An Approach to the Adaptive Flood Control by Multi-Reservoir Systems

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Abstract

The aim of this study is to establish a procedure for the adaptive flood control by multi-reservoir systems. The multi-reservoir systems have several reservoirs located in series, parallel, or mixed type and several flood defence points. The foundamental idea is to combine the typhoon simulation techniques with the optimal operation techniques based on Dynamic Programming. That is to say, at every control time, many typhoons are simulated according to the stochastic structures of a typhoon. And for each simulated typhoon, an optimal release flow at the control time will be decided in a sense of a probability of exceedance, or a safety rate based on the frequency distribution estimated from the set of the above optimal release flow. Of course, the simulated results antecedent to the control time will be compared with the observed; and through the feedback loop, the characteristic parameters will be corrected. From this point of view, we may consider this procedure as an approach to "the adaptive flood control".

1. Introduction

Recently, the flood runoff becomes large in volume and sharp in time because of the rapid development in the watershed and the progress of river improvement. Also, it has been difficult to construct a retarding basin and to extend the width of the river because of the requirement of the land use. Thus, it has become very necessary to construct many dam-reservoirs and control an increasing flood by those dam-reservoirs. Under these circumstances, the important problem is to establish an operational procedure for flood control by multi-reservoir systems, especially for synthetic control.

At present, with dam-reservoirs being operated on the basis of the simple and constant rule such as the method of releasing the constant volume or the volume at the constant rate to the inflow, there is no consideration for the prediction of the rainfall or the inflow into the dam-reservoir. So the storage capacity for the flood

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control is not used as effectively as possible.

However, about typhoons which bring flood damage every year, we know some characteristics through the stochastic analysis of the historical data. Thus, we may propose an adaptive flood control whereby, 1) at every control time courses of typhoon are generated, 2) for each generated typhoon the decided discharge is based on Dynamic Programming and 3) the actual release discharge is decided in a sense of a probability of exceedance or the safety rate based on the frequency distribution estimated from the set of the optimal release discharge in 2).

At first, we describe the procedure of this adaptive control and the typhoon simulation.

Next, we define the objective function for flood control, and under the simulated results, explain the methodology of the optimal operation by multi-reservoir systems.

Finally, as an example of this flood control, this method is applied to the Syorenji Dam in the Kizu River basin and compared with the existing method.

2. Procedure of the adaptive flood control

It is very important for any flood control to use the flood capacity of the reservoir as effectively as possible. Unfortunately, because of many uncertain elements being contained in the rainfall phenomena, it is very difficult to predict the inflow hydrograph with sufficient accuracy.

Firstly, at any control time step, many supposed courses on which a typhoon will go are generated, using the conditional transition probabilities among the courses estimated at the center point of the typhoon.

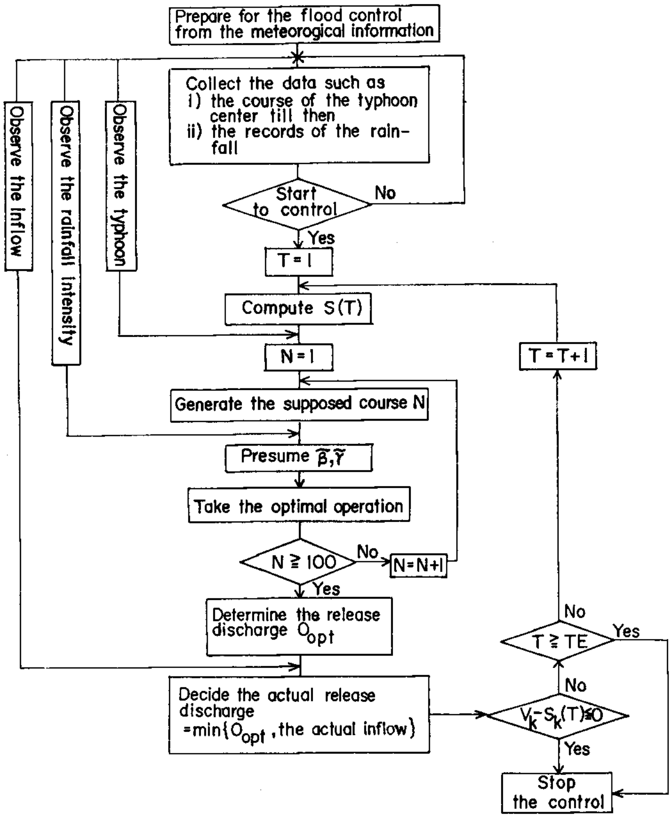
Secondly, for each supposed course, the hyetograph on the control area will be predicted by using the conditional transition probabilities among the rainfall intensities. As for the predicted hyetograph, the inflow hydrographs in the dam-reservoirs will be predicted through the runoff analysis such as the storage function method or the kinematic wave method. Then for these already-known hydrographs, the optimal release flows or the value of K described in section 4 are determined, based on Dynamic Programming. Of course, this second procedure is repeated till the number of the simulated typhoon equals the predetermined number.

Lastly, the release discharge at the control time step is decided as a basis of a probability of exceedance from the frequency distribution of the value K.

The above procedures are repeated till the typhoon has no effect on the control area, and their flow chart is shown in Fig. 1.

3. Typhoon simulation

At first, let us describe the stochastic structures for generating a typhoon.



T : the control time step
TE : the total number of the control time steps
$S_k(T)$: the storage level of reservoir k at the beginning of the control time step T
V_k : the maximum allowable storage capacity of reservoir k
$\tilde{\beta}$: described in the section 3
$\tilde{\gamma}$: described in the section 3

Fig. 1. The flow chart of the adaptive flood control

We divide the area from lat. 20°N to lat. 40°N and from long. 122°E to long. 146°E into a sub-area of 1° around. Then, for each sub-area through which a typhoon passes, we estimate the following characteristics from the historical data, such as i) the conditional transition probability with which a typhoon travels from one sub-area to another (Fig. 2), ii) the length of its stay for each course, iii) the probability whether it will rain or not on the control basin, and iv) if it rains there, the mean rainfall intensity. Moreover, we define α and β as follows.

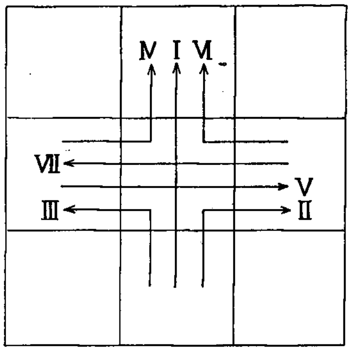


Fig. 2. The explanation of the conditional transition probability

$$\alpha = \frac{\sum_{t=1}^{TE} r(t)}{\sum_{i=1}^{J} r_m^i T_m^i} \tag{1}$$

$$\beta = \frac{TE}{\sum_{i=1}^{J} T_m^i} \tag{2}$$

We fine that the values of α and β are nearly constant (1).

Where

$r(t)$: the rainfall intensity at time t on the control basin

r_m^i : the mean rainfall intensity on the control basin when the typhoon passes through sub-area i

T_m^i : the mean length of its stay on subarea i

TE : the total number of the control time steps

J : the total number of the sub-areas through which the typhoon passes

Therefore, as for one supposed course, the future values of $\tilde{\alpha}$ and $\tilde{\beta}$ will be presumed, based on the observed data by control time T, as follows:

$$\tilde{\alpha} = \frac{\sum_{t=1}^{T} r(t)}{\sum_{i=1}^{j} r_m^i T_m^i} \tag{3}$$

$$\tilde{\beta} = \frac{T}{\sum_{i=1}^{j} r_m^i T_m^i} \tag{4}$$

where j is the sub-area number on which the typhoon stays.

Also, when we define $\gamma=\alpha/\beta$, $\tilde{\gamma}$ is presumed as follows:

$$\tilde{\gamma} = \frac{\sum_{t=1}^{T} r(t)}{\tilde{\beta} \sum_{i=1}^{j} r_m^i T_m^i} \tag{5}$$

Thus, for each sub-area on a supposed course, the length of the typhoon's stay is $\tilde{\beta} T_m^i$, and the rainfall intensity on the control area is $\tilde{\gamma} r_m^i$. Also, the hyetograph can be gained by transforming the unit of $\tilde{\gamma} T_m^i$ into the unit of the control time step. Furthermore, the inflow hydrograph can be gained from this hyetograph through a runoff analysis.

At every control time, we get new information from the observation such as the rainfall intensity or the course on which the typhoon passed. Consequently, $\tilde{\beta}$ and $\tilde{\gamma}$ are revised, becoming nearer to their true values respectively. This revision of $\tilde{\beta}$ and $\tilde{\gamma}$ at every control time is the key point for the adaptive flood control.

4. Optimal operation based on Dynamic Programming

Once the inflow hydrograph is predicted on any supposed course, the optimal operation by the dam-reservoirs must be determined for this hydrograph. We can get the optimal sequences of release flow by using the method of Dynamic Programming (2). At that time, however, it is very important to establish the objective function and the criterion function. Hence, in this section we define the objective function for the flood control and propose the criterion function which satisfies its object.

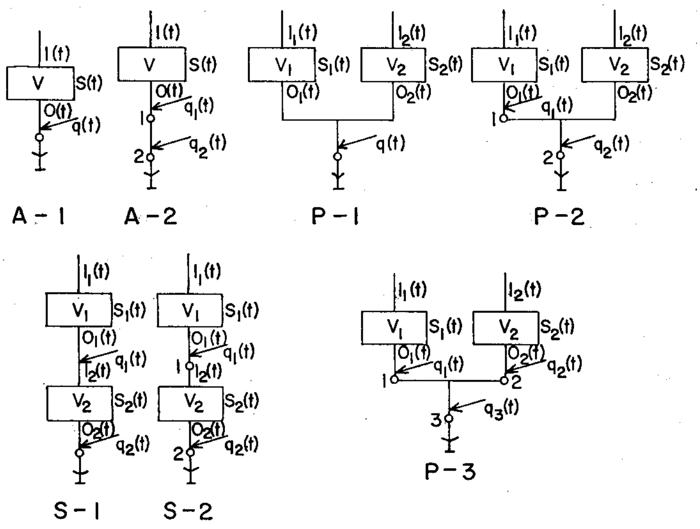


Fig. 3. The basic patterns composing of the complex system with multi-reservoirs and multi-defence points

Furthermore, in a complex system with multi-reservoirs and multi-defence points, we assume that the system may be composed of seven sub-systems as shown in Fig. 3. The reason is because of the hydrological influence in space and time, and that the optimal operation of the total system may be gained through their synthesis.

4.1 Objective function and criterion function

Let us define the objective function for the flood control as follows:

$$K = \max\{Q_{up}/Q_{ud}\} \longrightarrow \min.$$

and

$$K \leqq 1 \qquad (u=1,2,\ldots\ldots,\mathrm{m}) \tag{6}$$

where m is the total number of defence points, Q_{up}, and Q_{ud}, the peak flow and the allowable flow at u-th defence point, respectively.

Then we propose the criterion function that satisfies the above objective function as follows.

$$D_u\{Q_u(T)\} = \{(\omega+1)m\}^{a_u Q_u(T)-b}$$

and

$$a_u Q_{ud} = \text{constant} \qquad (u=1,2,\ldots\ldots,m) \tag{7}$$

Where $\omega = \min\{\max\limits_{1 \leq T \leq TE-1}(v_T),\ TE\}$, $Q_u(T)$, the pass flow at the control time T and u-th point after being controlled, a_u, positive integer, b, constant, v_T, the range of the state variable at time T, respectively. It is proved that these criterion functions give the optimal solution and have a character of minimizing the total number of ocurrences of the peak flow (3).

4.2 Some methods in computation

In the performance of Dynamic Programming with the above criterion function, there appear several problems such as over-flow or under-flow error in the computer, and the shortness of a capacity for memory attributed to multi-dimension or multi-time step. Those problems are solved approximately by the following methods.

i) The first approximate method is to use the following equation as the recursive equation $f_T(s_1(T), S_2(T),\ldots,S_K(T))$ of Dynamic Programming.

$$f_T(S_1(T), S_2(T),\ldots,S_K(T)) = \min_{\substack{0 \leq S_k \leq V_k\ (u=1,2,\ldots,\,m;\ k=1,2,\ldots,\,K)}}\{\max(Q_u(T)/Q_{ud}, f_{T+1}(S_1(T+1), S_2(T+1),\ldots$$
$$\ldots,S_K(T+1))\} \tag{8}$$

where $S_k(T)$ is the storage level of reservoir k at the beginning of control time step T and K is the total number of the reservoirs.

ii) The second method is to diminish the dimension of the system. For example, in the case of only one defence point we compose some reservoirs and distribute the release discharge of the composite reservoir based on the space ratio.

Space ratio:

for the parallel reservoirs,

$$\frac{V_k-S_k(T)}{\sum_{e=1}^{2}\{V_e-S_e(T)\}}=\frac{I_k(T)}{\sum_{e=1}^{2}I_e(T)} \qquad (k=1,2) \tag{9}$$

for the series reservoirs,

$$\frac{V_1-S_1(T)}{\sum_{e=1}^{2}\{V_e-S_e(T)\}}=\frac{I_1(T)}{\{I_1(T)+q_1(T)\}} \tag{10}$$

where V_k is the maximum allowable storage capacity, $I_k(T)$, the natural inflow into the reservoir k during the control time T and $q_1(T)$, the tributary inflow, respectively.

In the case of more than two defence points, we use the successive approximation

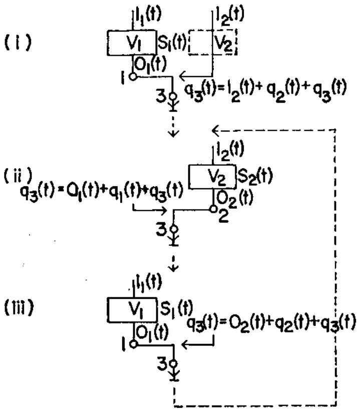


Fig. 4. The successive approximation algorithm for the parallel reservoirs

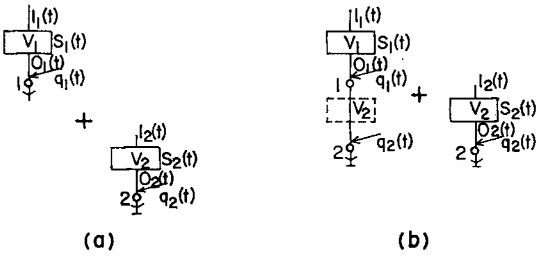


Fig. 5. The approximation algorithm for the series reserviors

algorithms as shown in Fig. 4 for the parallel reservoirs, and in Fig. 5 for the series reservoirs. It is one method to approach the optimal solution by computing the approximate solution on each reservoir iteratively.

iii) The third method is to use DDDP (Discrete Differential Dynamic Programming) algorithm (4). DDDP, in which the width of the corridor equals v_T, is the most available for the proposed criterion function. Both the problem of a capacity for memory and also the problem of over-flow and so on, can be solved by making the width of the corridor narrower.

iv) For the multi-time step problem, the control period is divided into several periods and repeated computation is performed among two adjacent periods.

Judging from the degree of approximation and computation time, the best procedure of computation for the complex system may be concluded as follows. We should find the trial trajectory by method i) or ii) and using its trial trajectory, gain the optimal solution by method iii) or iv).

5. Application to the Syorenji Dam and its results

In this section, we apply the procedure for the adaptive flood control mentioned in the previous sections to the Syorenji Dam in the Kizu River basin. The input data are four historical floods and the defence point is Nabari as shown in Fig. 6. The total number of the supposed courses generated according to the conditional transition probabilities was one hundred for computer convenience, and the runoff was analyzed by the kinematic wave method (5). Furthermore, at every control

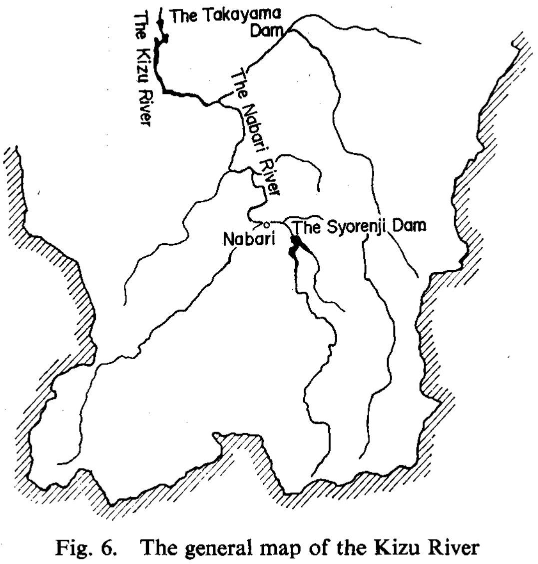


Fig. 6. The general map of the Kizu River

time the discharge corresponding to the probability of exceedance of 10% was decided as the release discharge. Their results are shown in Fig. 7, 8, 9 and 10. As the Syorenji Dam had been constructed in 1970, we could not compare the proposed

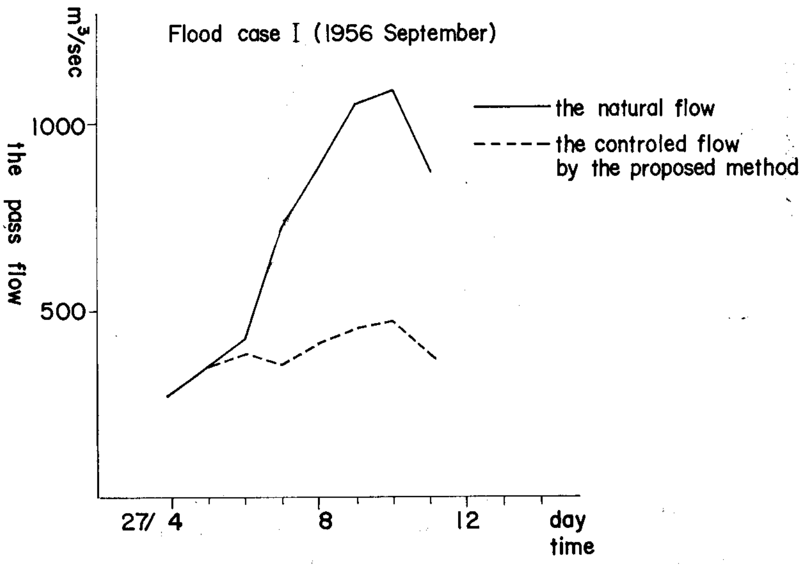


Fig. 7. Comparison between the natural flow and the controled flow passing through the defence point (Flood case I)

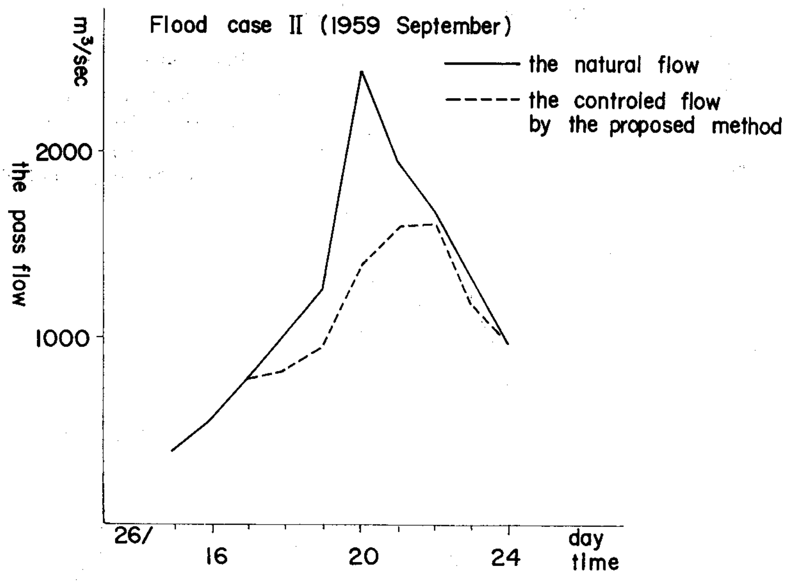


Fig. 8. Comparison between the natural flow and the controled flow passing through the defence point (Flood case II)

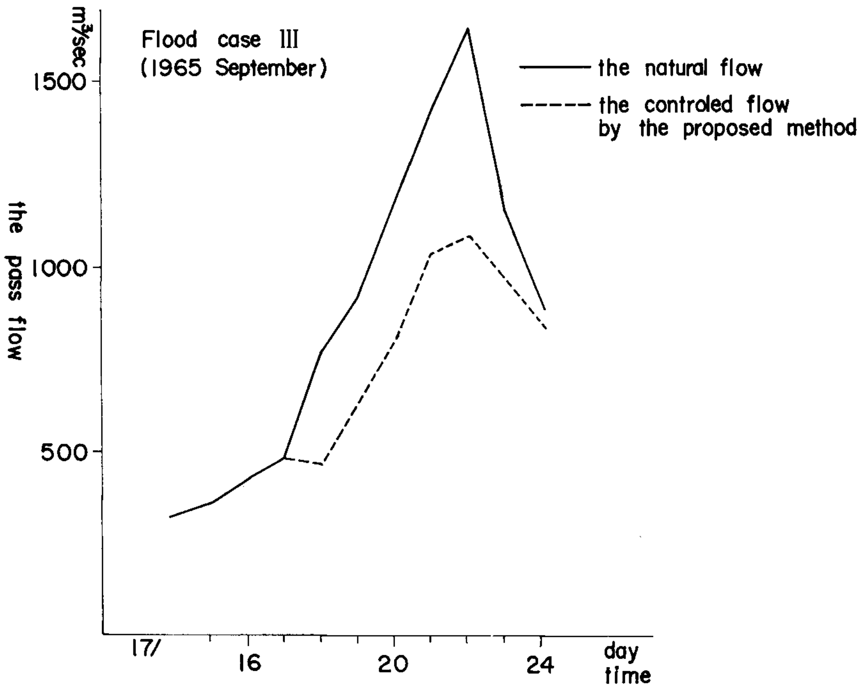


Fig. 9. Comparison between the natural flow and the controled flow passing through the defence point (Flood case III)

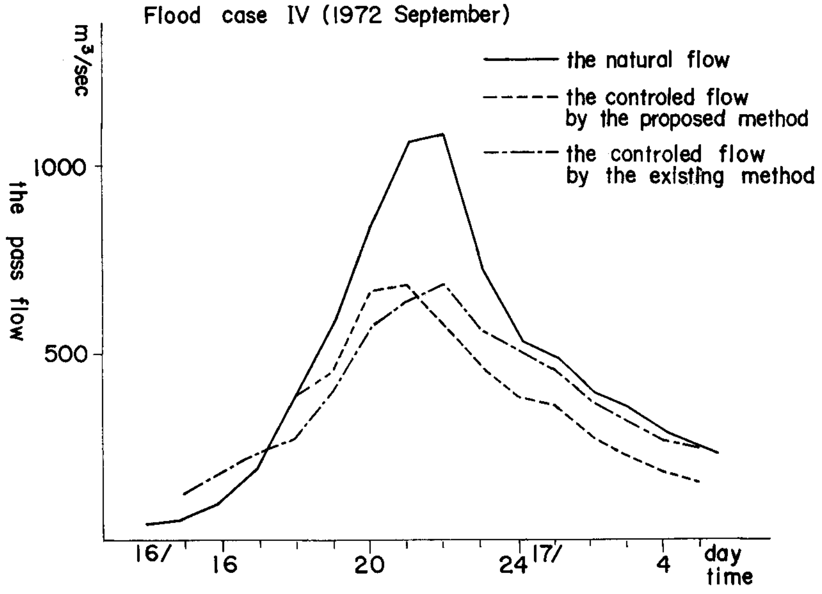
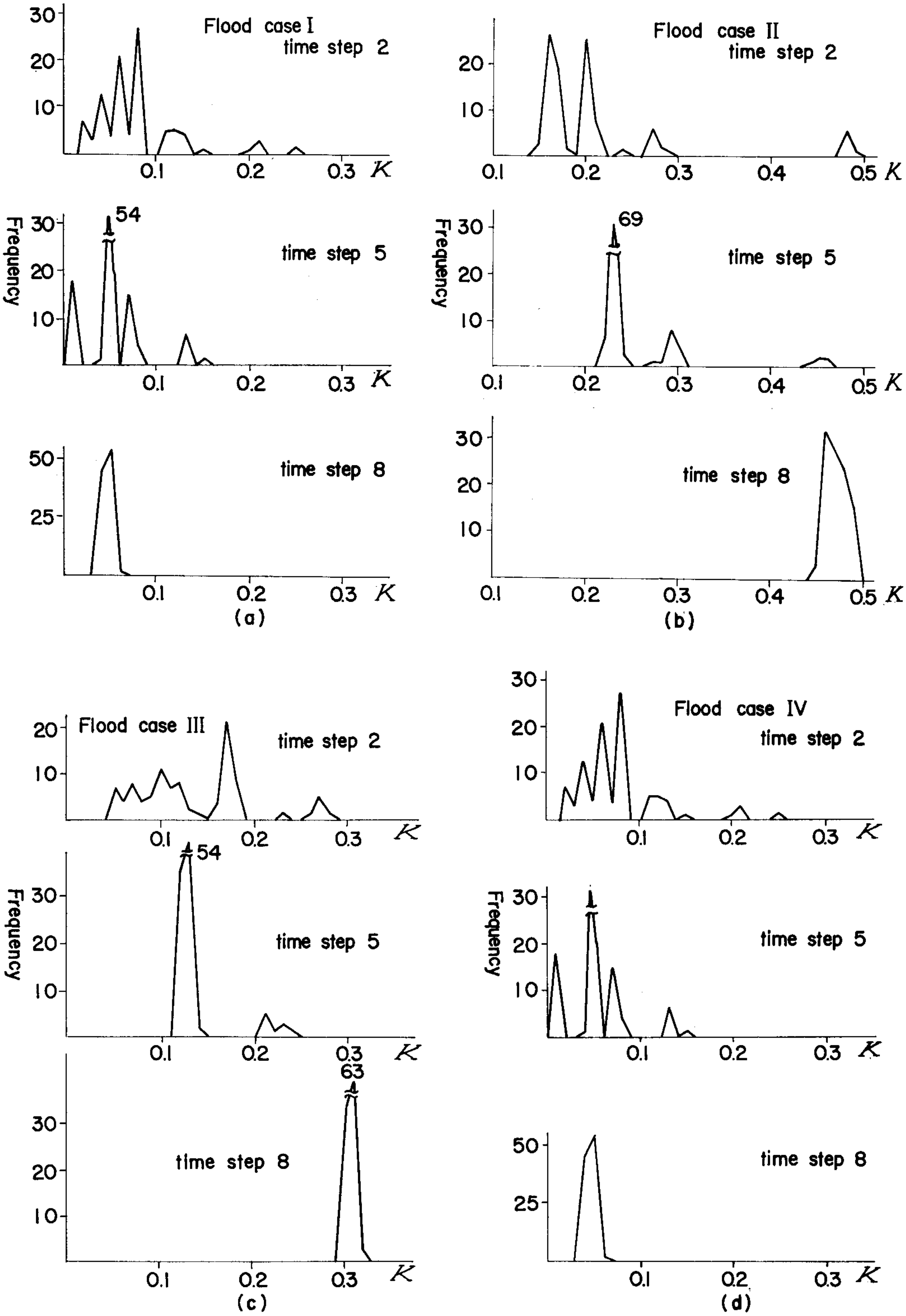


Fig. 10. Comparison between the natural flow and the controled flow passing through the defence point (Flood case IV)



(a) (b) (c) (d)

Fig. 11. The frequency distribution of the value *K* at the control time 2,5,8

method with the existing method except in the case of Fig. 10 (Flood case IV). These figures show that the peak of the pass flow at the defence point decreases and the objective function is satisfied.

However, when there is much precipitation at the beginning of the control period as in flood case IV, the computed result does not seem to be better than the result through the existing method. Fig. 11 shows the frequency distribution of the value K at the control time steps 2, 5 and 8, respectively. There is much variance at the control time step 2. This seems to be caused by the largeness of the sub-area, or the shortness of the statistical data. Fig. 12 and 13 show the change in the values of $\tilde{\beta}$ and $\tilde{\gamma}$ with the control time steps, respectively. The variance of $\tilde{\beta}$ is very small, but the variance of $\tilde{\gamma}$ is large. This seems to mean the influence of the prediction error at the beginning of the period of the rainfall and the necessity of limiting the maximum range where the value of $\tilde{\gamma}$ will vary.

In those examples, we use 10% as the probability of exceedance. It is very

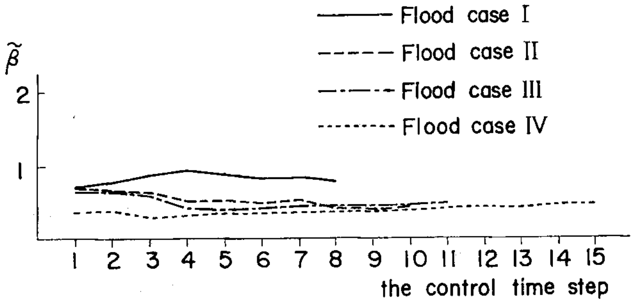


Fig. 12. The variation of the value $\tilde{\beta}$ with the control time step

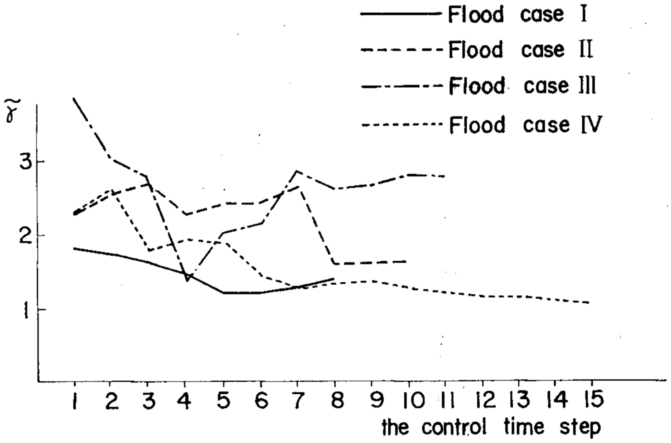


Fig. 13. The variation of the value $\tilde{\gamma}$ with the control time step

pessimistic to choose the maximum value of K. Also, it is not very efficient for the available storage capacity at the control time step to choose 50% as the probability of exceedance. Therefore, taking account of the typhoon simulation, the error in the runoff analysis and the risk of flood, it will be realistic to determine the release discharge by choosing about 10% as the probability of exceedance or the safety rate.

Though we applied the above method to the simplest type such as the A–1 type, it is a matter of course to be able to apply it to the other types in the same manner. At present, however, it takes a little more time in the decision of the pass flow at each defence point.

6. Conclusions

In this paper, we propose a flexible and effective method for the adaptive control by multi-reservoir systems in order to prevent and decrease flood damage caused by typhoons. Concretely, taking into account that the primary cause of uncertainty concerning the rainfall will be attributed to predicting the course of the typhoon, the risk rate after being controlled in the future will be computed from many supposed courses generated by the conditional transition probabilities. Then, the release discharge may be determined by using the probability of exceedance or the safety rate. The frequency distribution may be estimated from the set of the optimal release flow based on Dynamic Programming for each supposed course.

Though there are some problems in presuming the value $\bar{\gamma}$ at the beginning of the control period, better results will be had by limiting the maximum range of the variable $\bar{\gamma}$ with reference to the information observed by a radar or a satellite.

In the future, we will prove the above method to be very effective for flood control by applying it to many control areas, and develop the simulation method of the typhoon with a higher accuracy by taking account of more physical elements of typhoons.

7. Acknowledgements

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References

1) K. Mochizuki: System Approach to the Flood Control Planning in the Yodo River A thesis for a doctor's degree, Kyoto Univ. 1970.
2) Belleman, R: Dynamic Programming, Princeton Univ. Press, Princeton, New Jersey 1957.

3) T. Takasao, S. Ikebuchi and T. Kojiri: Dynamic Programming for Multi-Reservoir System Design based on the Water Quantity Control, Proceeding of JSCE No. 241 1975–9.
4) Heidari, M., V. T. Chow and D. D. Meredith: Water Resources Analysis by Discrete Differential Dynamic Programming, Civil Engineering Studies, Hydraulic Engineering Series No. 24 Illinois at Urabana-Champaign, Urabana, III, Jan., 1971.
5) T. Ishihara and T. Takasao: Fundamental Researches on the Unit Hydrograph Method and its Application, Transaction of JSCE No. 60 Extra Papers (3–3) 1959–3.