Assessment of Transient Stability via Stochastic Approach and its Application to Optimum Load Dispatching Problem in Power Systems

By

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Abstract

This paper describes a method of transient stability evaluation via the stochastic approach developed for the application to security control, and the method is applied to a model system. A comparison was made between the results obtained by the method here proposed and by the pattern recognition method proposed recently for the same purpose. The superiority of the former was ascertained. Furthermore, as an example of a practical application of this method, it was applied to the optimum load dispatching problem, considering not only economy but also transient stability.

1. Introduction

A stable supply of electric power of good quality is indispensable in the present-day life. Disturbances imposed on electric power systems are coming to have greater effects on the operation of the systems, since the composition of electric power systems is getting greater and more complicated, and the operation voltage is getting higher. In order to prepare for such disturbances, it is, of course, important to make the power systems well designed, leaving a margin. However, in fact, mainly from the economical point of view, the perfect preparation for disturbances can not be anticipated, and it is necessary to make up for it by operational control. Accordingly, security control, i.e. controls in order to improve the system's reliability, is becoming to play a more important role in the system operation.

In the security control, the most fundamental role is played by security evaluation using real-time data. Security evaluation makes it possible to foresee an interruption or deterioration of electric power supply and to make some control, if necessary. One

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of the methods of security evaluation is the simulation method on a computer. In a word, this method consists of assuming a number of faults in the present operation state of the system and subsequent criteria of the system such as the result of power-flow calculation, the transient stability calculation, whether the system can continue to operate normally, and so on. This method, however, is not suitable for applying it to the practical power systems in the on-line mode, since simulation takes much time. In particular, transient stability calculation requires much time even if fast methods, such as the direct method of Lyapunov, Popov's method and so on, are used.

The method of evaluating the transient stability proposed here by using a stochastic approach consists of an off-line simulation and an on-line evaluation of the transient stability, using the results of the simulation.

As mentioned before, security evaluation is the first step to security control. The aim is to use the result of the evaluation in determining the distribution of the power generation, which ensures a stable operation under some faults, or in applying some other control in order to improve the security. In the latter half of this paper, represented is the optimal load dispatching problem, taking into account not only economy but also transient stability using the stochastic transient stability evaluation method.

2. Transient Stability Evaluation via the Stochastic Approach

Recently, the method of pattern recognition has been proposed as one of the methods of evaluating the transient stability of electric power systems at high speed.[1),2)] This is to obtain hiperplane, which separates the stable domain from the unstable domain in the state space by an off-line calculation. After that, it is to judge by the on-line mode whether the present operating state is stable or unstable, by knowing on which side of the separating plane the present state is located. In order to carry out the above method, some kinds of functions have been used, for example, the function of the state variables, the value of which is positive when the system is stable and negative when unstable.[1)] Another example is the function which has the minimum error given the value 1 when the system is stable and 0 when unstable.[2)] In either case, however, only the alternative information is used whether the system is stable or unstable.

The method proposed in this paper is as follows. First of all, the measure of the transient stability is defined, and then its value is approximated by some function of operating variables, that is, the type and the coefficients of the approximating function are determined by an off-line calculation. When the system is under operation, the stability can be found by substituting the value of state variables into that function. By using such a method, it is expected to decrease the number of required sample operating states, as compared with the case when the alternative information only, i.e. stable or unstable, is used.

2-1 Measure of Transient Stability

In order to apply the method mentioned above, the measure of transient stability must be defined. Several kinds of stability measures of electric power system have been proposed, considering its particular property.[3)~7)] Especially, with regard to a steady-state and dynamic stability, quite a few kinds of measures, using such means as the Lyapunov function,[3)] integrated squared error of the state variables[4)] or the dominant root of the characteristic equation,[5)] have been developed. The number of measures of transient stability, however, is comparatively small.

Under the assumptions that each generator can be represented by its direct-axis transient reactance in a series with a constant voltage power source, and that the mechanical input into the generator can not be varied during the short time period under consideration, the swing equations of an n-machines system are as follows:

$$M_k \frac{d\omega_k}{dt} = P_{mk} - P_{ek} \tag{1}$$

$$\frac{d\delta_k}{dt} = \omega_k \tag{2}$$

$$P_{ek} = G_{kk} E_k^2 + \sum_{i=1, \neq k}^{n} E_k E_i \{G_{ki} \cos(\delta_k - \delta_i) + B_{ki} \sin(\delta_k - \delta_i)\}$$

$$k = 1, 2, \cdots, n \tag{3}$$

where E_k : induced voltage behind transient reactance x'_d of the k-th generator

δ_k : phase angle of the k-th generator

ω_k : angular velocity of the k-th generator

$G_{ij} + jB_{ij}$: transfer admittance between the induced voltage of the i-th generator and that of the j-th generator

M_k : inertia constant of the k-th generator

P_{mk} : mechanical power input into the k-th generator

P_{ek} : electric power output from the k-th generator

t : time (in radian)

For the transient behavior of electric power system described by the above equations, Teichgraeder et al.[6)] proposed the measure of transient stability based on the Lyapunov fucntion as follows:

$$\xi = 1 - (V_c / V_{\max}) \tag{4}$$

V_c : the value of V when the fault is cleared

$V_{\max}$: the value of V at the unstable equilibrium point corresponding to the step-out mode

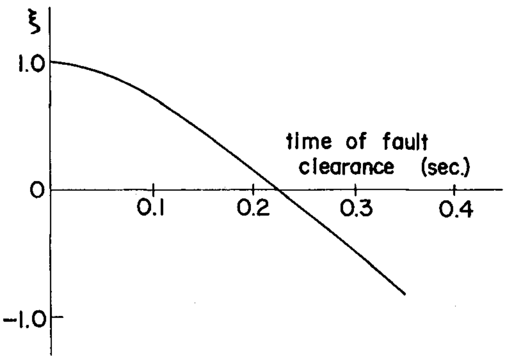

Fig. 1. Example of transient stability measure.

$$V(\omega, \delta)=\frac{1}{2M_T}\sum_{k=1}^{n-1}\sum_{j=k+1}^{n} M_j M_k \omega_{jk}^2+\frac{1}{M_T}\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}(-M_k P_{mj}$$
$$+M_j P_{mk}+M_k G_{jj} E_j^2-M_j G_{kk} E_k^2)(\delta_{jk}-\delta_{jk}^s)$$
$$-\sum_{k=1}^{n-1}\sum_{j=k+1}^{n} B_{jk} E_j E_k(\cos\delta_{jk}-\cos\delta_{jk}^s) \tag{5}$$

where $\omega_{jk}=\omega_j-\omega_k$

$\delta_{jk}=\delta_j-\delta_k$

δ_{jk}^s : the position of the stable equilibrium point after the clearance of the fault

$M_T=\sum_{i=1}^{n} M_i$

It might be said that $V_c/V_{\max}$ means the distance represented by means of the Lyapunov function from the state at the time of the fault clearance to the stable equilibrium point. The value of this measure ξ is positive and less than 1 when the system is stable, regardless of the construction of the system and the kind and location of the fault, that is, it is standardized divided by $V_{\max}$. The value is negative when the system is unstable. One example of this measure versus the time of the fault clearance is shown in Fig. 1. In this paper, this measure of transient stability is used.

2-2 Approximation of the Function via the Method of Least Square

For various operating states, the measure of transient stability defined in the preceding section is calculated in the off-line mode. After that, the measure is approximated by the function of the operating state vector. The determination of the approximating function is formulated as follows:

"$g(\boldsymbol{x})$, a function of vector $\boldsymbol{x}$, defined in a region is given its value only at n points $\{\boldsymbol{x}_i\}_{i=1}^{n}$ located in that region. Minimize the squared error

$$E=\sum_{i=1}^{n}[g(\boldsymbol{x}_i)-f(\boldsymbol{x}_i)]^2$$

by choosing the appropriate function $f(\boldsymbol{x})$."

If the function $f(\boldsymbol{x})$ is given by the linear combination of a set of known functions $\{f_k(\boldsymbol{x})\}$,

$$f(\boldsymbol{x})=\sum_{k=1}^{m} a_k f_k(\boldsymbol{x}) \qquad (m \leqslant n),$$

then the coefficients $a_k's$ which minimize the squared error E can be found by solving the following equation

$$\sum_{i=1}^{m} A_{ki} a_i = B_k \qquad (k=1, 2, \cdots, m)$$

where

$$A_{kj}=\sum_{i=1}^{n} f_k(\boldsymbol{x}_i) f_j(\boldsymbol{x}_i)$$

$$B_k=\sum_{i=1}^{n} g(\boldsymbol{x}_i) f_k(\boldsymbol{x}_i)$$

For example, if the function is taken as a linear function of the state variables, $f_k's$ may be taken as follows:

$$\begin{aligned} f_1(\boldsymbol{x}) &= x_1 \\ f_2(\boldsymbol{x}) &= x_2 \\ &\vdots \\ f_n(\boldsymbol{x}) &= x_n \\ f_{n+1}(\boldsymbol{x}) &= 1 \end{aligned}$$

2-3 Example

As an example, the method stated before is applied to a system consisting of four generators and six buses,[8] as shown in Fig. 2. Bus No. 1 is a slack bus, buses No. 2, No. 3 and No. 4 are assumed to be P-V specified buses, and buses No. 5 and No. 6 P-Q specified buses. Three-phase short curcuits at the six marked points in Fig. 2 are chosen as the faults assumed. The fault clearing time is assumed to be 0.2 sec. for the former four faults and 0.3 sec. for the latter two faults, so that we might get stable and unstable operating conditions with an appropriate ratio. Reclosing is not considered. As the operating state variables, we chose the effective power outputs P_{G1}, P_{G2}, P_{G3} of the three generators, except the slack generator, and the load powers P_{L1}, P_{L2} connected to the load buses. Therefore, the terminal voltage of each generator and the power factor of each load are assumed to be constant. These five state variables are changed at random between the upper and lower limits shown in Table 1 in order to get 200 operating states. The steady-state unstable ones among them were excluded, and then the proposed method was applied. The type of the approximating function chosen here was a quadratic function. In Table 2 are shown a number of samples for each fault, the total squared error of the transient stability measure

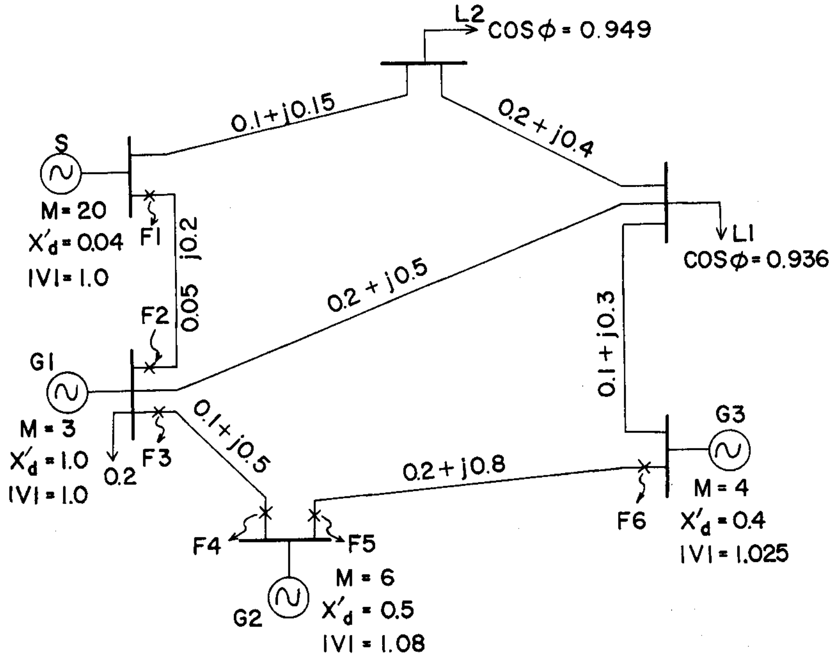

Fig. 2. Model system.

calculated, using the approximating function obtained via the method of least square, the error per one sample and the number of the samples judged wrong, i.e. the stable samples judged to be unstable and the unstable samples judged to be stable.

Although for the first and second faults the error and the number of false judgements are slightly greater than for the other faults, the results are on the whole satisfactory with the rate of correct judgement greater than 95%. In the same table is also shown the number of false judgements using the pattern recognition method mentioned before, that is, given a value 1 when the system is stable and 0 when unstable. The discriminating function was determined as a quadratic function which

Table 1. Upper and lower limits of the generator outputs and loads.

	Lower limit (p.u.)	Upper limit (p.u.)
P_{G1}	0.1	0.4
P_{G2}	0.2	0.5
P_{G3}	0.1	0.4
P_{L1}	0.1	0.5
P_{L2}	0.05	0.45

Table 2. Results of transient stability assessment for each fault.

Fault		F1	F2	F3	F4	F5	F6	Min.
Number of samples		188	158	200	200	200	200	158
(stable samples / unstable samples)		(161 / 27)	(74 / 84)	(115 / 85)	(172 / 28)	(105 / 95)	(122 / 78)	(41 / 117)
Total squared error of the measure		6.91	1.93	0.023	0.123	0.005	0.239	8.72
(average error per one sample)		(0.192)	(0.111)	(0.0107)	(0.0248)	(0.005)	(0.0346)	(0.235)
Number of misjudgements*	S→U	6(3)	4(7)	1(0)	0(0)	0(0)	1(2)	5(5)
	U→S	0(0)	2(0)	0(5)	0(1)	0(5)	2(5)	2(1)
Rate of misjudgements* (%)	S→U	3.2(1.6)	2.5(4.4)	0.5(0)	0(0)	0(0)	0.5(1.0)	3.2(3.2)
	U→S	0(0)	1.3(0)	0(2.5)	0(0.5)	0(2.5)	1.0(2.5)	1.3(0.6)
	Total	3.2(1.6)	3.8(4.4)	0.5(2.5)	0(0.5)	0(2.5)	1.5(3.5)	4.4(3.8)

* Values in the parentheses are obtained via the method of pattern recognition.
S→U denotes stable samples judged to be unstable, and U→S contrary.

has the least squared error. It can be seen that the number of misjudgements is less when using the method proposed here than the pattern recognition, except for the fault F1.

In this example six faults were assumed, and for each of them was found the function which approximated the stability measure. When we consider preventive controls, such as change of power flow in order to improve stability, it is very possible that some control against one fault will have a bad effect for another fault. In the case of the pattern recognition method, it is applied by regarding the system as stable if the system is stable for all of the assumed faults, and as unstable if it is unstable for at least one of the faults. In the case of the method proposed here, however, it is necessary to define a synthetic measure, taking all the assumed faults into account. As an example, the minimum value of the stability measures for each assumed fault was taken as the synthetic measure, and the stochastic method was applied. The results are also shown in Table 2. The error is comparatively great, but the rate of misjudgement is less than 5%.

In this example, the power at each bus was chosen as a state variable. If we are interested only in the assessment of stability, it is possible to choose other components, for example, the phase angle of the induced voltage of the generator as a state variable in order to simplify the form of the approximating function, or in order to reduce the number of samples, since it is expected that the phase angles are closely related to the stability. However, directly controllable elements, such as the output of the generator, are preferable when we consider not only stability but also coordination with other factors ,such as economy.

3. Economical Load Dispatch taking Transient Stability into Account

In the performance of the load dispatch among generators, there was recently pointed out the importance of the optimal load dispatch, considering not only economy as usual, i.e. minimization of the total fuel cost, but also system security such as stability. The method of the optimum load dispatch, considering a steady-state stability, was proposed as an example.[9]

In this chapter, a method of the optimum load dispatch considering transient stability is proposed, using the method of transient stability assessment mentioned in the preceding chapter, and a numerical example is shown.

3-1 Formulation of the Problem

The load dispatch problem, taking transient stability into account will be formulated below for convenience based on the example used in Chapter 2. This method, however, is applicable also to other systems which can not be assumed to have constant terminal voltages or constant load power factors.

Now, likewise in Chapter 2, the output power of each generator, except the slack generator, is denoted by P_{Gi}, $i=1, \cdots, n$, and the power consumption of each load by P_{Li}, $i=1, \cdots, m$. Then, the measure of transient stability defined in the previous chapter is given by the following approximating function

$$\Xi=\xi(P_G, P_L) \tag{6}$$

where
$$P_G=(P_{G1}, \cdots, P_{Gn})$$
$$P_L=(P_{L1}, \cdots, P_{Lm})$$

Given the load distribution P_L and the generating power distribution P_G, the power output of the slack generator is found by the following equation:

$$P_s=\sum_{i=1}^{m} P_{Li}+P_l-\sum_{i=1}^{n} P_{Gi}, \tag{7}$$

where P_l represents the transmission loss.

The amount of transmission loss for each sample can be found from the results of the power flow calculation performed at the beginning of the off-line computation. Therefore, it can be approximated also by a function of P_G and P_L like the stability measure,

$$P_l=l(P_G, P_L). \tag{8}$$

If the load distribution is known beforehand, the stability measure Ξ and the transmission loss P_l become functions of power outputs of the generators P_G only and can be represented as follows:

$$\Xi=\xi'(P_G), \quad P_l=l'(P_G). \tag{9}$$

Furthermore, we denote the fuel cost characteristic of each generator by $F_i(P_{Gi})$, $i=1, \cdots, n$ and the power cost characteristic through the slack bus by $F_s(P_s)$.

From the above consideration and definitions, the optimum load dispatching problem with a transient stability is formulated as follows:

"Minimize the objective function

$$\Phi(P_G)=C_1[F_s(P_s)+\sum_{i=1}^{n} F_i(P_{Gi})]-C_2\xi'(P_G) \tag{10}$$

under the constraints

$\underline{P_{Gi}} \leqslant P_{Gi} \leqslant \overline{P_{Gi}}$ (power output limits of the i-th generator)

$\xi'(P_G) \geqslant 0$ (transiently stable)

where P_s : electric power supplied through the slack bus as shown in Eq. (7)

$\underline{P_{Gi}}, \overline{P_{Gi}}$: lower and upper limit of the power output of the i-th generator respectively

C_1, C_2 : coefficients to determine which factor to emphasize, economy or stability

3-2 Method of the Solution

The optimum load dispatch problem formulated in the preceding section is a nonlinear optimization problem with inequality constraints. This is because the objective function and the constraint that the system is transiently stable are nonlinear if the measure of stability is approximated by a quadratic function.

The nonlinear optimization problem with constraints can be solved by including the constraints in the objective function, and reducing the problem to a nonlinear optimization without constraints. Several methods for that have been developed. In the numerical example of the following section, the method developed by Powell[10] was used, the details of which are shown in the Appendix.

3-3 Example

The above mentioned approach was applied to the system and the faults in it, used in Chapter 2.

First of all, the fuel cost characteristic of each generator is assumed as follows:

$F_1(P_{G1})=0.8+9P_{G1}+6P_{G1}^2$ (ten thousand yen/hour)

$F_2(P_{G2})=1.7+10P_{G2}+7P_{G2}^2$ (ten thousand yen/hour)

$F_3(P_{G3})=1.2+12P_{G3}+6P_{G3}^2$ (ten thousand yen/hour)

and the power cost characteristic of the slack bus as follows:

$$F_s(P_s)=\begin{cases} 14P_s+8P_s^2 & \text{for} \quad P_s>0 \\ 14P_s-8P_s^2 & \text{for} \quad P_s<0 \end{cases}$$

Table 3. Load patterns used for optimal load dispatching.

	P_{L1} (p.u.)	P_{L2} (p.u.)
Load pattern 1	0.4	0.3
Load pattern 2	0.45	0.1

Two load patterns were assumed, as shown in the Table 3, and for each of them the optimum solution was obtained, with the weighting coefficient C_2/C_1 varied.

First, the most economical load dispatch, disregarding stability, was obtained for the first load pattern, the result of which is tabulated in Table 4 (a). It can be seen from the table that the system is unstable because of the two faults, F2 and F3, when only economy is considered. Fig. 3 shows the optimum generating power distribution versus the value of C_2/C_1, with the transient stability for the fault F2 taken into account. It can be said that for C_2/C_1 less than about 0.35, the stability measure equals zero, and the optimum solution is restricted by the stability constraint. For C_2/C_1 greater than 0.35, the term concerning stability comes to have an appreciable effect, and with increasing C_2/C_1, P_{G1} decreases, the stability measure increases and the fuel cost increases accordingly. Furthermore, when the value of C_2/C_1 is greater than about 15, the stability term is predominant in the objective function, P_{G1} and P_{G2} are limited at their lower limits, and the stability measure and the fuel cost become almost constant. On the other hand, the optimum solution in the case considering transient stability for the fault F1 was similarly found as shown in Fig. 4. If the value of C_2/C_1 is small, the solution coincides with the most economical load dispatch obtained, without considering stability.

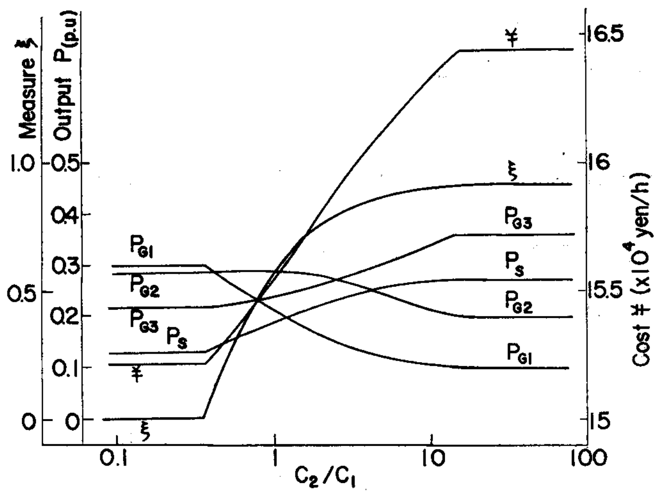

Fig. 3. Optimum load dispatch for load pattern 1 with transient stability for the fault F2 considered.

Table 4. Results of economical load dispatch without considering stability.

(a) Load pattern 1

	Outputs (p.u.)	Cost (10^4 yen/h.)	Stability measures
P_S	0.046	0.666	F1 0.300
P_{G1}	0.400	5.360	F2 −1.096 F3 −0.802
P_{G2}	0.284	5.110	F4 0.803
P_{G3}	0.204	3.897	F5 0.431 F6 0.526
Total	0.934	15.034	Min. −0.786

(b) Load pattern 2

	Outputs (p.u.)	Cost (10^4 yen/h.)	Stability measures
P_S	−0.487	−8.715	F1 −2.875
P_{G1}	0.400	5.360	F2 −5.800 F3 −1.391
P_{G2}	0.500	8.451	F4 −0.199
P_{G3}	0.400	6.961	F5 −1.099 F6 −0.647
Total	0.813	12.057	Min. −2.349

However, if C_2/C_1 increases, the generating power distribution becomes such that the stability measure is great, and the fuel cost increases accordingly.

Fig. 5 shows the optimum solution taking all of the six faults into account by defining the minimum value of the six measures as the stability measure. It shows a similar trend as in the case of fault F2.

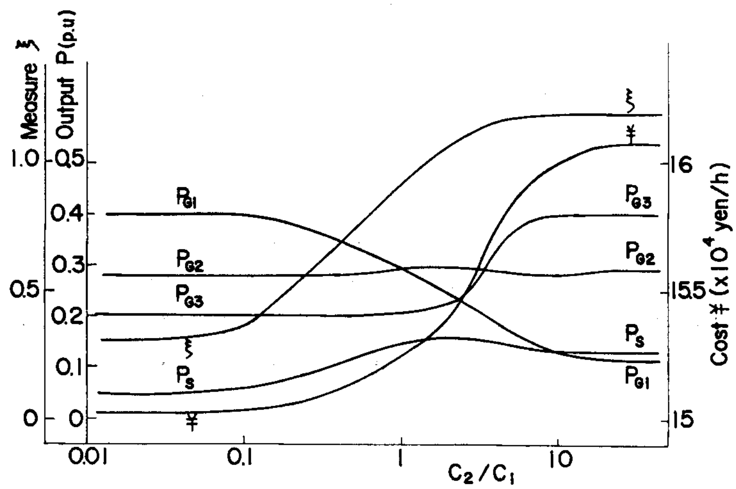

Fig. 4. Optimum load dispatch for load pattern 1 with transient stability for the fault F1 considered.

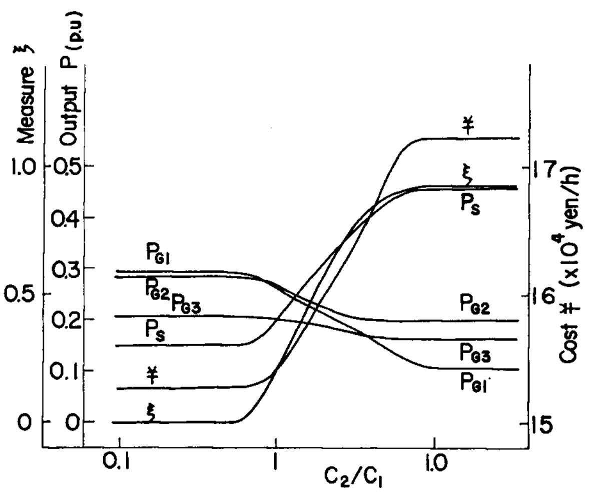

Fig. 5. Optimum load dispatch for load pattern 1 with transient stability for all of the six faults considered.

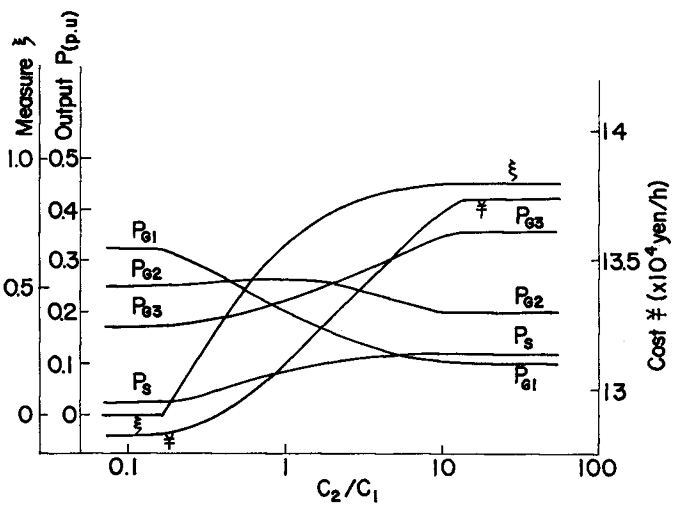

Fig. 6. Optimum load dispatch for load pattern 2 with transient stability for the fault F2 considered.

For the second load pattern, the system is unstable for all of the assumed faults, as shown in the Table 4 (b), if the most economical load dispatch is applied, disregarding the aspect of stability. Fig. 6 shows the optimum solution with stability for the fault F2 considered, and is found to have almost the same tendency as shown in Fig. 3 and Fig. 4.

4. Conclusion

In this paper, we showed a method of transient stability evaluation via a stochastic approach developed for the purpose of using it for security control. As an example, we applied it to a model system composed of four generators and six buses. A comparison was made between the results obtained by the method proposed here and by the pattern recognition method proposed recently for the same purpose. Furthermore, as an example of a practical application of this method, it was used to find the optimum load dispatching, considering not only economy but also transient stability. The results obtained in this paper are as follows:

1) Using the stochastic approach proposed in this paper, it is possible to assess the transient stability with sufficient accuracy at high speed.

2) Therefore, it can be used in security assessment which plays a fundamental part in security control.

3) Assuming that the number of samples are the same, this method has a little better accuracy than the pattern recognition method in the sense that the number of misjudgements is smaller.

4) It becomes possible to operate the system considering both economy and stability, since stability can be assessed by a simple function of the operating state variables.

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Appendix[10)]

A constrained optimization problem involving the minimization of a nonlinear function under nonlinear inequality constraints is formulated as follows:

minimize $f(x)$ subject to

$g_i(x) \geqslant 0, \quad i=1, \cdots, m$

where x is an n-dimensional vector.

This constrained problem can be solved by transforming it into a sequence of unconstrained problems. Powell introduced the following function

$$F(x, r, s)=f(x)+\sum_{i=1}^{m} \frac{(g_i(x)+s_i)^2}{r_i}, \quad g_i(x) \geqslant 0,$$

where s_i and r_i, $i=1, \cdots, m$, are constants during each sequential minimization, and inequalities are only included when violated. Supposing that a set of values r^j, s^j was available at the beginning of the j-th iteration, and that the minimum of $F(x, r^j, s^j)$ was found to be $x(r^j, s^j)$, then the problem that has been solved is the minimization of $f(x)$ subject to the constraints $g_i(x)=g_i(x^j(r^j, s^j))$, where i covers all equality constraints and violated inequality constraints. If $g_i(x^j(r^j, s^j))=0$ with some degree of precision, then the minimum of $f(x)$ subject to the required constraints would have been found, even though at the minimum points $x^j(r^j, s^j)$ the values of $F(x^j, r^j, s^j)$ and of $f(x^j)$ are not the same.

The remaining task is to find an adequate method of modifying r_i and s_i so that

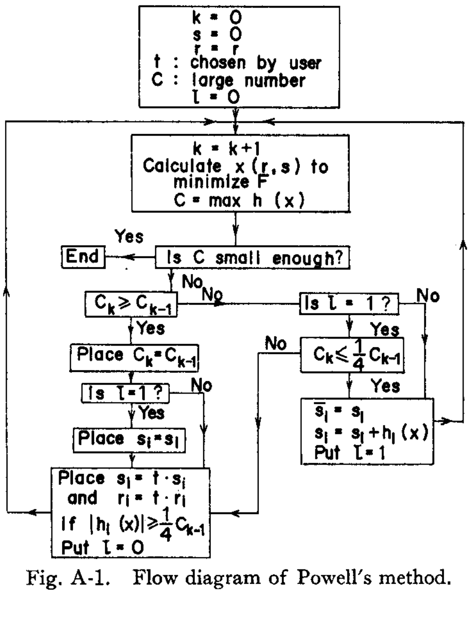

Fig. A-1. Flow diagram of Powell's method.

at each successive iteration the constraints $g_i(x)$ tend to be more satisfied. Powell has found that a convenient method for varying s, from iteration j to $i+1$, is

$$s_i^{j+1}=s_i^j+g_i.$$

The procedure to work on the s_i terms until their overall effect becomes small is recommended. Then, the r_i terms whose corresponding g_i constraints are not converging fast enough, are reduced. Following this, it is convenient to decrease the value of the s_i terms, corresponding to the r_i terms that are decreased, by the same factor used for r_i. The process starts with all s_i being set to zero and r_i chosen such that all g_i/r_i terms have the same order of magnitude as the initial $f(x)$. The definition of a fast enough rate of convergence and the choice of the factor to reduce r_i and corresponding s_i are two decisions that should be taken in the light of the problem at hand. For the problem discussed in this paper, it was found adequate to demand that the largest constraint violation should be reduced by a factor 4, and that should be decreased by factors of 100. Fig. A-1 presents a flow diagram of Powell's method summarizing all the above comments.