Modeling Multi-Objective, Interbasin, Surface Water Development System with Supplementary Use of Reclamation System

By

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This paper centers on the multi-objective, interbasin, surface water development system, whereby, if necessary, the conjunctive use of reclamation system is assumed to be available. The major objectives of the paper include (i) analysis of the quality of industrial waters blended with renovated waters; (ii) identification of the relation between the blended water demand and its blending ratio and (iii) modeling and analysis of the coordinated attainments of multiple goals involved in the stated system.

A case study is conducted at the interbasin system of the southern part of Hyogo Prefecture and some informative results obtained point to the need of this kind of alternative water utilization systems.

The major findings of the study include:

(1) The reclamation system is required to be implemented on a certain scale to supplement the industrial water supply.

(2) In cases where the permitted-levels of the collection goals related to each river basin are set to be relatively higher, the total amounts of water supply that must be covered by the reclamation systems should be allocated to the system of each basin roughly in proportion to the amounts of its industrial water demand.

(3) The increased attainments of the amounts of collection, and in consequence, the increased attainments of the improvement in the blended water quality are achievable only at the sacrifice of economic efficiency.

(4) The attainments of the goals are seen to be well-balanced and not biased to any of them, chiefly owing to the L-type utility function underlying the model.

1. Introduction

In this paper we shall confine ourselves to the problem where it is assumed that surface waters (fresh waters) tapped by dams are used as a main source. There is also an implicit assumption that the reclamation system is employed as a complementary means

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of supply in case the deviation arises between the attainable supply of fresh water and the needed water. The term "implicit" is used to mean that the costs associated with the complementary supply works are not included in the total construction costs. This kind of assumption seems to apply to cases where a water management agency on a national or regional level is planning to develop water by constructing dams and channels on an inter-basin-basis, and its main concern is to secure as much water as needed in the most economical manner. At the same time, its concern is also to meet the water demand of each locality as much as possible with fresh water, because excessive implementation of the reclamation system to be intensively located on a certain locality will extremely inflict disadvantages on it, such as a deteriorated quality of water supply.

In light of these considerations, one of the primary objectives of this study is to analyze the deterioration of water supply quality in cases where renovated waters are blended with industrial waters, and blended waters thus obtained are provided for industrial uses. Accordingly, we need to identify the amounts of those water demands for particular industrial uses which require a higher level of quality than that of the blended water. In this view, a detailed analysis of this kind of problem will also be given in this paper.

Another spectrum of the problem to which this study is addressed is the importance of coordinated attainments of multiple goals involved in the concerned system, which may roughly be identifed as follows:

(i) to secure a sufficiency of fresh water provisions for the water users on each river basin by means of developing dams on a cross-basin basis, and by diverting river flow from one stream to another,

(ii) to attain an efficiency of economy on a cross-basin, area-wide basis.

Therefore, in order to establish the inter-basin water resources development system, these two goals need to be attained. But more often than not, this involves conflicts of interests——a matter which complicates the choice of a desirable alternative system. That is to say, that the more an efficient economy is aimed for, the less amount of fresh water should be tapped. This is especially the case when some portion of fresh water provisions can be supplemented by reclamation systems.[1~4] Since both of the two goals are conflicting each other, and the attainments of both goals are pursued in practice, we need to find a properly-coorcinated alternative by some means. Therefore, we concern ourselves with this kind of coordination problem in this paper.

2. Goal Programming

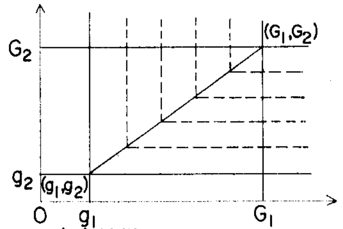
2-1 Preliminary Discussion

Goal programming is a special extension of linear programming. In the conventional linear programming method, the objective function is one-dimensional——either to maximize profits (effectiveness) or to minimize costs (sacrifice). The GP model handles multiple goals in multiple dimensions. Therefore, there is no dimensional limitation of the objective function. Though the interest for it is rather recent (It has its origin in the work by A. Charnes and W. W. Cooper.[5]), literature on the subject is already copious.

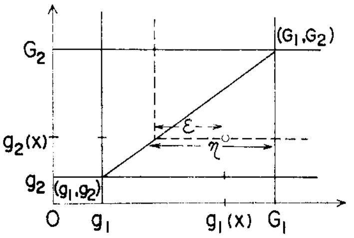
Generally speaking, conventional goal programming can be classified into two classes,[6] that is, (i) ordinary weights determined on the basis of the value judgment by the planner or manager as to the relative importance of each goal, (ii) preemptive weights determined on the basis of the value judgment by the planner or manager as to the priority of the desired goal.

The difference between the two classes lies, in short, in whether the planner or manager rank the goals in ordinal sequence, or not. One decided drawback is the explicit subjectivity involved in determining the weights.

Some elaborations of this point were attempted by G. Debreu,[7] H. Raiffa,[8] Luce[9], R. J. Aumann,[10] B. Roy[11] and others [12],[13]who introduced the notion of utility function (or indifference curves) in order to automatically weigh goals. This kind of approach seemed to have alleviated the difficulty, but there occurred another problem as to how to adequately obtain the utility function. In light of this point, one effective approach was explored by T. Fushimi and T. Yamaguchi[14] who instead of introducing a complex or sophisticated utility function, attempted to replace the unknown trade-offs by the L-type utility function (See Figure 1). This utility function can be conceived as a



(a) Utility Contours



(b) Deviational Variables

Fig. 1. L-type Utility Function.

simplified approximation to the unspecified utliity function and can be expected to reflect the qualitative characteristics of the utility function. The raison d'être behind the choice of the L-type utility function was developed in the light of the inability of obtaining the quantitative form of the unknown function.

They also demonstrated that the introduction of the L-type utility function in the formulation of the model leads to the problem of finding such a solution which can be thought of as well-balanced attainments of the desired goals. We might as well add one more point to the advantages of this approach. As will be seen later, this approach seems to be effective in cases where the set goals are mutually non-commeasurable, as is often the case with planning and management.

2-2 Permitted-level and Satisfied-level of Goal

The term "the permitted-level" is defined as such a level that the planner is determined to accept any alternative that assures the attainments of the concerned goals to the extent equal to, or larger than it, but otherwise he would never accept that alternative. The term "the satisfied-level" is used to mean that after taking account of various con-

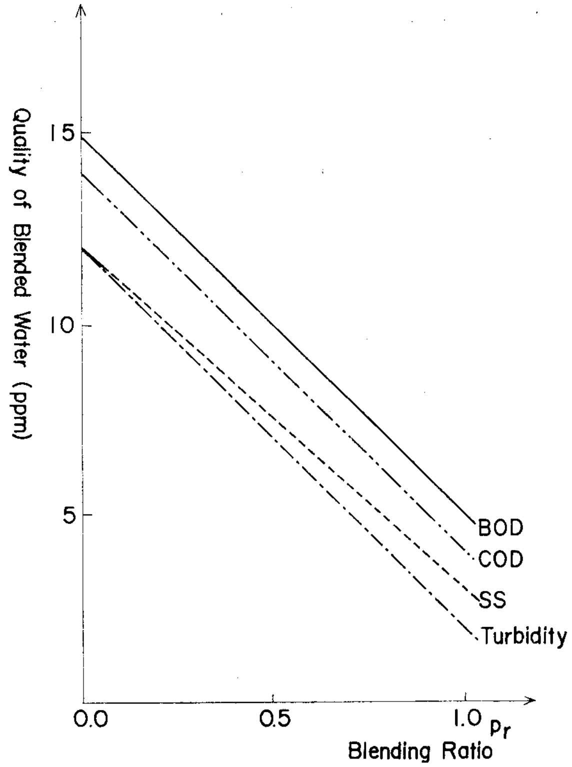


Fig. 2. Relation between Quality of Blended Water and Blending Ratio.

ditions, such as the case where one specified goal would be attempted to reach its full attainment by confining the remaining goals to be achieved at their permitted-levels, the planner becomes willing to accept that level of alternative as a satisfactory one, if not optimal.

2-3 L-type Utility Function

As described, we shall take the position that the utility function to be set takes the form of the L-type. It should be noted here that a given indifference curve of this function has its point of reflection on the goal vector as is defined by the line emanating from the point (g_i, g_j) to (G_i, G_j) (See Figure 2). This implies that any incremental movement parallel with the abscissa or the ordinate from a point (g_i, g_j) on the goal vector produces no increase in the utliity level, whereas such a move aslant and upward from the point leads to the increased attainments of both goals. It follows from this that behind the setting of the L-type utility function to formulate the problem lies our position that a set of goals should be attained at a well-balanced level.

2-4 Mathematical Formulation

For brevity of explanation, this will be deferred to the next section where the formulation of the model will be presented.

3. Modeling Multiple-goal, Inter-basin Surface Water Development System and Analysis of Supplementary Utilization of Reclamation System

3-1 Identification of the Problem

(i) Area-wide multiple river basins are treated where midstream and downstream areas in each basin are assumed to be grouped into one conceptual region as will be called hereafter "the demand region" of a given basin, and dams are assumed to be constructed at a number of sites in the upperstream valleys on each river.

(ii) The facilities to be explicitly considered are a set of dams to be constructed in each basin and an interbasin channel for streamflow diversions to be built between two adjacent basins.

(iii) Two kinds of goals are set, one being the minimization of the total construction costs for the above-identified facilities (cost-goal), the other being the maximization of the amount of water to be collected from each river for use in its own demand region (collection-goal).

(iv) The explicitly identified sources of water are those waters developed by dams to be constructed. (This type of water is referred to as "fresh water".)

(v) It is implicitly assumed that water usages in each demand region are classified into two types, i.e., domestic water and industrial water usages, both collecting water from the same soucre through the same intake. The point of collection in each basin is assumedly located downwards from the point of diversion.

(vi) The satisfied-levels of the collection-goals are taken to be identical to the full attainment of water supplies to meet the total demands of a given demand region.

(vii) Their permitted-levels are taken to be identical to a partial attainment of water supplies. This stands on the premise that the deviations between the satisfied- and permitted levels of a collection-goal, that is, the amount of water supply shortages in concerned-region, should be compensated by the supplementary provision of renovated waters. This is meant to imply that behind the explicitly established water resources system, i.e., the inter-basin system, there also exists such an implicitly established system of water reclamation. It is postulated that this system should be used exclusively to provide the industries with renovated waters by commonly utilizing the existing industrial water distribution systems where renovated waters are transported, being blended with industrial waters.

(viii) The satisfied or permitted-level of the cost-goal is set to be equal to that level which minimizes the total construction costs by confining each of the collection-goals to be equal to its permitted-level or satisfied-level, respectively.

Minor assumptions to be set will be referred to later when specifications become necessary.

3-2 Model Formulation

1) Notaion

(a) Variables

u_{rl}: amount of water developed by dam l to be constructed in the upperstream valley of river r.

y_{rs}, y_{sr}: amount of water (streamflow) to be diverted from river r (s) to adjacent river s (r).

S_r^T: total amounts of water to be collected from river r for use in its own demand region.

s_r^I: amount of industrial water supply in the demand region in river r.

s_r^D: domestic water supply.

s_r^N: renovated water supply.

d_r^I: industrial water demands in the water demand region on river r.

d_r^{DD}: domestic water demands for domestic use and those for industrial use.

d_r^N: renovated water demands.

E_c, η_c, E_r, η_r: deviational variables.

(b) Constants

C_{rl}: upper bound on u_{rl}

D_r^T: total water demands of the demand region on river r

D_r^I: water demands for industrial use

D_r^D: domestic use

α_{rl}: unit cost for construction of dam l on river r

$\beta_{rs}(\beta_{sr})$: diversion channel (rs) (or (sr))

G_c, g_c: satisfied- and permitted-levels of cost goal

G_r, g_r: collection-goal related to river r

T_r: set of those rivers adjacent to river r

Between D_r^T, D_r^I, D_r^D, it holds

$$D_r^T = D_r^I + D_r^D. \tag{3–1}$$

2) Technical and Physical Constraints

In view of the limited capacity of each dam, it follows

$$u_{rl} \leqq c_{rl} \quad (r=1, \cdots, v;\ l=1, \cdots, L). \tag{3–2}$$
$$(u_{rl} \geqq 0)$$

For each stream, water-quantity balance holds as

$$\sum_l u_{rl} + \sum_{s \in \Gamma_r} (y_{sr} - y_{rs}) - S_r^T = 0 \quad (r=1, \cdots, v). \tag{3–3}$$

Between the amount of collection from each river and the amounts of water supplies, it holds

$$S_r^T = s_r^I + s_r^D + s_r^N \quad (r=1, \cdots, v). \tag{3–4}$$
$$(S_r^T,\ s_r^I,\ s_r^D,\ s_r^N \geqq 0)$$

The amounts of water supplies and water demands of different kinds are required to satisfy the relations as

$$D_r^I = d_r^{DI} + d_r^N + d_r^I = d_r^{DI} + d_r^B \quad (r=1, \cdots, v), \tag{3–5}$$

because

$$d_r^B = d_r^N + d_r^I \quad (r=1, \cdots, v), \tag{3–6}$$
$$s_r^I = d_r^I \quad (r=1, \cdots, v), \tag{3–7}$$
$$s_r^N = d_r^N \quad (r=1, \cdots, v), \tag{3–8}$$
$$s_r^D = d_r^{DD} + d_r^{DI} = d_r^{DD} + D_r^D \quad (r=1, \cdots, v), \tag{3–9}$$
$$d_r^{DD} = D_r^D \quad (r=1, \cdots, v), \tag{3–10}$$
$$(d_r^{DD},\ d_r^N,\ d_r^I,\ d_r^{DI},\ d_r^B,\ s_r^I,\ s_r^N,\ s_r^D \geqq 0).$$

As will be seen, another relation holds between d_r^N and d_r^I.

3) Goal Constraints

The cost-goal is formulated as

$$\sum_r \sum_l a_{rl} u_{rl} + \sum_r \sum_{s \in \Gamma_r} \beta_{rs} y_{rs} + \sum_r \sum_{s \in \Gamma_r} \beta_{sr} y_{sr} - \varepsilon_c + \eta_c = G_c, \tag{3-11}$$

$$\sum_r \sum_l a_{rl} u_{rl} + \sum_r \sum_{s \in \Gamma_r} \beta_{rs} y_{rs} + \sum_r \sum_{s \in \Gamma_r} \beta_{sr} y_{sr} \leqq g_c. \tag{3-12}$$

For the collection-goal with respect to river r, it follows

$$S_r^T + \varepsilon_r - \eta_r = G_r \quad (r=1, \cdots, v), \tag{3-13}$$

$$(\varepsilon_r, \eta_r \geqq 0)$$

$$S_r^T \geqq g_r \quad (r=1, \cdots, v), \tag{3-14}$$

$$\frac{\varepsilon_c}{\lambda_c} = \frac{\varepsilon_r}{\lambda_r} \quad (r=1, \cdots, v), \tag{3-15}$$

where

$$\lambda_c = g_c - G_c, \tag{3-16}$$

$$\lambda_r = G_r - g_r. \tag{3-17}$$

4) Objective Function

On selecting any one, ε_c, say, from the set of $(\varepsilon_c, \varepsilon_1, \cdots, \varepsilon_v)$, we can formulate the objective function as

$$\text{Minimize } Z = \varepsilon_c. \tag{3-18}$$

3-3 Blended Water Demands As a Function of Blending Ratio

Let $\omega_{k,r}^I$ and $\omega_{k,r}^R$ denote the measurement values of industrial water and renovated water, respectively, for the demand region on river r, where $k=1, 2, 3, 4$ represent BOD ppm, COD ppm, SS ppm and turbidity ppm, respectively. We shall assume here that the blending of two distinct qualities of water causes no chemical and logical reactions, as is roughly true with the selected quality parameters. And let the blending ratio p_r be defined as

$$p_r = \frac{s_r^N}{s_r^I + s_r^N}. \tag{3-19}$$

Then, the estimated values for the selected quality parameters of the blended water, $\omega_{k,r}^B$ are given as follows:

$$\omega_{k,r}^B = \frac{\omega_{k,r}^I s^I + \omega_{k,r}^D s^D}{s_r^I + s_r^N} = \nu_{k,r} p_r + \omega_{k,r}^I, \tag{3-20}$$

or in terms of vectorial representation

$$\boldsymbol{\omega}_r^B = p_r \boldsymbol{\nu}_r + \boldsymbol{\omega}_r^I, \tag{3-21}$$

where

$$\nu_{k,r}=\omega_{k,r}{}^{I}-\omega_{k,r}{}^{N}, \tag{3-22}$$

$$\boldsymbol{\omega}_r{}^{B}=(\omega_{1,r}{}^{B},\ \omega_{2,r}{}^{B},\ \omega_{3,r}{}^{B},\ \omega_{4,r}{}^{B}), \tag{3-23}$$

$$\boldsymbol{\omega}_r{}^{I}=(\omega_{1,r}{}^{I},\ \omega_{2,r}{}^{I},\ \omega_{3,r}{}^{I},\ \omega_{4,r}{}^{I}), \tag{3-24}$$

$$\boldsymbol{\nu}_r=(\nu_{1,r},\ \nu_{2,r},\ \nu_{3,r},\ \nu_{4,r}). \tag{3-25}$$

The above relation, as is graphically shown in Figure 3, implies that vector $\omega_r{}^B$ representing a set of the estimated values for the water quality parameters of the blended water, is uniquely determined by a given blending ratio p_r. Hence, we shall employ hereafter this blending ratio as the comprehensive quality indicator, and these two terms are taken to be identical to each other.

It must be noted here that the blending of industrial waters with renovated waters results in a degraded quality of water which cannot be used for a certain type of industrial processes. Accordingly, we need to identify the amounts of those demands for particular industrial uses which require a higher level of quality than that of the blended water. We shall refer to this level of water quality as the "limiting quality level" or "limiting blending ratio", and "the amount of those demands which require a lower level of quality" as "the blended water demands with respect to its limiting quality level (or limiting blending ratio).

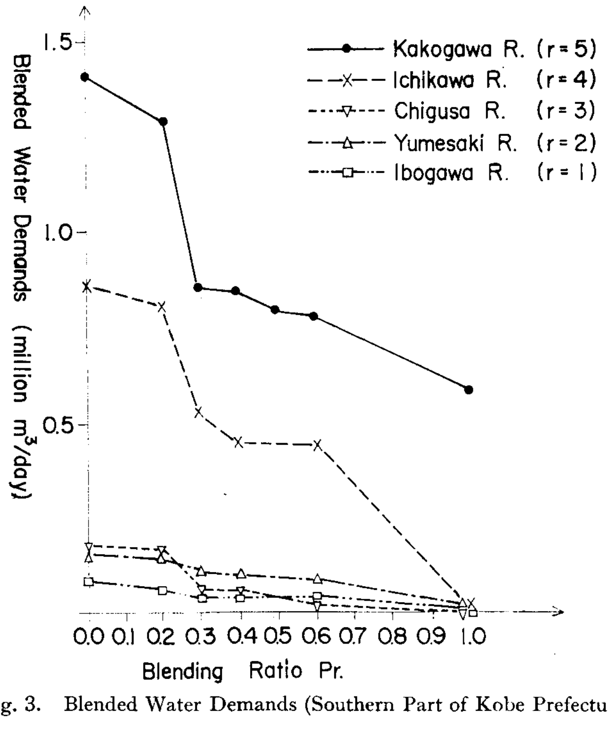


Fig. 3. Blended Water Demands (Southern Part of Kobe Prefecture).

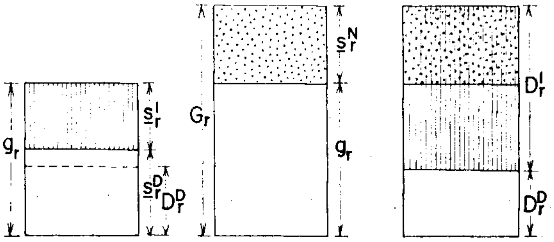


Fig. 4. Illustrated Relation between G_r, g_r, D_r^I, D_r^D, etc.

On the basis of the standards for different industrial processes which were developed by the Japan Committee for Water Quality Standards, the blended water demands, d_r^B were plotted against different limiting blending ratios, as shown in Figure 3. By use of this graph we can estimate those demands for particular industrial uses which require a higher level of quality than that of the blended water. Let us recall the assumption that those water demands are covered by the existing domestic-water distribution system. Accordingly, the following relation holds as was shown before.

$$d_r^{DI}+d_r^N+d_r^I=D_r^I \quad (r=1, \cdots, v), \tag{3-5}$$

$$d_r^B=d_r^N+d_r^I \quad (r=1, \cdots, v), \tag{3-6}$$

$$s_r^I=d_r^I \quad (r=1, \cdots, v), \tag{3-7}$$

$$s_r^N=d_r^N \quad (r=1, \cdots, v), \tag{3-8}$$

$$s_r^D=d_r^{DD}+d_r^{DI}=d_r^{DD}+D_r^D \quad (r=1, \cdots, v), \tag{3-9}$$

$$d_r^{DI}=D_r^D \quad (r=1, \cdots, v). \tag{3-10}$$

We shall proceed to find the functional relation between d_r^I, d_r^N and d_r^{DI} by utilizing the above-obtained graph (termed "blended-water demand curve" with respect to its blending ratio).

It is obvious from this curve that the amount of the blended water demands, d_r^B for a given blending ratio, p_r can be approximated by the following recurrence linear equations for a given interval (say t) p_r lies,

$$d_r^B=a_r p_r+b_r \quad (r=1, \cdots, v), \tag{3-26}$$

where

$$p_r=\frac{s_r^N}{s_r^I+s_r^N}, \tag{3-9}$$

and s_r and b_r are constant parameters for the above equation which are set as

$$d_r^B=d_{r,t}^B(>0) \text{ for } \overline{p_{r,t-1}} \leqq \overline{p_r} \leqq \overline{p_{r,t}} \quad (t=1, \cdots, \tau_0),$$

$$a_r=a_{r,t}(<0) \text{ for } \overline{p_{r,t-1}} \leqq \overline{p_r} \leqq \overline{p_{r,t}} \quad (t=1, \cdots, \tau_0),$$

$$b_r=b_{r,t}(>0) \text{ for } \overline{p_{r,t-1}} \leqq \overline{p_r} \leqq \overline{p_{r,t}} \quad (t=1, \cdots, \tau_0),$$

$$\overline{p_{r,0}}=0,\ \overline{p_{r,\tau_0}}=1,\ d_r{}^B=D_r{}^I \text{ for } p_r=\overline{p_{r,0}}=0.0\ (t=1, \cdots, \tau_0),$$
$$\overline{d_{r,t}{}^B}=a_{r,t}\overline{p_{r,t}}+b_{r,t}=a_{r,t+1}\overline{p_{r,t}}+b_{r,t+1}. \tag{3-27}$$

Substitution of Equation (3–19) into (3–26) gives

$$d_r{}^B=a_r\frac{s_r{}^N}{s_r{}^I+s_r{}^N}+b_r. \tag{3-28}$$

From Equations (3–6), (3–7) and (3–8) it holds that

$$d_r{}^B=s_r{}^I+s_r{}^N. \tag{3-29}$$

Accordingly, from Equations (3–28) and (3–29) we obtain the following quadratic equation with respect to $d_r{}^B$, that is,

$$(d_r{}^B)^2-b_rd_r{}^B-a_rs_r{}^N=0. \tag{3-30}$$

Given the value of $s_r{}^N$, the solution to this equation is easily found to be

$$d_r{}^B=\frac{b_r-\sqrt{b_r{}^2+4a_rs_r{}^N}}{2}. \tag{3-31}$$

Since Equations (3–26), (3–19) and (3–29) hold, $s_r{}^N$ is expressed in terms of p_r as

$$s_r{}^N=p_r(a_rp_r+b_r)=p_rd_r{}^B. \tag{3-32}$$

Since $a_r>0$ and if p_r increases, then $d_r{}^B$ decreases. Hence, when p_r lies in the interval, $\overline{p_{r,t-1}}\leqq p_r\leqq\overline{p_{r,t}}$ and moves from its lower bound, $\overline{p_{r,t-1}}$ toward its upper bound $\overline{p_{r,t}}$ and if $|a_r|$ is properly small, it can be expected that $s_r{}^N$ would increase. On the contrary, if $|a_r|$ is properly large, $s_r{}^N$ would decrease.

Then if p_r shifts from the interval, $\overline{p_{r,t-1}}\leqq p_r\leqq\overline{p_{r,t}}$ to the upward interval, $\overline{p_{r,t}}\leqq p_r\leqq\overline{p_{r,t+1}}$, will $s_r{}^N$ increase or decrease? Let us consider this question by comparing the values of $s_r{}^N$s ($s_{r,t}{}^N$ and $s_{r,t+1}{}^N$) for both intervals as follows.

$$\frac{s_{r,t+1}{}^N}{s_{r,t}{}^N}=\left(\frac{p_{r,t+1}}{p_{r,t}}\right)\left(\frac{d_{r,t+1}{}^B}{d_{r,t}{}^B}\right). \tag{3-33}$$

Since $p_{r,t+1}/p_{r,t}>1$ and $d_{r,t+1}{}^B/d_{r,t}{}^B<1$ (See Figure 4), we don't know whether $s_{r,t+1}{}^N/s_{r,t}{}^N>1$ or otherwise.

By utilizing Equations (3–26) and (3–27), we obtain

$$d_{r,t}{}^B=a_{r,t}p_{r,t}+b_{r,t}=a_{r,t}p_{r,t}-a_{r,t}\overline{p_{r,t}}+\overline{d_{r,t}{}^B}, \tag{3-34}$$

$$d_{r,t+1}=a_{r,t+1}p_{r,t+1}+b_{r,t+1}=a_{r,t+1}p_{r,t+1}-a_{r,t+1}\overline{p_{r,t}}+\overline{d_{r,t}{}^B}. \tag{3-35}$$

Hence

$$\frac{d_{r,t+1}{}^B}{d_{r,t}{}^B}=\frac{a_{r,t+1}p_{r,t+1}-a_{r,t+1}\overline{p_{r,t}}+\overline{d_{r,t}{}^B}}{a_{r,t}p_{r,t}-a_{r,t}\overline{p_{r,t}}+\overline{d_{r,t}{}^B}}. \tag{3-36}$$

Multiplying by $\left(-\frac{1}{a_{r,t}}\right)$ both the denominator and numerator of equation (3–36) gives

$$\frac{d_{r,t+1}{}^{B}}{d_{r,t}{}^{B}}=\frac{\frac{a_{r,t+1}}{a_{r,t}}(\overline{p_{r,t}}-p_{r,t+1})-\frac{\overline{d_{r,t}{}^{B}}}{a_{r,t}}}{(\overline{p_{r,t}}-p_{r,t})-\frac{\overline{d_{r,t}{}^{B}}}{a_{r,t}}}. \tag{3–37}$$

Moreover, if $a_{r,t+1}\ll a_{r,t}<0$ and since $\overline{p_{r,t}}-p_{r,t}>0$ and $\overline{p_{r,t}}-p_{r,t+1}<0$, it follows

$$\frac{d_{r,t+1}{}^{B}}{d_{r,t}{}^{B}}\ll\frac{-\frac{\overline{d_{r,t}{}^{B}}}{a_{r,t}}}{(\overline{p_{r,t}}-p_{r,t})-\frac{\overline{d_{r,t}{}^{B}}}{a_{r,t}}}<\frac{-\frac{\overline{d_{r,t}{}^{B}}}{a_{r,t}}}{-\frac{\overline{d_{r,t}{}^{B}}}{a_{r,t}}}=1. \tag{3–38}$$

This implies that if $a_{r,t+1}\ll a_{r,t}<0$ holds, then

$$\frac{s_{r,t+1}{}^{N}}{s_{r,t}{}^{N}}=\left(\frac{p_{r,t+1}}{p_{r,t}}\right)\left(\frac{d_{r,t+1}{}^{B}}{d_{r,t}{}^{B}}\right)<1. \tag{3–39}$$

In a likewise manner, if it can be demonstrated that the ratio of $a_{r,t+1}$ to $a_{r,t}$ is properly close to 1.0, then $s_{r,t+1}{}^{N}/s_{r,t}{}^{N}>1$ holds. That is, given that the followng condition holds for a properly small and positive value ϵ,

$$0<1-\epsilon<\frac{a_{r,t+1}}{a_{r,t}}<1, \tag{3–40}$$

$$(1-\epsilon)a_{r,t}>a_{r,t+1}>a_{r,t}. \tag{3–41}$$

or by utilizing Equation (3–36) and since $0<p_{r,t}<\overline{p_{r,t}}<p_{r,t+1}$ and it follows

$$\begin{aligned}\frac{d_{r,t+1}{}^{B}}{d_{r,t}{}^{B}}&=\frac{a_{r,t+1}p_{r,t+1}-a_{r,t+1}\overline{p_{r,t}}+\overline{d_{r,t}{}^{B}}}{a_{r,t}p_{r,t}-a_{r,t}\overline{p_{r,t}}+\overline{d_{r,t}{}^{B}}}\\&>\frac{a_{r,t}p_{r,t+1}-(1-\epsilon)a_{r,t}\overline{p_{r,t}}+\overline{d_{r,t}{}^{B}}}{a_{r,t}p_{r,t}-a_{r,t}\overline{p_{r,t}}+\overline{d_{r,t}{}^{B}}}\\&>\frac{(a_{r,t}p_{r,t}-a_{r,t}\overline{p_{r,t}}+\overline{d_{r,t}{}^{B}})+\epsilon a_{r,t}\overline{p_{r,t}}}{a_{r,t}p_{r,t}-a_{r,t}\overline{p_{r,t}}+\overline{d_{r,t}{}^{B}}}\\&=1+\frac{\epsilon a_{r,t}\overline{p_{r,t}}}{a_{r,t}p_{r,t}-a_{r,t}\overline{p_{r,t}}+\overline{d_{r,t}{}^{B}}}\\&=1+\epsilon\frac{a_{r,t}\overline{p_{r,t}}}{\overline{d_{r,t}{}^{B}}}.\end{aligned} \tag{3–42}$$

Since $\epsilon\frac{a_{r,t}\overline{p_{r,t}}}{\overline{d_{r,t}{}^{B}}}<0$ holds, if it is warranted that the term is negligibly small, then $\frac{d_{r,t+1}{}^{B}}{d_{r,t}{}^{B}}>1$ holds, consequently leading to

$$\frac{s_{r,t+1}{}^{N}}{s_{r,t}{}^{N}}>1. \tag{3–43}$$

For instance, let us assume $\frac{p_{r,t+1}}{p_{r,t}}=\frac{0.4}{0.2}=2.0$. Then, for such a small and positive value as

$$\epsilon < \left| \frac{\overline{d_{r,t}}^B}{2a_{r,t}\overline{p_{r,t}}} \right|, \tag{3-44}$$

it follows

$$\frac{s_{r,t+1}{}^N}{s_{r,t}{}^N} = \left(\frac{p_{r,t+1}}{p_{r,t}}\right)\left(\frac{d_{r,t+1}{}^B}{d_{r,t}{}^B}\right) > 2\left(1 - \frac{1}{2}\right) = 1. \tag{3-45}$$

We now summarize our above discussion.

(i) So far as a shift of p_r from one interval to its neighboring upward interval gives a small change to a_r, $s_r{}^N$ increases as p_r does.

(ii) Otherwise, if such a shift gives a drastic change to a_r, then $s_r{}^N$ decreases as p_r increases.

(iii) When p_r lies in the interval, $\overline{p_{r,t-1}} \leqq p_{r,t} \leqq \overline{p_{r,t}}$ and moves from its lower bound $\overline{p_{r,t-1}}$ toward its upper bound $\overline{p_{r,t}}$, and if $|a_r|$ is properly small, then $s_r{}^N$ increases. Otherwise it decreases.

It is of incidental interest to note the following three points.

(i) Since Equations (3–5) and (3–6) hold, the domestic water demands for industrial uses, $d_r{}^{DI}$ can be calculated as

$$d_r{}^{DI} = D_r{}^I - d_r{}^B. \tag{3-46}$$

(ii) The value for $d_r{}^B$ in the above equation and those for $d_r{}^I$, $d_r{}^{DI}$, $d_r{}^{DD}$, $d_r{}^N$, $s_r{}^I$ and $s_r{}^D$—all can be determined uniquely when the value for $S_r{}^T$ is found. This means that all these variables are those variables dependent on $S_r{}^T$ and, as a result, they can be precluded from the set of (independent) variables of our model.

(iii) The blending ratio, p_r must be strictly discriminated from the reuse ratio h_r which is defined as the ratio of the amount of renovated water supply to the total industrial water demand such that

$$h_r = \frac{s_r{}^N}{D_r{}^I} = \frac{s_r{}^N}{d_r{}^{DI} + d_r{}^B} = \frac{s_r{}^N}{d_r{}^{DI} + d_r{}^I + d_r{}^N} \leqq \frac{s_r{}^N}{d_r{}^I + d_r{}^N} = p_r. \tag{3-47}$$

Equality holds only for $d_r{}^{DI} = 00$, that is, $p_r = 0.0$. Hence, p_r is always larger than h_r except for $p_r = h_r = 0.0$.

(iv) As is illustrated in Figure 4, it is assumed

$$G_r = D_r{}^T = D_r{}^I + D_r{}^D = \overline{s_r{}^I} + \overline{s_r{}^D} + \overline{s_r{}^N}, \tag{3-48}$$

$$\overline{s_r{}^N} = 0.0, \tag{3-49}$$

$$g_r = G_r - \underline{s_r{}^N} = (D_r{}^I - \underline{s_r{}^N}) + D_r{}^D = \underline{s_r{}^I} + \underline{s_r{}^D}, \tag{3-50}$$

where $\overline{s_r{}^I}$, $\overline{s_r{}^D}$ and $\overline{s_r{}^N}$, represent the precalculated values for $s_r{}^I$, $s_r{}^D$ and $s_r{}^N$, respectively, for G_r, and s_r^I, s_r^D, and s_r^N, the precalculated values for s_r^I, s_r^D and s_r^N, respectively, for g_r.

Finally, let us summarize the formulation of our model.

3-4 Formulated Model

1) Major Model

[objective function]

$$\text{Minimize} \quad z=\varepsilon_c. \tag{3-18}$$

[technical and physical constraints]

$$u_{rl}\leqq c_{rl} \quad (r=1, \cdots, v;\ l=1, \cdots, L_r), \tag{3-2}$$

$$\sum_l u_{rl}+\sum_{s\in\Gamma_r}(y_{sr}-y_{rs})-S_r^T=0 \quad (r, s=1, \cdots, v;\ r\neq s). \tag{3-3}$$

[goal constraints]

$$\sum_r\sum_l a_{rl}u_{rl}+\sum_r\sum_{s\in\Gamma_r}\beta_{rs}y_{rs}+\sum_r\sum_{s\in\Gamma_r}\beta_{sr}y_{sr}-\varepsilon_c+\eta_c=G_c, \tag{3-11}$$

$$\sum_r\sum_l a_{rl}u_{rl}+\sum_r\sum_{s\in\Gamma_r}\beta_{rs}y_{rs}+\sum_r\sum_{s\in\Gamma_r}\beta_{sr}y_{sr}\leqq g_c, \tag{3-12}$$

$$S_r^T+\varepsilon_r-\eta_r=G_r \quad (r=1, \cdots, v), \tag{3-13}$$

$$S_r^T\geqq g_r \quad (r=1, \cdots, v), \tag{3-14}$$

$$\frac{\varepsilon_c}{\lambda_c}=\frac{\varepsilon_r}{\lambda_r} \quad (r=1, \cdots, v), \tag{3-15}$$

where

$$\lambda_c=g_c-G_c, \tag{3-16}$$

$$\lambda_r=G_r-g_r \quad (r=1, \cdots, v). \tag{3-17}$$

2) Submodel

$$S_r^T=s_r^I+s_r^D+s_r^N \quad (r=1, \cdots, v), \tag{3-4}$$

$$D_r^I=d_r^{DI}+d_r^N+d_r^I=d_r^{DI}+d_r^B \quad (r=1, \cdots, v), \tag{3-5}$$

$$d_r^B=d_r^R+d_r^I \quad (r=1, \cdots, v), \tag{3-6}$$

$$s_r^I=d_r^I \quad (r=1, \cdots, v), \tag{3-7}$$

$$s_r^N=d_r^N \quad (r=1, \cdots, v), \tag{3-8}$$

$$s_r^D=d_r^{DD}+d_r^{DI}=d_r^{DD}+D_r^D \quad (r=1, \cdots, v), \tag{3-9}$$

$$d_r^{DD}=D_r^D \quad (r=1, \cdots, v), \tag{3-10}$$

$$d_r^B=a_r p_r+b_r \quad (r=1, \cdots, v), \tag{3-26}$$

$$p_r=\frac{s_r^N}{s_r^I+s_r^N} \quad (r=1, \cdots, v). \tag{3-19}$$

$$(d_r^{DD}, d_r^{DI}, d_r^I, d_r^N, d_r^B, s_r^I, s_r^N, s_r^D\geqq 0)$$

4. Case Study on the Southern Part of Hyogo Prefecture

4-1 Preliminary Discussion

The southern part of Hyogo Prefecture is selected as the study area to which the above formulated model will be applied.

The conceptualized system of inter-basin water resources development is diagrammed in Figure 5, which shows that the inter-basin system consists of five intra-basin systems, i.e., the Chigusa, Ibogawa, Yumesaki, Ichikawa and Kakogawa Rivers.

4-2 Input Data

1) Water Demands

The projected water demands (B-type estimation) by the year 1985, are listed in Table 1.

These projected demands (D_r^T, $r=1, \ldots, 5$) of each demand region are set as the satisfi-

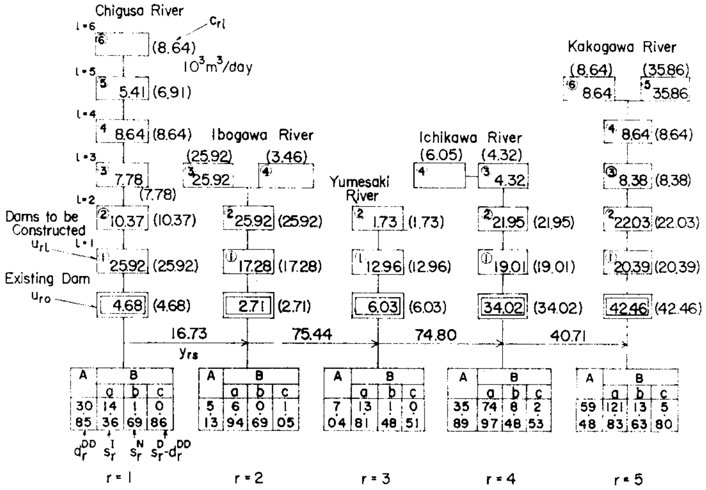


Fig. 5. Model Diagram and Illustrated Result for Case-1.

Table 1 Estimated Water Demands.

$(10^3m^3/day)$

Demand River Basin	Domestic Water Demand	Industrial Water Demand
Kakogawa R.	429	944
Ichikawa R.	281	650
Yumesaki R.	49	112
Ibogawa R.	51	36
Chigusa R.	219	98

ed-level of the collection goal for each region (basin).

2) Construction Costs

The unit construction costs for both dams and channels are all based on the data

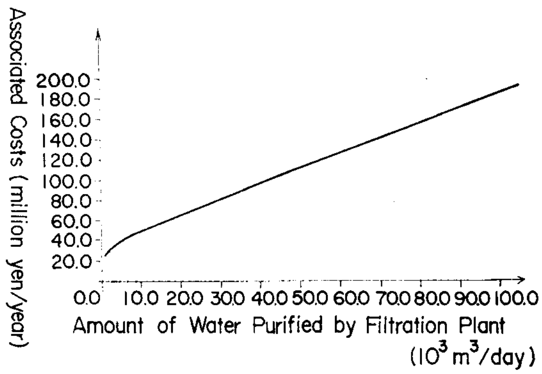


Fig. 6. Cost Curve for Filtration Plant.

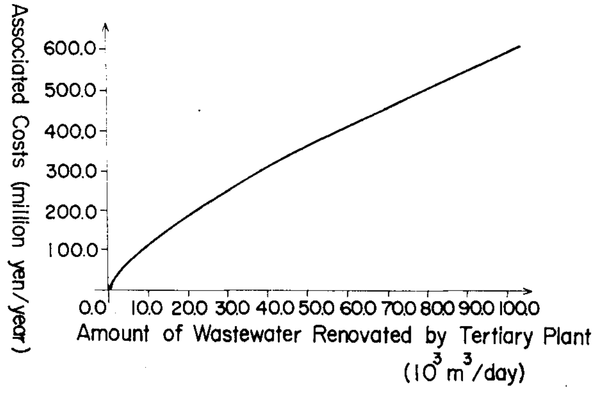


Fig. 7. Cost Curve for Tertiary Treatment Plant.

Table 2 Preplanned Cases

r	Case / River Basin	Case-1	Case-2	Case-3	$G_r(G_c)$
		$g_r(g_c)$	$g_r(g_c)$	$g_r(g_c)$	
1	Kakogawa R.	1, 323	1, 247	1, 119	(10^3 m^3/day) 1, 550
2	Ichikawa R.	716	702	613	858
3	Yumesaki R.	139	130	120	165
4	Ibogawa R.	98	93	91	109
5	Chigusa R.	398	408	414	427
c	Total Cost	31. 0	29. 1	24. 9	Mil-yen 45. 22

provided and authorized by a certain consulting company. Some of the cost functions are shown in Figures 6 and 7.

4-3 Calculation Cases

Prior to the computations on the model, we formulized three cases, i.e., Cases I, II and III according to the difference in the values set for the permitted-levels $g_r (r=1, \ldots, 5)$ concerned with the amount of water collected from river $r (r=1, \ldots, 5)$ for the use of its own demand region. That is,

(i) Case I where the quality of the blended industrial water is lowered to the physical limit in the sense that any degradation of quality added to that level would not be accepted by most, if not all, kinds of industries there. In light of these considerations, it is assumed that the permitted levels of the collection-goals, or in other terms the goals for the water utilization systems concerned with each of the basins, are characterized by higher values set for the blending ratios, p_r's $(r=1, \ldots, 5)$, that is, $p_r=0.6$ for all the basins $(r=1, \ldots, 5)$.

(ii) In a similar sense, Case II assumes that the permitted levels of the collection-goals are characterized by medium values for the blending ratios p_r, that is, $p_r=0.4$ $(r=1, \ldots, 5)$.

(iii) Case III assumes that they are characterized by lower values for p_r, that is, $p_r=0.2$ $(r=1, \ldots, 5)$.

As is clear from Figure 3, Case I corresponds to point M_r, Case II to N_r and Case III to L_r $(r=1, \ldots, 5)$.

And the permitted-levels, g_r $(r=1, \ldots, 5)$ are so established as shown in Table 2 which were obtained by making use of Figure 3. For each case, the satisfied-levels G_r are set fixed at D_r^T $(r=1, \ldots, 5)$.

So far as the permitted- and satisfied-levels of the cost-goal are concerned, they are so predetermined that they become equal to the optimal solutions to the linear programming problems.

Minimize

$$z=\sum_r \sum_l \alpha_{rl} u_{rl} + \sum_r \sum_{s \in \Gamma_r} \beta_{rs} y_{rs} + \sum_r \sum_{s \in \Gamma_r} \beta_{sr} y_{sr}, \tag{4–1}$$

subject to

$$u_{lr} \leqq c_{lr} \quad (r=1, \cdots, 5;\ l=1, \cdots, L_r), \tag{4–2}$$

$$\sum_l u_{rl} + \sum_{s \in \Gamma_r} (y_{sr} - y_{rs}) - S_r^T = 0, \tag{4–3}$$

and with

$$S_r^T = G_r - \overline{S_r}^N \quad (r=1, \cdots, 5), \tag{4–4}$$

or

$$S^T = G_r = g_r \quad (r=1, \cdots, 5) \quad (y_{sr}, y_{rs}, u_{rl}, S_r^T \geqq 0).$$

depending upon whether the permitted-level or satisfied-level is concerned, where

$$\sum_{r=1}^{5} G_r - \sum_{r=1}^{5} \overline{s_r}^N = \sum_r \sum_l c_{rl} \tag{4-5}$$

$$\overline{p}_r = \frac{s_r^N}{s_r^I + s_r^N} = 0.974 \tag{4-6}$$

Equation (4-5) implies that the total water demands $(\sum_{r=1}^{5} G_r = \sum_{r=1}^{5} D_r)$ exceed the maximal amounts of available fresh water $(\sum_r \sum_l c_{rl})$ by the amounts of $\sum_{r=1}^{5} \overline{s_r}^N$.

4-4 Calculation Results

To begin with, let us consider Case I as the standard case. After that, we shall make a comparative study on the results of all the cases.

1) Standard Case (Case I)

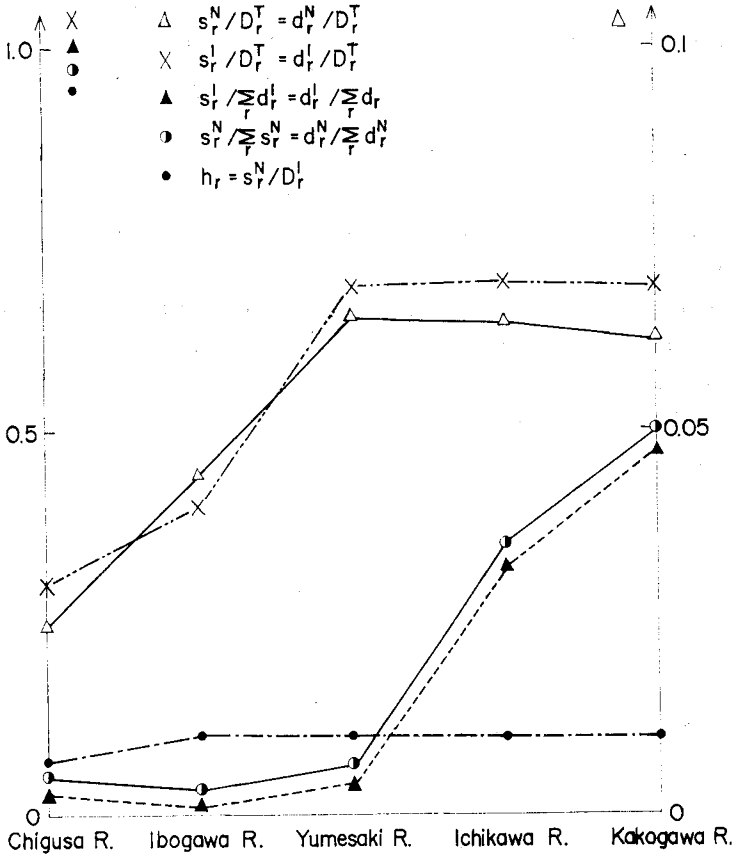


Fig. 8. Close Analysis of Case-1.

The following are immediately understood from Figure 5.

(i) From the Chigusa and Ibogawa Rivers, there are diverted 0.167, and 0.581 million m³/day of fresh water, of which 28 percent is diverted all the way to the Ichikawa and Kakogawa Rivers. On the Ibogawa River is developed total amounts of 0.432 million m³/day of fresh water, of which 99 percent of fresh water is further diverted to both rivers. On the other hand, the Ichikawa River has 28 percent of its total water demands met with the fresh water diverted from the above three rivers, and the Kakogawa River has 13 percent of its demands provided by them.

(ii) Attention needs to be given to the fact that the Ichikawa River has the potentials of another 0.061 million m³/day of fresh water to be developed which could partially cover the amounts of the fresh water that is owed to the other rivers. But we know from (i) that this is not the case with the above results. The reason for this seems to be the inter-basin development systems as such, chiefly owing to relatively economical developments of fresh water achievable on the Chigusa, Ichikawa and Yumesaki Rivers as well as to relatively economical diversion works to be carried out between the basins.

(iii) In each basin a water reclamation system is required to be implemented. The Reuse Ratios, p_r are found to range from 0.024 (for the Chigusa River) to 0.070 (for the Yumesaki, Ichikawa and Kakogawa River).

Through closer analysis of Figure 5, we obtained Figure 8, from which the following are understood.

(iv) Interestingly, in spite of the variations in the total water demand associated

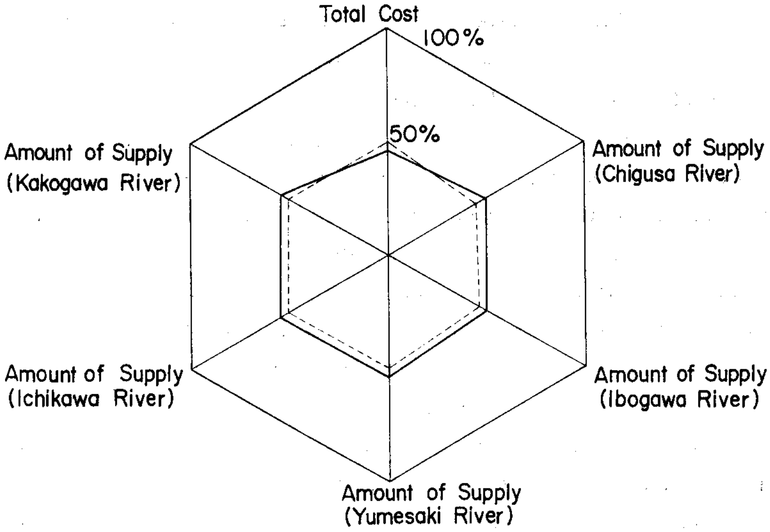


Fig. 9. Attained Levels of Goals.

with the water demand region in each river basin, the Reuse Ratios (h_3, h_4 and h_5) are found to be roughly the same. The reason for this can be found in that the ratio of the water demand for industrial uses ($D_r{}^I$, $r=1, \ldots, 5$) to the total water demand (termed hereafter as "the industrial-demand share") which can be considered to reflect the magnitude of the water-related industries' activities, proved to be approximately the same for the above-cited three regions. From this it might be presumed that the Reuse Ratio for a given region well corresponds to the industrial-demand share, which proved to be true, as shown in Figure 6. This implies that the total amounts of renovated water which are complementarily used to offset the shortage of supply are assigned to the five water-demand regions in proportion to their demands for industrial uses. This is well evidenced by the rough unison of the qualitative characteristics of the two curves, one related to the Reuse Ratios for the different regions and the other to the industrial-water shares for these regions.

(v) As is obvious from Table 2, the deviation λ_r between G_r and g_r happened to be roughly proportional to the demand for industrial uses, $D_r{}^I$ for any river basin r ($r=1, \ldots, 5$).

(vi) Let us define here the attainment ratio of each goal as the ratio of the deviation between the attained level and the permitted-level to the deviation, λ_r between the satisfied- and permitted-levels. Then it is clear from Figure 9 that the attainment ratios of all the goals proved to be around 0.5, which means that the selected alternative which can be conceived as the most efficient alternative for this case proved to be such that the attained-levels could be guaranteed to fall in a range midway between the permitted- and satisfied-levels.

This kind of well-balanced attainment of all the goals is considered to be due to the postulated L-type utility function as stated before.

2) Indepth Analysis of the Standard Case

Let us consider here what the model tells us about the intra-basin development system. In view of the fact that our model never precludes the choice of that alternative if the most efficient solution could hove been found to be a kind of inter-basin system, it can be fairly justified that the intra-basin system, is found to be far from the most efficient solution in the interests of the cost- and collection-goals.

To make our discussion more specified, let us solve a set of those goal programming problems, each representing the problem of the coordinated attainments of the cost- and collection- goals for each basin. These problems can be readily dealt with by neglecting the variable, y_{sr} and y_{sr}, and solving the goal programming problem for each basin.

The results lead to the following observations.

(i) The set of intra-basin systems costs 1.20 times as much as the inter-basin system.

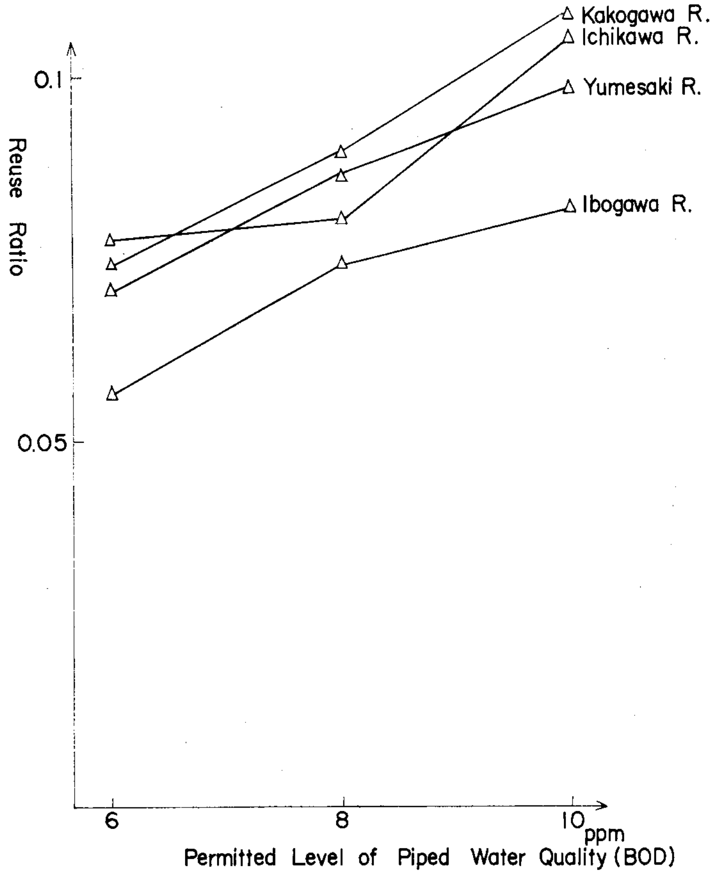


Fig. 10. Calculated Reuse Ratios for Different Water Quality Levels.

(ii) The attained quality of the blended water provided by the intra-basin (single-basin) systems tends to be 1.1 to 2.0 times as worse as the inter-basin system. This is particularly the case with the Chigusa River where available amounts of fresh water tapped by the dams are relatively small with regard to the total water demands as compared to the remaining river basins.

3) Comparative Study of Case I, II and III.

(i) The permitted-levels of the collection-goals are so determined that they implicitly represent those water-use patterns corresponding to the points denoted by (M_r, N_r and L_r) as was explained in **4-3**.

(ii) Stated otherwise, Case I is characterized by a lower quality (10 BOD ppm) of the blended water, Case II by a moderate quality (8 BOD ppm) and Case III by a higher quality (6 BOD ppm).

(iii) As is clear from Figure 10, the Reuse Ratios for the Kakogawa, Yumesaki

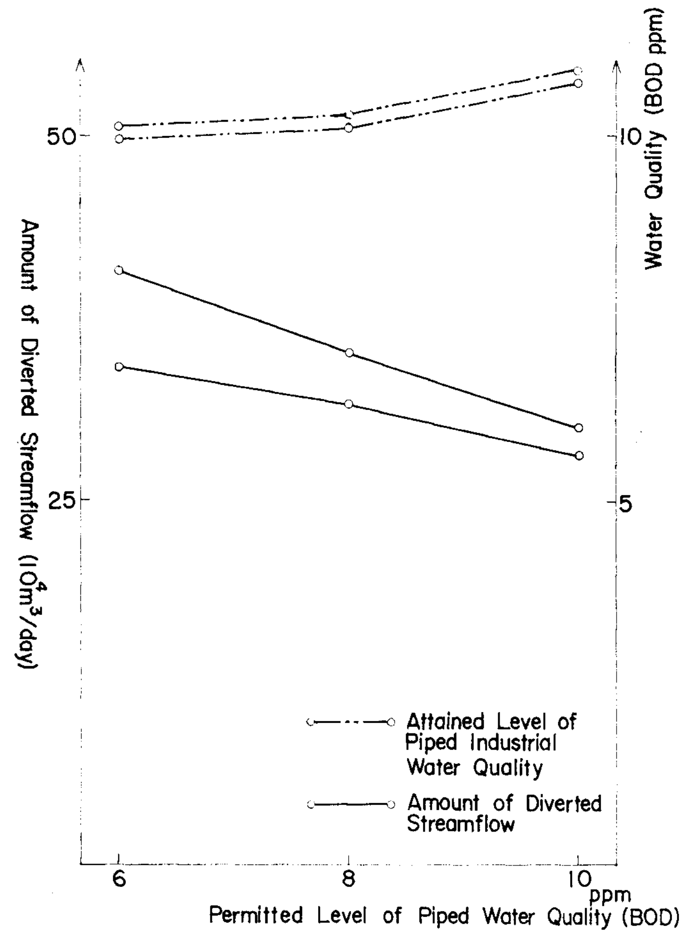


Fig. 11. Calculated Amounts of Diverted Streamflow for Different Water Quality Levels.

and Ichikawa Rivers tend to increase in proportion to the increase in the permitted BOD values. One particular exception to this trend is the Reuse Ratio for the Chigusa River which exhibits a decreasing pattern with an increase in the permitted BOD values. The reason for this is that, as was referred to in **4–3**, so far as the Chigusa River is concerned, the permitted amount of the renovated water, s_r^N decreases as the blending ratio p_r, namely a water degradation index, increases.

Notably this trend is contrary to those of the other basins where the s_r^N increases as the p_r decreases ($r=1, \ldots, 5$).

For this reason we may conclude that so far as the Chigusa River is concerned, it does not seem to be a good way to implement a reclamation system on large-scale. On the contrary, as regards the other river basins the amount of water needed for industrial uses can be saved at the expense of increased degradation in the quality of the blended industrial water.

(iv) In Figure 11, both the amounts of streamflow diversion and those of the attained water quality are plotted against different permitted values for the blended water quality represented by Cases I, II and III. This figure shows that the greater amount of river flow that is diverted from the Chigusa, Ibogawa and Yumesaki Rivers to the Ichikawa and Kakogawa Rivers, then the better becomes the attained quality of the blended water. In other terms, the implementation of the inter-basin streamflow diversion system can play an important role in improving the quality of the blended water.

(v) The intra-basin system is found to be a less efficient alternative, whether the quality of the blended water is required higher, medium or lower.

5. Conclusion

In this paper, we have concerned ourselves with the problem of the coordinated attainments of the multiple-goals involved in the planning of the inter-basin development system.

The major findings of this chapter are:

(i) So far as the proper attainments of the two distinct goals are called upon, a reclamation (recycling) system on a certain scale is required to be implemented to supplement the industrial water supply. (The Reuse Ratio ranges from 0.02 for the Chigusa River to 0.07 for the Ichikawa, Kagoshima and Yumesaki Rivers.)

(ii) In case the permitted-levels of the collection goals related to each river basin are set to be higher than 80 percent of the total water demands, the total amounts of water supply that must be covered by the reclamation systems should be allocated to the system of each basin roughly in proportion to the amounts of its industrial water demand.

(iii) This proved to apply equally to the other cases where the lower values were prescribed to those permitted-levels. One conspicuous exception, however, is the Chigusa River where relatively higher water quality is demanded by most of the industries located there as compared to the other basins. It does not seem to be a good way by any means to adopt a large-scale reclamation system in this basin.

(iv) The increased attainments of the amounts of collection, and in consequence, the increased attainments of the improvement in the blended water quality are achievable only at the sacrifice of the economic efficiency. This type of trade-off, which was quantitatively obtained from the goal programming model, exhibits roughly a proportional relation between the attainments of the goals.

(v) Interestingly, the increased costs attendant to the increased attainments of the water quality improvement of the blended water are derived from the extended developments of fresh water and the increased implementation of interbasin streamflow diversions.

(vi) The attainments of the goals are seen to be well-balanced and not biased to any of them, chiefly owing to the L-type utility function underlying the model.

(vii) The comparative analysis between the inter-basin system and the intra-basin system showed that in either of the two goals, the former system proved to be more efficient than the latter.

From the findings summarized above, it seems to be appropriate to conclude that the inter-basin system is the most efficient alternative in the interests of both the cost- and collection goals, if and only if the major assumptions on which the model building was made, are found to be reasonable.

One drawback to the model is the implicit assumption of the renovated water utilization system as a complementary means of supply in case the deviation arises between the attainable supply and the needed water. The term "implicit" is used to mean that the costs associated with the complementary supply works are not included in the total construction costs. The applicability of this assumption seems to be limited to a case where a certain water management agency on a national or regional level is planning to develop water by constructing dams and channels on an inter-basin-basis, and its main concerns are to secure as much water as needed in the most economical manner. At the same time, its concern is to meet the water demand of each locality as much as possible with fresh water, because excessive implementation of the reclamation system to be intensively concentrated on a certain locality will inflict disadvantages on it such as a deteriorated quality of water supply and an excessive financial investment on the system.

This discussion does not necessarily mean that such disadvantageous effects should be explicitly incorporated into the model and if not, our findings based on the model are not valid. In this connection, one is required to note that we set another goal such that the amounts of water to be collected from each river for its local use should cover its total water demand as much as possible. If the linearity holds between the scale of the reclamation system and its implementation costs, the above goals can be conceived as synonymous to that goal which prescribes the minimization of the implementation costs.

For this kind of discussion, the reader is invited to see the other works by the authors.15)

In conclusions it seems important to go as far as possible with the model presented in this paper, and it is believed that valuable information would come from this kind of analysis.

Acknowledgment

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