Random Fatigue Analysis of Structural Steel Bars Subjected to Plastic Bending

By

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(Received December 27, 1976)

Low-cycle fatigue life of structural steel bars subjected to random plastic flexural deformation is analyzed. Fatigue tests are performed on 100×100 SS41 H bars under constant-amplitude and randomly varying repeated loads. Fatigue life for random loads is estimated by using the linear cumulative damage law. Damage per unit time (or cycle) is predicted by (1) the equivalent amplitude factor and (2) peak-trough and plastic deforma-, tion criteria. Estimated results are compared with test results.

1. Introduction

Low-cycle fatigue is of primary concern when structures are subjected to several tens of large deformation excursions in dyanmic disturbances, including strong earthquakes. The fatigue behavior of structures under random dynamic loads remains a difficult engineering problem, because it is affected by material characteristics, structural geometry, load histrory and environment.

Despite the significant contributions made by Miner[8], Freudenthal[4], Munse[9], Yao[14] and others[2,7,12,15], the low-cycle fatigue problem under dynamic random loads still requires extensive studies. One reason for this would be that the stress-life relation obtained from single-stress level laboratory tests cannot be directly applied to the resulting load history of the corresponding inelastic structural response. Another reason would be that there are not enough experimental data to determine which is the rational criterion for the fatigue accumulation law. In this regard, Yao and Munse[13] proposed a fatigue criterion based on the plastic strain amplitude and the ultimate plastic strain prior to fracture in simple tension-compression tests.

The objective herein is to find a simple rule for predicting the low-cycle fatigue life

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under a dynamic random flexural deformation on the basis of fatigue tests on structural steel bars. When low-cycle fatigue failure of civil engineering structures is of concern, it will take place rather in flexure or shear than in pure tension and compression. In this regard, low-cycle fatigue tests were performed on flexural steel bars with a 100 mm × 100 mm H section. Use of such specimens for fatigue tests should be helpful for establishing the actual relation between fatigue strength and structural response.

In practical application, fatigue life under random loads is usually estimated on the basis of the SN relation obtained from constant-amplitude tests. Therefore, random loads must be evaluated relative to constant amplitude loads. For this purpose, various rules for evaluating cumulative damage have been proposed. In paricular, the Plamgren-Miner criterion[8] corresponds to the linear cumulative damage rule, and is often used to compare predicted fatigue life with test results. It is applicable to any material whose SN relation is available. This means that it can also be applied to the low-cycle fatigue on the basis of the deflection amplitude and fatigue life relation.

Although more refined cumulative damage rules[10,13] have been proposed, the Palmgren-Miner criterion will be used as the theoretical basis, since it is not yet possible to evaluate explicitly the nonlinear effects of the cumulative damage for general sections of the flexural structural members. For example, the low-cycle fatigue life of wide flange H section beams is affected by the behavior of the flanges, including local buckling which usually does not take place in idealized tension-compression tests. Moreover, from the results of the programmed loading tests on H-section beams[5,6], no method for evaluating the nonlinear cumulative damage with a sound physical basis seems to be deductable.

Within the framework of the linear cumulative damage, another important problem in random fatigue analysis is the evaluation of damage per unit time (or per cycle) of random load processes. Appropriateness of such a procedure would be a key question in the estimation of random fatigue life on the basis of the constant-amplitude SN relation.

A simple and convenient method is to use the peak amplitude distribution which, in the case of the stationary narrow-band Gaussian process, is given as the Rayleigh distribution. However, the mean peak amplitude may not necessarily correspond to the mean fatigue life due to the potential nonlinearity in the damage accumulation law. This study first develops a method for this calibration by comparing the test results for constant-amplitude tests. The result is given in terms of the equivalent amplitude factor.

For estimating the fatigue life under nonstationary random deflection, a step-by-step evaluation of cumulative damage per cycle is required. Two methods for this purpose are tested herein. One is the conventional peak-trough criterion, and the other is the cumulative damage depending on plastic deformation. They are applied to the test results and checked for their appropriateness.

Test results of this type are necessarily subject to uncertainties of various kinds. In this regard, the variability of the fatigue test results and the corresponding analytical results are estimated.

2. Method of Experiment

2.1 Test specimens

The test specimens are SS41 wide flange H-section $100\times100\times6\times8$ (mm) steel bars as rolled. Fig. 1 shows a sketch of a specimen. H-section steel bars of 2 m length with a span of 1.4 m were alternately loaded at the midspan. Since the alternating plastic bending may develop local buckling in the flange, stiffners were inserted at the midspan as shown in Fig. 1.

2.2 Test setup

(1) Method of loading—. The loading and supporting frames with a specimen are shown in Fig. 2. In order to form a simple support system, both ends of the specimen are bound by a pair of cylinders. Cyclic and random loads are applied through the crosshead of a hydraulic loading system with a capacity of ±15 tons. Fluctuation of the applied load is measured by a load cell of tension-compression type installed between the loading frame and the crosshead.

The loading device transmits only a horizontal force. Vertical displacement of the specimen is restrained at the midspan loading frame. The specimen is allowed to rotate about the axis of the loading ram, so that second-mode lateral torsional buckling with an inflection point at the midspan may take place.

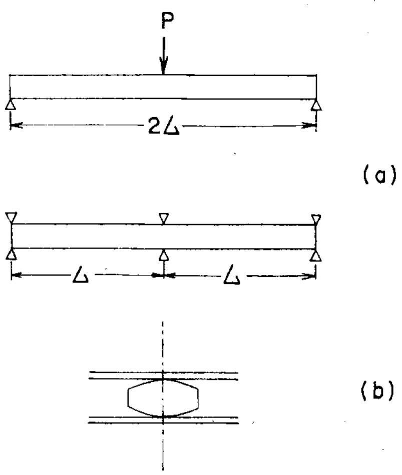

Fig. 1. Tested Specimen.

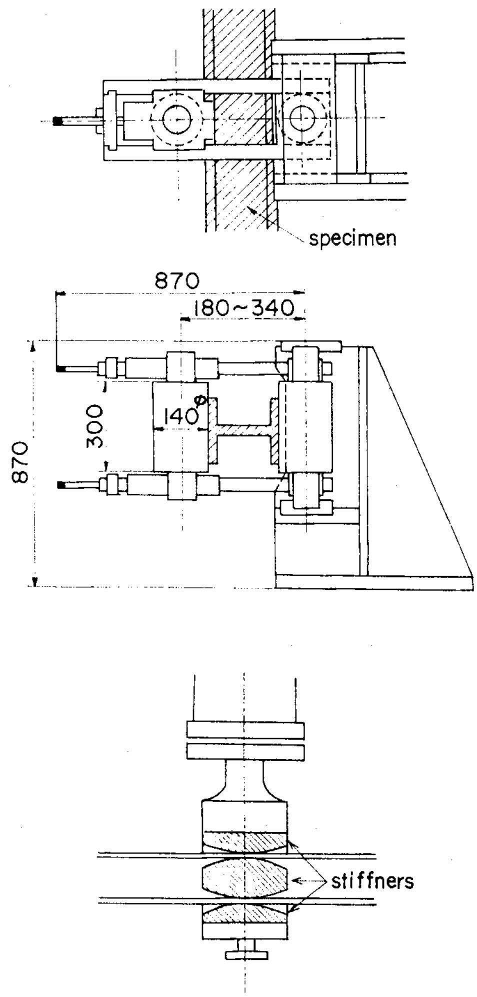

Fig. 2. Test Setup.

(2) Control conditions—. In the constant amplitude test, cyclic displacement is applied at the midspan of a specimen. The displacment pattern in this test is sinusoidal with a frequency of 0.5 Hz. The random load test is controlled through random excitation generated in a white noise generator.

2.3 Test procedure and basic results

(1) Constant amplitude test—. This is the most common fatigue test used to estimate the fatigue life. The relation between the load cycles to failure and the dimensionless deflection amplitude is treated as the SN relation.

The number of load cycles to failure is usually counted up to the flange-crack

Table 1 SN Regression for Constant Amplitude Tests

regression parameter				conditional COV of N	correlation between ln a and ln N	sample mean of ductility factor
b		c				
mean μ_b	COV δ_b	mean μ_c	COV δ_c	δ_N	ρ	μ_a; $\left(a=\frac{X}{X_y}\right)$
2.502	0.075	1.184×10^4	0.207	0.206	−0.981	2.857

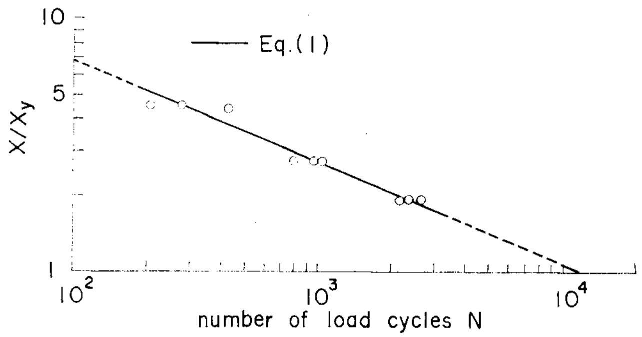

Fig. 3. Constant-Amplitude Fatigue Test Result.

initiation. This procedure was also used in the writers' previous study[4,5]. However, it is difficult to detect a flange-crack initiation during random load tests, whereas a web-crack initiation is much easier to find. Therefore, a web-crack initiation was adopted as the measure of fatigue failure. The SN regression line determined for the web-crack initiation is represented by

Table 2 Results of Random Fatigue Tests

	input level σ_X/X_y	time to failure T, sec
	1.42	4476 3403 3777
	1.58	1449 1454 1957
	2.33	921 918 1040
	2.85	481
sample mean	1.884	1988

$$N(X/X_y)^b = c \tag{1}$$

where N is the number of load cycles to failure, X is the displacement amplitude, X_y is the yield displacement, and b and c are regression parameters. The mean values μ_b, μ_c and coefficients of variation (COV) δ_b, δ_c of the regression parameters b, c are shown in Table 1 along with COV of N conditional on this regression line, δ_N, and the correlation coefficient between ln N and ln a. The regression lines thus determined are shown in Fig. 3. They are to be compared with the results of the random load tests.

(2) Random load Test—. Random load tests were carried out by using a stationary band-limited white noise with a zero mean and a frequency range of 0.125 to 1.0 Hz. The random signal was applied directly on a specimen as a midspan displacement.

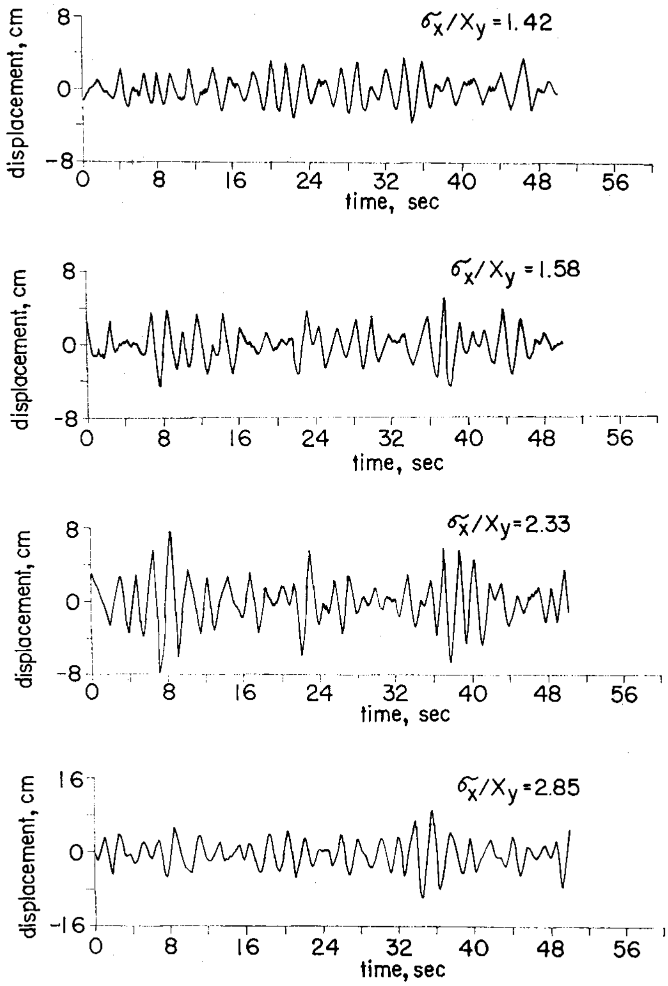

Fig. 4. Random Deformation Time Histories.

Ten specimens were tested for four different values of the rms intensity σ_X of the input random signal as indicated in Table 2. As described above, the time to failure T was measured at the web-crack initiation, which is also shown in Table 2. Fig. 4 shows examples of the displacement time history. Random load levels are relatively high, since the low-cycle fatigue failure due to the high level random excitation is of primary concern in this study.

The results of the random fatigue tests are represented by the following regression equation:

$$T(\sigma_X/X_y)^{b^*}=c^* \tag{2}$$

in which b^* and c^* are regression parameters. In Table 3, there are shown the statistics of regression for Eq. (2) based on the random fatigue tests. They may be compared with the corresponding results of the constant amplitude tests. Fig. 5 shows the relation between the dimensionless rms amplitude σ_X/X_y and the fatigue life T along with the regression line. It may be emphasized that the results of the constant amplitude tests and the random load tests are in good agreement in the slope of the regression lines. This result is reflected in the close values of μ_b in Table 1 and μ_{b^*} in Table 3.

Table 3 SN Regression for Random Load Tests

regression parameter				conditional COV of T	correlation between	sample mean of ductility factor
b^*		c^*				
mean μ_{b^*}	COV δ_{b^*}	mean μ_{c^*}	COV δ_{c^*}	δ_T	$\ln \frac{\sigma_X}{X_y}$ and $\ln T$	μ_{a^*}; $\left(a^*=\xi\,\frac{\sigma_X}{X_y}\right)$
2.509	0.141	0.715×10^4	0.231	0.296	−0.926	2.901 (1.884×1.54)

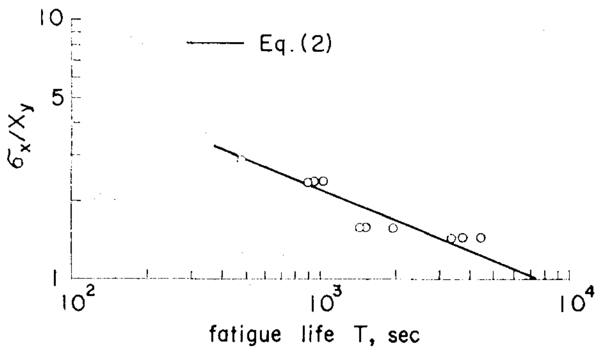

Fig. 5. Random-Deformation Fatigue Test Result.

3. Estimation of Random Fatigue Life

3.1 Equivalent amplitude factor

In predicting the actual fatigue life under stationary random loads on the basis of the constant-amplitude test results, it would be useful to define an equivalent amplitude of sinusoidal excitation corresponding to the rms intensity of the random loads.

Note that the input excitation is a stationary band-limited random process. Moreover, close values of μ_b and μ_b* encourage one to establish a relation between the results of the two types of tests in the following manner. If we assume that the deflection X in Eq. (1) corresponds to ξ times σ_X in Eq. (2), and that the number of cycles to failure N in Eq. (1) corresponds to ν_0 times T in Eq. (2), we then have

$$\left.\begin{array}{l} X=\xi\,\sigma_X \\ N=T\,\nu_0 \end{array}\right\} \tag{3}$$

in which ν_0 is the mean frequency of the random deflection process, that assumes a value of $(0.125+1.0)/2=0.5625$Hz in this study. The parameter ξ may be referred to as an equivalent amplitude factor. Then, on substitution from Eq. (3), Eq. (1) yields

$$T\,\nu_0(\xi\,\sigma_X/X_y)^b=c \tag{4}$$

Since we can set $\mu_b \simeq \mu_b*=2.50$ from Tables 1 and 3, we may assert that on the average $b=b^*=2.50$. Hence from Eqs. (2) and (4), we have

$$\xi=\left[\frac{c}{\nu_0\,c^*}\right]^{1/2.5}=1.54 \tag{5}$$

The peak amplitude of stationary narrow-band Gaussian processes follows the Rayleigh distribution[3] with a mean of $\sqrt{\pi/2}\ \sigma_X=1.25\,\sigma_X$. This value is sometimes used to predict the fatigue life, that is, the equivalent amplitude factor ξ' corresponding to the Rayleigh model is 1.25. However, the result in Eq. (5) demonstrates that the equivalent amplitude must assume a larger value than the mean peak value of the stationary band-limited Gaussian processes, since if $\xi'\sigma_X$ is adopted instead of $\xi\sigma_X$, the fatigue life would be overestimated on the average by $(\xi/\xi')^b-1=68\%$.

Evaluation of errors arising from using the equivalent amplitude factor instead of the actual random processes is of interst. This may be done by estimating the coefficient of variation (COV) of the equivalent amplitude factor, δ_ξ. Applying the first order approximation in the evaluation of statistical parameters[1] and assuming statistical independence between the variates, the COV of N and T, δ_N and δ_T, respectively, are represented by

$$\delta_N{}^2 \simeq \delta_c{}^2+\mu_b{}^2\,\delta_{X_y}{}^2+(\mu_b\,\ln\mu_a)^2\delta_b{}^2 \tag{6}$$

$$\delta_T^2 \simeq \delta_{c^*}^2 + \mu_{b^*}^2 (\delta_{X_y}^2 + \delta_\xi^2)^2 + \left(\mu_{b^*} \ln \frac{\mu_{a^*}}{\mu_\xi}\right)^2 \delta_{b^*}^2 \tag{7}$$

From Eqs. (6) and (7), we obtain

$$\delta_\xi^2 \simeq \frac{\delta_T^2 - \delta_N^2}{\mu_{b^*}^2} - \frac{\delta_{c^*}^2 - \delta_c^2}{\mu_{b^*}^2} - \left[1 - \frac{\mu_b^2}{\mu_{b^*}^2}\right] \delta_{X_y}^2 - \left[\ln \frac{\mu_{a^*}}{\mu_\xi}\right]^2 \delta_{b^*}^2 + \left[\frac{\mu_b}{\mu_{b^*}} \ln \mu_a\right]^2 \delta_b^2 \tag{8}$$

By virtue of the results in Table 1 and 3, let us assume that $\mu_b = \mu_{b^*} = 2.5$. Then the third term on the right-hand side of Eq. (8) vanishes. Substituting the numerical values in Table 1 and 3 into Eq. (8), COV δ_ξ is obtained as

$$\delta_\xi = 0.061 \tag{9}$$

Thus Eq. (4) may be used for estimating the time to failure T under random excitation, in which b and c are regression parameters in Eq. (1) obtained from constant-amplitude fatigue tests, whereas ξ is a variable with a mean of 1.54 and COV of 0.061. A relatively small value of δ_ξ (that is 0.061) would give prospects for an application of the equivalent amplitude factor ξ. Its validity should be tested through experimental results for wider parameter ranges.

3.2 Damage estimation of random deflection processes

Random fatigue life may be estimated by using the equivalent amplitude factor developed in the previous section when the loads are stationary random processes. However, a single value for ξ does not exist when the load process is nonstationary, and in such cases a step-by-step evaluation of cumulative damage is required. In the following, two such methods are compared and evaluated.

As mentioned in **1.**, the linear cumulative damage criterion is adopted in this study. Damage per cycle defined on the basis of the dimensionless deflection amplitude is available in the low-cycle fatigue failure. Fig. 6 is a part of random deflection $X(t)$.

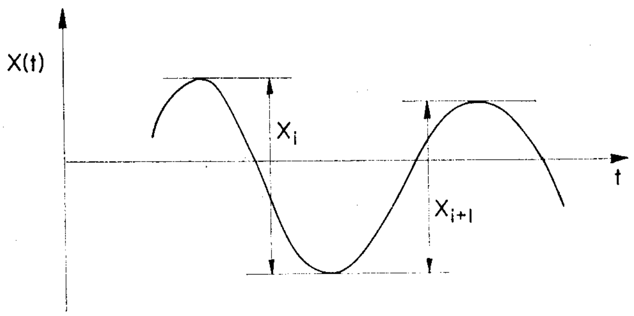

Fig. 6. Peaks and Troughs of X.

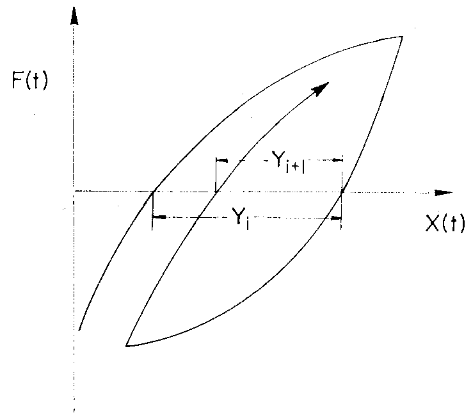

Fig. 7. Plastic Deformation.

X_i and X_{i+1} are the local minima (trough) and local maxima (peak), respectively. Damage d_i due to half cycle X_i may be half the linear-law damage for one cycle obtained from Eq. (1); that is, $d_i=1/(2N)$, which is represented by

$$d_i=\frac{1}{2}\frac{\left\{\dfrac{X_i}{2X_y}\right\}^b}{c} \tag{10}$$

in which X_y is the yield deflection and b and c are parameters of the SN line in Eq. (1). Eq. (10) gives the damage based on the *peak-trough* criterion.

When a hysteretic force-deformation model is used in the analysis, the damage may be related to the plastic deformation Y_i shown in Fig. 7. If a simple elasto-plastic model is used, X_i and Y_i are related as $X_i=Y_i+2X_y$. Substituting this into Eq. (10) yields

$$d_i=\frac{1}{2}\frac{\left\{1+\dfrac{Y_i}{2X_y}\right\}^b}{c} \tag{11}$$

Eq. (11) gives the damage based on the *plastic deformation* criterion. Note that the damage defined in Eq. (11) is nonconservative since the values of Y_i for general hysteretic models would be smaller than those for the elasto-plastic model; that is, $1+Y_i/(2X_y) < X_i/(2X_y)$.

Fig. 8 shows the occurrence pattern of the peaks and troughs in a part of the time histories of the random deflection process. The cumulative damage shown in Fig. 9 shows the damage accumulation based on Eq. (10). Fig. 10 shows the occurrence pattern of the plastic deformation and the fluctuation of the neutral axis obtained from Eq. (11).

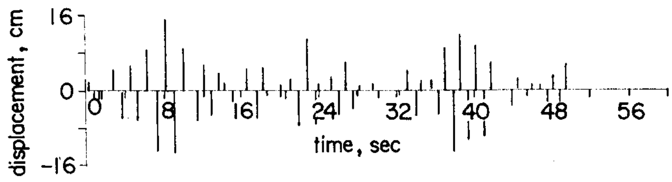

Fig. 8. Occurrence of Peaks and Troughs.

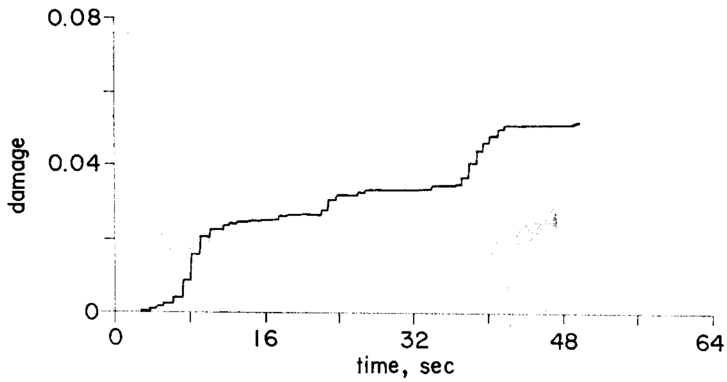

Fig. 9. Fatigue Damage Accumulation Based on Eq. (10).

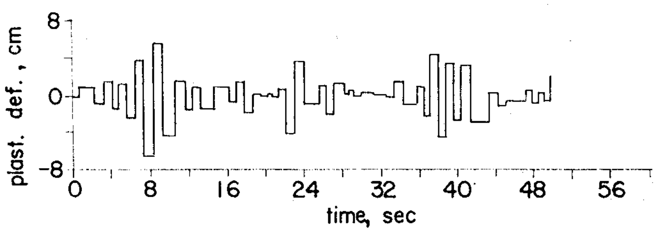

Fig. 10. Occurrence of Plastic Deformation.

The estimated fatigue life T can be defined as the time until the cumulative damage reaches a critical value D_{cr}. Then T may be evaluated from the following relation.

$$T=\frac{D_{cr}}{\nu_D} \tag{12}$$

in which ν_D is the damage rate defined by

$$\nu_D=\frac{D^*}{T^*} \tag{13}$$

where D^* is the damage accumulated during the loading period of T^*.

Experimental results of damage rate ν_D are listed in Table 4. They have been obtained for the averaging time of $T^*=50$ sec. The values of ν_D are also plotted in

Table 4 Damage Rate

σ_X/X_y	peak-trough		plastic deformation	
	D^* ($T^*=50$ sec)	ν_D, (sec^{-1})	D^* ($T^*=50$ sec)	ν_D, (sec^{-1})
1.42	1.13×10^{-2}	2.26×10^{-4}	1.03×10^{-2}	2.06×10^{-4}
1.58	1.81×10^{-2}	3.62×10^{-4}	1.68×10^{-2}	3.36×10^{-4}
2.33	5.20×10^{-2}	10.40×10^{-4}	4.13×10^{-2}	8.25×10^{-4}
2.85	7.63×10^{-2}	15.25×10^{-4}	6.34×10^{-2}	12.67×10^{-4}

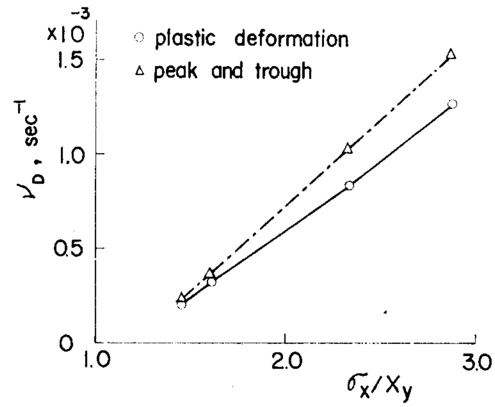

Fig. 11. Damage Rate vs. Deformation Level.

Fig. 11. If we assume that the value of ν_D in Table 4 applies to all test cases, the mean predicted fatigue life μ_T is represented by

$$\mu_T = \frac{\mu_{D_{cr}}}{\nu_D} \tag{14}$$

in which $\mu_{D_{cr}}$ is the mean critical damage. Under the assumption of linear cumulative damage, $\mu_{D_{cr}}$ may be presumed to be unity. Then Eq. (14) yields

$$\mu_T = \frac{1}{\nu_D} \tag{14'}$$

The mean estimated life calculated from Eq. (14′) using the values of ν_D in Table 4 is shown in Fig. 12 along with the actual observed life. Fig. 12 demonstrates that the estimated life μ_T tends to overestimate the actual random fatigue life. Thus in estimating the mean fatigue life, it will be necessary to use Eq. (14) instead of (14′) and adopt a value smaller than unity for the mean critical damage $\mu_{D_{cr}}$. The value of $\mu_{D_{cr}}$ can be evaluated by using the statistics of D_{cr} represented by

$$D_{cr} = \nu_D T \tag{15}$$

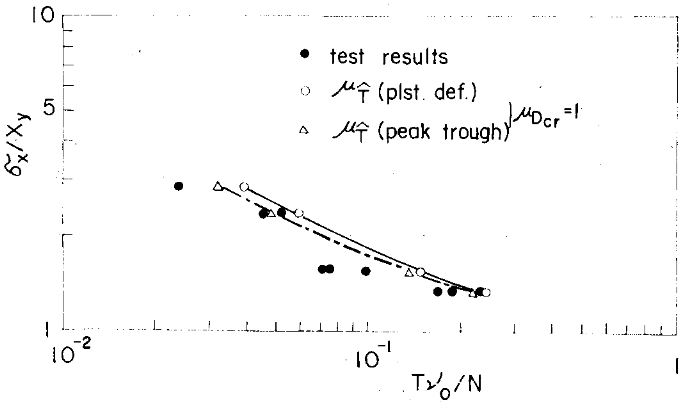

Fig. 12. Estimated life for $\mu_{D_{cr}}=1$.

Table 5 Critical Damage in Random Fatigue

σ_X/X_y	T (sec)	critical damage D_{cr}	
		peak-trough	plastic deformation
1.42	4476	1.01	0.921
	3403	0.769	0.700
	3777	0.854	0.777
1.58	1449	0.525	0.486
	1454	0.527	0.488
	1957	0.709	0.657
2.33	921	0.957	0.760
	918	0.954	0.757
	1040	1.08	0.858
2.85	481	0.733	0.610
sample mean		0.812	0.701
sample COV		0.238	0.206

which has been reduced from Eq. (12) through replacing T by observed life T. The values of D_{cr} corresponding to the specific test results of T are listed in Table 5. Within these test results, there seems to exist no definite dependence of D_{cr} on the deflection level σ_X/X_y. At this stage, therefore, the sample mean for all results may be adopted as an estimator for $\mu_{D_{cr}}$ and the sample coefficient of variation for the uncertainty of the estimated life T based on Eq. (12).

Thus, based on Table 5, we may adopt following the values for the mean critical damage $\mu_{D_{cr}}$ and the coefficient of variation $\delta_{D_{cr}}$:

peak-trough criterion:

$\mu_{D_{cr}}=0.812$, $\delta_{D_{cr}}=0.238$

plastic deformation criterion:

$\mu_{D_{cr}}=0.701$, $\delta_{D_{cr}}=0.206$

This result suggests, within the range of the present test results, that the critical damage D_{cr} in random fatigue be reduced compared to constant amplitude tests by about 20% when evaluated from the peak-trough criterion, and 30% when evaluated from the plastic deformation criterion. The value of some 20–24% for the coefficient of variation $\delta_{D_{cr}}$ may be compared with δ_N and δ_T in Tables 1 and 3.

4. Conclusions

This study presented the results of low-cycle fatigue tests of H-section SS41 steel bars in plastic bending in order to investigate the effectiveness of the cumulative damage law as the criterion of the low-cycle random flexural fatigue failure. Statistical estimates of random parameters used in this study will be useful in the uncertainty analysis of fatigue life prediction.

The hollowing conclusions may be derived from the results.

(1) Constant-amplitude tests and random deflection tests resulted in SN regression lines with very close values of the slope parameter b. This would imply that the random fatigue life may be estimated closely from the constant-amplitude test results. Statistical properties of regression parameters and fatigue life estimates were also discussed.

(2) The fatigue life under a stationary random deflection may be estimated from constant-amplitude test results by using the equivalent amplitude factor ξ. Its mean value of $\mu_\xi=1.54$ and the coefficient of variation of $\delta_\xi=0.061$ were obtained from the test results.

(3) Peak-trough criterion and plastic deformation criterion were introduced for evaluating the cumulative damage in random fatigue. The estimated mean random fatigue life agrees with the experimental results for the mean critical damage $\mu_{D_{cr}}=0.812$ for the peak-trough criterion and $\mu_{D_{cr}}=0.701$ for the plastic deformation method. The coefficient of variation of the critical damage is some 20%.

References

1) Ang, A. H-S., and Tang, W. H., "Probability Concepts in Engineering and Design, Volume I," John Wiley, 1975.

2) Corten, H. T., and Dolan, T. J., "Cumulative Fatigue Damage," *International Conference on Fatigue of Metals,* IME-ASME, 1956.

3) Crandall, S. H. and Mark, W. D., "Random Vibration in Mechanical Systems," Academic Press, 1963.

4) Freudenthal, A. M., and Heller, R. A., "On Stress Interaction in Fatigue and a Cumulative Damage Rule," *Journal of Aerospace Science,* Vol. 26, No. 7, 1959.
5) Goto, H., Kameda, H., Koike, T., Izunami, R., Wakita, K., and Sugihara, Y., "A Consideration on Failure Process of Structural Steel under Repeated Flexural Loads," *Bulletin of the Disaster Prevention Research Institute,* Kyoto University, No. 17B, April, 1974, pp. 157–169, (in Japanese).
6) Goto, H., Kameda, H., Koike, T., Aoyama, I., and Wakita, K., "Probabilistic Considerations on Plastic Fatigue Failure of Structural Steel," *Bulletin of the Disaster Prevention Research Institute,* Kyoto University, No. 18B, April, 1975, pp. 377–393, (in Japanese).
7) Kasiraj, I., and Yao, J. T. P., "Fatigue Damage in Seismic Structures," *Journal of the Structural Division,* ASCE, Vol. 95, No. ST8, Aug., 1969, pp. 1673–1692.
8) Miner, M. A., "Cumulative Damage in Fatigue," *Journal of Applied Mechanics,* ASME, Vol. 12, 1945, pp. A159–A164.
9) Stefaniuk, A. R., and Munse, W. H., "Fatigue Behavior of Butt Welded Joints under a Random Loading," *Technical Report,* Dept. of Civil Engineering, Univ. of Illinois, 1975.
10) Suidan, M. T., and Eubanks, R. A., "Cumulative Fatigue Damage in Seismic Structures," *Journal of the Structural Division,* ASCE, Vol. 99, No. ST5, May, 1973, pp. 923–943.
11) Tang, J.-P., and Yao, J. T. P., "Expected Fatigue Damage of Seismic Structures," *Journal of the Engineering Mechanics Division,* ASCE, Vol. 98, No. EM3, June, 1972, pp. 695–709.
12) Yang, J.-N., "Statistics of Random Loading Relevant to Fatigue," *Journal of the Engineering Mechanics Division,* ASCE, Vol. 100, No. EM3, June, 1974, pp. 469–475.
13) Yao, J. T. P., and Munse, W. H., "Low-cycle Axial Fatigue Behavior of Mild Steel," *Special Technical Publication,* No. 338, ASTM, 1962, pp. 5–24.
14) Yao, J. T. P., "Fatigue Reliability and Design," *Journal of the Structural Division,* ASCE, Vol. 100, No. ST9, 1974, pp. 1827–1836.
15) Yokobori, T., "Strength, Fracture and Fatigue of Materials," Gihodo Co., Tokyo, 1955, (in Japanese).