Improvement of Transient Performance of Synchronous Generator via Excitation Control

By

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Abstract

This paper investigates the improvement of a transient performance of a synchronous generator in a one-machine system via the excitation control. First, in regard to the optimal control of the excitation, the effects of the weighting coefficients on the optimal response are examined. The optimal control is approximately realized by the feedback from the state variables, and the obtained responses are compared with the optimal responses. Furthermore, the system responses, in a case of being installed with a fast acting excitation control system with a stabilizer, are calculated and compared with those by the optimal control system.

1. Introduction

In recent electric power systems, generating stations are inclined to be constructed collectively in places remote from the load center due to the environmental problems etc. Also, the extension of transmission lines is getting difficult because of lack of sites. Consequently, the transmission lines have become longer and of larger capacity, and accordingly, the problem of transient stability is that it has resulted in limiting the transmission capacity of the system.

On the other hand, an application of the modern optimal control theory to the control of the power systems has been proposed for these several years in order to improve the transient stability. Many theoretical considerations have been made, for example, the optimal control of excitation voltage and mechanical power input[1)-3)], the bang-bang control of line reactance and damping resistance[4)] etc. Optimal control of the excitation voltage or the mechanical power input is usually obtained by using the maximum principle, which minimizes the performance index of quadratic form of the deviations of state variables and control variables. The weighting coefficients in the performance index can be chosen freely. In this paper, the optimal excitation voltage control of a one-

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machine system is considered as an example. At first, the relationship between weighting coefficients and optimal solutions is numerically investigated, and a set of weighting coefficients which gives the best system performance is selected. Next, the obtained optimal control is approximately realized with the feedback control using the method of [11], and its results are shown.

The above mentioned optimal control is still at the stage of theoretical investigation. Among the practical, effective and economical methods for improving transient stability, the applications of series condensers, damping resistances and a fast acting excitation control system are investigated in the real system[5]. In the last part of this paper, the effect of a fast acting excitation system for improvement of stability is shown and is compared with the results of optimal control.

2. System Equations

In this paper the control of excitation for a single synchronous generator connected through a transmission line to an infinite busbar is considered. A synchronous machine is represented as a simple 3rd-order model, i.e., damper windings, armature resistances, time derivatives of stator flux linkages and speed deviation in the expressions of the stator voltages are neglected. Then Park's equations are written as follows:[3]

$$\begin{array}{ll}
\text{voltages} & e_d = -\omega_0\psi_q \\
 & e_q = \omega_0\psi_d \\
 & e_{fd} = p\psi_{fd}+r_{fd}i_{fd} \\
\text{flux linkages} & \omega_0\psi_d = -(x_{md}+x_a)i_d+x_{md}i_{fd} \\
 & \omega_0\psi_q = -(x_{mq}+x_a)i_q \\
 & \omega_0\psi_{fd} = -x_{md}i_d+(x_{md}+x_f)i_{fd} \\
\text{torque} & T_e = P_e = e_di_d+e_qi_q \\
 & M\ddot{\delta} = P_i-P_e-D\dot{\delta}
\end{array} \tag{1}$$

Assuming that the transients in transmission lines are negligible, the equations relating the machine voltages to the infinite-busbar voltage can be written as

$$\begin{aligned}
e_d &= E\sin\delta-x_ei_q \\
e_q &= E\cos\delta+x_ei_d
\end{aligned} \tag{2}$$

Eq. (1) and (2) can be rewritten as the following three equations,

$$\begin{aligned}
\dot{\delta} &= s \\
\dot{s} &= B_1-A_1s-A_2\psi_f\sin\delta-(B_2/2)\sin 2\delta \\
\dot{\psi}_f &= u-C_1\psi_f+C_2\cos\delta
\end{aligned} \tag{3}$$

where

$$A_1 = D/M \qquad B_1 = P_i/M \qquad C_1 = (x_d+x_e)/(x_d'+x_e)T_{d'0}$$
$$A_2 = E/M(x_d'+x_e)T_{d'0} \qquad B_2 = E^2(x_d'-x_q)/M(x_d'+x_e)(x_q+x_e)$$
$$C_2 = (x_d-x_d')E/(x_d'+x_e) \qquad u = e_{fd}x_{md}/r_{fd}$$

Eq. (3) forms a nonlinear 3rd-order system with δ, s and ψ_f as the state variables and u as the control variable.

3. Optimal Control

When the system is subjected to a transient disturbance, optimal control with respect to the chosen performance index is determined. Here we make use of the Pontryagin's maximum principle.[3),6)] The form of the performance index is chosen as

$$I = \int_0^T [A_\delta(\delta-\delta_f)^2+A_s s^2+A_f(\psi_f-\psi_{ff})^2+A_u(u-u_f)^2]dt$$

where δ_f, ψ_{ff} and u_f are the final steady-state values of the variables, and A_δ, A_s, A_f and A_u are the weighting coefficients, which must be determined depending upon the relative emphasis to be placed on each variable. Using a set of adjoint variables p_1, p_2 and p_3, the Hamiltonian function H can be written as follows:

$$\begin{aligned} H = & -A_\delta(\delta-\delta_f)^2-A_s s^2-A_f(\psi_f-\psi_{ff})^2-A_u(u-u_f)^2 \\ & +p_1 s+p_2(B_1-A_1 s-A_2\psi_f \sin\delta-\frac{B_2}{2}\sin 2\delta)+p_3(u-C_1\psi_f+C_2\cos\delta) \end{aligned}$$

The optimal control u^* is determined from

$$u^* = \{u\,|\,H(u) = \max_{u_{\min}\leqslant u\leqslant u_{\max}} H\}$$

and when u is between $u_{\min}$ and $u_{\max}$, u^* is given by the following equation,

$$u = u_f+p_3/2A_u$$

The optimal time variation of the variables can be obtained by solving the following differential equations:

$$\begin{aligned} \dot{p}_1 &= -\partial H/\partial\delta = 2A_\delta(\delta-\delta_f)+A_2p_2\psi_f\cos\delta+B_2p_2\cos 2\delta+C_2p_3\sin\delta &&\equiv f_1 \\ \dot{p}_2 &= -\partial H/\partial s = 2A_s s-p_1+A_1p_2 &&\equiv f_2 \\ \dot{p}_3 &= -\partial H/\partial\psi_f = 2A_f(\psi_f-\psi_{ff})+A_2p_2\sin\delta+C_1p_3 &&\equiv f_3 \\ \dot{\delta} &= \partial H/\partial p_1 = s &&\equiv f_4 \\ \dot{s} &= \partial H/\partial p_2 = B_1-A_1s-A_2\psi_f\sin\delta-\frac{B_2}{2}\sin 2\delta &&\equiv f_5 \\ \dot{\psi}_f &= \partial H/\partial p_3 = u-C_1\psi_f+C_2\cos\delta &&\equiv f_6 \end{aligned} \qquad (4)$$

under the boundary conditions which are introduced from the transversality conditions.

$$p_1(T) = p_2(T) = p_3(T) = 0\,, \tag{5}$$

and the initial conditions of the system,

$$\delta(0) = \delta_i,\quad s(0) = 0,\quad \psi_f(0) = \psi_{fi}\,. \tag{6}$$

4. Quasi-linearisation

In the preceding section, the optimisation problem was reduced to the 2-point boundary value problem by the application of Pontryagin's maximum principle. In this paper, this 2-point boundary value problem is solved by the method of quasi-linearisation.[3),7)] This technique solves nonlinear differential equations by recursively solving a series of linear differential equations, and it can be said to be Newton-Raphson's method in a functional space. The principal advantage of this method is that it converges quadratically to the solution of the original equations, if the procedure converges, and consequently has a rapid convergence.

Eq. (4) can be written as

$$dX/dt = F(X, u) \tag{7}$$

where $X=(p_1, p_2, p_3, \delta, s, \psi_f)^t$, $F=(f_1, f_2, f_3, f_4, f_5, f_6)^t$, and the boundary conditions are given by eq. (5) and (6). u can be expressed as a function of X in the range $(u_{\min}, u_{\max})$. Expanding the righthand side of eq. (7), and neglecting the terms of order higher than first, eq. (7) is approximated as follows:

$$dX/dt = F(X_0, u_0) + J(X_0)(X - X_0) \tag{8}$$

where each element of $J(X_0)$ can be written as

$$J_{ij} = \partial f_i/\partial x_j + (\partial f_i/\partial u)(\partial u/\partial x_j),\quad i, j = 1, 2, \cdots, 6$$

and when u equals $u_{\min}$ or $u_{\max}$,

$$\partial u/\partial x_j = 0$$

X_0 and u_0 are functions of time t, and approximate solutions of eq. (7). Since eq. (8) is linear, the solution which satisfies eq. (8), (5) and (6) can be determined numerically by using the principle of superposition.

If $X_p(t)$ is a solution of the equation

$$dX_p/dt = F(X_0, u_0) + J(X_0)(X_p - X_0) \tag{9}$$

which satisfies eq. (6), and the vectors $X_{hj}(t), j=1, 2, 3$ are solutions of the homogeneous equations given by

$$dX_{hj}/dt = J(X_0)X_{hj} \tag{10}$$

which satisfies the conditions

$$X_{hj}(0) = [\Delta_{1j}, \Delta_{2j}, \Delta_{3j}, 0, 0, 0]$$
$$\Delta_{ij}: \text{ Kronecker's delta}$$

then, by the principle of superposition, the general solution of eq. (8), which satisfies the initial conditions of eqn. (6) is given as follows:

$$X(t) = X_p(t) + \sum_{j=1}^{3} a_j X_{hj}(t) \tag{11}$$

Eq. (11) contains three integration constants $a_j, j=1,2,3$, which can be determined by use of the three final conditions given by eq. (5). With a_j's known, the complete solution of eq. (8) can be determined from eq. (11). Then using this $X(t)$ as the new $X_0(t)$, the above described procedure is continued until the following inequality is satisfied,

$$|X(t)-X_0(t)| < \varepsilon, \quad 0 \leqslant t \leqslant T$$

where ε is a predetermined value for the criterion of convergence.

5. Preliminaries

The system initially in the steady state is disturbed by a step change of power input P_i, which is brought up to 1.0 p.u. from 0.725 p.u., and is again brought down to the original magnitude after the time lapse of 0.35 sec.

The system parameters are as follows:

synchronous machine	$x_d = 1.25$	$x_d' = 0.3$
	$x_q = 0.7$	$T_{d'0} = 9.0$ sec.
	$M = 0.0185$	$D = 0.005$
transmission line	$x_e = 0.2$	
infinite-busbar	$E = 1.0$	

Assuming the initial values of u and P_i to be 1.1 p.u. and 0.725 p.u. respectively, the initial and final steady-state values of the system variables are obtained as follows:

$$\delta_i = 0.7461 \text{ rad.}, \quad s_i = 0, \quad \psi_{fi} = 7.7438$$
$$\delta_f = 0.7461 \text{ rad.}, \quad s_f = 0, \quad \psi_{ff} = 7.7438\,.$$

The first approximation $X_0(t)$ described in the preceding section is assumed as follows:

$$p_{10}(t) = 0 \qquad p_{20}(t) = 0 \qquad p_{30}(t) = 0$$
$$\delta_0(t) = 0.7461 \qquad s_0(t) = 0 \qquad \psi_{f0}(t) = 7.7438$$
$$u_0(t) = 1.1$$

and the convergence criterion ε is 0.001.

With the above parameters and conditions, the differential equations (9) and (10) are integrated by the R.K.G. method. The integration step length and the total time period are 0.01 sec. and 2.0 sec. respectively.

6. Relationship between Weighting Coefficients and Optimal Solution

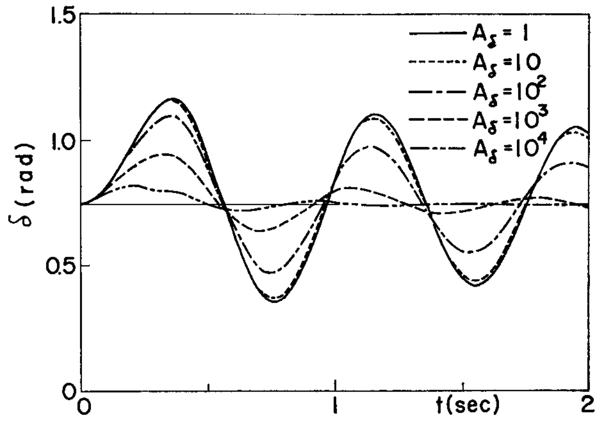
In the formulation of the optimal control problem, the performance index is chosen as a quadratic form of the deviations of the state variables and the control variable from their final values as shown in Section 3. The optimal solution is optimal with respect to this particular performance index. Hence, if different performance index is chosen, i.e., the weighting coefficients in the performance index are varied, different optimal solutions result.

Although some results are reported on the relationship between weighting coefficients and optimal system performance for linear systems,[8)-10)] they are not applicable to nonlinear systems nor to the case of constrained control. The system considered in this paper is nonlinear. Hence, it is difficult to analytically clarify the influence that weighting coefficients have upon the optimal solution.

In this paper, the motion of a synchronous machine is described by the 3rd-order differential equation, so the performance index includes four weighting coefficients A_δ, A_s, A_f and A_u, which correspond to three state variables and one control variable. Of course, the behavior of rotor angle δ is most important for the stability of synchronous generator. Therefore, first of all, we investigate in 6–1 how the optimal solution is varied when A_δ is varied and $A_s=A_f=0$. In 6–2 and 6–3 we examine the effects of A_s and A_f respectively.

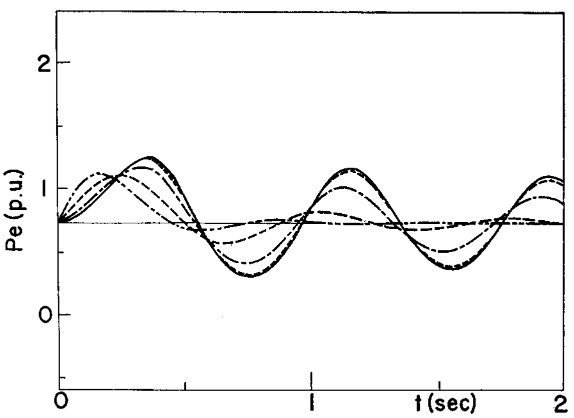
6-1 Case 1 $A=[A_\delta, 0, 0, 1]$

In this case we investigate how the optimal solution is varied as A_δ is varied with $A_s=0$, $A_f=0$ and $A_u=1$. The results are shown in Fig. 1. Fig. 1 (a) shows the time variations of the rotor angle δ taking A_δ as a parameter. When $A_\delta=1$, the control hardly varies, and almost equals the case of noncontrol. In this case, $\delta-\delta_f\equiv\Delta\delta$ oscillates about zero. $\Delta\delta$ reaches its peak at t=0.35 sec., and after this time $\Delta\delta$ damps down with the damping coefficient of the system. As seen from the figure, this damping is not very fast. As the magnitude of A_δ gets larger,



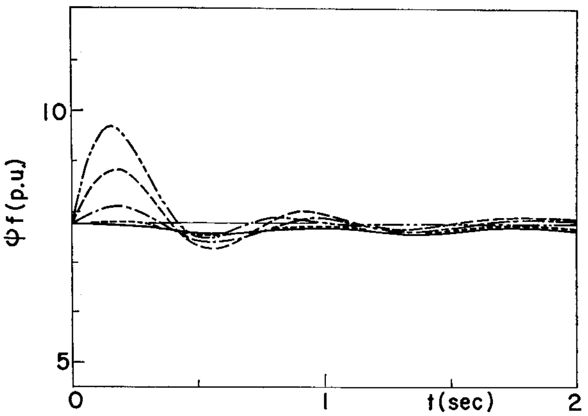
1.5
1.0
0.5
0
δ (rad)
0
1
t(sec)
2
$A_\delta = 1$
$A_\delta = 10$
$A_\delta = 10^2$
$A_\delta = 10^3$
$A_\delta = 10^4$

(a)



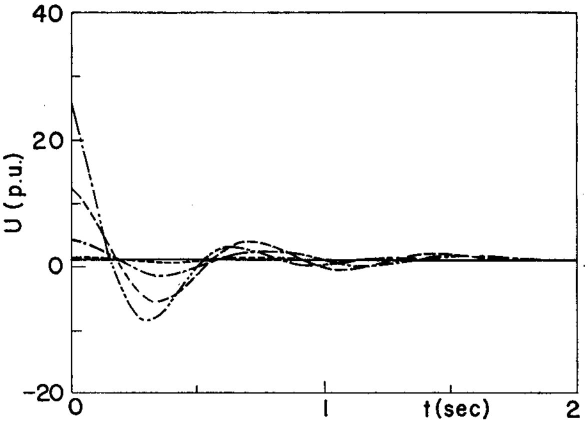
2
1
0
Pe (p.u.)
0
1
t (sec)
2

(b)



10
5
Φf (p.u)
0
1
t (sec)
2

(c)



(d)

Fig. 1. Optimal responses for $\Lambda=[\Lambda_\delta, 0, 0, 1]$ with Λ_δ as a parameter.

the value of $\Delta\delta$ at t=0.35 sec. gets smaller and the damping becomes faster. Thus, the optimal control acts so as to make the value of $\Delta\delta$ small at time t=0.35 sec. when the input disturbance is cleared off.

Fig. 1 (b) shows the time variations of power output P_e of the synchronous machine. When $\Lambda_\delta=1$, its variation equals the noncontrol case. In this case, $\Delta P=P_e-P_i$ oscillates with an amplitude 0.275 p.u., because P_i changes from 0.725 p.u. to 1.0 p.u. in a stepwise manner at $t=0$. At $t=0.35$ sec., P_i again changes from 1.0 p.u. to 0.725 p.u., so ΔP oscillates with the amplitude of 0.55 p.u. thereafter. This variation is similar to that of $\Delta\delta$, which can be explained from the fact that P_e is linearized to the form of $\Delta P=a\Delta\delta+b\Delta\psi_f$, and $\Delta\psi_f$ nearly equals zero. Next, as Λ_δ gets larger, the peak which was initially at $t=$ 0.35 sec. begins to appear at an earlier time, and its value gets smaller as a whole. The parts where ΔP is positive or negative are more subdivided in the case of $\Lambda_\delta=10^4$ than in the case of $\Lambda_\delta=1$. In other words, the variation of P_e becomes similar to that of P_i. As P_e is controlled in this manner, $\Delta\delta$ is suppressed to the small value. Hence, the variation of ΔP_e is mainly caused by $\Delta\psi_f$.

Fig. 1 (c) shows the variations of the field flux linkage ψ_f with Λ_δ as a parameter. When $\Lambda_\delta=1$, ψ_f oscillates a little. This oscillation is caused by δ, and not by u, because u is constant. As the value of Λ_δ gets larger, ψ_f is varied to have its peak between $t=0.15$ sec. and 0.2 sec., and its peak value gets larger. As mentioned above, when the value of Λ_δ is small, the effects of $\Delta\psi_f$ on ΔP_e are not large, but when Λ_δ is large, especially $\Lambda_\delta=10^4$, $\Delta\psi_f$ and ΔP_e vary in a similar manner. Such variations are caused by $\Delta\dot{\psi}_f\cong u-u_f$.

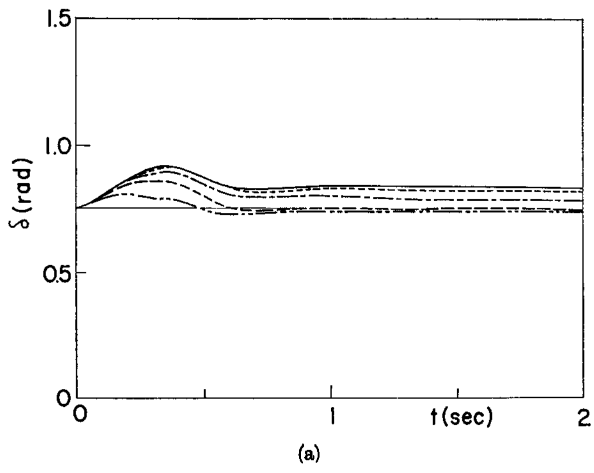
Fig. 1 (d) shows the variations of the control variable u with Λ_δ as a parameter.

When $A_\delta=1$, u is the same as u_f, and equals the noncontrol case. As the value of A_δ gets larger, the value of u at $t=0$ gets larger. u decreases with time to reach $u_f=1.1$ at $t\cong 0.15$ sec., and further decreases to the minimum at $t=0.3\sim 0.35$ sec. After this time, u continues to make a small oscillation about u_f, and approaches u_f at $t=2.0$ sec. Therefore, large changes of u appears in the first time period of 1.0 sec. The large value of u at $t=0$ corresponds to the steep increase in ψ_f just after $t=0$. The smaller value of u than u_f in the time interval $t=0.15\sim$ 0.55 sec. corresponds to the fact that ψ_f again decreases to ψ_{ff} after arriving at the peak.

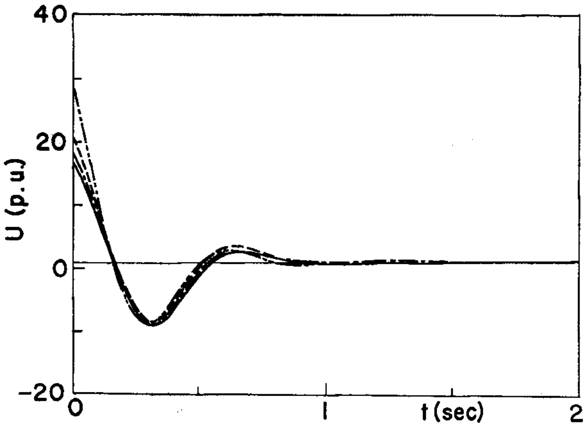
As considered above, the change of the rotor angle δ becomes suppressed, as the value of A_δ gets larger. It is concluded that the control u controls ψ_f which acts on the power output P_e of the synchronous machine so as to make the variation of δ and $\dot{\delta}$ optimal.

6-2 Case 2 $A=[A_\delta, 10^2, 0, 1]$

Here the optimal solutions are obtained with A_δ varied and $A_s=10^2$, $A_f=0$ and $A_u=1$. The results are shown in Fig. 2. Fig. 2 (a) shows the time variations of the rotor angle δ with A_δ as a parameter. A_s acts so as to make the change of s small, and therefore the oscillation of δ damps fast. In the cases of $A_\delta=1$, 10 and 10^2, however, δ does not approach δ_f in two seconds, i.e. the change of s is regarded as more important than that of δ. With a large value of A_δ, δ returns to δ_f in two seconds. Among these solutions, the case of $A_\delta=10^3$, $A_s=10^2$, $A_f=0$ and $A_u=1$ is noteworthy, i.e. δ increases monotonously from δ_f to 0.85 rad. in $t=0\sim 0.2$ sec., remains constant in $t=0.4\sim 0.6$ sec., decreases again monotonously to δ_f in $t=0.4\sim 0.6$ sec., and makes only a small oscillation there-



(a)



(b)

Fig. 2. Optimal responses for $A=[A_\delta, 10^2, 0, 1]$ with A_δ as a parameter.

after. It is thought that A_δ and A_s balance well to produce such a curve as this. Hence, it is concluded that it is possible to suppress the change of δ not only by making the value of A_δ large, but also by giving an appropriate value to A_s.

Fig. 2 (b) shows the time variations of the control variable u with A_δ as a parameter. The curves of this figure are considerably different from those of Fig. 1 (d). The values of u at $t=0$ are larger as a whole in this case. The values of u only at the earlier time get larger as the value of A_δ gets larger, and the change after $t=0.15$ sec. is almost invariable for any A_δ. In the case of $A_\delta=1$, $u(0)$ already equals 17 p.u. and A_δ changes only the values of u in $t=0\sim0.15$ sec. which determine the peak value of ψ_f.

6-3 Case 3 $A=[A_\delta, 0, 10^2, 1]$

In this case, the optimal solutions are investigated with A_δ varied and $A_s=0$, $A_f=10^2$ and $A_u=1$. The results are shown in Fig. 3. Fig. 3 (a) shows the time variations of the rotor angle δ. The changes of δ with A_δ are similar to those in Fig. 1 (a). Namely, δ has its peak at $t=0.35$ sec., and its peak value is a little larger than that of case 1, when the magnitude of A_δ is large. This is because ψ_f cannot change as freely as in case 1 due to the effect of A_f. When A_δ is small, the change of ψ_f is small, so the influence of A_f is also small.

Fig. 3 (b) shows the time variations of the control variable u. When A_δ is large, the value of u is smaller compared with case 1. Due to the effect of A_f, the change of ψ_f is suppressed as described above, and accordingly u is influenced in this manner.

From the above considerations we can conclude that the effect of A_f does

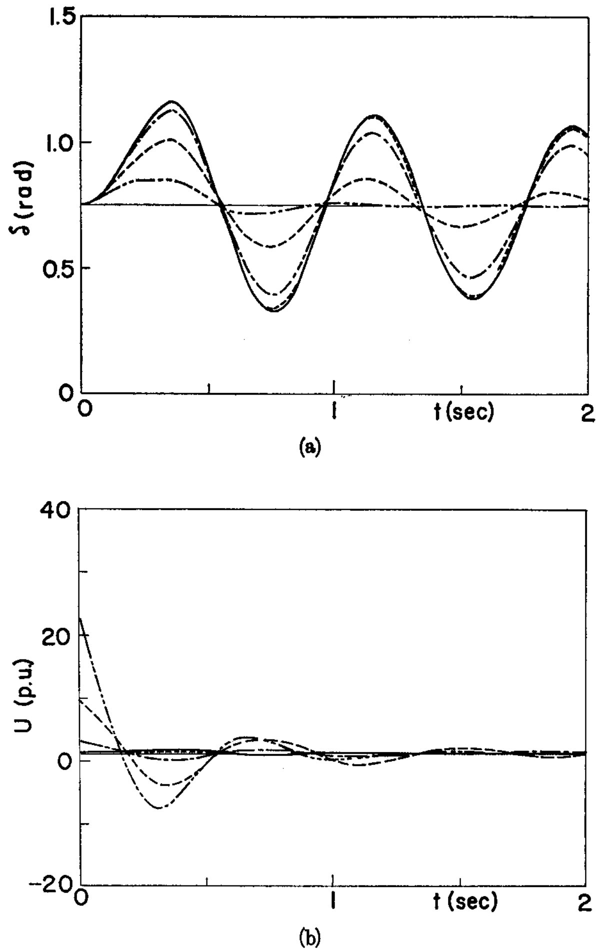


Fig. 3. Optimal responses for $A=[A_\delta, 0, 10^2, 1]$ with A_δ as a parameter.

not have its own particular feature, but suppresses the change of u as a result of suppressing the change of ψ_f, and increases the magnitude of A_u substantially.

7. Feedback Control

The optimal control u^* which is obtained in Section 3, and so calculated hitherto, is expressed as a function of time instead of as a function of the state variables. Hence, even if the state variables are measurable, it will be difficult to realize the optimal control. In this section, a method for approximately constituting the optimal control by the feedback from the state variables is described,[11] and the numerical results are given.

It is usually desirable to generate the control u by the linear combination of observable variables of the system, i.e.,

$$u-u_f = \sum_{i=1}^{l} k_i(x_i-x_{if})$$

where l is the number of feedback variables and x_{if} is the final steady state value of the variable. k_i, $i=1\sim l$ are determined, which minimize

$$J = \int_0^T [(u^*-u_f)-\sum_{i=1}^{l} k_i(x_i^*-x_{if})]^2 dt$$

in order to make u as similar to u^* as possible. Setting the partial derivatives $\partial J/\partial k_1$, $\partial J/\partial k_2$, $\cdots$, $\partial J/\partial k_l$ equal to zero, the following linear equation is obtained:

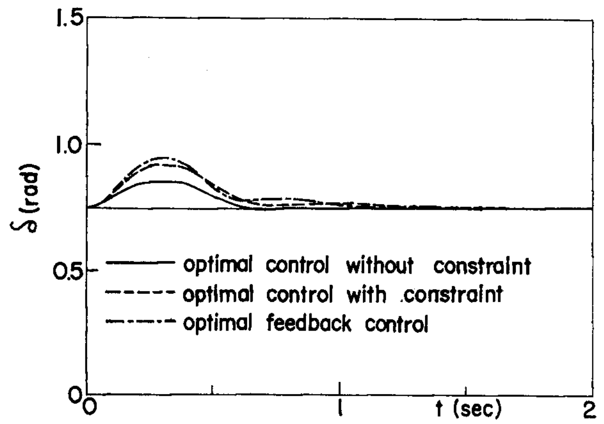
$$CK = D \tag{12}$$

where C, K and D are matrix or vectors as follows:

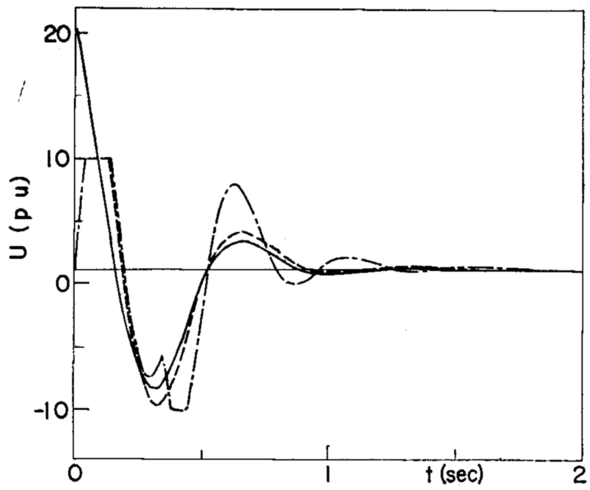
$$\begin{aligned} c_{ij} &= c_{ji} = \int_0^T (x_i^*-x_{if})(x_j^*-x_{jf})dt \quad i,j = 1, \cdots, l \\ K &= [k_1, k_2, \cdots, k_l]^t \\ D &= [d_1, d_2, \cdots, d_l]^t \\ d_i &= \int_0^T (u^*-u_f)(x_i^*-x_{if})dt \end{aligned}$$

The solution of eq. (12) yields the vector K. This method is simple and easy to apply to a practical case.

By the above described method, the optimal control for the case $A=[10^3, 10^2, 0,1]$ was approximated by the feedback control. The control u is here restricted between $u_{\min}=-10$ p.u. and $u_{\max}=10$ p.u., while it was not constrained in the last section. The results are shown in Fig. 4. Fig. 4 (a) shows the time variations of the rotor angle δ. In this figure, the solutions for the unlimited and the limited



(a)



(b)

Fig. 4. System responses for i) optimal control without constraint, ii) optimal control with constraint on control, and iii) optimal feedback control system.

optimal controls are also shown. Although the variation of δ in the limited case is larger than in the unlimited case, the aspect of change is similar. From eq. (12), the feedback coefficients of δ, s and ψ_f are determined to be $k_\delta=-8.488$, $k_s=20.363$ and $k_f=-11.622$, respectively. The change of δ by feedback control is considerably similar to that by the optimal control. Therefore, it is concluded that the feedback control from the state variables is satisfactory.

Fig. 4 (b) shows the time variations of the control u. When the control is not limited, it assumes a large value at the early time of control. When the control is restricted to 10 u.p., the change of the rotor angle in Fig. 4 (a) becomes larger. As the feedback control u is determined so as to approximate the optimal control with limits, their changes are similar to each other, especially for $t=0\sim0.3$ sec.

8. Fast Acting Excitation Control System

It was made clear in the preceding sections that the optimal control of the excitation of synchronous generator is very effective for the improvement of stability. However, some problems must be solved before the optimal control is applied to real power systems, for example, how to solve the optimal solution in the on-line mode and how to realize the obtained control. One of the power system stabilizing methods, which are realistic and economic under the present state of arts, is the application of a fast acting excitation system. In this section a comparison is made between the optimal excitation control and a fast acting excitation control.

8-1 Improvement of Transient Stability by Fast Acting Excitation System and Stabilizing Singal

When some fault occurs in an electric power system, the electrical output power of the generator usually decreases, while the mechanical input power to the generator hardly changes. Consequently, the difference between input and output power makes the rotor of the generator accelerated. At the same time the terminal voltage of the generator decreases. By increasing the excitation, the decrease of the terminal voltage can be made small, the electrical power output increases and the acceleration of the rotor can be suppressed. If we control the excitation voltage by a fast acting excitation system, we can thus improve the transient stability. The faster the response of the ecxitation system, the more suppressed is the first swing of the rotor. A very fast acting excitor, however, makes the damping of the following swings slow and, in the worst case, makes the generator go out of step. This is because the terminal voltage changes later than the excitation voltage due to the time constant of the field circuit, and the terminal voltage increases even after the peak of the first swing, when the rotor decelerates. The second and the following swings can damp rapidly by adding the supplementary signal (stabilizing signal) to the excitation system in order to compensate for the delay and strengthen the damping torque. The angular speed deviation of the rotor Δs, and the output power deviation of the generator ΔP can be used as the supplementary signal. It is ascertained that both of them are effective for suppressing the changes of the electric power and the rotor angle, provided that appropriate gain and phase compensations are used.[5),12)]

8-2 Example and Comparison with the Optimal Control

The excitation voltage of the previous model system is here controlled by the fast acting automatic voltage regulator (AVR), the block diagram of which is shown in Fig. 5. ΔP is used as the supplementary signal. The system equations can be obtained by adding the following equation of AVR to eq. (3),

$$\Delta\dot{u} = -\frac{\Delta u}{T_f} - \frac{K_f}{T_f}\Delta e_t - \frac{K_p K_f}{T_f}\Delta P$$

A three phase short circuit is assumed to occur at the generator bus, to continue for 0.25 sec. and then to be cleared. The excitation voltage u is constrained between 10 p.u. and -10 p.u. as in Section 7, and the system equations are solved under the same initial condition as Section 6. Fig. 6 shows the change of the rotor angle δ when the AVR does not have the stabilizing signal, i.e., $K_p=0$.

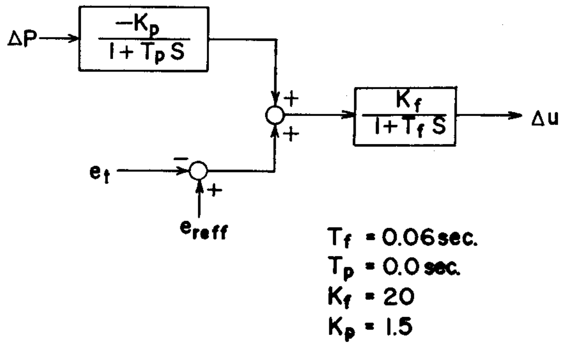


Fig. 5. Block diagram of fast acting excitation control system with stabilizer.

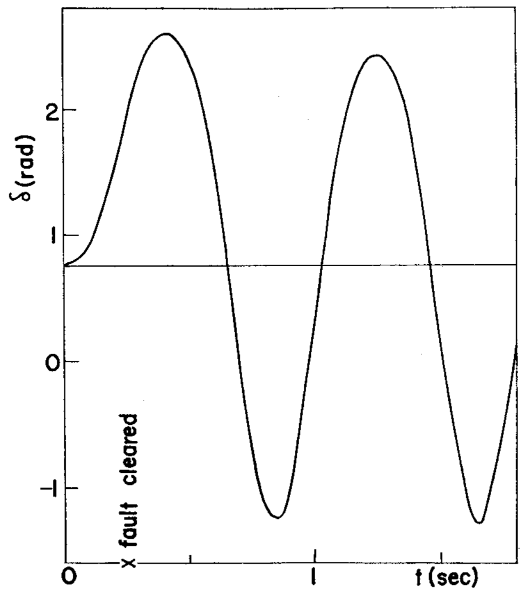
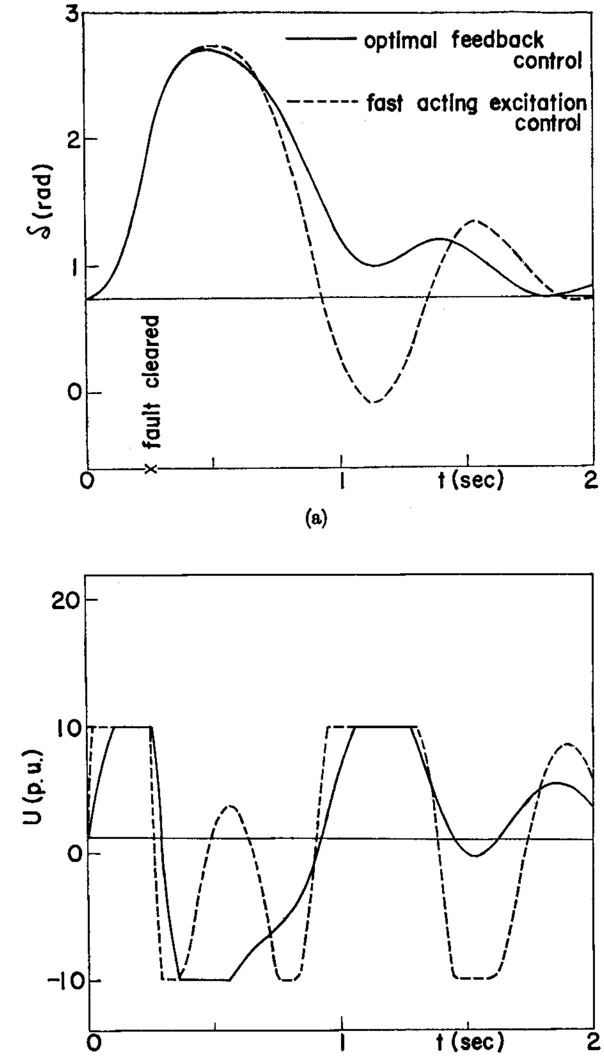


Fig. 6. System response for fast acting excitation control system without stabilizer.

The system, which is unstable with a constant excitation voltage, becomes stable by using a fast acting excitation control system. It is shown, however, that a large oscillation continues.

The changes of δ and control u in the case of applying supplementary signals are shown in Fig. 7 (a) and (b), respectively. The magnitude of the first swing is slightly greater than Fig. 6, but the second and the following swings damp quickly. The results of the feedback control obtained by approximating the optimal control as described in Section 7 are also shown in these figures. The weighting



(a)

(b)

Fig. 7. System responses for i) optimal feedback control system and ii) fast acting excitation control system with stabilizer.

coefficients were chosen to be $[10^3, 10^2, 0, 1]$ and the gains of the feedback control were $k_\delta = -8.9947$, $k_s = 2.9166$ and $k_f = -3.9996$. The damping of the rotor angle δ is of course a little more rapid in the case of the optimal feedback control. However, the difference is rather small. The changes of control u also show a similar tendency in the two cases. Consequently, it has become clear that a fast acting excitation control system is very effective for the stabilization of a power system.

9. Concluding Remarks

In this paper, we first investigated the relationship between the weighting coefficients of the performance index and the corresponding optimal solutions in the optimal control of the excitation of a one-machine power system in response to a step change of the input power into the generator. The dynamic characteristic of the generator was represented by a 3-rd order model, and so the performance index includes four weighting coefficients A_δ, A_s, A_f and A_u. From the calculations of the optimal control for various values of weighting coefficients, the following results were obtained:

1) When $A_s=A_f=0$, the deviation of the rotor angle from its steady-state value becomes small, and its damping becomes rapid with the increasing value of A_δ.

2) When $A_s=10^2$, the oscillation of δ damps quickly.

3) When $A_f=10^2$, the same effect was seen as the case when A_u is increased or equivalently when A_δ is decreased.

4) For this system and disturbance, the performance of the system seems to be best when $A=[10^3, 10^2, 0, 1]$.

When the control variable is constrained between the upper and the lower limits, the system performance becomes a little worse. However, the optimal solution has the same characteristics as the case without constraint. Moreover, we approximated the optimal control by feedback control, the results of which were quite satisfactory.

When the disturbance is assumed to be a three phase short circuit on the transmission line, the optimal solution for the weighting coefficients $[10^3, 10^2, 0, 1]$ also has the same feature as the case of input power disturbance where δ decreases to the steady-state value montonously. Consequently, it can be said that the weighting coefficients determine the optimal performance of the system to some extent regardless of the kind of the disturbance. It should be noted, however, that the gains of the feedback control depend on the disturbance even if the weighting coefficients are the same.

Last of all, we examined the effect of a modern fast acting excitation control system on the improvement of the dynamic performance of the power system in order to compare with the optimal control of the excitation. ΔP, the deviation of the generator output power from its steady-state value, was used as the supplementary signal to suppress the oscillation. It was made clear from the results of the calculations that the change of the rotor angle in the case of using a fast acting excitor is considerably similar to that obtained from optimal control. Therefore, the fast acting excitation control system is sufficiently effective for the stabilization of power systems.

Nomenclature

General

e_d, e_q	$=d$- and q-axis voltages
ψ_d, ψ_q	$=d$- and q-axis flux linkages
i_d, i_q	$=d$- and q-axis currents
e_t	=armature voltage
e_{fd}	=field voltage
i_{fd}	=field current
ψ_{fd}	=field flux linkage
ω_0	=synchronous speed
T_e	=energy-conversion torque
P_e	=power of energy conversion
P_i	=mechanical power input
δ	=rotor angle (radians)
s	=speed deviation, $p\delta$ (rad/sec)
p	$=d/dt$
ψ_f	=a field flux linkage (see Fig. N-1)

System parameters

r_{fd}	=field resistance
x_{md}, x_{mq}	$=d$- and q-axis magnetizing reactances
x_a	=armature-leakage reactance
x_f	=field-leakage reactance

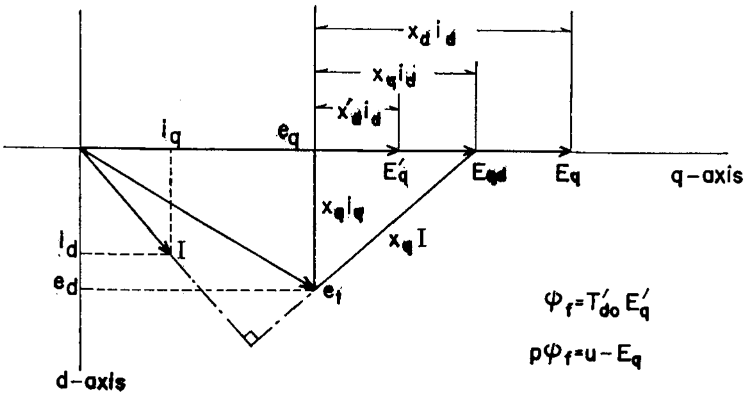


Fig. N-1. Vector diagram of synchronous generator.

x_d	$=d$-axis synchronous reactance
x_d'	$=d$-axis transient reactance
x_q	$=q$-axis synchronous reactance
M	=inertia constant
D	=damping coefficient
T_{d0}'	$=d$-axis transient open-circuit time constant
T_d'	$=d$-axis transient short-circuit time constant
E	=infinite-busbar voltage
x_e	=transmission-line reactance

Optimal control

I	=performance index
H	=Hamiltonian function
A_δ, A_s, A_f, A_u	=weighting coefficients in performance index
u	=control variable
p_1, p_2, p_3	=adjoint variables
T	=final time
$u_{\max}, u_{\min}$	=upper and lower limits of u
u^*	=optimal control
δ_i, s_i, ψ_{fi}	=initial steady-states of state variables
δ_f, s_f, ψ_{ff}	=final steady-states of state variables

Feedback control

x^*	=optimal response of feedback variable
k_δ, k_s, k_f	=feedback gains (see eqn. (11))
K_f, K_p, T_f, T_p	=gains and time constants of AVR
e_{ref}	=excitor reference voltage
$\Delta\delta$	$=\delta-\delta_f$
$\Delta\psi_f$	$=\psi_f-\psi_{ff}$
Δs	$=s-s_f$
Δu	$=u-u_f$
ΔP	$=P_e-P_i$
Δe_t	$=e_t-e_{\mathrm{ref}}$

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