.

Case C: $i+j \leq c-2$.

We consider the case where $i+j \leq c-2$. There are idle trunks (besides the trunk in reserve), and there isn't a buffer queue. Therefore, when a new direct or S/F traffic call arrives, it is permitted to have access to one of the trunks, and this increases the number of trunks occupied $i+j$ by one. When the transmission service of direct or S/F traffic is over, this termination decreases $i+j$ by one.

Arranging the above transition mechanism of states, we obtain Table I. In Table I, the first column shows the conditions on state (i, j, k) before the transition. The second column shows the neighboring states after the transition from state (i, j, k). The third column shows the transition coefficients. For example, in the case $0 \leq i+j \leq c-2$ and $k=0$, Table I shows that the transition coefficient from state $(i, j, 0)$ is: λ_1 through a new arrival of direct traffic (transition to state $(i+1, j, 0)$), λ_2 through a new arrival of S/F traffic (transition to state $(i, j+1, 0)$), $i\mu_1$ through a termination of one of i direct traffic calls in progress (to state $(i-1, j, 0)$) and $j\mu_2$ through a termination of one of j S/F traffic calls in progress (to state $(i, j-1, 0)$).

By equating the transition probability from state (i, j, k) to the transition probability to state (i, j, k), we have the equilibrium state probability equation. The mathematical derivation of these equations is given in Appendix I. One of these equations is de-

Table I Transition Mechanism for The System with One Trunk Held in Reserve

Conditions on i, j, k	Neighbouring States	Transition Coefficients
$i+j \leq c-2,\ k=0$	$i+1, j, 0$	λ_1
	$i, j+1, 0$	λ_2
	$i-1, j, 0$	$i\mu_1$
	$i, j-1, 0$	$j\mu_2$
$i+j=c-1,\ k \geq 0$	$i+1, j, k$	λ_1
	$i, j, (k+1)^*$	λ_2
	$i-1, j+\delta_{ok}, (k-1)^+$	$i\mu_1$
	$i, j-1+\delta_{ok}, (k-1)^+$	$j\mu_2$
$i+j=c,\ k \geq 0$	i, j, k	λ_1
$i \neq 0$	$i, j, (k+1)^*$	λ_2
	$i-1, j, k$	$i\mu_1$
	$i, j-1, k$	$j\mu_2$

where $\delta_{ok}=0$, if $k=0$
$=1$, otherwise,
$x^+=\max(0, x)$
$x^*=\min(x, M)$

pendent on the other equations. Therefore we must add the following condition, that is, the condition of total probability:

$$\sum_{m=0}^{c-2}\sum_{i=0}^{m}P(i, m-i, 0)+\sum_{k=0}^{M}\sum_{i=0}^{c-1}P(i,c-1-i, k)$$
$$+\sum_{k=0}^{M}\sum_{i=1}^{c}P(i, c-i, k)=1. \tag{1}$$

These equations are solved by a sweep out method.

Important quantities for this system can be represented by the following: The average number of direct traffic calls in progress is given by

$$\sum_{m=0}^{c-2}\sum_{i=0}^{m}iP(i, m-i, 0)+\sum_{k=0}^{M}\sum_{i=0}^{c-1}iP(i, c-1-i, k)$$
$$+\sum_{k=0}^{M}\sum_{i=1}^{c}iP(i, c-i, k). \tag{2}$$

The average number of S/F traffic calls in trunks and in the queue is:

$$\sum_{m=0}^{c-2}\sum_{i=0}^{m}(m-i)P(i, m-i, 0)+\sum_{k=0}^{M}\sum_{i=0}^{c-1}(c-1-i+k)P(i, c-1-i, k)$$
$$+\sum_{k=0}^{M}\sum_{i=1}^{c}(c-i+k)P(i, c-i, k). \tag{3}$$

The blocking probability for direct traffic calls is:

$$\sum_{k=0}^{M}\sum_{i=1}^{c}P(i, c-i, k). \tag{4}$$

The blocking probability for S/F traffic calls is:

$$\sum_{i=0}^{c-1}P(i, c-1-i, M)+\sum_{i=1}^{c}P(i, c-i, M). \tag{5}$$

3.2 No Trunk in Reserve: s=0

We consider the transition mechanism and the important quantities for the system under a dual use of trunks with no trunk held in reserve. Table II shows the transition mechanism from the equilibrium state (i, j, k) for this system. The meaning of this Table is the same as that of Table I. The differences of the transition mechanisms between this system and the system with one trunk held in reserve are: the queue of S/F traffic in state (i, j, k), $(i+j=c-1)$ cannot exist in the former system, but can exist in the latter system; state $(0, c, k)$, $(k=0, 1, 2, \ldots, M)$ cannot exist in the latter system but can exist in the former system; the transition from state $(i, j, 0)$, $(i+j=c-1)$ to the state $(i, j+1, 0)$, $(i+j+1=c)$ through a new arrival of S/F traffic is not permitted in the latter system, but is permitted in the former system. The mathematical derivation of the equilibrium state probability equation is given in Appendix II. These equations are solved by the same method used for the system with one trunk held in reserve.

Table II Transition Mechanism for The System with No Trunk Held in Reserve

Conditions on i, j, k	Neighbouring States	Transition Coefficients
$i+j\leq c-1,\ k=0$	$i+1, j, 0$	λ_1
	$i, j+1, 0$	λ_2
	$i-1, j, 0$	$i\mu_1$
	$i, j-1, 0$	$j\mu_2$
$i+j=c,\ k\geq 0$	i, j, k	λ_1
	$i, j, (k+1)^*$	λ_2
	$i-1, j+\delta_{ok}, (k-1)^+$	$i\mu_1$
	$i, j-1+\delta_{ok}, (k-1)^+$	$j\mu_2$
where	$\delta_{ok}=0$, if $k=0$	
	$=1$, otherwise,	
	$x^+=\max(0, x)$	
	$x^*=\min(x, M)$	

Important quantities for this system can be represented by the following: The average number of direct traffic calls in progress is given by

$$\sum_{m=0}^{c-1}\sum_{i=0}^{m} iP(i, m-i, 0)+\sum_{k=0}^{M}\sum_{i=0}^{c} iP(i, c-i, k). \tag{6}$$

The average number of S/F traffic calls in trunks and in the queue is:

$$\sum_{m=0}^{c-1}\sum_{i=0}^{m}(m-i)P(i, m-i, 0)+\sum_{k=0}^{M}\sum_{i=0}^{c}(c-i+k)P(i, c-i, k). \tag{7}$$

The blocking probability for direct traffic calls is:

$$\sum_{k=0}^{M}\sum_{i=0}^{c} P(i, c-i, k). \tag{8}$$

The blocking probability for S/F traffic calls is:

$$\sum_{i=0}^{c} P(i, c-i, M). \tag{9}$$

4. Results and Conclusions

The blocking probabilities derived above were computed by a sweep out method* on a FACOM 230-60 computer system of the Kyoto University Computing Center.

The values of the blocking probability of direct traffic are given in Fig. 1 for the system in which the buffer size is 2, the total offered load is 0.5 c erlangs, r is 100 and one or no trunk is held in reserve with A1 as a parameter. Here A1 represents the percentage of the total offered load which is direct traffic. The blocking probability

* The convergency of solution obtained by the Gauss Seidel method is not suitable for this problem.

of direct traffic in the system with one trunk held in reserve is less than that in the system with no trunk held in reserve. Especially, in the case $c=8$ and A1=10, the ratio of the blocking probability in the system with one trunk held in reserve to that in the system with no trunk held in reserve is 10^{-3}. The blocking probability of direct traffic in the system under the dual use of trunks with no trunk held in reserve decreases as Al increases. The reason for this is that the grade of service to direct traffic with no trunk held in reserve is strongly affected by S/F traffic. The blocking probability in the system with direct traffic only, where the Lost-Calls-Cleared assumption holds, is computed by the Erlang B formula and also shown in Fig. 1. The impairment in

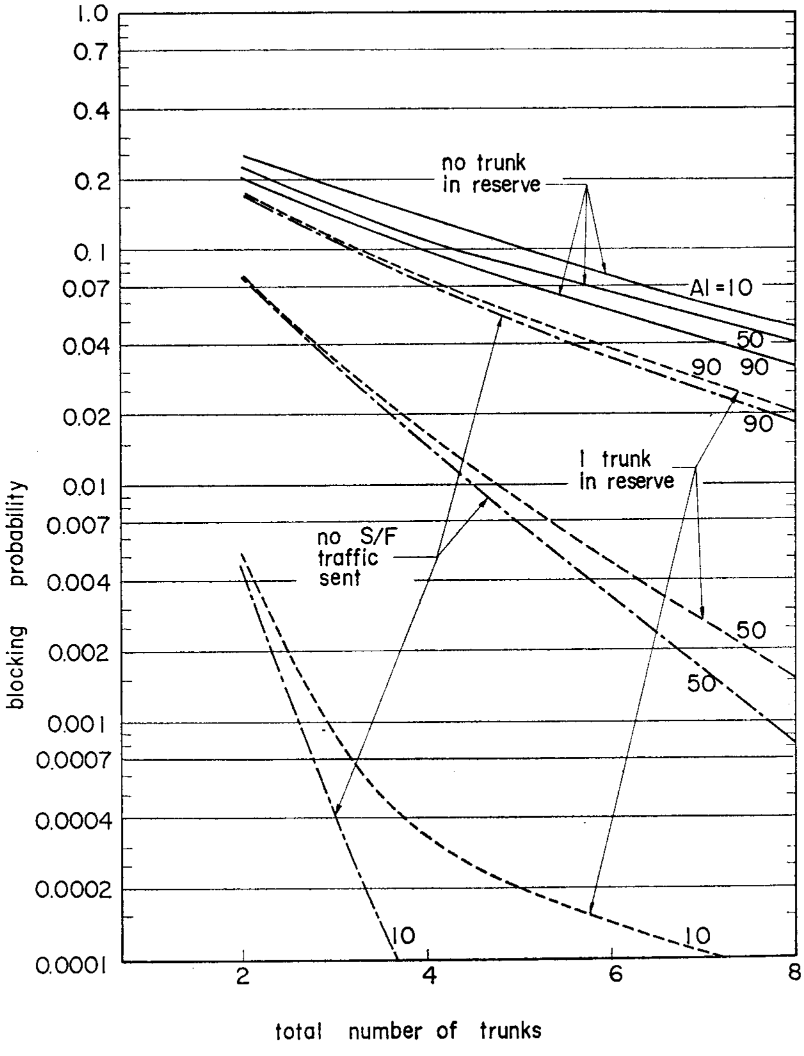

Fig. 1. Blocking probability of direct traffic (0.5 c erlangs total offered load, $r=100$, $M=2$).

the grade of service to direct traffic due to mixing S/F traffic, i.e., the difference between the blocking probability in the system with direct traffic only, and that in the system with one trunk held in reserve, is rather small when one trunk is held in reserve.

The blocking probability of direct traffic is plotted in Fig. 2 for a total offered load of 0.9 c. The blocking probability of S/F traffic is plotted in Fig. 3, using the parameters of Fig. 1. The blocking probability of S/F traffic in the system with one trunk held in reserve is greater than that in the system with no trunk held in reserve. This is because the trunk held in reserve decreases the number of trunks available for S/F traffic. The impairment in the grade of service to S/F traffic is rather small; for example it is only 0.015 in the case Al=10 and c=8. The blocking probability

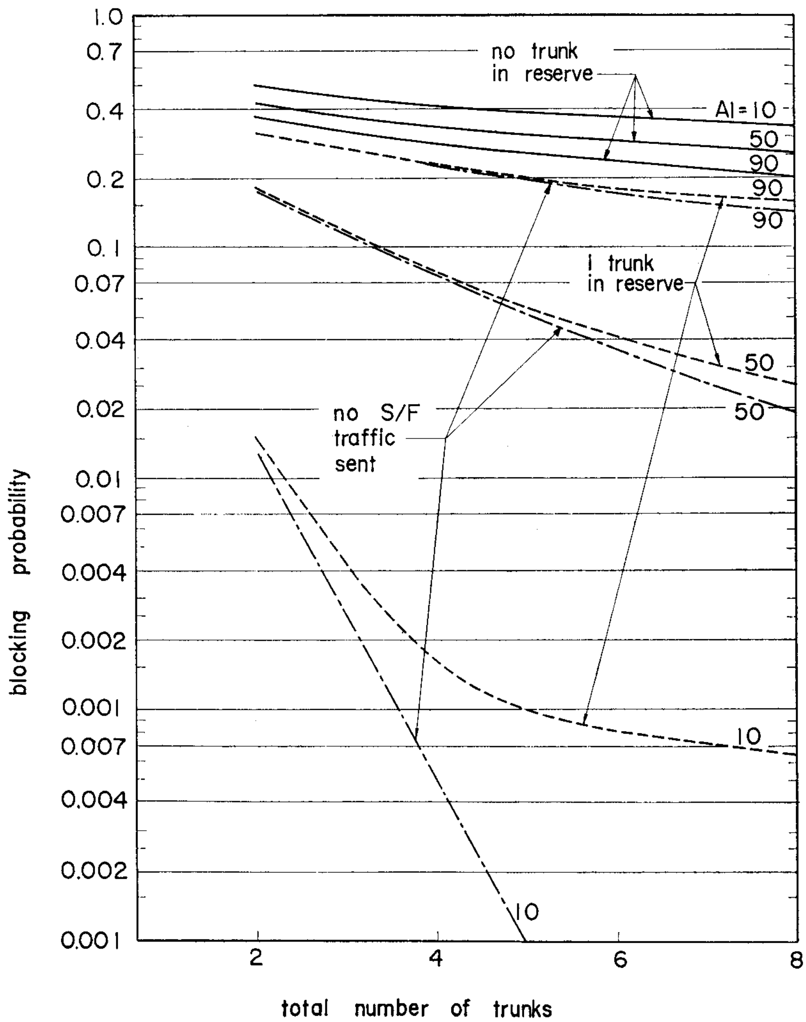

Fig. 2. Blocking probability of direct traffic (0.9 c erlangs total offered load, r=100, M=2).

of S/F traffic in the system with one trunk held in reserve increases as Al increases.

Furthermore, for some values of the parameter Al, the blocking probability of S/F traffic in the system with no trunk held in reserve increases with an increasing number of trunks in some range. The reason for this is that the buffer size for S/F traffic is fixed, while the offered load of S/F traffic is proportional to the number of trunks. For some other values of the parameter Al, the same probability decreases with an increasing number of trunks. The reason for this is that the effect of the increasing number of trunks overcomes the above mentioned effect on the fixed buffer size. In Fig. 4, the blocking probability of S/F traffic, similar to that in Fig. 3, is plotted for the total offered load 0.9 c.

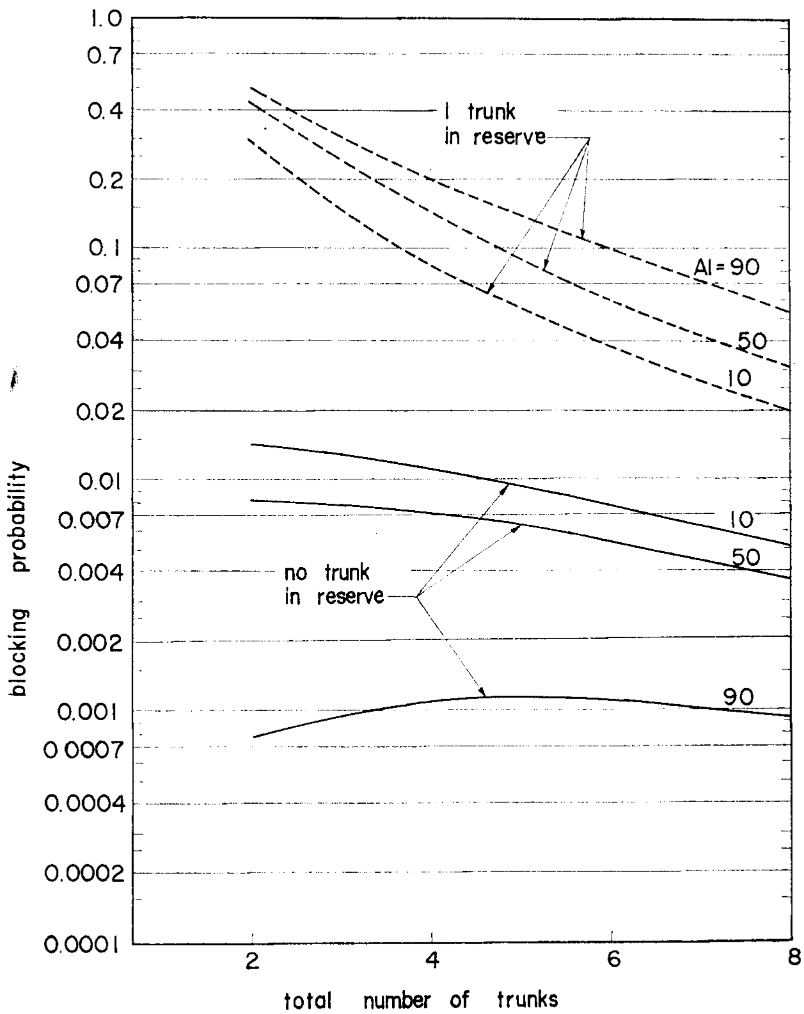

Fig. 3. Blocking probability of S/F traffic (0.5 c erlangs total offered load, $r=100$, $M=2$).

The blocking probabilities similar to those in Figs. 1~4 are plotted in Figs. 5~8 for $r=10$. For all parameters, the blocking probability of direct traffic at $r=10$ is greater than the corresponding probability at $r=100$. This is because the holding time of S/F traffic relative to that of direct traffic is greater at $r=10$ than at $r=100$. At $r=100$, the blocking probability of S/F traffic in the system with one trunk held in reserve increases with increasing values of the parameter Al. However, at $r=10$ the values Al=90, 10 and 50 yield increasing values of the blocking probability. The reason for this is that since at $r=10$ direct traffic, which has a shorter holding time than that of direct traffic at $r=100$, does not affect the grade of service to S/F traffic

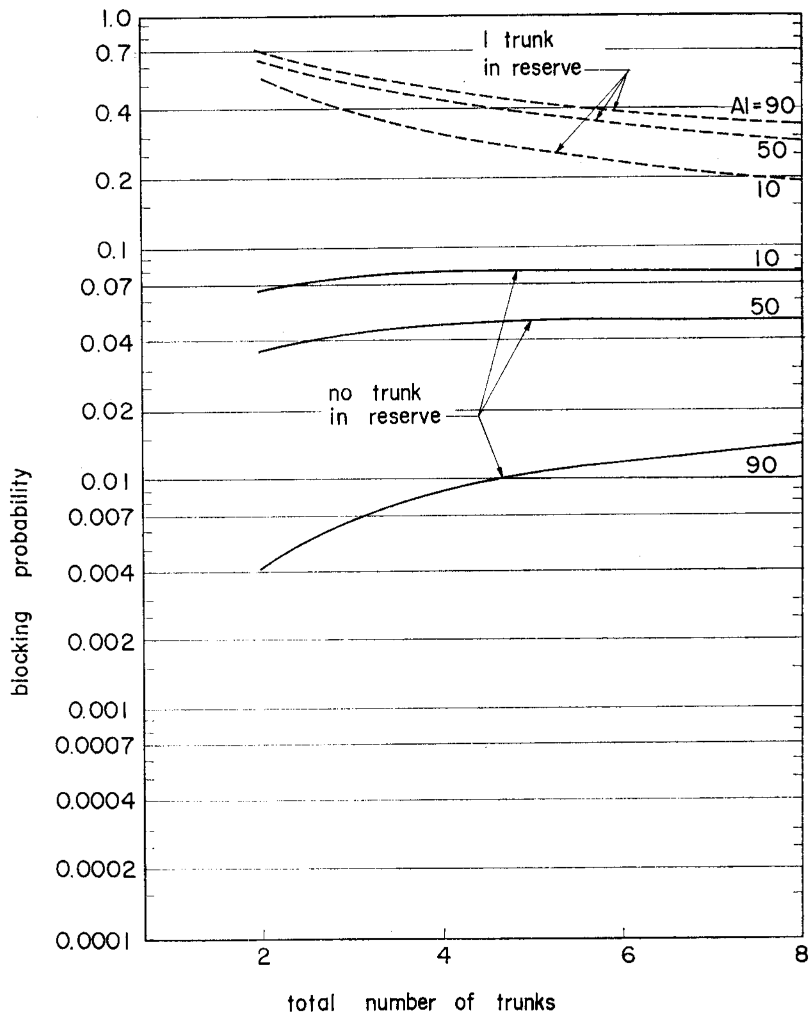

Fig. 4. Blocking probability of S/F traffic (0.9 c erlangs total offered load, $r=100$, $M=2$).

very much; and the blocking probability of S/F traffic depends upon the offered load of S/F traffic. Thus the blocking probability should increase in the order of Al=90, Al=50 and Al=10. However, the strong interference between both types of traffic at Al=50 makes the blocking probability at Al=50 greater than that at Al=10. Because of the interference, the blocking probability increases in the order of Al=90, Al=10 and Al=50.

The trunk efficiency (TE) is the carried load divided by the number of trunks. The trunk efficiencies of the system with one trunk held in reserve and the system with no S/F traffic sent are plotted in Fig. 9. The trunk efficiency of the system with one trunk in reserve is given by:

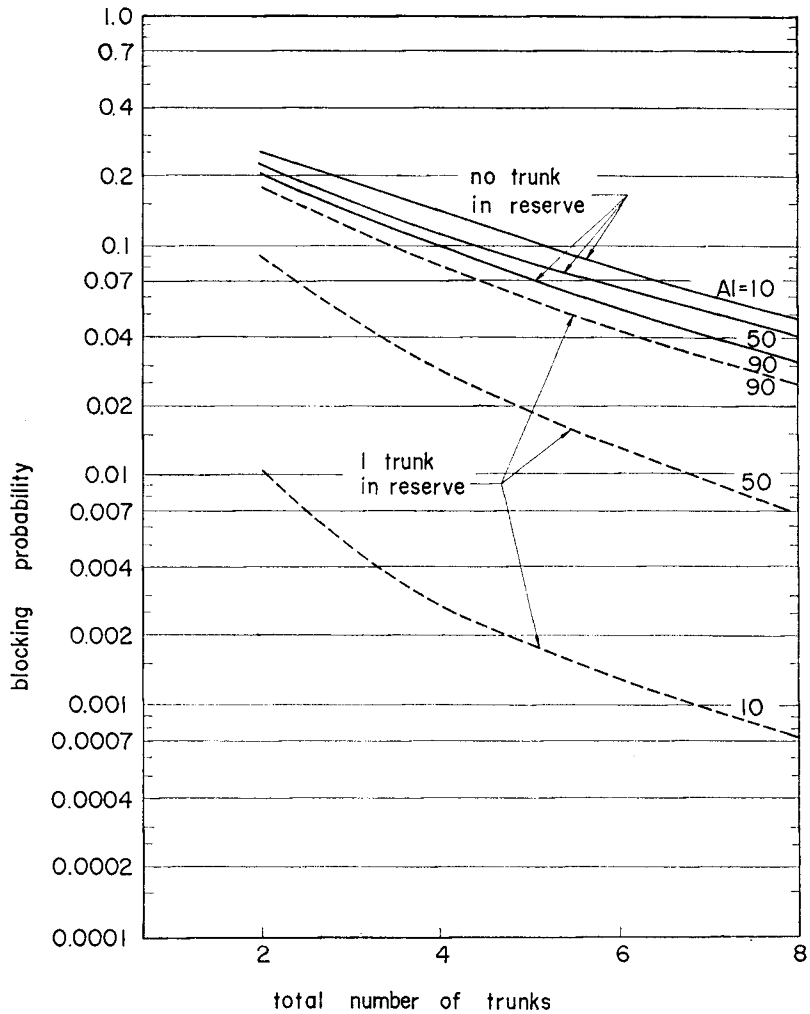

Fig. 5. Blocking probability of direct traffic (0.5 c erlangs total offered load, $r=10$, $M=2$).

$$TE=\frac{(1-\mathrm{Bl})a+b}{c}=\frac{c-1+\mathrm{Bl}}{c},$$

where B1 is the blocking probability of direct traffic, a is the offered load of direct traffic $(=a_1)$, b is the amount of S/F traffic in erlangs that can be accomodated, and c is the number of trunks (5). Moreover the trunk efficiency of the system with no S/F traffic sent is given by:

$$TE=\frac{(1-B(c,\, a))a}{c},$$

where $B(c,\, a)$ is the Erlang B blocking probability (5). The trunk efficiency of the

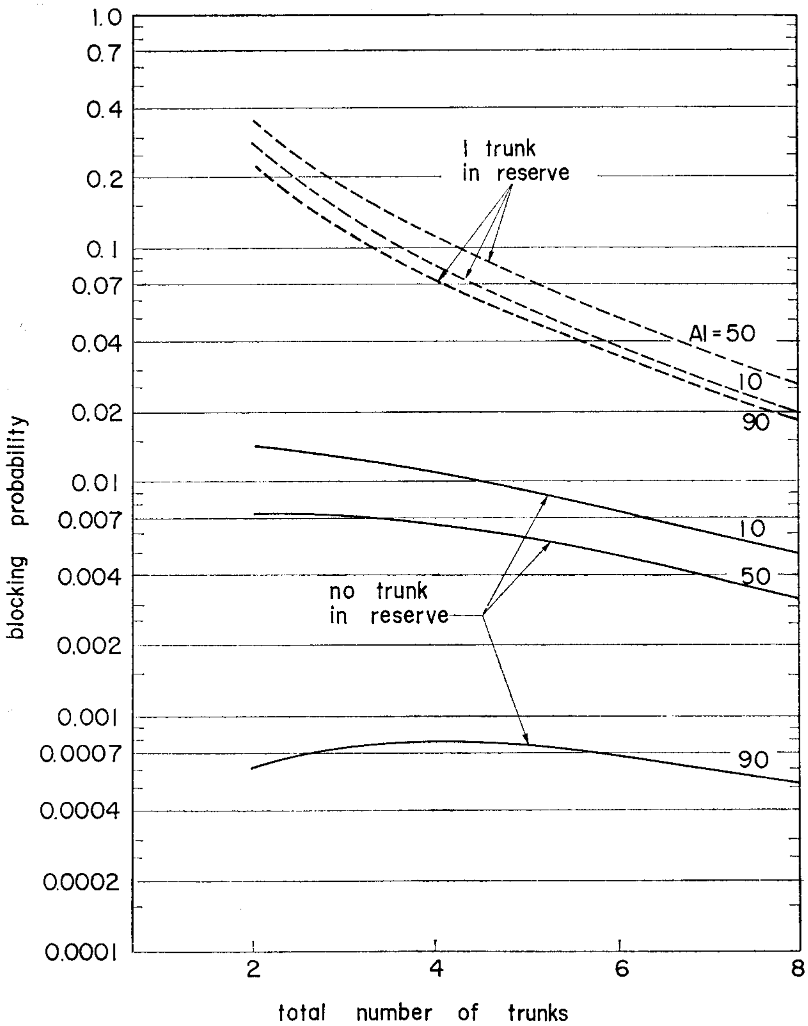

Fig. 7. Blocking probability of S/F traffic (0.5 c erlangs total offered load, $r=10$, $M=2$).

system with one trunk held in reserve is about twice as high as that in the system with no S/F traffic sent. The effects of the buffer size are shown in Figs. 10~13.

The following conclusions can be drawn from the numerical results. By holding one trunk in reserve for direct traffic, the grade of service to direct traffic is much improved with a rather small impairment in grade of service to S/F traffic—provided that the parameters for the number of trunks and the percentage of direct traffic to the total offered load are properly selected. When one trunk is held in reserve for the direct traffic whose offered load is small and whose holding time is long relative to that of S/F traffic, the system with one trunk held in reserve is especially effective. For

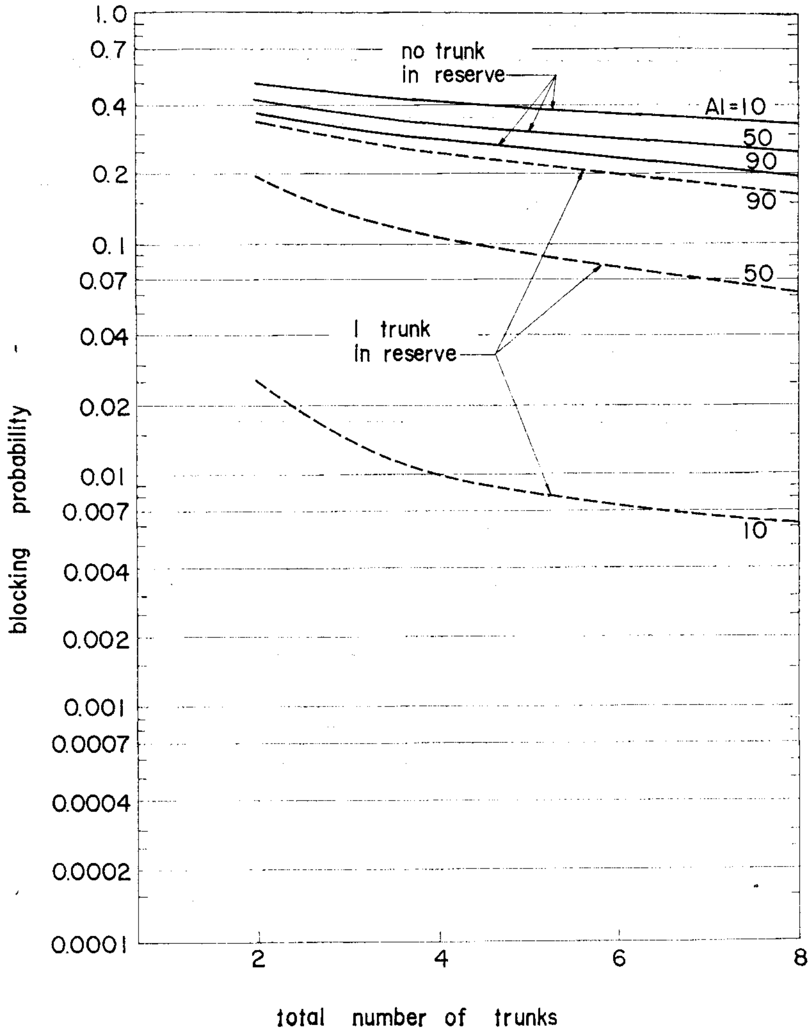

Fig. 6. Blocking probability of direct traffic (0.9c erlangs total offered load, $r=10$, $M=2$).

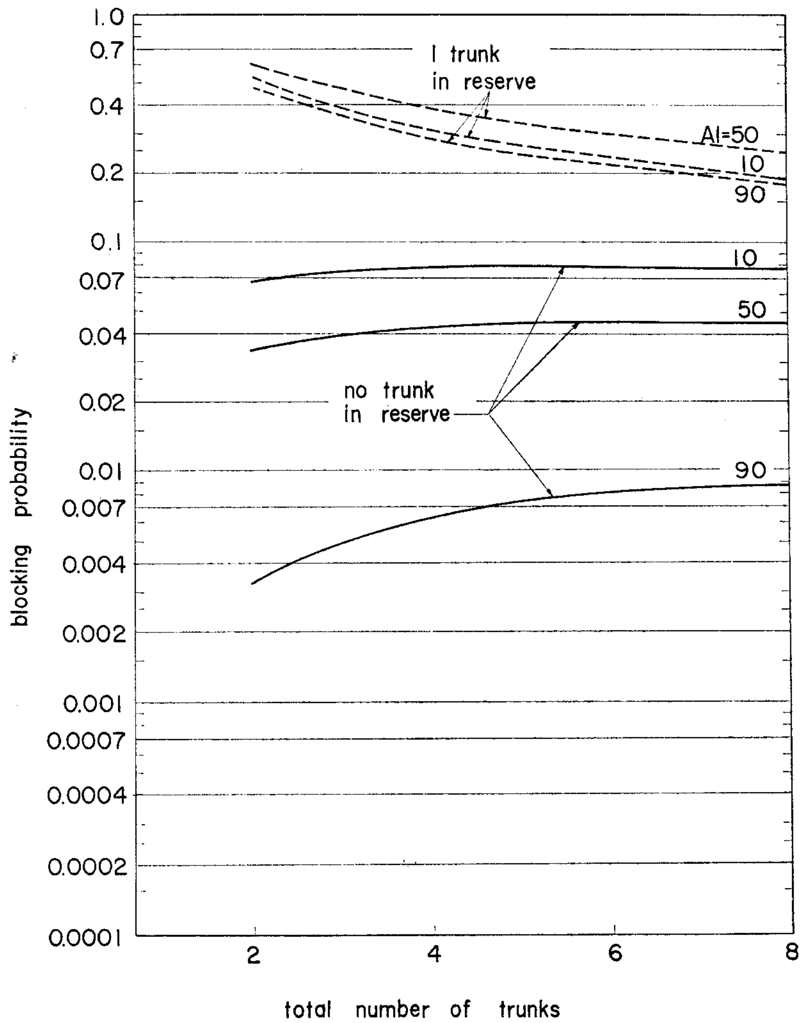

Fig. 8. Blocking probability of S/F traffic (0.9c erlangs total offered load, $r=10$, $M=2$).

large values of the ratio (r) of the average holding time of the direct traffic to that of S/F traffic, the difference between the blocking probability of the direct traffic in the system with one trunk held in reserve and that in the system with no S/F traffic sent can be small. In the system under the dual use of trunks with one trunk held in reserve, the trunk efficiency is about twice as high as in the system with no S/F traffic sent; and there is no significant impairment of the grade of service to the direct traffic.

Appendix I

This Appendix lists the equilibrium state equations for the system under the dual

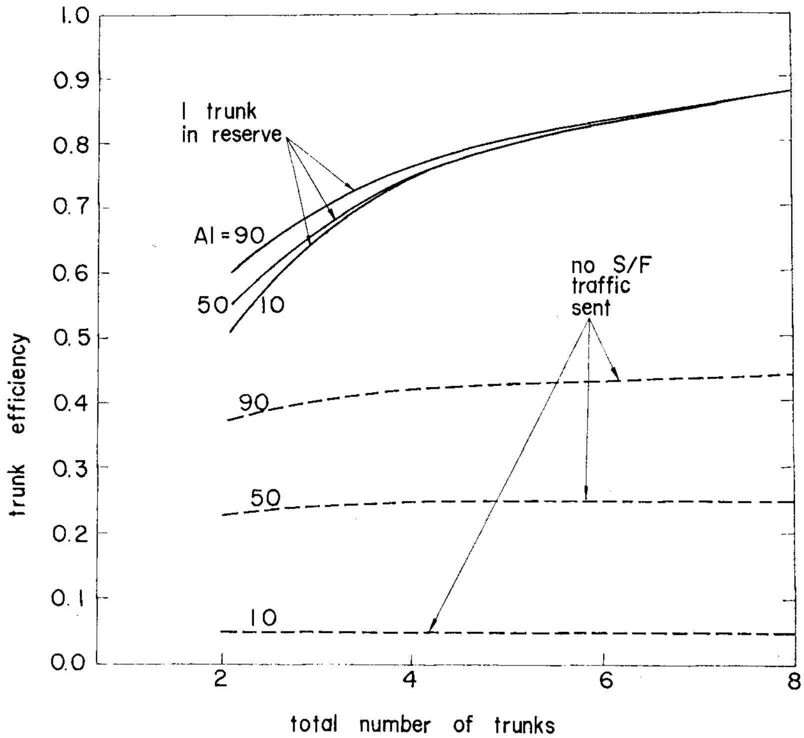

Fig. 9. Trunk efficiency (0.5 c erlangs total offered load, $r=100$, $M=2$).

use of trunks with one trunk held in reserve.

(1) For $0 \leq i+j \leq c-2$, $k=0$,

the transition probabilities from state $(i, j, 0)$ during dt are: $\lambda_1 dtP(i, j, 0)$ through a new arrival of direct traffic (transition to state $(i+1, j, 0)$); $\lambda_2 dtP(i, j, 0)$ through a new arrival of S/F traffic (transition to state $(i, j+1, 0)$); $i\mu_1 dtP(i, j, 0)$ through a termination of one of i direct traffic calls in progress (to state $(i-1, j, 0)$); $j\mu_2 dtP(i, j, 0)$ through a termination of one of j S/F traffic calls in progress (to state $(i, j-1, 0)$).

The transition probabilities to state $(i, j, 0)$ are: $\lambda_1 dtP(i-1, j, 0)$ through a new arrival of direct traffic in state $(i-1, j, 0)$; $\lambda_2 dtP(i, j-1, 0)$ through a new arrival of S/F traffic in state $(i, j-1, 0)$; $(i+1)\mu_1 dtP(i+1, j, 0)$ through a termination of one of $(i+1)$ direct traffic calls in progress in state $(i+1, j, 0)$; $(j+1)\mu_2 dtP(i, j+1, 0)$ through a termination of one of $(j+1)$ S/F traffic calls in progress in state $(i, j+1, 0)$.

Equating the transition probability from state $(i, j, 0)$ to the transition probability to state $(i, j, 0)$ results in the following:

$$(\lambda_1+\lambda_2+i\mu_1+j\mu_2)P(i, j, 0)=\lambda_1 P(i-1, j, 0)+\lambda_2 P(i, j-1, 0)$$
$$+(i+1)\mu_1 P(i+1, j, 0)+(j+1)\mu_2 P(i, j+1, 0),$$

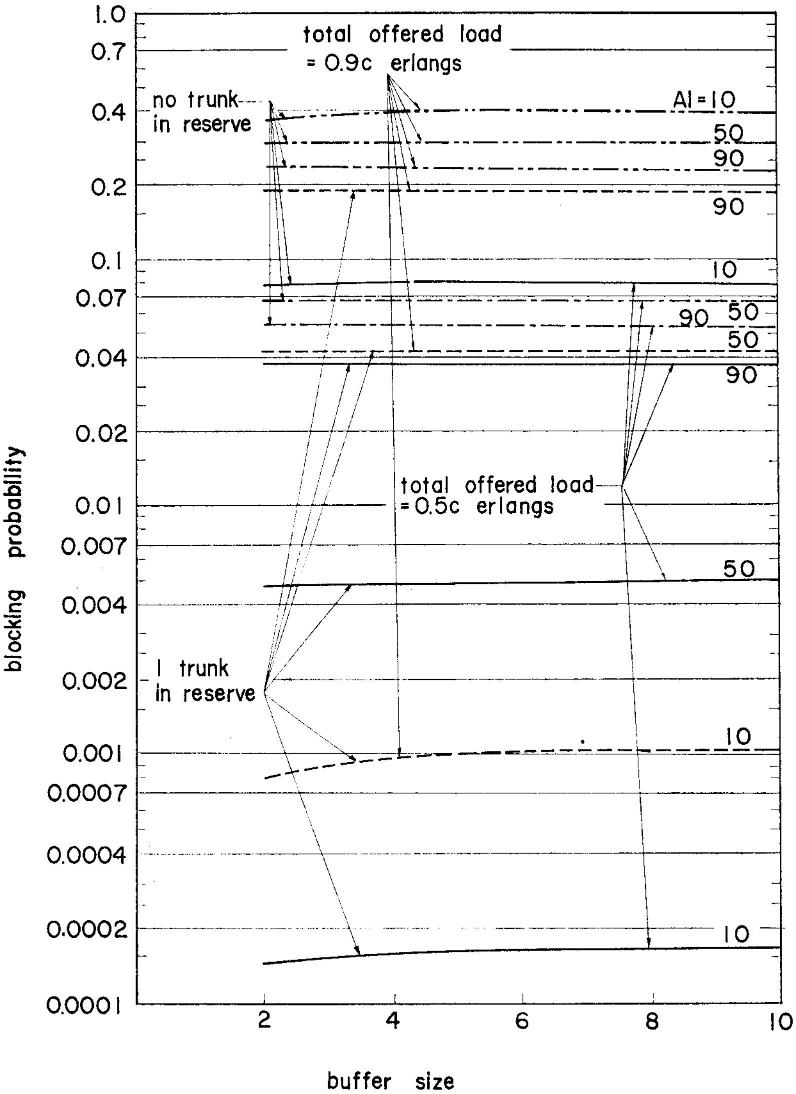

Fig. 10. Blocking probability of direct traffic vs buffer size M (six trunk group, $r=100$).

where if $i<0$ or $j<0$, $P(i, j, k)=0$; and this requirement is met in the following equations.

(2) For $i+j=c-1$, $k=0$, $0 \leq i \leq c-1$,

in a manner similar to the above,

$$\begin{aligned}(\lambda_1+\lambda_2+i\mu_1+j\mu_2)P(i, j, 0)&=\lambda_1 P(i-1, j, 0)+\lambda_2 P(i, j-1, 0)\\&+(i+1)\mu_1 P(i+1, j-1, 1)+(i+1)\mu_1 P(i+1, j, 0)\\&+(j+1)\mu_2 P(i, j+1, 0)+j\mu_2 P(i, j, 1).\end{aligned}$$

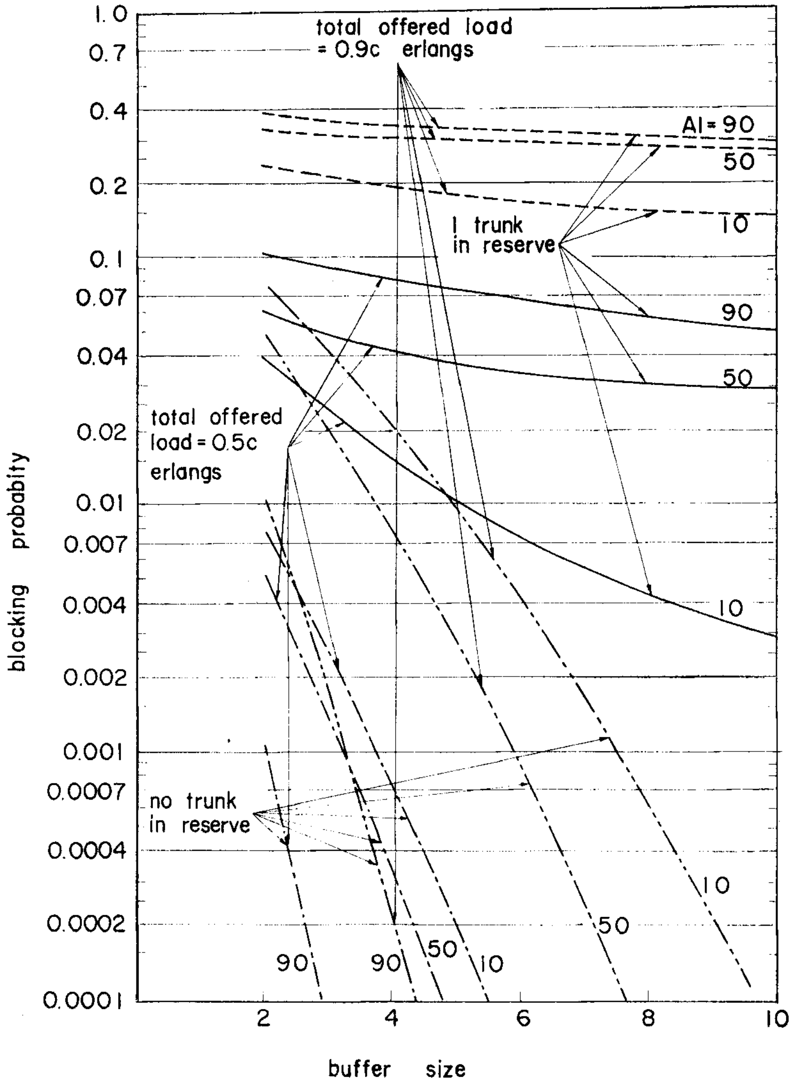

Fig. 11. Blocking probability of S/F traffic vs buffer size M (six trunk group, $r=100$).

(3) For $i+j=c-1$, $0<k<M$, $0\leq i\leq c-1$,

$$
\begin{aligned}
(\lambda_1+\lambda_2+i\mu_1+j\mu_2)&P(i,\ j,\ k)\\
&=\lambda_2 P(i,\ j,\ k-1)+(i+1)\mu_1 P(i+1,\ j-1,\ k+1)\\
&\quad+(i+1)\mu_1 P(i+1,\ j,\ k)+j\mu_2 P(i,\ j,\ k+1)\\
&\quad+(j+1)\mu_2 P(i,\ j+1,\ k).
\end{aligned}
$$

(4) For $i+j=c-1$, $k=M$, $0\leq i\leq c-1$,

$$
\begin{aligned}
(\lambda_1+i\mu_1+j\mu_2)P(i,\ j,\ M)&=\lambda_2 P(i,\ j,\ M-1)+(i+1)\mu_1 P(i+1,\ j,\ M)\\
&\quad+(j+1)\mu_2 P(i,\ j+1,\ M).
\end{aligned}
$$

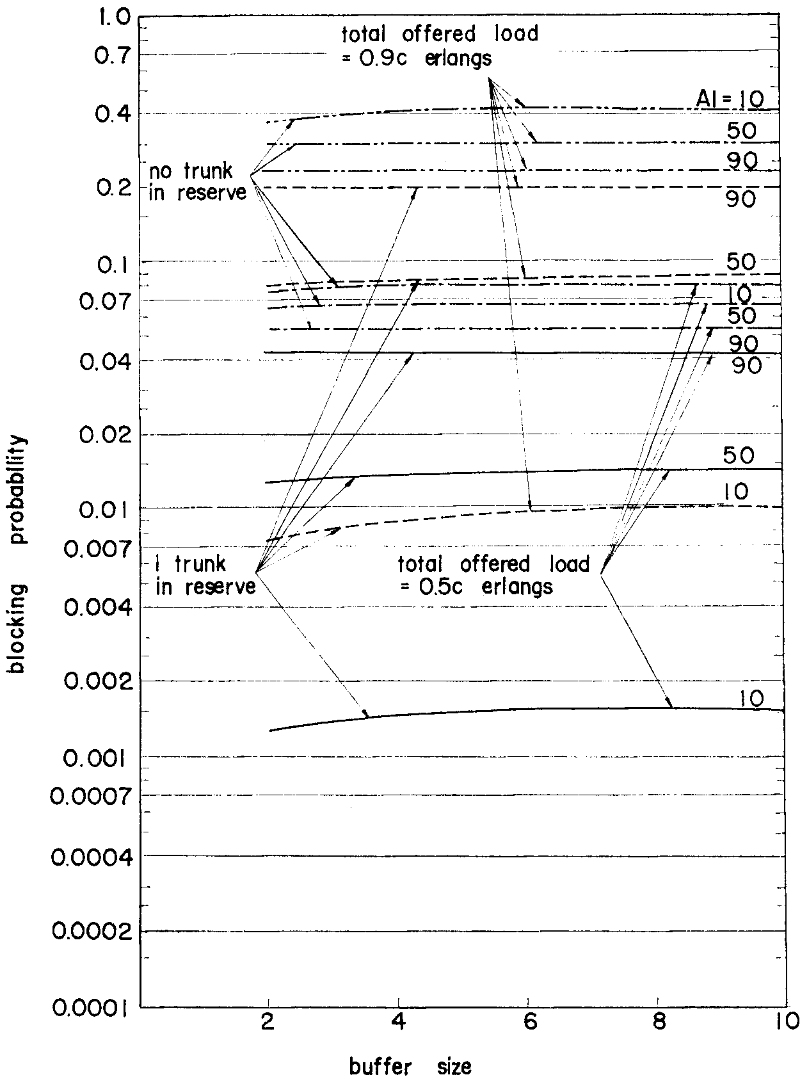

Fig. 12. Blocking probability of direct traffic vs buffer size M (six trunk group, $r=10$).

(5) For $i+j=c$, $k=0$, $1 \leq i \leq c$,

$$(\lambda_2+i\mu_1+j\mu_2)P(i,\ j,\ 0)=\lambda_1 P(i-1,\ j,\ 0).$$

(6) For $i+j=c$, $0<k<M$, $1 \leq i \leq c$,

$$(\lambda_2+i\mu_1+j\mu_2)P(i,\ j,\ k)=\lambda_1 P(i-1,\ j,\ k)+\lambda_2 P(i,\ j,\ k-1).$$

(7) For $i+j=c$, $k=M$, $1 \leq i \leq c$,

$$(i\mu_1+j\mu_2)P(i,\ j,\ M)=\lambda_1 P(i-1,\ j,\ M)+\lambda_2 P(i,\ j,\ M-1).$$

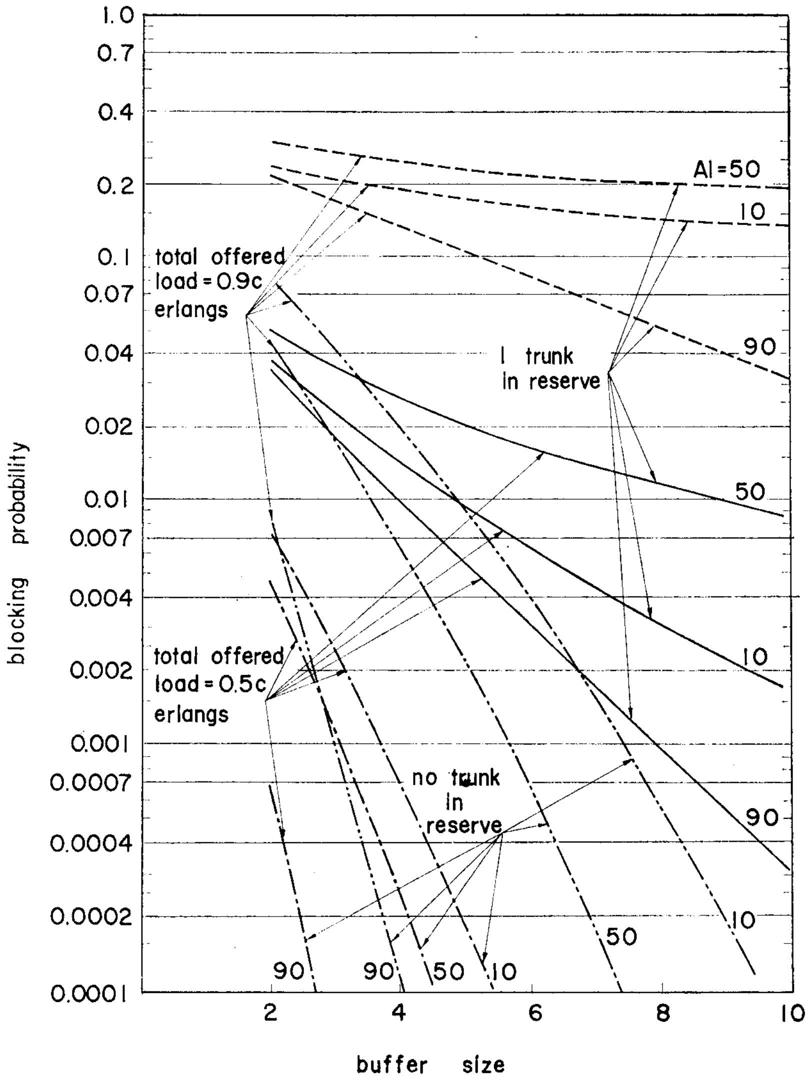

Fig. 13. Blocking probability of S/F traffic vs buffer size M (six trunk group, $r=10$).

Appendix II

This Appendix lists the equilibrium state equations for the system under the dual use of trunks with no trunk held in reserve.

(1) For $0 \leq i+j \leq c-1$, $k=0$,

$$(\lambda_1+\lambda_2+i\mu_1+j\mu_2)P(i, j, 0)=\lambda_1 P(i-1, j, 0)+\lambda_2 P(i, j-1, 0)$$
$$+(i+1)\mu_1 P(i+1, j, 0)+(j+1)\mu_2 P(i, j+1, 0),$$

where if $i<0$ or $j<0$, $P(i, j, k)=0$; and this requirement is met in the following equations.

(2) For $i+j=c,\ k=0,\ 0\leq i\leq c,$

$$(\lambda_2+i\mu_1+j\mu_2)P(i,\ j,\ 0)=\lambda_1 P(i-1,\ j,\ 0)+\lambda_2 P(i,\ j-1,\ 0)$$
$$+(i+1)\mu_1 P(i+1,\ j-1,\ 1)$$
$$+j\mu_2 P(i,\ j,\ 1).$$

(3) For $i+j=c,\ 0<k<M,\ 0\leq i\leq c,$

$$(\lambda_2+i\mu_1+j\mu_2)P(i,\ j,\ k)=\lambda_2 P(i,\ j,\ k-1)$$
$$+(i+1)\mu_1 P(i+1,\ j-1,\ k+1)$$
$$+j\mu_2 P(i,\ j,\ k+1).$$

(4) $i+j=c,\ k=M,\ 0\leq i\leq c,$

$$(i\mu_1+j\mu_2)P(i,\ j,\ M)=\lambda_2 P(i,\ j,\ M-1).$$

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