Analysis of the Effects of Buffer Storage Capacity

By

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Abstract

The purpose of this paper is to gain insight into the problem on the role of buffer stocks in production lines, and to present the results of a theoretical study of the problem. After the formulation of the problem is given, a Markovian process model of production buffer systems is proposed. It is shown that this model is a useful tool for analyzing the role of in-process inventory banks in production lines for improving the line efficiency. A two stage line consists of two groups of stations separated by a buffer. The reasons for installing buffers can be best illustrated by condsidering two stages. Based on this analysis, some reflections about two stage lines are presented: (1) Should buffer stocks be used ? (2) What is the effect of a given buffer capacity on the line efficiency, or on the number of mean buffer stocks ? (3) What is the effect of a variation of breakdown rates, repair rates, or stage efficiencies ? Cost analysis of a production line is made to help the system designer make a better decision so as to maximize the system profit.

1. Introduction

The complete specification of a production line design contains a rather large number of decisions and considerations. The production line consists of a number of interconnected stations or stages (groups of stations) at which operations are performed on workpieces in order to convert the inputs to the system into outputs of the system. The operations in the system are performed by some equipment which is liable to failure or breakdown. Breakdowns must be repaired, and production from the station is lost during repairs. By linking the stages to form a line, the efficiency is decreased significantly, compared to the use of an individual machine, with the consequence that if any one station stops, all other stations in the line are forced to shut down.

One way of improving the line efficiency is to provide buffer stocks between certain stages or sections of the line. A storage facility between two successive stages is called a buffer. A group of stations located between two in-process inventory

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banks, but having no in-process inventory storage within the group, is regarded as a stage. The in-process buffers decouple the production stages and diminish the forced down effect caused by a stage breakdown. Such buffer storage occupies valuable space, and the workpieces kept in it have a high storage cost. Handling the unit into and out of these in-process inventory banks is the storage facility cost. The purpose of this paper is to give better guidance on how much interstage storage capacity should be provided, what is the effect of a given buffer capacity on the line efficiency, and how the stages should be placed.

Each of the above decisions is subject to technical and economic constraints. Within all of them, these decisions should be made so as to maximize the profit margin realized from the line, or minimize the capital expenditure and operating costs of the production line.

It will be assumed throughout this paper that the in-process inventroy storage has a limited capacity for operational and economic reasons. It will be assumed further that the system is processing only one commodity.

2. Problem Statement

While production lines can take various forms, the following presents a picture of actual production lines: The line produces one kind of commodity, consisting of a number of stages, at each of which an operation is carried out on a workpiece. The stages are arranged serially so that each workpiece enters the line at the same stage, and transfers from one stage to the next till it has passed through the final stage. All workpieces begin to transfer from one stage to the next at the same instant. The interval between successive transfers is called the cycle time. There is always a supply of workpieces available to the first stage of the production line. The final stage will deposit the completed workpiece into a storage area which has an infinite capacity. Each storage point has a fixed capacity: the capacity of the storage point between stage i and stage $i+1$ is denoted by N_i, while the total capacity of the (n$-$1) storage points is denoted by N.

In the following analysis, unit production time (cycle time) is taken as a time unit, and transport time between stages is assumed to be negligible or subsumed by the unit production times.

The performance of a particular station or stage is described by whether it is: Operationg: in working order and carrying out its function [abbrev.: 1]. Broken down and under repair: Each stage in the line is subject to breakdowns, which are random in both occurrence and duration. These breakdowns may be the result of a malfunction, or time required to change or adjust tools, settings, and so forth [abbrev. : 0].

Forced down (type 1) : in working order but unable to operate because it has no workpiece to process. The stage is said to be idle or starved [abbrev. : I]. It is assumed that if a stage can not transfer its completed workpiece to the next station, or has no place to eject it into the buffer store, then the stage holds the workpiece.

Forced down (type 2): the stage is physically able to produce but it can not transfer its completed workpiece to the next station or into the buffer store. The stage is then said to be blocked [abbrev. : B].

The role that a buffer plays is to diminish or eliminate the transmission of forced breakdown by means of its storing and replenishing functions. Forced breakdowns transmit either forward or backward. Forward transmission occurs in the case in which a stage cannot operate because it has no workpiece to process. The forced breakdown of type 1 transmission occurs in the case in which a stage cannot transfer its completed workpiece to the next station or into the buffer store. The forced breakdown of type 2 transmits simultaneously to the preceding stages. The transmission speed of the former, or conversely speaking, the buffer effect of the former, depends on the number of stocks present in the buffer space. The transmission speed of the latter relates to the spare space in the buffer space. The stages under forced breakdowns of type 1 begin operating successively as repair of the broken down stage is completed. All the stages under forced breakdowns of type 2 begin operating simultaneously as soon as repair of the broken down stage is completed.

The following states describing the line behavior are defined as follows:

Up: The line is considered to be producing whenever the last stage is turning out finished workpieces.

Down : The line is considered down, either because it has had a breakdown or because some other station in the line has had a breakdown and the last station is forced down.

The efficiency and the mean buffer stocks are defined in the following way: The efficiency of the line is the probability that at the steady state the last stage is up. The mean buffer stocks are the expected number of workpieces in the buffer space at the steady state.

On the characteristics of breakdown and repair of the stages, the following fundamental assumptions are made:

(1) On the characteristic of breakdown of the stages: It is assumed that the probability that stage i breaks down in a cycle, given that it was working at the end of the previous cycle is λ_i, which is called breakdown rate. The breakdown rate

of a forced down stage is assumed to be zero.

(2) On the characteristic of repair of the stages: It is assumed that the probability that repair of the broken down stage i is completed in a cycle, given that it broke down and was under repair at the previous cycle is μ_i, which is called repair rate.

(3) It is assumed that stage i does not hold its workpiece when it is broken down.

3. Analysis

3.1. Efficiency of a single stage line

There are two states for a single stage line, namely, an operating state, and a broken down and under repair state. As the stationary probabilities of the two states of the line in the long run are sought (the steady state), the initial state of the line is immaterial.

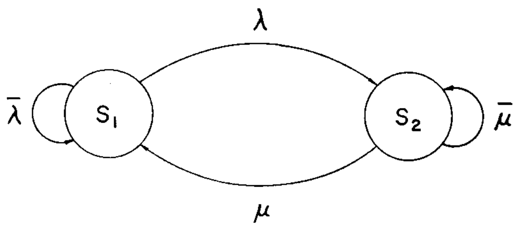

Fig. 1. Transition diagram of a single stage line.

Fig. 1 shows a schematic transition diagram of the single stage line, where S_1 and S_2 show the states that the stage is operating, broken down and under repair, respectively. Let $\pi=(\pi_1, \pi_2)$ denote the stationary probabilities of the states S_1 and S_2. The matrix of transition probabilities assumes the simple form:

$$T=\begin{pmatrix}\bar{\lambda} & \lambda \\ \mu & \bar{\mu}\end{pmatrix}, \qquad \begin{matrix}\lambda+\bar{\lambda}=1 & 0<\lambda<1 \\ \mu+\bar{\mu}=1 & 0<\mu<1\end{matrix} \tag{1}$$

Then,

$$T^t=\frac{1}{\lambda+\mu}\begin{pmatrix}\mu & \lambda \\ \mu & \lambda\end{pmatrix}+\frac{(1-\lambda-\mu)^t}{\lambda+\mu}\begin{pmatrix}\lambda & -\mu \\ -\mu & \lambda\end{pmatrix} \tag{2}$$

where factors common to all four elements have been taken out as factors to the matrices. Since $|1-\lambda-\mu|<1$, the second matrix tends to zero as $t\rightarrow\infty$. Therefore,

$$\lim_{t\rightarrow\infty} T^t=\frac{1}{\lambda+\mu}\begin{pmatrix}\mu & \lambda \\ \mu & \lambda\end{pmatrix} \tag{3}$$

For an arbitrary initial distribution

$$\pi^{(0)} = (\pi_1^{(0)}, \pi_2^{(0)}), \qquad \pi_1^{(0)}+\pi_2^{(0)} = 1$$

$$\lim_{t\to\infty} \pi^{(t)} = \lim_{t\to\infty} \pi^{(0)} T^t = \left(\frac{\mu}{\lambda+\mu}, \frac{\lambda}{\lambda+\mu}\right) \tag{4}$$

Therefore, the limiting probabilities of the states do not depend on the initial distribution. Eventually, the efficiency of the single line is

$$E = \frac{\mu}{\lambda+\mu} = \frac{1}{1+\rho} \tag{5}$$

where $\rho=\lambda/\mu$.

3.2 Efficiency of a two stage line

(1) $N_1=0$.

There are six possible states which are illustrated in Fig. 2 for a two stage line without a buffer.

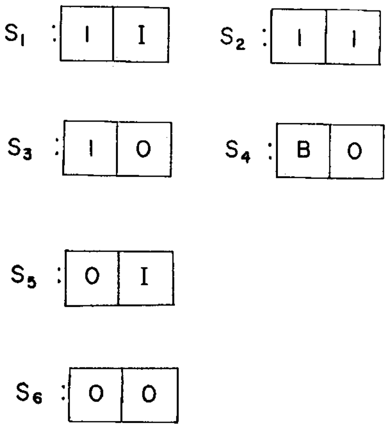

Fig. 2. Six possible states for a two stage line without a buffer.

The matrix of transition probabilities assumes the following form:

$$T = \begin{pmatrix} 0 & \bar{\lambda}_1 & 0 & 0 & \lambda_1 & 0 \\ 0 & \bar{\lambda}_1\bar{\lambda}_2 & 0 & \bar{\lambda}_1\lambda_2 & \lambda_1\bar{\lambda}_2 & \lambda_1\lambda_2 \\ 0 & \bar{\lambda}_1\mu_2 & 0 & \bar{\lambda}_1\bar{\mu}_2 & \lambda_1\mu_2 & \lambda_1\bar{\mu}_2 \\ 0 & \mu_2 & 0 & \bar{\mu}_2 & 0 & 0 \\ \mu_1 & 0 & 0 & 0 & \bar{\mu}_1 & 0 \\ \mu_1\mu_2 & 0 & \mu_1\bar{\mu}_2 & 0 & \bar{\mu}_1\mu_2 & \bar{\mu}_1\bar{\mu}_2 \end{pmatrix} \tag{6}$$

Since the Markov process is normal, there exist stationary probabilities $\pi=(\pi_1, \pi_2, \cdots, \pi_6)$ of $S_1, S_2, \cdots$, and S_6. The solution of the equation:

$$\pi = \pi T \tag{7}$$

or

$$\left.\begin{aligned} &\pi_j = \sum_{\nu=1}^{6} \pi_\nu p_{\nu j}, \quad j = 1, 2, \cdots, 6 \\ &\sum_{j=1}^{6} \pi_j = 1 \,, \end{aligned}\right\} \tag{8}$$

is

$$\begin{aligned} \pi_1 &= \frac{\lambda_1(r-\rho_1 s^2-qs)}{(1+\rho_1)(r-\rho_1 s^2)+(1-\lambda_1)^2\rho_2 r}\,, \\ \pi_2 &= \frac{(1-\lambda_1)(r-\rho_1 s^2)}{(1+\rho_1)(r-\rho_1 s^2)+(1-\lambda_1)^2\rho_2 r}\,, \\ \pi_3 &= \frac{\lambda_1 qs}{(1+\rho_1)(r-\rho_1 s^2)+(1-\lambda_1)^2\rho_2 r}\,, \\ \pi_4 &= \frac{(1-\lambda_1)^2\rho_2 r}{(1+\rho_1)(r-\rho_1 s^2)+(1-\lambda_1)^2\rho_2 r}\,, \\ \pi_5 &= \frac{\rho_1(r-\rho_1 s^2)-\lambda_1 q}{(1+\rho_1)(r-\rho_1 s^2)+(1-\lambda_1)^2\rho_2 r}\,, \\ \pi_6 &= \frac{\lambda_1 q}{(1+\rho_1)(r-\rho_1 s^2)+(1-\lambda_1)^2\rho_2 r}\,, \end{aligned} \tag{9}$$

where

$$\begin{aligned} p &= 1-(1-\lambda_1)(1-\lambda_2), \quad q = (1-\lambda_1)\lambda_2, \\ r &= 1-(1-\mu_1)(1-\mu_2), \quad s = \mu_1(1-\mu_2)\,. \end{aligned} \tag{10}$$

The efficiency of the line is

$$E = \frac{(1-p)(r-\rho_1 s^2)}{(1-\rho_1)(r-\rho_1 s^2)+(1-\lambda_1)^2\rho_2 r} \tag{11}$$

and the mean buffer stocks are

$$M = 0\,.$$

(2) $N_1 \in (0, \infty)$.

First of all, consideration is given as to what kinds of states and how many states are possible in a case where the buffer capacity is N_1. In the previous case in which the buffer capacity is zero, there are six possible states. When there are buffer stocks between the stages, the following stage can continue operating with the buffer stocks, and therefore, there are seven basic states which are schematic-

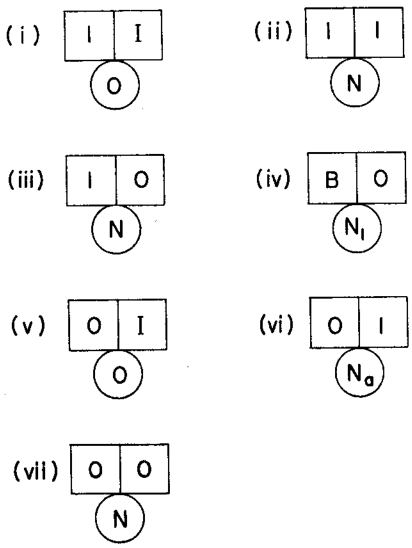

Fig. 3. Seven basic states of a two stage line with buffer capacity N_1.

ally shown in Fig. 3.

Lemma 1. If a production line consists of n stages, the number of basic states of the line $B(n)$ is given by the following:

$$B(n) = \left(\frac{3}{4}\sqrt{2}+1\right)(2+\sqrt{2})^{n-1} - \left(\frac{3}{4}\sqrt{2}-1\right)(2-\sqrt{2})^{n-1}. \qquad (12)$$

(Proof) Note that three states 1, B, and 0 are possible for the first stage, four states I, 1, B, and 0 for the second $\sim(n-1)$ stages, and three states I, 1, and 0 for the last stages and that the combinations (B, I) and (B, 1) are prohibited for two consecutive stages. Let $B(n-1)$ denote the total number of the basic states for an $(n-1)$ stage line. Suppose that an n stage line consists of the $(n-1)$ stages and in addition to them the n th stage. There exist at least $3B(n-1)$ states since stages I, 1 and 0 can be added as the n th stage to each of the B $(n-1)$ states of the $(n-1)$ stages. Now the $(n-1)$ stage can be blocked in the n stage line. In this case the n th stage is not operating. The above value $3B(n-1)$ does not include this case. The state in which the $(n-1)$ stage is blocked and the n th stage is not operating can be added to each of $B(n-2)$ states of the $(n-2)$ stages. But the value $B(n-2)$ does not include the case in which the $(n-2)$ stage is blocked, the $(n-1)$ stage is also blocked and the n th stage is not operating. Continuing in the same manner results in the following recurrence formula:

$$B(n) = 3B(n-1)+B(n-2)+\cdots+B(1)+1\,. \tag{13}$$

The value in the right hand side of the above equation corresponds to the case in which the first $\sim(n-1)$ stages are blocked and the last stage is not operating. From (13),

$$\begin{aligned} B(n)-(2-\sqrt{2})B(n-1) &= (2+\sqrt{2})\{B(n-1)-(2-\sqrt{2})B(n-2)\} \\ &= \cdots = (2+\sqrt{2})^{n-2}(3-2\sqrt{2})\,, \end{aligned} \tag{14}$$

where $B(1)=2$. From (14) the above equation (12) can be easily obtained.

Taking the number of buffer stocks into consideration, in addition to the above basic states, results in the construction of various possible states. The case of a two stage line with a buffer capacity N_1 will be considered next. In Fig. 3, the cases (i) and (v), in which the second stage is starved, occur only when there exists no buffer stock. The case (iv) in which the first stage is blocked happens only when the buffer space is full of buffer stocks. The cases (ii), (iii), and (vii) arise no matter how many buffer stocks there are. The case (vi) also occurs no matter how many buffer stocks there are, except for the case where the buffer is full of buffer stocks. Therefore, the total number of possible states is $2(2N_1+3)$ for the two stage line case with a buffer capacity N_1. It is quite easy to obtain the transitional matrix of the case. For example, the possible states and the transitional matrices for the cases of buffer capacity 1 and 2 are shown in Fig. 4 and Fig. 5, respectively. The purpose of displaying these two cases is to show that construction of the possible states for the case of a two stage line with an arbitrary buffer capacity N and its transitional matrix are made systematically. After checking the normality of the Markov process and looking for the stationary probabilities $\pi=(\pi_1, \pi_2\cdots, \pi_{4N+6})$,

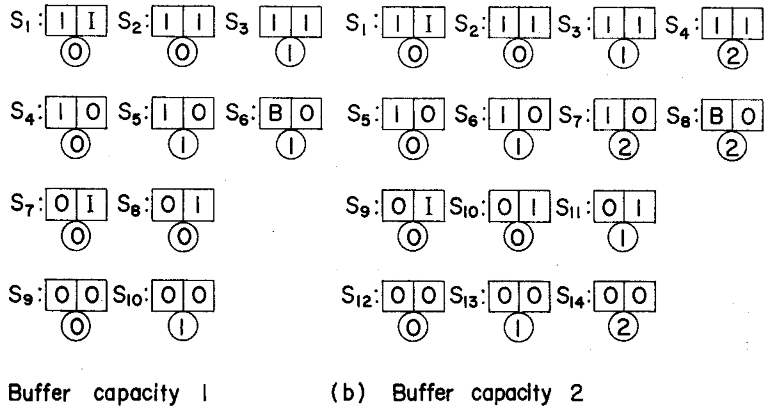

Fig. 4. Possible states of two stage lines with buffers.

$$
T=\begin{bmatrix}
0 & \bar\lambda_1 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0\\
0 & \bar\lambda_1\bar\lambda_2 & 0 & 0 & \bar\lambda_1\lambda_2 & 0 & \lambda_1\bar\lambda_2 & 0 & \lambda_1\lambda_2 & 0\\
0 & 0 & \bar\lambda_1\bar\lambda_2 & 0 & 0 & \bar\lambda_1\lambda_2 & 0 & \lambda_1\bar\lambda_2 & 0 & \lambda_1\lambda_2\\
0 & \bar\lambda_1\mu_2 & 0 & 0 & \bar\lambda_1\bar\mu_2 & 0 & \lambda_1\mu_2 & 0 & \lambda_1\bar\mu_2 & 0\\
0 & 0 & \bar\lambda_1\mu_2 & 0 & 0 & \bar\lambda_1\bar\mu_2 & 0 & \lambda_1\mu_2 & 0 & \lambda_1\bar\mu_2\\
0 & 0 & \mu_2 & 0 & 0 & \bar\mu_2 & 0 & 0 & 0 & 0\\
\mu_1 & 0 & 0 & 0 & 0 & 0 & \bar\mu_1 & 0 & 0 & 0\\
\mu_1\bar\lambda_2 & 0 & 0 & \mu_1\lambda_2 & 0 & 0 & \bar\mu_1\bar\lambda_2 & 0 & \bar\mu_1\lambda_2 & 0\\
\mu_1\mu_2 & 0 & 0 & \mu_1\bar\mu_2 & 0 & 0 & \bar\mu_1\mu_2 & 0 & \bar\mu_1\bar\mu_2 & 0\\
0 & \mu_1\mu_2 & 0 & 0 & \mu_1\bar\mu_2 & 0 & 0 & \bar\mu_1\mu_2 & 0 & \bar\mu_1\bar\mu_2
\end{bmatrix}
$$

(a) Buffer capacity 1

$$
T=\begin{bmatrix}
0 & \bar\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0\\
0 & \bar\lambda_1\bar\lambda_2 & 0 & 0 & 0 & \bar\lambda_1\lambda_2 & 0 & 0 & \lambda_1\bar\lambda_2 & 0 & 0 & \lambda_1\lambda_2 & 0 & 0\\
0 & 0 & \bar\lambda_1\bar\lambda_2 & 0 & 0 & 0 & \bar\lambda_1\lambda_2 & 0 & 0 & \lambda_1\bar\lambda_2 & 0 & 0 & \lambda_1\lambda_2 & 0\\
0 & 0 & 0 & \bar\lambda_1\bar\lambda_2 & 0 & 0 & 0 & \bar\lambda_1\lambda_2 & 0 & 0 & \lambda_1\bar\lambda_2 & 0 & 0 & \lambda_1\lambda_2\\
0 & \bar\lambda_1\mu_2 & 0 & 0 & 0 & \bar\lambda_1\bar\mu_2 & 0 & 0 & \lambda_1\mu_2 & 0 & 0 & \lambda_1\bar\mu_2 & 0 & 0\\
0 & 0 & \bar\lambda_1\mu_2 & 0 & 0 & 0 & \lambda_1\bar\mu_2 & 0 & 0 & \lambda_1\mu_2 & 0 & 0 & \lambda_1\bar\mu_2 & 0\\
0 & 0 & 0 & \bar\lambda_1\mu_2 & 0 & 0 & 0 & \bar\lambda_1\bar\mu_2 & 0 & 0 & \lambda_1\mu_2 & 0 & 0 & \lambda_1\bar\mu_2\\
0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & \bar\mu_2 & 0 & 0 & 0 & 0 & 0 & 0\\
\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar\mu_1 & 0 & 0 & 0 & 0 & 0\\
\mu_1\bar\lambda_2 & 0 & 0 & 0 & \mu_1\lambda_2 & 0 & 0 & 0 & \bar\mu_1\bar\lambda_2 & 0 & 0 & \bar\mu_1\lambda_2 & 0 & 0\\
0 & \mu_1\bar\lambda_2 & 0 & 0 & 0 & \mu_1\lambda_2 & 0 & 0 & 0 & \bar\mu_1\bar\lambda_2 & 0 & 0 & \bar\mu_1\lambda_2 & 0\\
\mu_1\mu_2 & 0 & 0 & 0 & \mu_1\bar\mu_2 & 0 & 0 & 0 & \bar\mu_1\mu_2 & 0 & 0 & \bar\mu_1\bar\mu_2 & 0 & 0\\
0 & \mu_1\mu_2 & 0 & 0 & 0 & \mu_1\bar\mu_2 & 0 & 0 & 0 & \bar\mu_1\mu_2 & 0 & 0 & \bar\mu_1\bar\mu_2 & 0\\
0 & 0 & \mu_1\mu_2 & 0 & 0 & 0 & \mu_1\bar\mu_2 & 0 & 0 & 0 & \bar\mu_1\mu_2 & 0 & 0 & \bar\mu_1\bar\mu_2
\end{bmatrix}
$$

(b) Buffer capacity 2

Fig 5. Transition matrices of two stage lines with buffers.

the line efficiency and the mean buffer stocks can be obtained;

$$E=(1-\lambda_1)\left(\sum_{i=0}^{N}\pi_{i+2}+\sum_{i=1}^{N}\pi_{2N+5+i}\right), \tag{15}$$

$$M=\sum_{i=1}^{N} i\pi_{i+2}+\sum_{i=1}^{N} i\pi_{N+3+i}+N\cdot\pi_{2N+4}+\sum_{i=1}^{N-1} i\pi_{2N+6+i}+\sum_{i=1}^{N} i\pi_{3N+6+i}\,. \tag{16}$$

For the case of buffer capacity 1,

$$
\begin{aligned}
E(1)&=(1-\lambda_1)(\pi_2+\pi_3+\pi_8)\\
&=\frac{(1-p)\{r-\rho_1 q^2)u^{(1)}+\mu_2 q v^{(1)}+\alpha_1 q r\}}{(1+\rho_1)\{(r-\rho_1 q^2)u^{(1)}+\mu_2 q v^{(1)}+\alpha_1 q r\}+\alpha_1 t(1-\lambda_1)^2\rho_2 r},
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
M(1)&=1\cdot\pi_3+1\cdot\pi_5+1\cdot\pi_6+1\pi_{10}\\
&=\frac{\alpha_1 q r+\{(1-\lambda_1)^2 r+\rho_1\mu_1\mu_2(1-\lambda_1)\}\alpha_1 t\rho_2}{(1+\rho_1)\{(r-\rho_1 q^2)u^{(1)}+\mu_2 q v^{(1)}+\alpha_1 q r\}+\alpha_1 t(1-\lambda_1)^2\rho_2 r},
\end{aligned}
\tag{18}
$$

where the value in the parentheses represent the buffer capactiy. For the case of buffer capacity 2,

$$E(2)=\frac{(1-p)\{(r-\rho_1 s^2)u^{(2)}+\mu_2 q v^{(2)}+\alpha_1(sqr+\mu_1 qt)\}}{(1+\rho_1)\{(r-\rho_1 s^2)u^{(2)}+\mu_2 q v^{(2)}+\alpha_1(\delta qr+\mu_1 qt)\}+\alpha_1 t(1-\lambda_1)^2\rho_2 r}, \tag{19}$$

$$M(2)=\frac{\alpha_1 qr\{\delta+2-(1-\lambda_1)v^{(1)}\}-\mu_1 q^2 v^{(1)}+\{2(1-\lambda_1)^2 r+2\rho_1\mu_1\mu_2(1-\lambda_1)+(1-\lambda_1)\rho_1\mu_1\mu_2\delta\}\,\alpha_1 t\rho_2}{(1+\rho_1)\{(r-\rho_1 s^2)u^{(2)}+\mu_2 q v^{(2)}+\alpha_1(\delta qr+\mu_1 qt)\}+\alpha_1 t(1-\lambda_1)^2\rho_2 r} \tag{20}$$

where

$$\left.\begin{aligned}
\alpha_i &= \frac{1-\lambda_i}{\lambda_i}, \\
\beta_i &= \frac{1-\mu_i}{\mu_i}, \\
t &= \rho_1\{\beta_2+\alpha_1\rho_2\}\mu_1\mu_2 = \lambda_1+\lambda_2-\lambda_1(\lambda_2+\mu_2), \\
u^{(1)} &= 1-\frac{s\cdot t}{r}, \\
v^{(1)} &= 1-\frac{\mu_1\cdot t}{r}, \\
\delta &= \frac{1}{1-\lambda_1}\left\{\frac{\lambda_1 u^{(1)}+qv^{(1)}}{t}\right\} = \frac{1}{1-\lambda_1}\left\{\frac{p}{t}-\frac{\mu_1 t}{r}\right\}, \\
u^{(2)} &= \left\{\delta-\frac{\mu_1\lambda_2}{\beta_2 r}\right\}u^{(1)}+\frac{\mu_1\mu_2\lambda_2}{\beta_2 r}, \\
v^{(2)} &= \left\{\delta-\mu_1\left(1-\frac{\mu_1\lambda_2}{r}\right)v^{(1)}\right.
\end{aligned}\right\} \tag{21}$$

The efficiency E and mean buffer stocks M of a two stage line can be expressed as functions of buffer capacity in this manner.

Given buffer capacity, breakdown probabilities and repair probabilities of a two stage line, looking for the line efficiency and the mean buffer stocks is to solve a set of simultaneous linear equations. In the case of buffer capacity N, a set of simultaneous linear equations of $(4N+6)$ variables must be solved. By FACOM 236–60, up to $N=36$ can be solved from the restriction $4N+6\leqq 150$. If the line efficiency for a buffer capacity more than 36 is wanted, it should be obtained by means of extrapolation.

3.3. Efficiency of a three stage line

First, consider how many possible states can be constructed.

Lemma 2. The total number of possible states for an n stage line with N_i $(i=1, \cdots, n-1)$ is given by the following:

$$P(n) = 2(2N_1+3)(2N_2+3)\cdots(2N_{n-1}+3) \tag{22}$$

(Proof by mathematical induction) Suppose that an n stage line consists of the $(n-1)$ stages and the n th stage. Let $P_I(k)$, $P_1(k)$, $P_B(k)$, and $P_0(k)$ denote the number of possible states up to the k th stage in an n stage line, when the k th stage is I, 1, B, and 0, respectively.

Then, the following expression is obtained:

$$\begin{aligned}
P(n) &= (2N_{n-1}+3)P_1(n-1)+(2N_{n-1}+3)P_1(n-1) \\
&\quad +(2N_{n-1}+2)P_0(n-1)+P_B(n-1),
\end{aligned} \tag{23}$$

where the coefficients of $P_I(n-1)$, $P_1(n-1)$, $P_0(n-1)$, and $P_B(n-1)$ are obtained by considering how the possible number is increased by adding I, 1 and 0 states as the n th stage to the $(n-1)$ stages when the $(n-1)$ stage is I, 1 and 0, respectively. By mathematical induction, the following may easily be shown:

$$\left.\begin{aligned}
P_I(n-1) &= (2N_1+3)\cdots(2N_{n-3}+3)\cdot 2\\
P_1(n-1) &= (2N_1+3)\cdots(2N_{n-3}+3)\cdot(2N_{n-2}+1)\\
P_0(n-1) &= (2N_1+3)\cdots(2N_{n-3}+3)\cdot(2N_{n-2}+3)\\
P_B(n-1) &= (2N_1+3)\cdots(2N_{n-3}+3)\cdot(2N_{n-2}+3)
\end{aligned}\right\}, \qquad (24)$$

By substituting these into (23), the equation (22) is obtained.

Now, for a three stage line, there are $2(2N_1+3)(2N_2+3)$ possible states from the above analysis. It is an easy matter to seek the transition matrix for the case. In case of a three stage line with a buffer, the line efficiency and the mean buffer stocks up to buffer capacity of 11 can be obtained by means of FACOM 230-60. The same argument can be applied to lines with more than 4 stages. Therefore, from an analytical point of view, it is an easy (but tiresome) job to obtain the line efficiency and the mean buffer stocks for a line provided that the number of stages, the probability of breakdown, the probability of repair of each stage, and buffer capacities between stages are known. But from a computational point of view, the task becomes quite horrendous even by means of a modern computer as the number of stages and buffer capacity increase.

4. Optimum Buffer Installation Policy for Two Stage Lines

Based on the above analysis, this section will address itself mainly to the following problems about two stage lines.

(1) Should buffer stocks be used?

(2) What is the effect of a given buffer capacity on the line efficiency, or on the number of mean buffer stocks?

(3) What is the effect of a variation of breakdown rates with identical repair rates or repair rates with identical breakdown rates?

4.1 The line efficiency curve and the mean buffer stock curve as functions of buffer capacity

In order to get a primary knowledge of the effects of system parameters on the line efficiency and the mean buffer stocks, balanced systems ($\lambda_1=\lambda_2=\lambda$, $\mu_1=\mu_2=\mu$ and therefore $\rho_1=\rho_2=\rho$) are investigated first, and then unbalanced systems are considered. For the sake of explanation, the following qualitative adjectives are used to express the values of λ, μ and ρ,

Table 1. Qualitative adjectives to express λ, μ, ρ.

		1/10000		1/1000		1/100		1/10		1.0	
λ:	very low		low		medium		high		very high		
μ:	very low				low		medium		high		very high
ρ:	very low				low		medium		high		very high

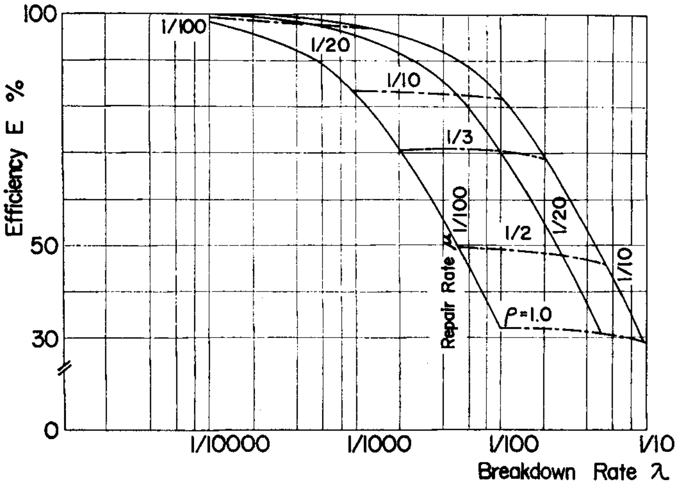

Fig. 6. Effect of breakdown rates on the line efficiency of two stage lines with no buffer.

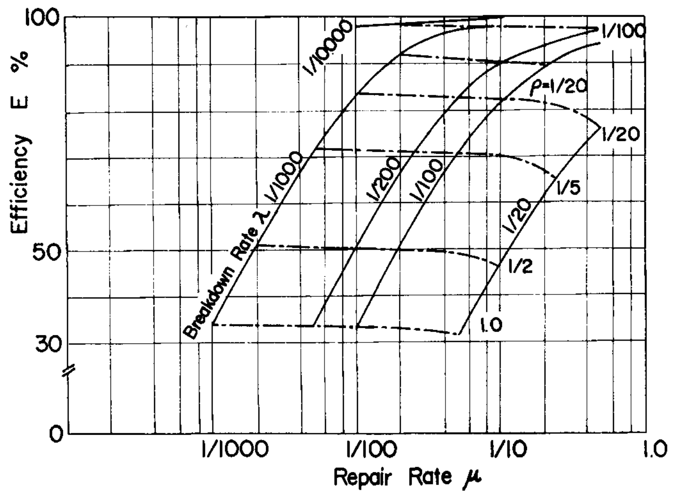

Fig. 7. Effect of repair rates on the line efficiency of two stage lines with no buffer.

4.1.1 The line efficiency for the case $N_1=0$.

The line efficiency of a balanced two stage line with no buffer can be calculated by equation (11). The results for various breakdown rates and repair rates are shown in Fig. 6, and Fig. 7. Fig. 6 shows the effect of breakdown rates on the line efficiency, while Fig. 7 shows the effect of repair rates on the line efficiency. From the figures, the line efficiency shows a slight falling tendency as the repair rates become high. Particularly when the breakdown rates are high and are greater than about 1/50, this tendency becomes pronounced. But excluding such a case where the line has high or very high breakdown rates, the line efficiency of a balanced two stage line with no buffer may be determined according to the values of ρ as in Table 2. The effect of installing a buffer on such a line will be investigated in the following.

Table 2. Efficiency of two stage lines with no buffer.

$\rho=1/1000$	more than 99%	1/10	more than 80%
1/100	more than 97%	1/5	around 70%
1/50	around 95%	1/2	around 50%
1/20	around 90%	1.0	more than 30%

Table 3. System parameters.

No.	Conditions		
	B. R.	R. R.	ρ
(1)	1/10000	1/10	1/1000
(2)–1	1/10000	1/100	1/100
–2	1/1000	1/10	1/100
–3	1/200	1/2	1/100
(3)–1	1/1000	1/100	1/10
–2	1/200	1/20	1/10
–3	1/50	1/5	1/10
–4	7/100	7/10	1/10
(4)–1	1/1000	1/200	1/5
–2	1/200	1/40	1/5
–3	1/100	1/20	1/5
–4	1/50	1/10	1/5
–5	1/20	1/4	1/5
(5)–1	1/1000	1/1000	1.0
–2	1/100	1/100	1.0
–3	1/20	1/20	1.0
–4	1/10	1/10	1.0

4.1.2 The effect of installing a buffer between the stages.

The line efficiency and the mean buffer stocks of a two stage line with a buffer can be calculated by equations (15) and (16). Selecting as the values of ρ, 1/1000, 1/100, 1/10, 1/5 and 1, and choosing as the values of λ and μ the values shown in Table 3, and then calculating the line efficiency and the mean buffer stocks

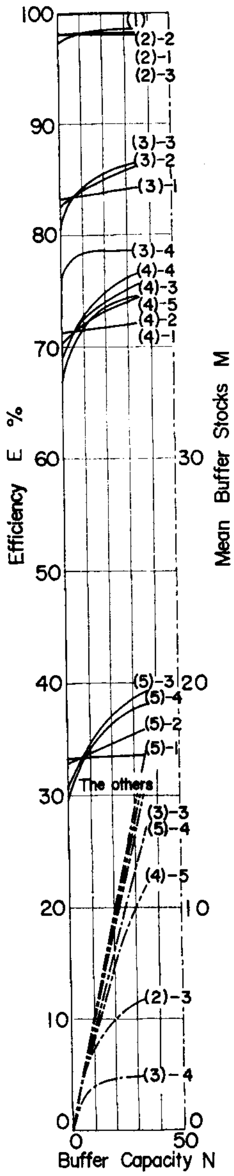

Fig. 8. The line efficiecy curves and the mean buffer stock curves whose system parameters are shown in Table 3.

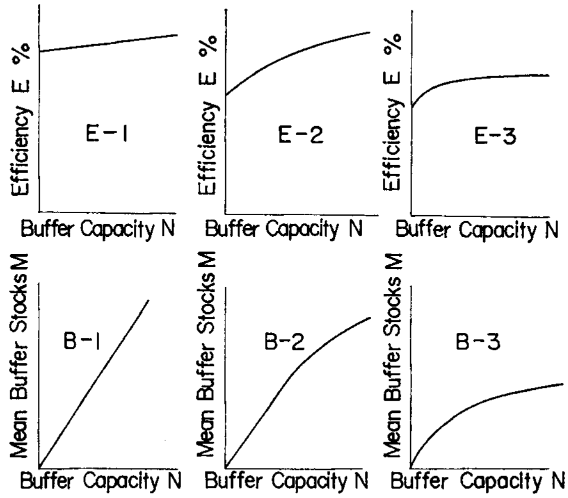

Fig. 9. Classification of line efficiency curves and mean buffer stock curves.

by the equations, result in Fig. 8. The curves which show the line efficiency, and the mean buffer stocks as functions of buffer capacity are termed the line efficiency curve, and the mean buffer stock curve, respectively.

From this investigation, the following two useful results are obtained.

(1) There are three kinds of line efficiency curves and two kinds of mean buffer stock curves in the case of balanced two stage lines. In Fig. 9, type E–1 represents the case in which the line efficiency curve is approximately linear at first, then tends to turn to be smooth concave and finally converges to a certain value. In Fig. 8, (1), (2)–1, –2, (3)–1, (4)–1, (5)–1, and (5)–2 belong to this type. In such a line it is almost impossible to improve the line efficiency by providing a buffer. Type E–2 is the case in which the line efficiency curve shows clear concavity. For such a line, the question of buffer capacity arises. In Fig. 8, (3)–2, (4)–2, –3, –4, and (5)–3 are of this type. Type E–3 shows the case in which at some initial increase of buffer capacity, the line efficiency improve markedly, but thereafter the improvements get progressively smaller. The initial buffer capacity which brings about the remarkable efficiency increase depends on the breakdown rates and the repair rates of the two stages. Generally speaking, the initial buffer capacity should equal five times the mean repair time in case the breakdown rates are medium ((2)–3 in Fig. 8) and almost eight to ten times the mean repair time in case the breakdown rates are high ((3)–3, (4)–5, (5)–4 in Fig. 8). Although the case (3)–4 in Fig. 8 also belongs to this type, buffer stocks are hardly saved due to the high breakdown rates of the two stages. The mean buffer stocks for a buffer capacity of 30 are around

2 to 3 (units), which is quite low. The efficiency improvement can not be achieved in this case. The stage efficiency of both stages is 90.9 (%), but linking the two stages causes a remarkable efficiency decrease and gains only 78.6 (%) even by providing a buffer storage. The effect of efficiency improvement is due mainly to the provision of an initial buffer capacity of 10. From the viewpoint of efficiency, linking stages for such a case should be definitely avoided.

The mean buffer stock curves which appear in Fig. 8 belong to either type B-2 or type B-3 in Fig. 9. It will be shown later that type B-1 exists, but for the sake of consistency, type B-1 is explained here with B-2 and B-3.
Type B-1 appears when the efficiency of stage 2 is lower than that of stage 1. The mean buffer stocks increase linearly as the buffer capacity increases. Type B-2 shows the case in which the mean buffer stock curve is almost linear at first, then turns to be concave and finally tends to converge to a certain value. In the linear portion of the line, the mean buffer stocks are about half of the buffer capacity. In case the mean buffer stock curve exceeds this half buffer capacity line, it tends to be of type B-1. If the mean buffer stock curve goes down the half buffer capacity line, it tends to be of type B-3. Type B-3 represents the case in which at each increase of buffer capacity the rate of increase of the mean buffer stocks gets progressively smaller, and the mean buffer stock curve tends to converge to a certain value. Cases (1), (2)-1, (2)-2, (3)-1, (3)-2, (4)-1, (4)-2, (4)-3, (4)-4, (5)-1, (5)-2, (5)-3 in Fig. 8 belong to type B-2, while the others belong to B-3.

The line efficiency curves and the mean buffer stock curves for buffer capacities up to 35 have been presented before. These calculations revealed that there are three kinds of line efficiency curves and two kinds of mean buffer stock curves, presuming that both stages have identical system parameters.

In what follows, a buffer capacity of 30 is taken as a measure to investigate the effect of installing a buffer, since it suffices to get the line efficiency curve up to a buffer capacity 30 in order to judge which type the two stage line concerned belongs to.

(2) Subject to the identical ρ's of two stages, if there is no buffer between the two stages, sophisticated lines having low breakdown rates and long mean repair time have higher line efficiency than unsophisticated lines having higher breakdown rates and shorter mean repair time. By providing buffer between the stages, unsophisticated lines may have higher line efficiency than sophisticated lines. However, when it comes to cases such as (3)-4 and (4)-5 in Fig. 8 which have high breakdown rates and high repair rates, the line efficiency without a buffer is constitutionally low, and therefore an expected line efficiency improvement is hardly achieved even providing much buffer capacity.

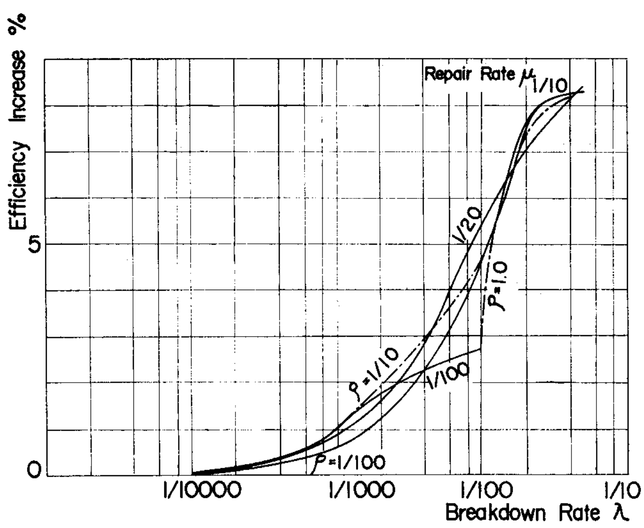

Fig. 10. Effect of the breakdown rates on the efficiency increase by providing buffer capacity of 30.

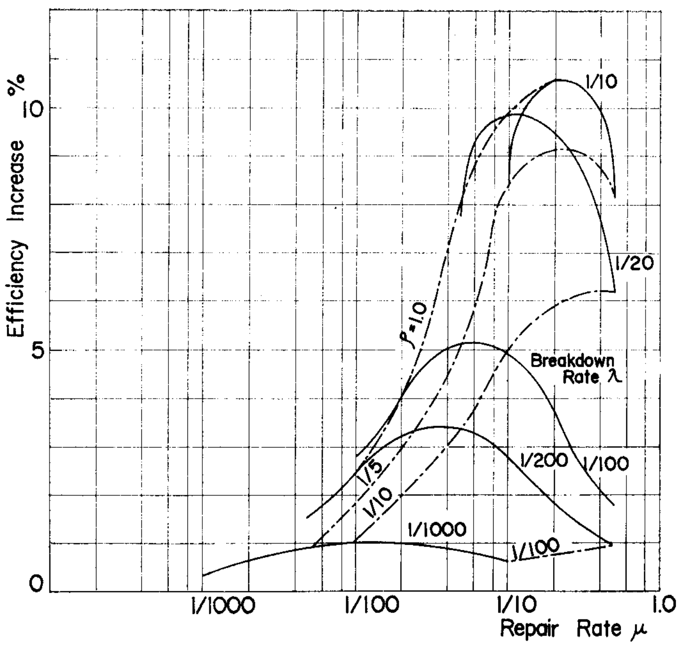

Fig. 11. Effect of the repair rates on the efficiency increase by providing buffer capacity of 30.

Fig. 10 shows the effects of repair rates on the efficiency increase by a buffer capacity of 30. Fig. 11 shows the effects of breakdown rates on the efficiency increase by a buffer capacity of 30. From Fig. 10, the effect of providing a buffer is brought about in case breakdown rates are high. From Fig. 11, the buffer effect seems to appear mostly when the repair rates are around 1/10, although it depends on the breakdown rates.

4.1.3 The effect of variation of repair rates with identical breakdown rates.

Setting the breakdown rates to be 1/200, and changing the repair rates from 1/2 to 1/200 (Table 4. (a)), and calculating the line efficiency curves and the mean buffer stock curves, provides the results in Fig. 21 (a). As the value of μ decreases, type E-3 [(1), (2), (3)], then type E-2 [(4), (5)] and finally type E-1 [(6), (7)] characteristically appear. The efficiency increase resulting from providing a buffer capacity of 30 gets the maximum value 3.34 (%) when the repair rates are around 1/20. The mean buffer stock curve E-3 shows up, correspondingly with line efficiency curve B-3; and E-2 and E-1 appear, correspondingly with B-2.

4.1.4 The effect of variation of breakdown rates with identical repair rates.

The results by setting the repair rates to be 1/20, and changing the breakdown rates from 1/2000 to 1/20 (Table 6.4. (b)) are shown in Fig. 12 (b). As the value λ increases, type E-1 [(1), (2)] appears first and then type E-2 [(3), (4), (5), (6), (7)] shows up characteristically. In this case, type E-3 does not turn up. The mean

Table 4. System parameters for variations of repair rates (a) and breakdown rates (b).

	No.	Conditions		
		B. R.	R. R.	ρ
(a)	(1)	1/200	1/2	1/100
	(2)		1/4	1/50
	(3)		1/10	1/20
	(4)		1/20	1/10
	(5)		1/40	1/5
	(6)		1/100	1/2
	(7)		1/200	1.0
(b)	(1)	1/2000	1/20	1/100
	(2)	1/1000		1/50
	(3)	1/400		1/20
	(4)	1/200		1/10
	(5)	1/100		1/5
	(6)	1/40		1/2
	(7)	1/20		1.0

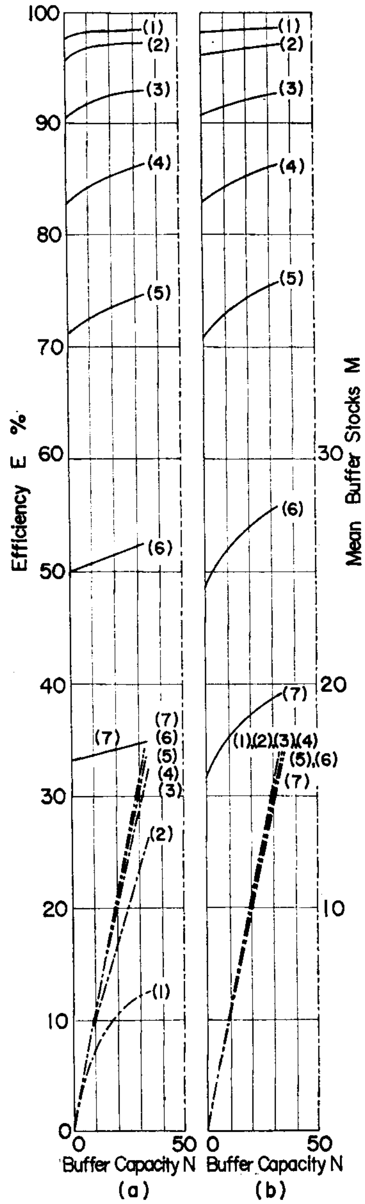

Fig. 12. Variations of repair rates (a) and breakdown rates (b).

buffer stock curves B-1, and B-2 appear correspondingly with E-1, and E-2, respectively.

4.1.5 The system parameters which classify line efficiency curves into three types.

It was shown that there are three kinds of line efficiency curves. It should be clarified how the system parameters determine these three types. The previous

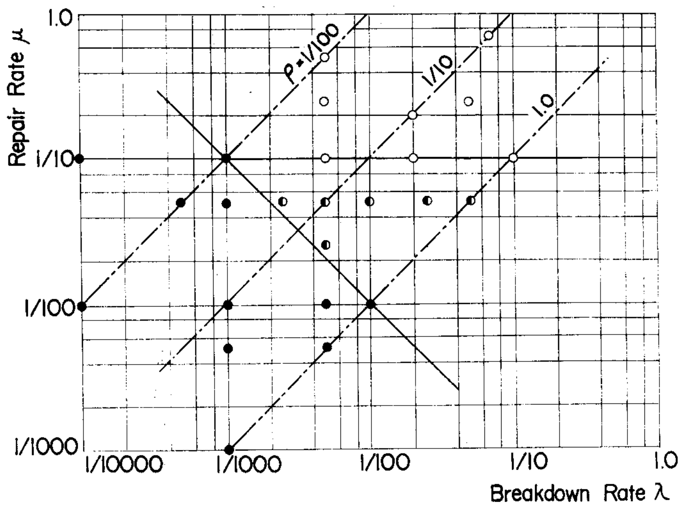

Fig. 13. Classification of line efficiency curves by system parameters, ●: type E–1 ◐: type E–2 ○: type E–3

data after classifying them into three types are plotted in Fig. 13. In the figure, the abscissa represents the breakdown rates, the ordinate, the repair rates. It can be said from the figure that type E-1 (● mark) shows up in case the product of λ and μ is less than $1/10^4$. Type E-3 (○ mark) appears in case the repair rates are less than 1/10 and the breakdown rates are relatively high. Otherwise, type E-2 (◐ mark turns up. When a two stage line has breakdown rates 1/1000 and repair rates 1/10, it becomes type E-1. In this case, as shown in Fig. 8, the efficiency increases, by providing buffer capacities of 10, 20, and 30, are 0.04 (%), 0.05 (%), and 0.06 (%), respectively. The effect of improving the line efficiency is mostly due to the provision of an initial buffer capacity of 10. From the viewpoint of efficiency increase, the line is of type E-1, but from a relative point of view, it has the characteristic of type E-3 also. This is due to low repair rates 1/10 of the line.

4.1.6 The effect of variation of stage efficiencies.

The primal knowledge on the line efficiency curve and the mean buffer stock curve in the case of a balanced two stage line was obtained in the above. Now the line efficiency curve and the mean buffer stock curve in the case of an unbalanced two stage line will be investigated.

Fig. 14(a) shows an example in which both the stages have identical repair rates 1/20 but different breakdown rates 1/400 and 1/200. The detailed system parameters are shown in Table 5(a). For the sake of comparison, the cases where both stages have identical breakdown rates 1/400, and 1/200 are also shown in the figure.

Table 5. System parameters for the variations of different breakdown rates.

		Stage No.	Conditions		
			B. R.	R. R.	ρ
(a)	(1)	1 2	1/400	1/20	1/20
	(2)	1 2	1/200	1/20	1/10
	(3)	1 2	1/400 1/200	1/20 1/20	1/20 1/10
	(4)	1 2	1/200 1/400	1/20 1/20	1/10 1/20
(b)	(1)	1 2	1/200	1/20	1/10
	(2)	1 2	1/100	1/20	1/5
	(3)	1 2	1/200 1/100	1/20 1/20	1/10 1/5
	(4)	1 2	1/100 1/200	1/20 1/20	1/5 1/10
(c)	(1)	1 2	1/200	1/20	1/10
	(2)	1 2	1/20	1/20	1.0
	(3)	1 2	1/200 1/20	1/20 1/20	1/10 1.0
	(4)	1 2	1/20 1/200	1/20 1/20	1.0 1/10

It can be observed first that from the viewpoint of line efficiency, the effects of interchanging stage 1 and stage 2 having different breakdown rates are almost negligible. On the other hand, the mean buffer stock curve in the case (3), in which stage 2 has a higher breakdown rate than stage 1, shows type B-1, the mean buffer stocks by the provision of a buffer capacity of 30 being about 20 which is quite high. In the converse case (4), the mean buffer stock curve shows type B-3, the mean buffer stocks by the provision of a buffer capacity of 30 being around 9 which is quite

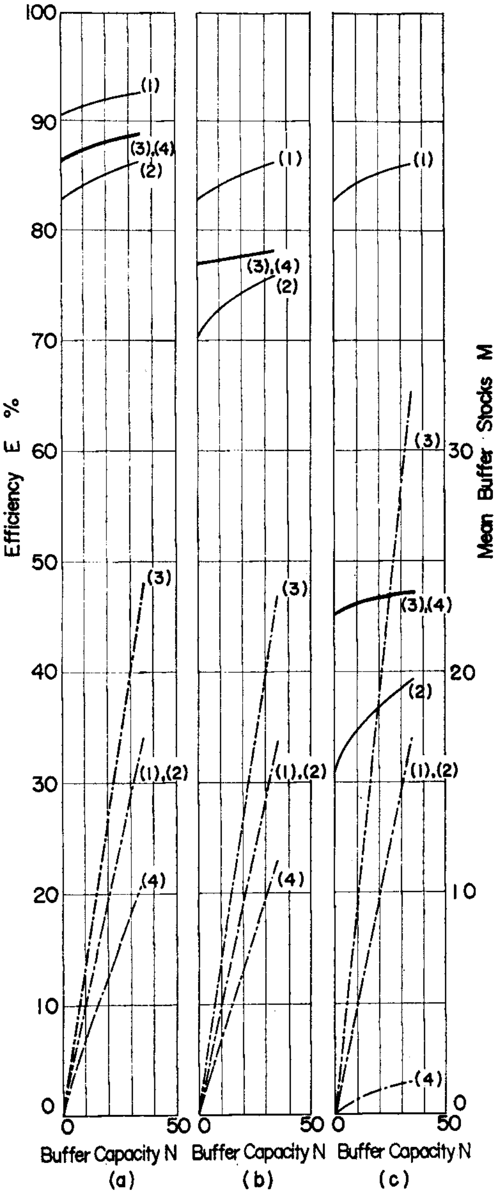

Fig. 14. Variations of different breakdown rates.

low. It can be said that in case both stages with identical repair rates have different breakdown rates, the difference between not being large, the line efficiency takes a median value between the line efficiency gained in the case where both stages have lower breakdown rates and that gained in the case where both stages have higher breakdown rates.

Fig. 14(b) shows another example, whose system parameters are shown in Table 5. (b). For reference, the case where both stages have identical breakdown rates 1/200, and 1/100 are also shown in the figure. From the figure, the difference between the cases (3) and (4) can be neglected. If there is no buffer the line efficiency of the cases (3) and (4) takes a median value between the line efficiencies of the case (1) and (2). If there is a buffer between the stages, the line efficiency of the case (3) or (4) is affected to a greater extent by the higher breakdown rates. It is quite interesting that the both line efficiency curves of the case (1) and (2) are of type E-2, but the line efficiency curves of the cases (3) and (4) are of type E-1. According to the classification of line efficiency curves, the breakdown rates and repair rates concerned do not enter into the region that produces E-1 type. It seems to be reasonable that combining two stages, which have different breakdown rates, results in reducing the effects of installing a buffer. To investigate this matter in detail, the case in which two stages with different breakdown rates, the difference between which is large, will be examined next. The system parameters are shown in Table 5. (c). The results are shown in Fig. 14. (c). It can be read from the figure that in case there is a large difference between the two breakdown rates, the line efficiency is influenced to a greater extent by the higher breakdown rate. Moreover, the effects of providing a buffer are reduced due to different breakdown rates. The line efficiency curves of the cases (3) and (4) tend to be of type E-3, the gradient of the curve being more smooth than those of the efficiency curves of the cases (1) and (2). Furthermore, it is better for such a line to locate the stage with the lower breakdown rate first.

Finally, for a two stage line with identical repair rates, if the difference of breakdown rates is small, the line efficiency takes a median value between the line efficiency gained in the case where both the stages have lower breakdown rates and that gained in the case where both the stages have higher breakdown rates. The mean buffer stock curve shows type B-1 in the case where stage 1 has the lower breakdown rate, and type B-3, otherwise. As the difference increases, the line efficiency is affected to a greater extent by the higher breakdown rate, and the effects of installing a buffer are reduced.

Fig. 15(a), (b), and (c) show three examples in which both stages have identical breakdown rates 1/200 but different repair rates. The system parameters are shown in Table 6. (a), (b), and (c), respectively. From the figure, the following can be concluded for a two stage line with identical breakdown rates: If the difference of the repair rates is small, the efficiency takes a median value between the efficiency gained in the case where both stages have lower repair rates and that gained in the case where both stages have higher repair rates. As the difference increases,

Table 6. System parameters for variations of different repair rates

		Stage No.	Conditions		
			B. R.	R. R.	ρ
(a)	(1)	1 2	1/200	1/10	1/20
	(2)	1 2	1/200	1/20	1/10
	(3)	1 2	1/200 1/200	1/10 1/20	1/20 1/10
	(4)	1 2	1/200 1/200	1/20 1/10	1/10 1/20
(b)	(1)	1 2	1/200	1/20	1/10
	(2)	1 2	1/200	1/40	1/5
	(3)	1 2	1/200 1/200	1/20 1/40	1/10 1/5
	(4)	1 2	1/200 1/200	1/40 1/20	1/5 1/10
(c)	(1)	1 2	1/200	1/20	1/10
	(2)	1 2	1/200	1/200	1.0
	(3)	1 2	1/200 1/200	1/20 1/200	1/10 1.0
	(4)	1 2	1/200 1/200	1/200 1/20	1.0 1/10

the line efficiency tends to be affected to a greater extent by the lower rate, but the effects of installing a buffer are not reduced, as is seen in the case of two stages with different breakdown rates.

From the above, it can be also said that a variation of breakdown rates affects the line efficiency more strongly than that of repair rates.

4.1.7 Interchanging the two stages which have different parameters.

As has been seen in Fig. 14. (a), (b), and (c), Fig. 15. (a), (b), and (c) in Section 4.1.6, from the viewpoint of the line efficiency, the effects of interchang-

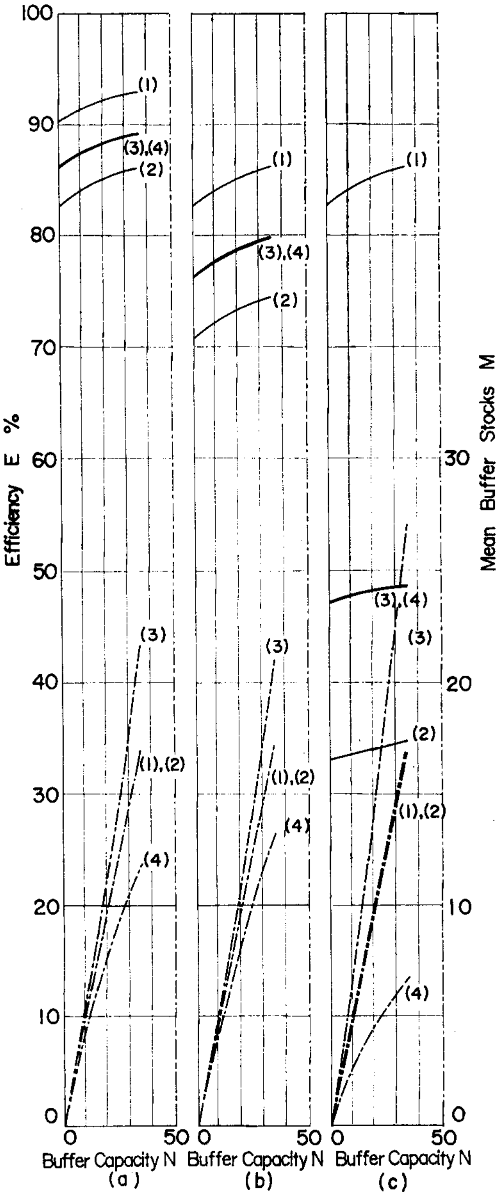

Fig. 15. Variations of different repair rates.

ing the two stages which have different parameters are almost negligible. When there is a large difference between two breakdown rates or between two repair rates, it is better to locate the stages with the higher stage efficiency first. On the other hand, the mean buffer stock curve is of type B-1, in the case where stage 1 has the higher stage efficiency, and type B-3 in the converse case. The investigation on the line efficiency curve and the mean buffer stock curve for a two stage line is now concluded and an optimal buffer capacity model is to be developed.

4.2 Cost analysis of a two stage line

The purpose of this section is to analyze the problem of determining the optimal buffer capacity with respect to the appropriate costs for a two stage line. In order to formulate an optimal buffer capacity model, the following assumptions are introduced.

(1) The revenue from the system is proportional to the line efficiency.

(2) The following two costs are incurred by providing a buffer:

(i) The inventory holding cost, which is proportional to the mean buffer stocks in the buffer storage.

(ii) The storage facility cost, which is the cost to initially install a storage facility plus the cost proportional to the buffer capacity.

(3) The profit from the system is given by the following equation:

Profit=Revenue−Inventory holding cost−Storage facility cost.

In order to find the optimal buffer capacity, the line efficiency curve and the mean buffer stock curve, as functions of buffer capacity, must be known. In the case of two stage lines they are easily obtained as shown in the previous section. That is, the line efficiency curve and the mean buffer stock curve up to $N_1=36$ are obtained and thereafter they are easily extrapolated, assuming that the line efficiency curve is concave and that the mean buffer stock curve is linear or approximately concave, depending upon the mean buffer stock curve up to $N_1=36$.

Given the line efficiency curve and the mean buffer stock curve, the revenue $f(N)$ and the inventory holding cost $h(N)$ are expressed by the following:

$$f(N) = A\cdot E(N) \text{ [}N\text{ is buffer capacity]} \tag{25}$$

$$h(N) = B\cdot MW(N) \tag{26}$$

where, A: the fixed revenue in releasing a completed workpiece from the system,

B: the fixed inventory carrying cost per workpiece per time unit,

$F(N)$: the line efficiency as a function of buffer capacity N,

$MW(N)$: the mean buffer stocks as a function of buffer capacity N.

The storage facility cost $s(N)$ is expressed by the folowing:

$$s(N) = \begin{cases} 0 & \text{for } N = 0 \\ C+D\cdot N & \text{for } N > 0 \end{cases} \tag{27}$$

where, C: the fixed cost per unit time of initially installing an inventory storage facility,

D: the fixed cost per workpiece per unit time of maintaining an inventory storage facility.

Therefore, the system profit per unit time $k(N)$ is then

$$k(N) = f(N) - h(N) - s(N) \tag{28}$$

$$\begin{cases} = A \cdot E(0) - B \cdot MW(0) & \text{for } N=0 \\ = A \cdot E(N) - B \cdot MW(N) - C - D \cdot N & \text{for } N>0 \end{cases} \tag{29}$$

The profit curve as a function of the buffer capacity can be drawn as shown in Fig. 16. The buffer capacity N_0 which gives the maximum of the profit function is the optimal buffer capacity. The problem of installing a buffer comes into consideration only in the case $k(N) - B(0) > C$.

Now assume further that the inventory holding cost is a linear function of the buffer capacity, viz., the mean buffer stocks are proportional to the buffer capacity, which is estimating the mean buffer stocks higher than the actural values in case the

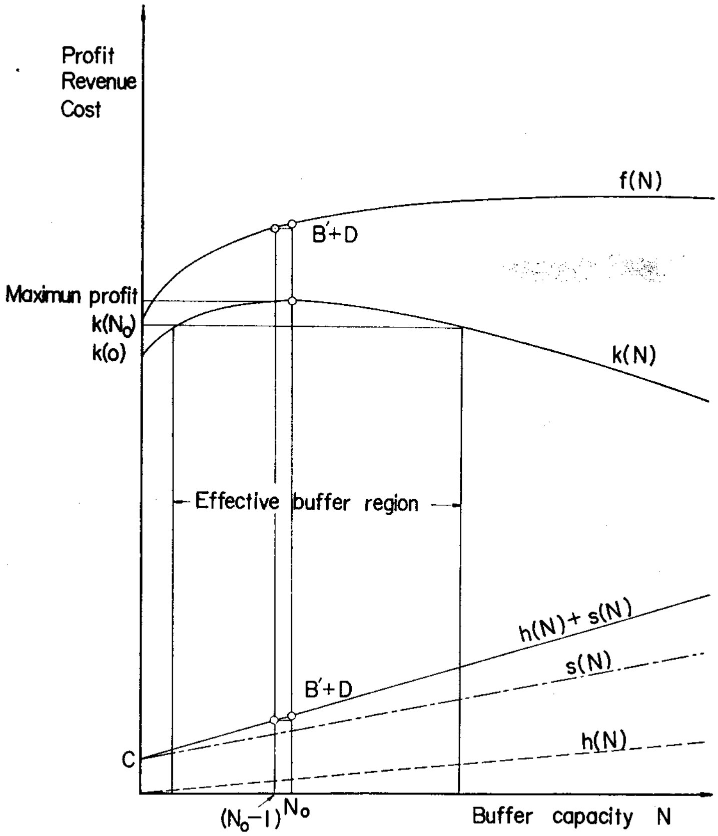

Fig. 16. Profit curve as a function of buffer capacity.

mean buffer stock curve is either type B-2 or type B-3.

$$k(N) = B' \cdot N \tag{27$'$}$$

Then, $$\Delta k(N) = \Delta f(N) - B' - N \quad \text{for } N > 0 \tag{29}$$

Since function $f(N)$ can be assumed to be concave,

$$\Delta f(N-1) > \Delta f(N) \tag{30}$$

Therefore, the buffer capacity which satisfies the following:

$$\Delta k(N_0 - 1) > 0 > \Delta k(N_0) \tag{31}$$

$$\Delta f(N_0) \doteqdot B' + D \tag{32}$$

gives the optimal buffer capacity. In other words, the value N_0 which satisfies the following on the line efficiency curve

$$E(N_0) \doteqdot \frac{B'+D}{A} \tag{33}$$

is the optimal buffer capacity. The equation (33) gives a sort of measure up to how much buffer capacity the line efficiency curve should be calculated.

5. Conclusions

By carrying out an investigation on the line efficiency and the mean buffer stocks for various system parameters of two stage lines, the following were obtained:

(1) In what case should buffer stocks be used?

There are three kinds of line efficiency curves E-1, E-2, and E-3. In case of two stage lines with identical breakdown rates and identical repair rates, type E-1 appears when the product of λ and μ is less than $1/10^4$, and it is impossible to improve the line efficiency by providing a buffer in this case. Type E-3 shows up when the repair rates are less than 1/10 and the breakdown rates are relatively high. In this case, the line efficiency improves remarkably at some initial increase of buffer capacity, but thereafter the improvements get progressively much smaller. The initial buffer capacity, which brings about the remarkable efficiency increase, depends on the breakdown rates and the repair rates of the two stages. However, generally speaking, the initial buffer capacity should be five times the mean repair time in case the breakdown rates are medium, and about eight to ten times the mean repair time in case the breakdown rates are high. Otherwise, type E-2 turns up, which is clearly concave. In this case, the question of how much buffer capacity should be prepared arises and a careful cost analysis is required.

(2) As to the mean buffer stocks,

There are three kinds of mean buffer stock curves B-1, B-2, and B-3. Type B-1 represents the case in which the mean buffer stocks increase linearly as the buffer capacity increases. Type B-2 represents the case in which the mean buffer stock curve is almost linear at first, then turns to be smooth concave and finally tends to converge to a certain value. In the linear portion of the line the mean buffer stocks are about half of the buffer capacity. Type B-3, the case in which at each increase of buffer capacity the increase rate of the mean buffer stocks gets progressively smaller, and the mean buffer stock curve tends to converge to a certain value.

(3) System parameters which demand in-process inventory banks.

Installing buffer storage is most effective when the breakdown rates are high and the mean repair time is short.

(4) The effects of variation of repair rates and breakdown rates.

A variation of breakdown rates affects the line efficiency more than that of repair rates. Therefore, the stages should be designed to have approximately the same breakdown rate.

(5) The effects of variation of stage efficiency.

If the difference of breakdown rates (repair rates) is small, the line efficiency takes a median value between the line efficiency gained in the case where both stages have lower breakdown rates (repair rates) and that gained in the case in which both stages have higher breakdown rates (repair rates). As the difference increases, the line efficiency is affected to a greater extent by a higher breakdown rate (lower repair rate). The variation of breakdown rates reduces the effects of installing a buffer, while that of repair rates does not.

Cost analysis has revealed the following:

(6) Under the assumption that the mean buffer stock curve is approximately linear, the buffer capacity at which the line efficiency improvement by a unit increase of buffer capacity is equal to: (the fixed inventory carrying cost per workpiece per time unit + the fixed cost per workpiece per unit time of maintaining an inventory storage facility)/(the fixed revenue in releasing a completed workpiece from the system) gives the optimum buffer capacity to maximize the system profit.

(7) Since the line efficiency curve is assumed to be a concave function, the system does not demand in-process inventory banks if the line efficiency improvement by providing a buffer capacity of 1 does not exceed the above value in (6).