On Estimation of the Maximum Structural Response to Random Earthquake Motion from Response Envelope

By

Hiroyuki KAMEDA*

(Received June 29, 1974)

Synopsis

A method of analysis is discussed to obtained the probability distribution of the maximum response of structures subjected to a non-stationary random earthquake motion. The envelope of the narrow band response of structures is analyzed, from which an approximate result for the probability distribution of the maximum response is derived. The accuracy of the analytical results is chekced by means of a numerical simulation.

1. Introduction

It is frequently the case in structural design for dymaic loads that sefety of the structure is stipulated as the requirement that the maximum response be not in excess of a prescribed allowable value. When the design for random seismic loads is considered, the concept of the structural reliability takes an important role, and in this case, the probability distribution of the maximum response should be a response parameter of primary importance since it represents the load distribution in the reliability analysis.

Because of its role in the discussion of the structural behavior in random vibration, the analysis of the probability distribution of the maximum response has received considerable attention from many researchers. Its basic formulation involves a kind of the pure-birth-process equation[1),6),8)], or the first passage time density[2),3),7),9),12),13),16)]. However, no exact solution for this problem has been presented since the random response of structures is usually a narrow band process which consequently causes a high correlation between consecutive response peaks. Therefore, the methods of analysis so far developed provide approximate procedures. They include a simple Poisson process approximation[1)], taking account of the correlation effects between the response peaks in a direct way[8)], or in terms of the mean clump size[2),12)], or a renewal process approximation[9),13)], and obtaining

* Dept. of Transportation Engineering

the upper and lower bounds of the exact solution[15].

The method of analysis in this study deals with the envelope of a narrow band response of structures. Since the envelope of the structural response varies relatively slowly, a Poisson process approximation can be applied to it. Lin[11] applied this idea to the case of a statioary response. However, this method has not been checked from the aspect of its accuracy. Morevoer, it is necessary to treat random earthquake motions as non-stationary processes. In the subsequent chapters, the non-stationary response of structures is discussed as an amplitude-modulated narrow band random process, and the probability distribution of the maximum response is obtained from the response envelope. The accuracy of the method is surveyed by comparing the analytical result with the result of numerical simulation.

2. Basic Properties of Random Earthquake Response of Simple Structures

(1) Random Earthquake Motion and r.m.s. Response

A statistical model of earthquake acceleration $\ddot{z}(t)$ proposed by the author[8] is employed:

$$\ddot{z}(t) = \beta f(t;\tau) g(t) \qquad \cdots\cdots(1)$$

where $g(t)$ is a stationary Gaussian process with a zero mean value, the variance of unity and the power spectral density $S_g(\omega)$, β is a constant with the dimension of acceleration, and $f(t;\tau)$ is a deterministic positive function with the maximum value of unity. The parameter τ is the equivalent duration[8] defined as the duration of a finite portion of a stationary process $\beta g(t)$ cut out so that its mean maximum acceleration may be equal to that of the non-stationary motion in Eq. (1).

Representing the earthquake ground acceleration in the form of Eq. (1) seems to be appropriate in some cases[10),17]. However, it does not mean that Eq. (1) is capable of representing any type of accelerogram. Eq. (1) should be understood as a first order approximation of accelerograms which should be replaced by a more precise model, especially with non-stationary spectral characteristics beased on future developments in the strong motion seismology[18),19].

For $f(t;\tau)$, the following form is used[8]:

$$f(t;\tau) = \frac{(1+\xi)^{(1+1/\xi)}}{\xi} e^{-st}(1-e^{-\xi st}) \qquad \cdots\cdots(2)$$

Eq. (2) is modification of the type employed earlier[15] to give a linear initial set up and an exponential subsiding tail of the earthquake motion. When $f(t;\tau)$ in Eq. (2) is used, and if $g(t)$ has a single predominant frequency ω_0, the equivalent duration τ is approximately related to s as[8)]

$$\frac{s}{\omega_0} = C\left(\frac{\tau}{T_0}\right)^{-1.09} \quad \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

where $T_0=2\tau/\omega_0$ is the predominant period of $g(t)$, and C is dependent on ξ.

The maximum value $f(t;\ \tau)=1$ is attained at

$$t = t_m = \frac{1}{s\xi}\log(1+\xi) \quad \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

Sets of numerical values for st_m, ξ, and C are given in Table 1.

The relation between the equivalent duration τ and the acceleration parameter β has been obtained[4),5)], as shown in Fig. 1, in which α_m is the expected value of the maximum ground acceleration.

Table 1. Non-stationarity Parameters of Earthquake Motion

st_m	ξ	C
1.0	0.0	0.166
0.8	0.539	0.136
0.6	1.579	0.108
0.4	4.047	0.077
0.2	13.30	0.050

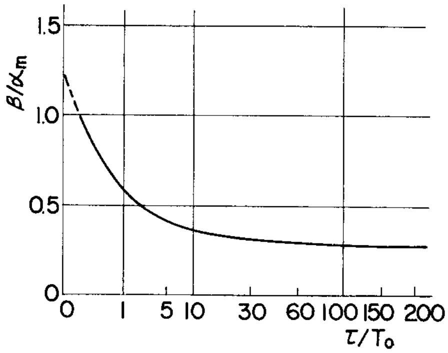

Fig. 1. Intensity Parameter β (cubic-root scale for abscissa).

With the assumptions described above, the statistical characteristics of the earthquake acceleration has been specified. Hence, it is possible to deal with the structural response $y(t)$ given by

$$y(t) = -\frac{1}{\sqrt{1-h_n^2}\,\omega_n}\int_0^t e^{-h_n\omega_n(t-\tau)}\sin\sqrt{1-h_n^2}\,\omega_n(t-\tau)\ddot{z}(\tau)d\tau \quad \cdots\cdots\cdots (5)$$

which represents the relative displacement of a simple structure with the natural circular frequency ω_n and the damping factor h_n.

A close approximation to the r.m.s. response $\sigma_y(t)$ of $y(t)$ for the model of earthquake acceleration discussed above can be obtained from[8]

$$\sigma_y^2(t) \cong \frac{\pi\beta^2 A^2}{4\omega_n^3} S_g(\omega_n) e_1(t) \qquad \cdots\cdots(6)$$

where

$$e_1(t) = \frac{e_0(s, s, t)}{\lambda(s)} + \frac{e_0((1+\xi)s, (1+\xi)s, t)}{\lambda((1+\xi)s)} - \frac{2}{\lambda(s)+\lambda((1+\xi)s)} \{e_0(s, (1+\xi)s, t) + e_0((1+\xi)s, s, t)\}$$

$$e_0(s_1, s_2, t) = e^{-(s_1+s_2)t} - e^{-2h_n\omega_n t}$$

$$\lambda(s) = h_n - s/\omega_n, \quad A = \frac{(1+\xi)^{(1+1/\xi)}}{\xi}$$

The power spectral density $S_g(\omega)$ of $g(t)$ is assumed to take the following form[8]:

$$S_g(\omega) = \frac{4h_0}{\pi\omega_0} \frac{(\omega/\omega_0)^2}{\{1-(\omega/\omega_0)^2\}^2 + 4h_0^2(\omega/\omega_0)^2} \qquad \cdots\cdots(7)$$

For the numerical computations in this study, the following values are assigned to the basic parameters which appeared in the preceding discussions[8]:

$$st_m = 0.4$$
$$\tau/T_0 = 3, 10, 20$$
$$h_n = 0.02, 0.05, 0.1, 0.2$$
$$h_0 = 0.9$$

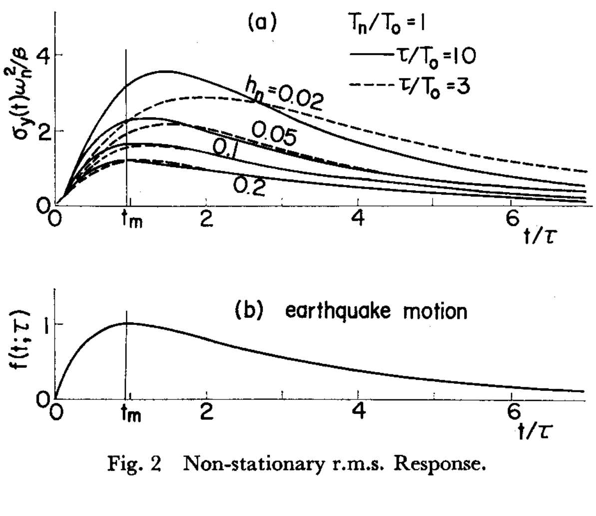

Fig. 2 Non-stationary r.m.s. Response.

Some examples of the r.m.s. intensities $\sigma_y(t)$ and $f(t;\tau)$ of $y(t)$ and $\ddot{z}(t)$, respectively, are shown in Fig. 2, in which $\sigma_{y\max}$ is the maximum value of $\sigma_y(t)$. Delay of the time of appearence of $\sigma_{y\max}$ from t_m exhibits the transient effect.

(2) Spectral Characteristics of Response

The structural response $y(t)$ in Eq. (5) is a random process which is non-stationary both in intensity and spectral characteristics. However, since the spectral non-stationarity is significant only at an early stage of response which has little effect upon the maximum response, it would be appropriate to represent $y(t)$ in the following form analogous to Eq. (1):

$$y(t) = \sigma_y(t)r(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots (8)$$

where $\sigma_y(t)$ is given by Eq. (6) and $r(t)$ is a stationary Gaussian process with a zero mean value, the variance of unity and the power spectral density $S_r(\omega)$. Eq. (8) differs from the exact expression in that it neglects the initial non-stationary frequency contents. However, this spectral non-stationarity vanishes very rapidly before $\sigma_y(t)$ grows significantly, since the correlation coefficient between $y(t)$ and $\dot{y}(t)$ approaches zero very quickly[6),8)]. Hence, Eq. (8) would be justified in the analysis of the maximum response which is the main subject of this study.

The stationary process $r(t)$ in Eq. (8) can be represented in the following spectral form:

$$r(t) = \int_0^\infty \cos(\omega t - p(\omega))dc(\omega) \quad \cdots\cdots\cdots\cdots\cdots\cdots (9)$$

Here $p(\omega)$ is a random phase angle distributed uniformly over $[0, 2\pi]$ and forms a white noise on the ω-axis. The random amplitude $dc(\omega)$ has a Gaussian distribution with

$$E[dc(\omega)] = 0\,, \quad \text{and} \quad E[\{dc(\omega)\}^2] = 2S_r(\omega)d\omega \quad \cdots\cdots\cdots\cdots (10)$$

The random process $c(\omega)$ is of uncorrelated increments on the ω-axis.

The power spectral density $S_r(\omega)$ of $r(t)$ for a slightly damped simple structure can be represented by

$$S_r(\omega) = \frac{4h_n}{\pi\omega_n} \frac{1}{\{1-(\omega/\omega_n)^2\}^2 + 4h_n^2(\omega/\omega_n)^2} \quad \cdots\cdots\cdots\cdots\cdots\cdots (11)$$

The auto-correltation function of $r(t)$ is obtained as

$$R_r(\tau') = \int_0^\infty S_r(\omega)\cos\omega\tau' d\tau'$$
$$= e^{-h_n\omega_n\tau'}\left(\cos\sqrt{1-h_n^2}\,\omega_n\tau' + \frac{h_n}{\sqrt{1-h_n^2}}\sin\sqrt{1-h_n^2}\,\omega_n\tau'\right) \quad \cdots\cdots (12)$$

From Eqs. (8) and (9), the response $y(t)$ can also be represented by

$$y(t) = \sigma_y(t)(I_c(t)\cos\omega_n t - I_s(t)\sin\omega_n t) \quad \cdots\cdots(13)$$

where

$$\left.\begin{aligned} I_c(t) &= \int_0^\infty \cos(\omega t - \omega_n t - p(\omega))dc(\omega) \\ I_s(t) &= \int_0^\infty \sin(\omega t - \omega_n t - p(\omega))dc(\omega) \end{aligned}\right\} \quad \cdots\cdots(14)$$

Here $I_c(t)$ and $I_s(t)$ are slowly-varying random amplitudes of the cosine and sine members of the response in Eq. (13). It is noted that $I_c(t)$ and $I_s(t)$ are stationary Gaussian processes with a zero mean value.

It can be verified[14] that $I_c(t)$ and $I_s(t)$ have a same auto-correlation function represented by

$$R_I(\tau') = \int_0^\infty S_r(\omega)\cos(\omega - \omega_n)\tau' d\omega \quad \cdots\cdots(15)$$

The cross-correlation function between $I_c(t)$ and $I_s(t)$ is obtained as

$$R_{cs}(\tau') = \int_0^\infty S_r(\omega)\sin(\omega - \omega_n)\tau' d\omega \quad \cdots\cdots(16)$$

If we set

$$\beta_k = \int_0^\infty \left(\frac{\omega}{\omega_n} - 1\right)^k S_r(\omega)d\omega \quad \cdots\cdots(17)$$

the variances of $I_c(t)$, $I_s(t)$, $\dot{I}_c(t)$, $\dot{I}_s(t)$ are obtained as

$$\left.\begin{aligned} \sigma_{I_c}^2 &= \sigma_{I_s}^2 = R_I(O) = 1 \\ \sigma_{\dot{I}_c}^2 &= \sigma_{\dot{I}_s}^2 = -\frac{d^2R_I}{d\tau'^2}\bigg|_{\tau'=0} = \int_0^\infty (\omega-\omega_n)^2 S_r(\omega)d\omega = \omega_n^2\beta_2 \end{aligned}\right\} \quad \cdots\cdots(18)$$

Likewise, the correlation coefficients between $I_c(t)$, $I_s(t)$, $\dot{I}_c(t)$, $\dot{I}_s(t)$ are obtained from

$$\left.\begin{aligned} \rho_{I_s\dot{I}_s} &= \rho_{I_c\dot{I}_c} = \frac{1}{\omega_n\sqrt{\beta_2}}\frac{dR_I}{d\tau'}\bigg|_{\tau'=0} = 0 \\ \rho_{I_cI_s} &= R_{cs}(0) = 0 \\ \rho_{I_c\dot{I}_s} &= -\rho_{\dot{I}_cI_s} = \frac{1}{\omega_n\sqrt{\beta_2}}\frac{dR_{cs}}{d\tau'}\bigg|_{\tau'=0} = \frac{\beta_1}{\sqrt{\beta_2}} \end{aligned}\right\} \quad \cdots\cdots(19)$$

Since both $R_I(\tau')$ and $R_r(\tau')$ decrease as $e^{-h_n\omega_n\tau'}$, it can be stated that $I_c(t)$ and $I_s(t)$ have the same degree of auto-correlation as $r(t)$.

Fig. 3 shows $R_I(\tau')$ and $R_{cs}(\tau')$ for $S_r(\omega)$ in Eq. (11). It is noted in this figure that $R_{cs}(\tau')$ almost vanishes for $\tau' > T_n$, and that $R_I(\tau')$ has an appearence similar to the envelope of $R_r(\tau')$ in Eq. (12). Indeed, $R_I(\tau')$, shown in Fig. 3 can be closely approximated by

$$R_I(\tau') \fallingdotseq e^{-h_n \omega_n \tau'}(1 + h_n e^{-\zeta \omega_n \tau'} \sin \omega_n \tau') \qquad \cdots\cdots(20)$$

in which ζ assumes the values listed in Table 2.

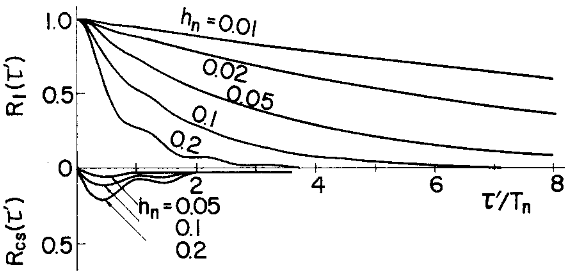

Fig. 3, Correlation Characteristics of $I_c(t)$ and $I_s(t)$.

Table 2. Parameter ζ

h_n	ζ
0.01	0.2655
0.02	0.2555
0.05	0.2259
0.1	0.1776
0.2	0.0824

3. Analysis of Response Envelope

(1) Envelope Formulation

Let the structural response $y(t)$ be represented in the following form:

$$\begin{aligned} y(t) &= W(t) \cos(\omega_n t + \phi(t)) \\ &= \sigma_y(t) R(t) \cos(\omega_n t + \phi(t)); \qquad \cdots\cdots(21) \\ & R(t) \geqq 0, \quad 0 \leqq \phi(t) \leqq 2\pi \end{aligned}$$

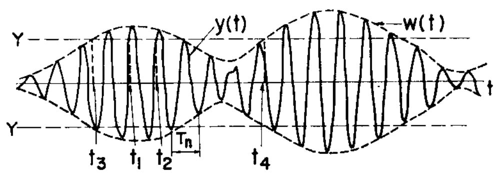

Fig. 4. Illustration of Structural Response and Its Enveloe.

where $W(t)$ is the response envelope of $y(t)$ shown in Fig. 4, $R(t)=W(t)/\sigma_y(t)$ is the envelope of the stationary process $r(t)$, and $\phi(t)$ is the slowly-varying phase angle.

From Eqs. (13) and (21), $R(t)$ and $\phi(t)$ can be expressed in terms of $I_c(t)$ and $I_s(t)$ as

$$\left.\begin{aligned} R(t) &= \sqrt{I_c^2(t)+I_s^2(t)} \\ \phi(t) &= \tan^{-1}(I_s(t)/I_c(t)) \end{aligned}\right\} \qquad \cdots\cdots(22)$$

By virtue of Eq. (22), it is expected that the auto-correlation characteristics of $R(t)$ are similar to those of $I_c(t)$ and $I_s(t)$, and hence to those of $r(t)$. This statement verifies the appropriateness of employing the response envelope in analyzing the probability distribution of the maximum response in the next chapter.

(2) Joint Probability Density of $W(t)$, $\phi(t)$, and Their Derivatives

The joint probability densities of $W(t)$, $\phi(t)$ and their derivatives can be obtained by the method analogous to that used by Lin[11] for stationary processes.

First, introduce four random processes $y_1(t)\sim y_4(t)$ defined as

$$\left.\begin{aligned} y_1(t) &= \sigma_y(t)I_c(t) \\ y_2(t) &= \frac{d}{dt}(\sigma_y(t)I_s(t)) = \dot{\sigma}_y(t)I_s(t)+\sigma_y(t)\dot{I}_s(t) \\ y_3(t) &= \sigma_y(t)I_s(t) \\ y_4(t) &= \frac{d}{dt}(\sigma_y(t)I_c(t)) = \dot{\sigma}_y(t)I_c(t)+\sigma_y(t)\dot{I}_c(t) \end{aligned}\right\} \qquad \cdots\cdots(23)$$

By using Eqs. (18) and (19), the variances and the covariances of $y_1(t)\sim y_4(t)$ are obtained as

$$\left.\begin{aligned} \sigma_{y_1}^2 &= \sigma_{y_3}^2 = \sigma_y^2(t) \\ \sigma_{y_2}^2 &= \sigma_{y_4}^2 = \omega_n^2\beta_2\sigma_y^2(t)+\dot{\sigma}_y^2(t) \\ \sigma_{y_1}\sigma_{y_2}\rho_{y_1y_2} &= -\sigma_{y_3}\sigma_{y_4}\rho_{y_3y_4} = \omega_n\beta_1\sigma_y^2(t) \\ \sigma_{y_1}\sigma_{y_4}\rho_{y_1y_4} &= \sigma_{y_2}\sigma_{y_3}\rho_{y_2y_3} = \sigma_y(t)\dot{\sigma}_y(t) \\ \sigma_{y_1}\sigma_{y_3}\rho_{y_1y_3} &= \sigma_{y_2}\sigma_{y_4}\rho_{y_2y_4} = 0 \end{aligned}\right\} \qquad \cdots\cdots(24)$$

Since $I_c(t)$, $I_s(t)$, $\dot{I}_c(t)$ and $I_s(t)$ are Gaussian processes, so are $y_1(t)\sim y_4(t)$. Hence, the joint probability desity of $y_1(t)\sim y_4(t)$ is represented by the multi-dimensional Gaussian distribution with the null mean value vector and the covariance matrix given by

$$[\Lambda] = \begin{pmatrix} \sigma_y^2 , & \omega_n\beta_1\sigma_y^2 , & 0 , & \sigma_y\dot{\sigma}_y \\ \omega_n\beta_1\sigma_y^2 , & \omega_n^2\beta_2\sigma_y^2+\dot{\sigma}_y^2 , & \sigma_y\dot{\sigma}_y , & 0 \\ 0 , & \sigma_y\dot{\sigma}_y , & \sigma_y^2 , & -\omega_n\beta_1\sigma_y^2 \\ \sigma_y\dot{\sigma}_y , & 0 , & -\omega_{n1}\sigma_y^2, & \omega_n^2\beta_2\sigma_y^2+\dot{\sigma}_y^2 \end{pmatrix} \quad \cdots\cdots\cdots(25)$$

The determinant of $[\Lambda]$ is obtained as

$$|\Lambda| = \omega_n^4 B^2 \sigma_y^8(t) \quad \cdots\cdots\cdots\cdots\cdots\cdots(26)$$

where

$$B = \beta_2 - \beta_1^2$$

The inverse matrix of $[\Lambda]$ is

$$[\Lambda]^{-1} = \frac{1}{\omega_n^2 B \sigma_y^2} \begin{pmatrix} \omega_n^2\beta_2+\gamma^2, & -\omega_n\beta_1, & 0 , & -\omega_n\gamma \\ -\omega_n\beta_1 , & 1 , & -\omega_n\gamma , & 0 \\ 0 , & -\omega_n\gamma , & \omega_n^2\beta_2+\gamma^2, & \omega_n\beta_1 \\ -\omega_n\gamma , & 0 , & \omega_n\beta_1 , & 1 \end{pmatrix} \quad \cdots\cdots\cdots(27)$$

where

$$\gamma = \gamma(t) = \frac{\dot{\sigma}_y(t)}{\omega_n \sigma_y(t)}$$

From these results, the joint probability density of $y_1(t) \sim y_4(t)$ is obtained as

$$\phi_n(y_1, y_2, y_3, y_4) = \frac{e^{-Q/2}}{4\pi^2\omega_n^2 B\sigma_y^4} \quad \cdots\cdots\cdots\cdots\cdots\cdots(28)$$

where

$$Q = \frac{1}{B\sigma_y^2}\left\{(\beta_2+\gamma^2)(y_1^2+y_3^2) + \frac{1}{\omega_n^2}(y_2^2+y_2^4) - \frac{2\beta_1}{\omega_n}(y_1y_2 - y_3y_4) - \frac{2\gamma}{\omega_n}(y_1y_4+y_2y_3)\right\}$$

Next, Eqs. (13), (21) and (23) lead to the following expressions:

$$\left.\begin{aligned} y_1(t) &= W(t)\cos\phi(t) \\ y_2(t) &= \dot{W}(t)\sin\phi(t) + W(t)\cos\phi(t)\cdot\dot{\phi}(t) \\ y_3(t) &= W(t)\sin\phi(t) \\ y_4(t) &= \dot{W}(t)\cos\phi(t) - W(t)\sin\phi(t)\cdot\dot{\phi}(t) \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots(29)$$

Hence, we have

$$\begin{aligned} dy_1\,dy_2\,dy_3\,dy_4 &= \left|\frac{\partial(y_1, y_2, y_3, y_4)}{\partial(W, \phi, \dot{W}, \dot{\phi})}\right| dW\,d\phi\,d\dot{W}\,d\dot{\phi} \\ &= W^2(t)\,dW\,d\phi\,d\dot{W}\,d\dot{\phi} \end{aligned} \quad \cdots\cdots\cdots\cdots\cdots\cdots(30)$$

With the aid of Eqs. (28)~(30), the joint probability density of $W(t)$, $\phi(t)$, $\dot{W}(t)$ and $\dot{\phi}(t)$ is obtained as

$$\phi_c(W, \phi, \dot{W}, \dot{\phi}) = \frac{W^2}{4\pi^2 \omega_n^2 B \sigma_y^4} \exp\left[-\frac{1}{2B\sigma_y^2}\left\{(\beta_2+r^2)W^2 - \frac{2r}{\omega_n} W\dot{W} - \frac{2\beta_1}{\omega_n} W^2\dot{\phi} + \frac{1}{\omega_n^2}(\dot{W}^2+W^2\dot{\phi}^2)\right\}\right] \quad \cdots\cdots(31)$$

The joint probability density of $W(t)$, and $\dot{W}(t)$ is represented by the marginal density function of $\phi_c(W, \phi, \dot{W}, \dot{\phi})$ with respect to ϕ and $\dot{\phi}$ i.e.,

$$\begin{aligned}\phi_c(W, \dot{W}) &= \int_0^{2\pi} d\phi \int_{\infty}^{\infty} \phi_c(W, \phi, \dot{W}, \dot{\phi}) d\dot{\phi} \\ &= \frac{W}{\omega_n\sqrt{2\pi B}\,\sigma_y^3} \exp\left[-\frac{1}{2\sigma_y^2}\left\{\left(1+\frac{r^2}{B}\right)W^2 - \frac{2r}{\omega_n B} W\dot{W} + \frac{\dot{W}^2}{\omega_n^2 B}\right\}\right]\end{aligned} \quad \cdots\cdots\cdots\cdots\cdots\cdots(32)$$

For the form of $S_r(\omega)$ in Eq. (11), β_1, β_2 and B are obtained as

$$\left.\begin{aligned}\beta_2 &= -2\beta_1 = \frac{2}{\sqrt{1-h_n^2}}\left(\frac{\delta}{\pi}+\sqrt{1-h_n^2}-1\right) \\ B &= 1-\frac{1}{1-h_n^2}\left(1-\frac{\delta}{\pi}\right)^2\end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots(33)$$

where

$$\delta = \tan^{-1}\frac{2h_n\sqrt{1-h_n^2}}{1-2h_n^2}$$

4. Probability Distribution of the Maximum Response

If Y denotes the maximum absolute value of the response $y(t)$ during the earthquake, its probability distribution is represented by[6),8)]

$$\begin{aligned}\Phi(Y) &= P[\max|y(t)| \leqq Y;\ 0 \leqq t < \infty] \\ &= a_0(Y) \exp\left\{-\int_0^{\infty} c_0(Y, t)dt\right\}\end{aligned} \quad \cdots\cdots\cdots\cdots\cdots\cdots(34)$$

where*a*

$$\left.\begin{aligned}a_0(Y) &= P[|y(0)| \leqq Y] \\ c_0(Y, t)dt &= P[|y(t+dt)| > Y \,|\, \max|y(t')| \leqq Y;\ 0 \leqq t' \leqq t]\end{aligned}\right\} \quad \cdots\cdots(35)$$

in which $P[A]$ is the probability of event A, and $P[A|B]$ is the conditional probability of event A on the hypothesis of event B. No success has been made in obtaining the exact solution for $c_0(Y, t)$ in Eq. (35) in an explicit form, and therefore, many approximate solutions have been proposed as indicated in **1**.

The significance of $c_0(Y, t)$ is the rate of the upward crossing of $|y(t)|=Y$ under the condition that no such crossing took place in the past response. From discusions in the foregoing chapters, an approximation is made herein by equating $c_0(Y, t)$ to the unconditional crossing rate $N_W(Y, t)$ of the envelope $W(t)$ at the response level Y; i.e.,

$$\Phi(Y)\cong a_0(Y)\exp\left\{-\int_0^\infty N_W(Y, t)dt\right\} \quad\cdots\cdots\cdots(36)$$

The crossing rate $N_W\ (Y, t)$ is obtained as

$$\begin{aligned} N_W(Y, t) &= \int_0^\infty \dot{W}\phi_e(Y, \dot{W})d\dot{W} \\ &= \omega_n\frac{Y}{\sigma_y}\exp\left\{-\frac{1}{2}\left(\frac{Y}{\sigma_y}\right)^2\right\}\left[\sqrt{\frac{B}{2\pi}}\exp\left\{-\frac{r^2}{2B}\left(\frac{Y}{\sigma_y}\right)^2\right\}\right. \\ &\quad \left.+\frac{r}{2}\frac{Y}{\sigma_y}\left\{1+\mathrm{erf}\left(\frac{r}{\sqrt{2B}}\frac{Y}{\sigma_y}\right)\right\}\right] \end{aligned} \quad\cdots\cdots\cdots(37)$$

If the maximum or the minimum value of $y(t)$ is related to the structural design, Eq. (34) is modified as

$$\left.\begin{aligned} &\Phi_1(Y) = P[\max y(t)\leqq Y;\ 0\leqq t<\infty] \\ \text{or}\quad &\Phi_2(Y) = P[\min y(t)>-Y;\ 0\leqq t>\infty] \end{aligned}\right\} \quad\cdots\cdots\cdots(37)$$

In the response of structures with symmetrical dynamic properties in the positive and negative domains of response, an approximation can be made by using $N_W(Y, t)/2$ instead of $N_W(Y, t)$ in Eq. (36). Hence we have

$$\left.\begin{aligned} &\Phi_1(Y)\cong P[y(0)\leqq Y]\left[\exp\left\{-\int_0^\infty N_W(Y, t)dt\right\}\right]^{1/2} \\ &\Phi_2(Y)\cong P[y(0)>-Y]\left[\exp\left\{-\int_0^\infty N_W(Y, t)dt\right\}\right]^{1/2} \end{aligned}\right\} \quad\cdots\cdots\cdots(39)$$

Fig. 5 shows the numerical results of the probability distribution of the maximum response given by Eq. (36), computed for the model of the erathquake discussed in **2**. (1). It also shows the experimental values obtained from a numerical simulation made by means of the Monte Carlo method. It is noted that the agreement between the theoretical and simulated values are fairly good, except for high response levels of very slightly damped oscillators. For such cases where the method of this study fails to be accurate, the pure-birth-process method and the peak envelope method[8] give a better approximation. However, whereas these two former methods are applicable only in a limited range of parameters, Fig. 5 would demonstrate that the analytical method discussed in this paper

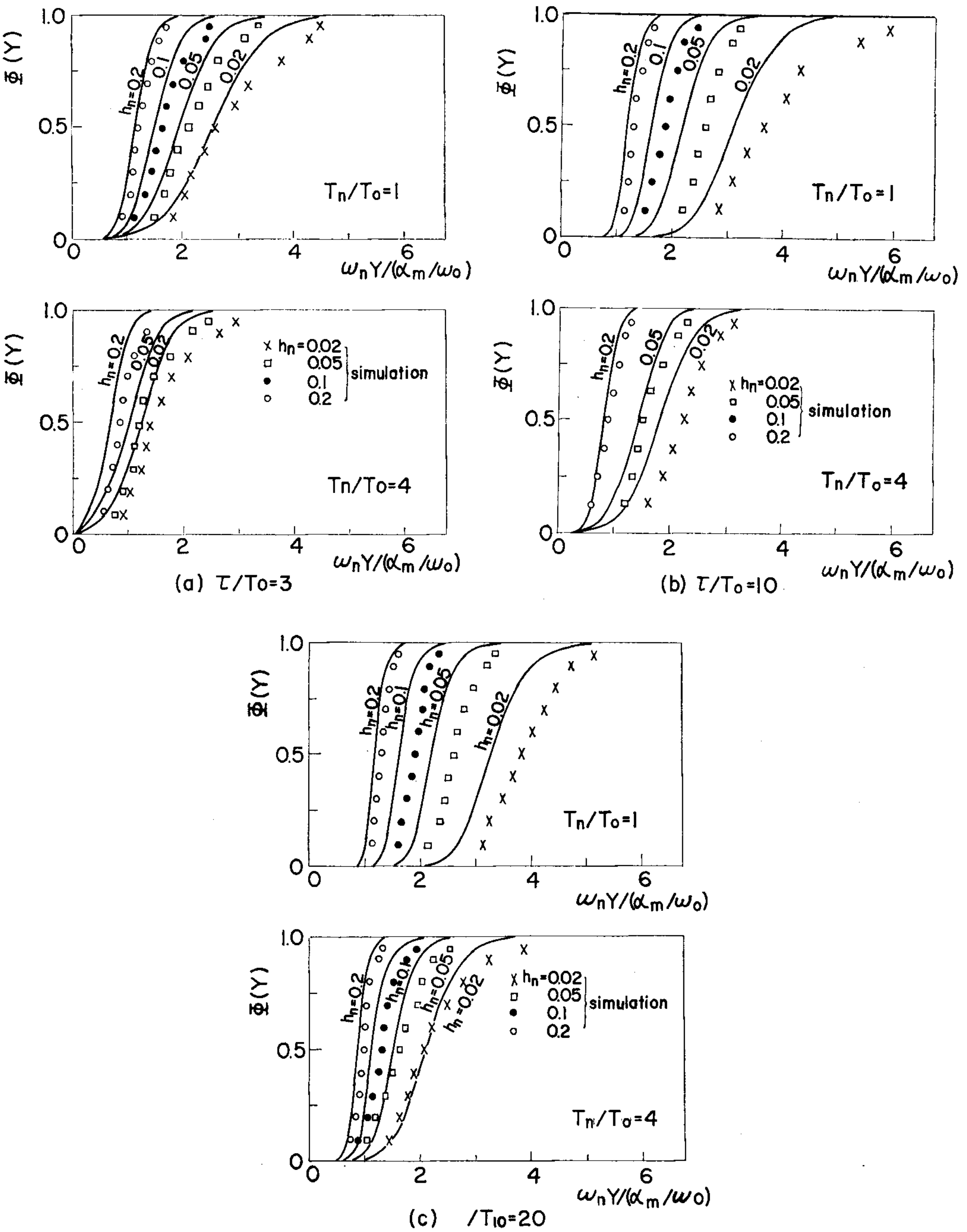

Fig. 5. Probability Distribution of the Maximum Response.

furnishes a fairly good approximation over a wide range of response level and other structural parameters. From these results, it could be recommended that the method of analysis adopted herein be used for obtaining the distribution of the maximum response in the intermediate response levels.

5. Moments of the Maximum Response

(1) Mean Response Spectra

From the probability distribution given by Eq. (36), the mean maximum response S_D can be obtained as

$$S_D = E[Y] = \int_0^\infty \{1 - \Phi(Y)\} dY \qquad \cdots\cdots\cdots\cdots\cdots\cdots(40)$$

Fig. 6 shows plots of S_D in the form of response spectra. The ordinate of this figure shows the pseduo-velocity response spectra. The dotted lines are the results of numerical simulation. It is observed that the analytical and simulated results

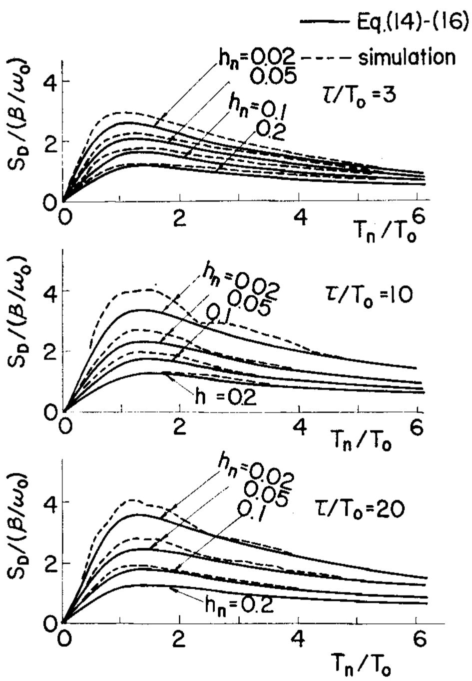

Fig. 6. Mean Response Spectra.

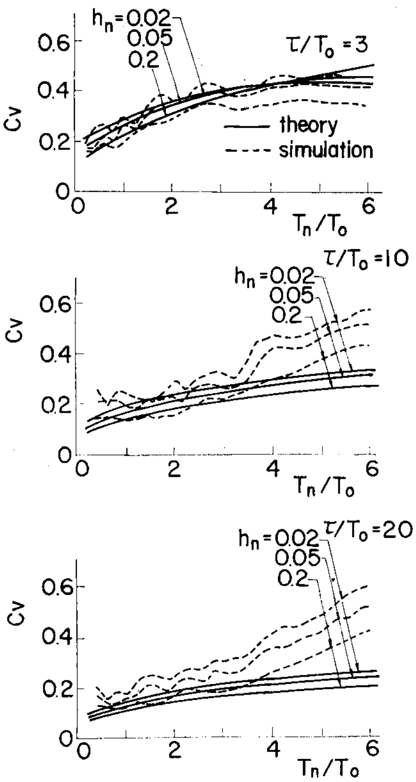

Fig. 7. Coefficient of Variation of the Maximum Response.

are in fairly good agreement. Especially for the damping ratio larger than 0.05, the agreement is satisfactory. This agreement is considered as a consequence of the relatively good approximation of the analytical results on the probability distribution in intermediate response levels.

(2) Coefficient of Variation

The coefficient of variation C_v of the maximum response has also been obtained on the basis of the results of the preceding discussions. C_v is represented by

$$C_v = \frac{\{E[Y^2]-S_D^2\}^{1/2}}{S_D} \qquad \cdots\cdots(41)$$

The mean square of the maximum response has been computed from[8]

$$E[Y^2] = 2\int_0^\infty dY \int_Y^\infty \{1-\Phi(Y')\} dY' \qquad \cdots\cdots(42)$$

The numerical results are shown in Fig. 7 along with the simulated results. It can be stated from this figure that the method of analysis in this study fails to give a good approximation to the coefficient of the variation of the maximum response as the duration τ of earthquake motion or the natural period T_n of the structure increases.

6. Concluding Remarks

In this study, an approximate method of analysis has been discussed to obtain the probability distribution of the maximum response of structures subjected to non-stationary random earthquake motion by using the response envelope. From the numerical results, it has been confirmed that the method in this study is a good approximation in an intermediate range of the response level. And as a consequence, the mean response spectra based on this method shows a fairly good agreement with the result of numerical simulation. However, when a high level of the maximum response is discussed, the method in this study is less accurate.

Since no exact method has been developed to deal with the maximum response statistics of structures subjected to continuous random process excitations, some suitable approximate method should be adopted according to the objective of the analysis. In this sense, the method of analysis discussed in this study is to be incorporated into the group of other approximate methods so far developed.

References

1) Coleman, J. J.; "Reliability of Aircraft Structures in Resisting Chance Failure", Operations Research, Vol. 7, 1959, pp. 639–645.
2) Corotis, R. B., Vanmarcke, E. H., and Cornell, C. A.; "First Passage of Nonstationary Random Processes", Proc. ASCE, Vol. 98, EM2, April, 1972, pp. 401–414.
3) Crandall, S. H., Chandiramani, K. L., and Cook, R. G.; "Some First-Passage Problems in Random Vibration", Jour. Appl. Mech., Trans. ASME, 1966, pp. 532–538.
4) Goto, H., and Kameda, H.; "A Statistical Study of the Maximum Ground Motion in Strong Earthquakes", Mem. Fac. Eng., Kyoto Univ., Vol. XXIX, Part 4, Oct., 1967, pp. 389–419.
5) Goto, H., and Kameda, H.; "Statistical Inference of the Future Earthquake Ground Motion", Proc. IV WCEE, Vol. 1, 1969, pp. 39–54.
6) Goto, H., and Kameda, H.; "On the Probability Distribution of the Maximum Structural Response in Random Vibration, —Transient Response to Stationary Input—", Ann. Disaster Prevention Res. Inst., Kyoto Univ., No. 12A, March, 1969, pp. 289–299, (in Japanese).
7) Gray, A. H. Jr.; "First-Passage Time in a Random Vibrational System", Jour. Appl. Mech., Trans. ASME, Vol. 33, Ser E, March, 1966, pp. 187–191.
8) Kameda, H.; "Probability Distribution of the Maximum Response of Structures Subjected to Nonstationary Random Earthquake Motion", Mem. Fac. Eng., Kyoto Univ., Vol. XXXIII, Part 4, Oct., 1971, pp. 243–280.
9) Kameda, H.; "On the Probability Distribution of the Number of Crossings of a Certain Response Level in Random Vibration", Mem. Fac. Eng., Kyoto Univ., Vol. XXXIV, Part 1, Jan., 1972, pp. 68–83.
10) Kameda, H., Izunami, R., and Yamada, Y.; "On Evaluation of the Duration of Strong Earthquake Motions", Proc. 27th Ann. Conf., JSCE, Vol. I, Oct., 1972, pp. I–31–1∼I–31–2.
11) Lin, Y. K.; "Probabilistic Theory of Structural Dynamics", McGraw-Hill, 1972.
12) Racicot, R. L., and Moses, F.; "A First-Passage Approximation in Random Vibration", Jour. Appl. Mech., Trans. ASME, Vol. 38, March, 1971, 143–147.
13) Rice, J. R., and Beer, F. P.; "First-Occurrence Time of High-Level Crossings in a Continuous Random Process", Jour. Acoust. Soc. Amr., Vol. 39, No. 2, 1966, pp. 323–335.
14) Rice, S. O.; "Mathematicl Analysis of Random Nosise", Selected Papers on Noise and Stochastic Processes, N. Wax, ed., Dover, 1954, pp. 133–294.
15) Shinozuka, M.; "Probability of Structural Failure under Random Loading", Proc. ASCE, Vol. 90, EM5, Oct., 1964, pp. 147–170.
16) Shinozuka, M. and Yang, J.-N.; "On the Bound of First Excursion Probability", Proc. ASCE, Vol. 95, EM2, April, 1969, pp. 363–377.
17) Toki, K.; "Earthquake Response of Structure and Its Foundation", Doctoral Thesis submitted to Kyoto Univ., March, 1969, pp. 151–156, (in Japanese).
18) Trifunac, M. D.; "Response Envelope Spectrum and Interpretation of Strong Earthquake Ground Motion", Bull. Seism. Soc. Amr., Vol. 61, No. 2, April, 1971, pp. 343–356.
19) Trifunac, M. D,; "A Method for Synthesizing Realistic Strong Ground Motion", Bull. Seism. Soc. Amr., Vol. 61, No. 6, Dec., 1971, pp. 1739–1753.