Stability of D. C. Arc Welding System

By

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This paper deals with the stability of the D. C. arc welding system in which the electrode wire is fed with a constant speed. The system under consideration is described by nonlinear differential equations. The solutions of the equations are studied for various values of system parameters. The stability of solutions is investigated by considering the behavior of small variations from the steady solutions. From the analytical results, it may be inferred that the arc length is readily held almost constant by making use of a constant voltage power source, even if the feeding speed varies. It is useful, to some extent, to utilize the phase -plane analysis in investigating the transient states of arc. The methods of analysis presented in this paper may also be applicable to other welding systems which are described by differential equations of a like form.

1. Introduction

The problem of stability in the arc discharge is important in arc welding systems. It should first be noticed that the problems discussed in this paper are concerned with the welding arc in a constant feeding speed system. In arc welding, the melting speed of the electrode wire increases or decreases as the arc length decreases or increases. Thus, the welding arc has a self-regulating action by which the arc tends to keep its length constant in itself. Several papers have been published concerning experimental results in arc welding systems[1)~5)]. However, very few theoretical investigations have been reported on the self-regulating characteristic of the welding arc.

The circuit equations take the form of simultaneous nonlinear differential equations. We find the steady solutions of the equations and consider the stability of the equilibrium state of the system. It may be discussed by solving the variational equations which have small deviations from the equilibrium state.

The transient states of the welding arc are treated, and then the phase-plane analysis is utilized for this purpose. The integral curves of the autonomous equations are studied

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with the basic idea that singular points are correlated with steady states, and integral curves are correlated with transient states. Then, typical examples of a phase-plane diagram are illustrated and the illustrations provide a general view of arc discharges in both transient and steady state conditions. In this way, the relationship between initial conditions and resulting steady states is discussed.

2. Fundamental Equations

The fundamental connection diagram in Fig. 1 shows an arc welding system in which the electrode wire is fed with a constant speed v_0. In the figure, one of rollers is revolved by a motor, and another roller revolves freely. "L" is the internal inductance of the welder having the no-load voltage "E", and also "R" is a resistance inserted in series to the arc. The arc voltage being "V" in the case of the arc current "i", the following equation is established.

$$\frac{di}{dt}=\frac{1}{L}(E-Ri-V)\equiv X(l,\,i) \tag{1}$$

Since the melting speed of the electrode wire is proportional to the arc current when the arc current is not very large, we have the following equation with regard to the arc length:

$$\frac{dl}{dt}=\alpha i-v_0\equiv Y(l,\,i), \tag{2}$$

where α is a proportionality coefficient. It is noted that the effect of the welding speed is neglected in Eqs. (1) and (2).

The voltage-ampere characteristic of the welding arc may be approximated by the form;

$$V=al+b+(cl+d)i+(el+f)/i, \tag{3}$$

where $a, b, \ldots, f$ are constants* characterizing the welding arc. The characteristic of Eq.

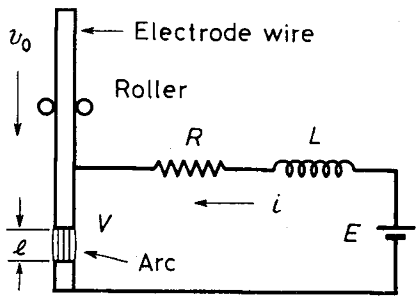

Fig. 1. Welding system in which the electrode wire is fed with constant speed.

(3) shows a fairly good approximation to the arc in steady states(6). As long as we deal with cases where the arc current and the arc length vary slowly, Eq. (3) may be considered to be legitimate5).

Equations (1) and (2) play a significant role in our investigation, since they serve as the fundamental equations in studying the transient state as well as the steady state of welding arc.

3. Steady Solutions and Their Stability

We consider the steady state in which the current $i(t)$ and the arc length $l(t)$ are constant, that is,

$$\frac{di}{dt}=0 \text{ and } \frac{dl}{dt}=0. \tag{4}$$

By substituting these conditions into Eqs. (1) and (2), the steady current i_0 and length l_0 of the arc are obtained.

$$\left.\begin{aligned} i_0&=\frac{v_0}{a} \\ \text{and} \qquad & \\ l_0&=\frac{E-b-(R+d)i_0-f/i_0}{a+ci_0+e/i_0}. \end{aligned}\right\} \tag{5}$$

The equilibrium states determined by Eqs. (5) are not always realized, but are actualy able to exist only so long as those are stable. We investigate the stability of the equilibrium states and find the solutions which are sustained in the stable state. For this end, we consider the small variations ξ and η respectively from the steady state values i_0 and l_0, and observe whether these variations approach zero or not, with an increase of the time t. From Eqs. (1), (2) and (3), we obtain

$$\frac{d\xi}{dt}=a_1\xi+a_2\eta, \qquad \frac{d\eta}{dt}=b_1\xi+b_2\eta \tag{6}$$

Coefficients a_1, etc. in Eqs. (6) are as follows:

$$\left.\begin{aligned} a_1&=\left[\frac{\partial X}{\partial i}\right]_0=\frac{1}{L}\{(el_0+f)/i_0^2-(cl_0+d-+R)\}, \\ a_2&=\left[\frac{\partial X}{\partial l}\right]_0=-\frac{1}{L}(a+ci_0+e/i_0), \\ b_1&=\left[\frac{\partial Y}{\partial i}\right]_0=a, \end{aligned}\right\} \tag{6a}$$

* These coefficients should be determined by experiments.

$$b_2=\left[\frac{\partial Y}{\partial l}\right]_0=0,$$

where $(\partial X/\partial i)_0$, ... and $(\partial Y/\partial l)_0$ denote the values of $\partial X/\partial i$, ... and $\partial Y/\partial l$ at $i=i_0$ and $l=l_0$ respectively. The characteristic equation of the system defined by Eqs. (1), (2) and (3) is given by

$$\begin{vmatrix} a_1-\lambda & a_2 \\ b_1 & b_2-\lambda \end{vmatrix}=0. \tag{7}$$

The variations ξ and η approach zero with the time t, provided that the real part of λ is negative. In this case, the corresponding steady solution is stable. The conditions of stability are given by the Routh-Hurwitz criterion, that is,

$$a_1b_2-a_2b_1>0 \text{ and } -a_1-b_2>0. \tag{8}$$

Substituting Eqs. (6a) into (8),

$$\left.\begin{aligned} &\alpha(a+ci_0+e/i_0)>0 \\ \text{and} \quad & \\ &cl_0+d-(el_0+f)/i_0^2>-R \end{aligned}\right\} \tag{9}$$

result, We shall consider what these conditions mean physically. From Eq. (3), we obtain

$$\left.\begin{aligned} &\left[\frac{\partial V}{\partial l}\right]_0=a+ci_0+e/i_0 \\ \text{and} \quad & \\ &\left[\frac{\partial V}{\partial i}\right]_0=cl_0+d-(el_0+f)/i_0^2. \end{aligned}\right\} \tag{10}$$

Therefore, the conditions of stability (9) may be written as

$$\left.\begin{aligned} &\alpha\left[\frac{\partial V}{\partial l}\right]_0>0 \\ \text{and} \quad & \\ &\left[\frac{\partial V}{\partial i}\right]_0>-R \end{aligned}\right\} \tag{11}$$

Evidently, the first condition is fulfilled from the fact that the arc voltage becomes higher as the arc length increases. Fig. 2 shows the characteristic curve of Eq. (3), i.e., the relationship between i and V in the case where the current i varies while the arc length l is held constant. Also, the dashed line in the figure shows the line obtained from

$$\frac{di}{dt}=\frac{1}{L}(E-Ri-V)=0. \tag{12}$$

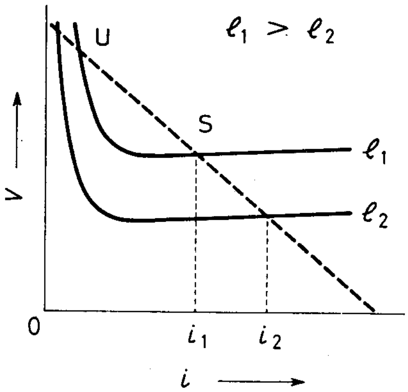

Fig. 2. Arc characteristic curves and a welder characteristic line.

The intersection S(i_1, V_1) of the line of Eq. (12) with the curve of Eq. (3) represents an equilibrium state, since the point S satisfies Eqs. (4). The condition of Eq. (11) shows that the equilibrium state is stable under such conditions that the slope of the arc characteristic curve of Eq. (3) is larger than that of the line of Eq. (12) at the intersection. We consider the stability of the equilibrium state in the intersection S. In this case, suppose that the arc length decreases from l_1 to l_2. Then, the arc length tends to increase spontaneously, since the melting speed of the electrode wire becomes larger as the arc current increases. Hence, the corresponding equilibrium state is stable. It is decided in the same manner as above that the equilibrium state represented by the intersection U is unstable.

In the next step, we carry out a numerical analysis of the system Eqs. (1) and (2) for the parameters* as given by the following table:

Table 1. System parameters in Eqs. (1), (2) and (3)

L	0.02 H	E	80.0 V	α	0.05 mm/A·sec
a	3.25 V/mm	b	17.0 V	c	0.40 V/m·A
d	0.0035 V/A	e	30.0 V·A/mm	f	10.0 V·A

The current and arc length are calculated from Eqs. (1), (2) and (3); and the iso-length curves are plotted on the v_0—R plane in Fig. 3. The stable region is hatched in Fig. 3. In the case where a value of R is small, the arc length is held almost constant even if the feeding speed v_0 changes.

* The characteristic coefficients of arc a, b,..., f are obtained by the experimental results in Ref. (5).

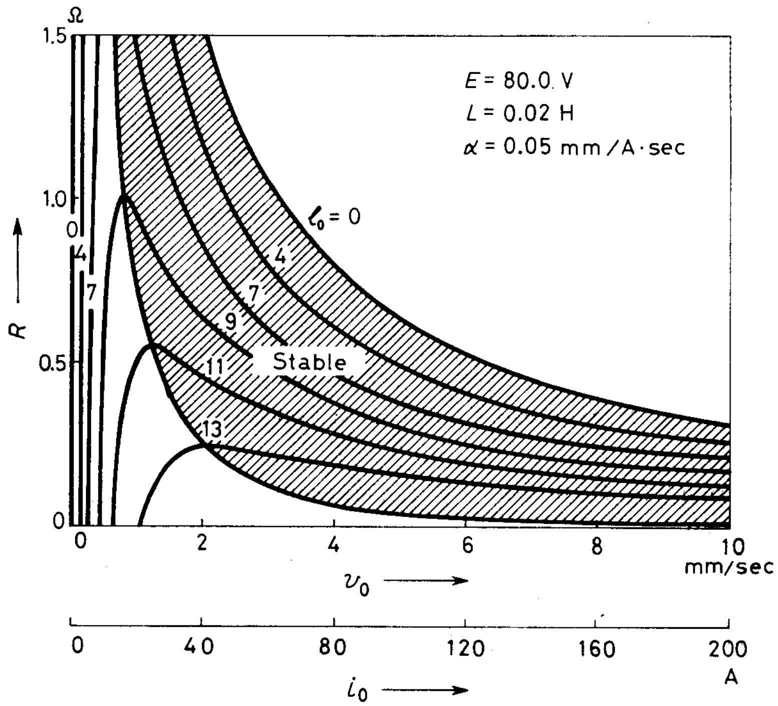

Fig. 3. Iso-length curves and region in which the welding arc is sustained in the stable state.

4. Analysis of The Welding Arc by Means of Integral Curves

In the preceding section, we have concentrated our attention on steady solutions and discussed their stability. We shall next investigate the transient state concerned with the arc until it gets to the steady state. Then the relationship between the initial conditions and the resulting steady states will become clear. To study the solution of Eqs. (1), (2) and (3), it is useful to investigate, with reference to the theories of Poincaré[7] and Hayashi[8], the integral curves of the following equation derived from Eqs. (1) and (2), that is,

$$\frac{di}{dl}=\frac{X(l, i)}{Y(l, i)}. \tag{13}$$

Since the time t does not appear explicitly in this equation, we can draw the integral curves on the l-i plane with the aid of isocline. A digital computer may be used for numerical integration. Once the integral curves of Eq. (13) are obtained on the phase-plane, it is not difficult to find the solutions $l(t)$ and $i(t)$ of Eqs. (1) and (2). Thus, the behavior of the system may be described by the movement of the representative point

$(l(t),\ i(t))$ along the integral curves of Eq. (13). The point $(l_0,\ i_0)$ at which $X(l_0,\ i_0)=Y(l_0,\ i_0)=0$ is called a singular point. Physically, a singular point represents an equilibrium point, since both $l(t)$ and $i(t)$ are constant under such a condition. We discuss the types of singular points of Eq. (13) which are correlated with the steady solutions of Eqs. (1) and (2). These singularities are classified according to the natures of the roots of the characteristic equation (7).

We infer a singularity to be stable or unstable according as a representative point on any integral curve moves toward the singular point or not, with increasing t, that is, according as the real part of λ is negative or positive.

The numerical analysis used for the above is as follows. Let us consider an example in which the system parameters are given by Table 1. The current i_0 and arc length l_0 are first calculated from Eqs. (5) for the various values of v_0 and R. We now distinguish these equilibrium states according to the types of singularity. By varying the values of v_0 and R, we can determine the regions for the different types of singularity. The analytical results are shown in Fig. 4.

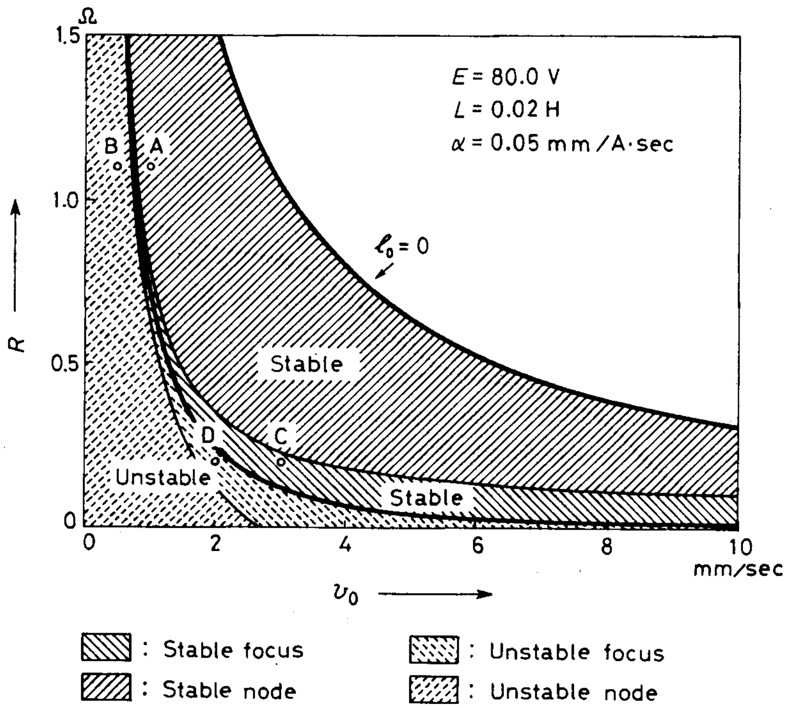

Fig. 4. Singularities which correlate with the steady solutions.

5. Phase-plane Analysis of The Welding Arc

In the preceding section, we have briefly referred to the transient solutions which are correlated with the integral curves of Eq. (13). It is useful to consider the integral curves for certain typical cases. The special cases considered are those stemming from use of the following values of v_0 and R in Eqs. (1) and (2), namely,

Case 1: $v_0=1.0$ mm/sec, $R=1.1\,\Omega$
Case 2: $v_0=0.5$ mm/sec, $R=1.1\,\Omega$
Case 3: $v_0=3.0$ mm/sec, $R=0.2\,\Omega$

and

Case 4: $v_0=2.0$ mm/sec, $R=0\ 2\,\Omega$.

These parameters are located at the points A, B, C and D in Fig. 4, respectively. The integral curves for these cases are, respectively, shown in Figs. 5a~5d. The singularities are determined by Eqs. (5); the corresponding details are listed in Table 2.

Table 2. Singular points in Fig. 5.

Singular point	l_0 [mm]	i_0 [A]	$\lambda_1,\ \lambda_2$	Classification
a	8.50	20	$-0.548,\ -21.68$	Stable node
b	8.15	10	$0.218,\ 71.68$	Unstable node
c	13.4	60	$-2.357\pm 1.969\,i$	Stable focus
d	13.6	40	$1.307\pm 2.887\,i$	Unstable focus

The integral curves in Fig. 5 are drawn with aid of the isoclines represented by dotted lines, the numbers on which indicate the values of di/dl for the respective isoclines. As seen from Eqs. (1) and (2), a representative point $(l(t),\ i(t))$ moves, with the increase of time t, to the direction of the arrows along the integral curve. The singularities in Figs. 5a and 5c are stable, and the corresponding equilibrium states are realized. From the integral curves of Figs. 5a and 5c, the relationship between the initial conditions and the resulting phenomena is apparent: an arc discharge started with any initial conditions in the shaded region tends to the singularity, resulting in the stable welding arc, whereas an arc discharge started from the unshaded region tends to the axis of abscissa on which the current becomes zero, resulting in no arc discharge.

The current and the length of an arc discharge in the neighborhood of the stable state approach, with the increase of time, the final state of damped sinusoids when the corresponding singularity is a stable focus, whereas they approach the final state with damped exponentials when the singularity is a stable node.

In Figs. 5b and 5d, we have an unstable node and an unstable focus, respectively. The corresponding equilibrium state cannot be sustained, since a representative point

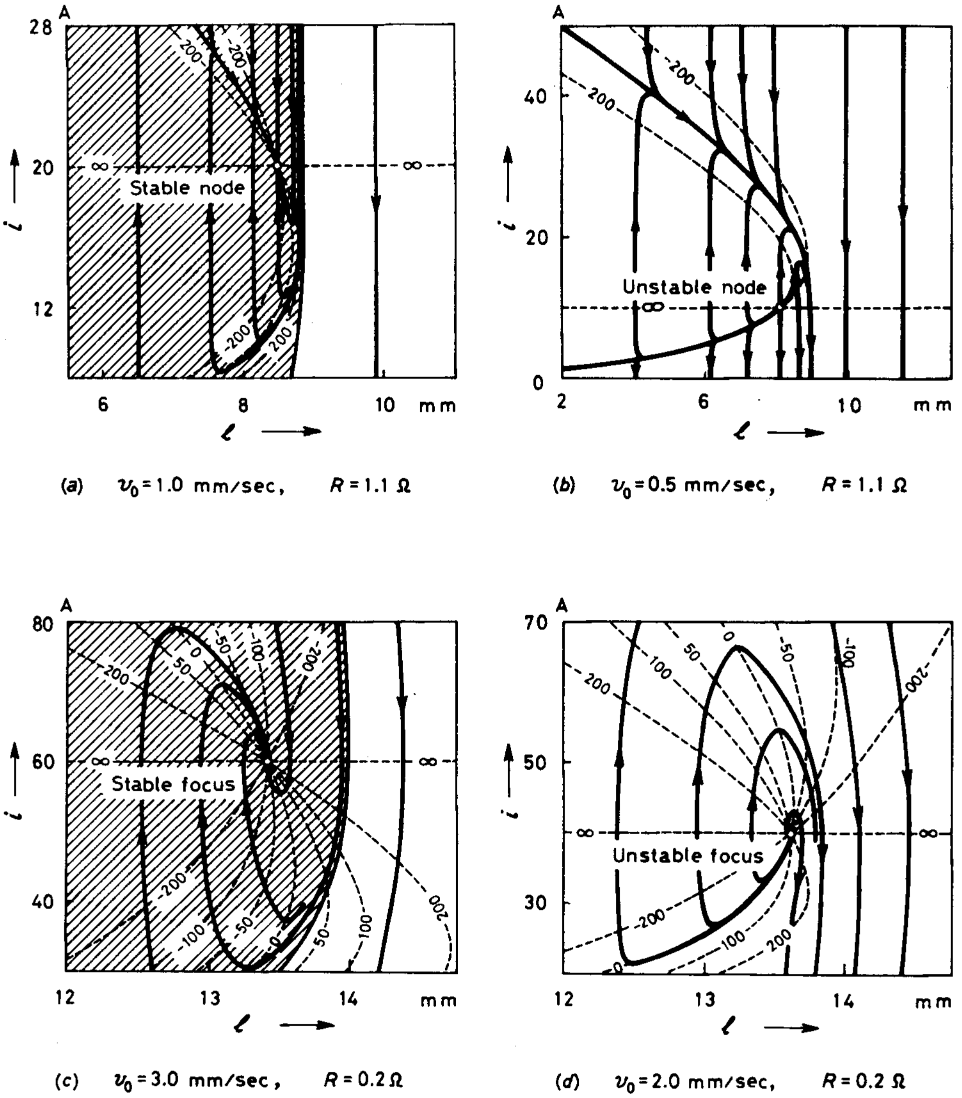

Fig. 5. Integral curves of Eqs. (1) and (2) in the l–i plane.

moves away from the singularity with the increase of time. Therefore, we have no welding arc for any of the initial conditions prescribed.

6. Concluding Remarks

The essential aspect of the welding system has been described in this paper. The subject of investigation is limited to the field of a constant feeding speed system. The differential equations which govern the system take the form of simultaneous nonlinear differential equations. The phase-plane analysis has been used for the investigation on the stability of the welding arc. The solutions in the steady state, which are correlated with singular points on the phase-plane, have been first sought for the various combinations of system parameters. The stability of solutions has been investigated according to Routh-Hurwitz criterion. It is preferable to use the power supply of constant voltage rather than that of drooping characteristic in order to keep the arc length constant.

Particular attention has been directed to the relationship between the initial conditions and the resulting arc responses, and examples illustrating this relationship have been given. The welding arc is sustained only when the initial condition is properly chosen.

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