An Application of Stochastic Control Theory to the Gust-Alleviation System for a Transport Airplane

By

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Abstract

The optimal control law that minimizes the gust responses of an airplane's longitudinal motion is obtained, making assumptions that the airplane is approximated as a point and that aeroelastic problem is ignored. The airplane gust-alleviation problem has been treated mainly in the frequency domain because of the simplicity of the input-output relations for the power spectra. But in the optimal control problem, the approach in the time domain, applying the optimal stochastic control theory, seems to have more advantages for investigating such a complex control as gust-alleviation. The system state equations consist of the short-period equation of an airplane and the gust shaping filter. The optimal linear control law is derived by applying the Matrix Minimum Principle which minimizes the R.M.S. values of the normal acceleration and the pitch rate at the center of gravity. The results of the numerical calculation for two types of control systems, one being the linkage-control system and the other the independent-control system, are shown in the case of a conventional transport. The latter system is ascertained to have a fairly better performance. The optimal system is also ascertained to have very low sensitivity to the change of the scale of turbulence.

Nomenclature

u_0 : flgit velocity
$\bar{c}$: mean aerodynamic chord
m : mass of airplane
S : wing area
I_B : moment of inertia of airplane about y-axis
ρ : air density
g : acceleration of gravity
$\mu = \dfrac{m}{\rho S(\bar{c}/2)}$: relative mass parameter

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$i_B=\dfrac{I_B}{\rho S(\bar{c}/2)^3}$: non-dimensional moment of inertia
t : time
$t^*=\bar{c}/2\mu$
$\hat{t}=t/t^*$: non-dimensional time
α : angle of attack
θ : pitch angle
$q=\dot{\theta}$: pitch rate
δ_f : flap deflection angle
δ_e : elevator deflection angle
n_z : normal acceleration factor
w_g : vertical gust velocity
ω : circular frequency
$k=\dfrac{\omega\bar{c}}{2u_0}$: reduced frequency
L : representative scale of atmospheric turbulence
σ_g : R.M.S. value of the gust velocity
$\wedge$: non-dimensional quantity
suffix g: quantities due to gusts
Φ : power spectral density
$\boldsymbol{x}$: state vector
$\boldsymbol{u}$: control vector
v : white noise
$\boldsymbol{d}$: response vector
$\boldsymbol{F}, \boldsymbol{G}, \boldsymbol{C}, \boldsymbol{A}, \boldsymbol{B}$: coefficient matrices
J : criterion functional (performance index)
$\boldsymbol{W}, \boldsymbol{Z}$: weighting matrices
$\boldsymbol{K}$: feed-back gain matrix
$\boldsymbol{X}$: state second-moment matrix
$\boldsymbol{P}$: co-state matrix associated with $\boldsymbol{X}$
H : Hamiltonian function
^{T} : transpose of vectors and matrices

I. Introduction

The purpose of a gust-alleviation system for an airplane is the reduction of oscillating motion which is generated by gusty air; and it is important from the viewpoint of passenger comfort and structual fatigue. Since atmospheric turbulence affects the airplane as a continuous random disturbance, this problem should be treated with methods of statistical reasoning.

There have been many analytical investigations of this problem. For example, K. Nakagawa et al. obtained the performances of several gust-alleviation systems for a typical conventional transport, analyzed by the classical control theory and Chan's optimal control theory in the frequency domain[1,2]. Also, R.A. Hess advanced the analysis in the time domain and investigated the feasibility of the optimal gust-alleviation system[3].

In this paper, the gust is considered as the stationary random disturbance to an airplane; and a gust-alleviation system for the airplane's longitudinal motion is designed by using the stochastic control theory.

II. Gust Response of an Airplane

In order to make the analysis simple, the following assumptions and approximations are introduced.

i) The airplane is rigid. And the aeroelastic effects on the airplane are ignored.

ii) The gust is normal and one-dimensional, so that the span-wise change of the gust is neglected.

Under these assumptions, we can derive the longitudinal equations of the motion of an airplane in a horizontal flight condition[4].

Between the two kinds of modes which occur in a longitudinal motion, it may be considered that the phugoid mode is not so important for the mean square gust response. Therefore, we will treat only the short-period mode here-after.

In addition to the elevator system, the wing-flaps should also be used for control, because it is obvious that the elevator system alone cannot be expected to control the airplane response sufficiently. In other words, to alleviate the oscillating motion induced by gust, a more powerful direct lift control system is required.

For the analysis of the gust response of an airplane, some additional assumptions and approximations are made as follows:

If the wave-length of the gust, λ, is considerably large relative to the length of airplane, (i.e. $\lambda \geqq 8l$ or the reduced frequency $k \leqq k_0 = \pi \bar{c}/8l$, $\bar{c}$; mean aerodynamic chord l; airplane length) the wave-form of the gust can be approximated by the linear segment. Then, the aerodynamic effect of gust velocity w_g is equivalent to the change of angle of attack α_g, and the gradient in w_g is equivalent to the change of the non-dimensional pitch-rate $\hat{q}_g$,

$$\alpha_g = -w_g/u_0 \qquad (1)$$

$$\hat{q}_g = t^* \frac{\partial w_g}{\partial x} = -t^* u_0 \frac{\partial \alpha_g}{\partial x} \tag{2}$$

Furthermore, it is assumed that the gust field is "frozen", i.e. the statistical characteristics of the turbulence do not depend on time during the flight through this patch of turbulence. Accordingly, the space derivative may be transformed into the time derivative as follows;

$$\hat{q}_g = -\frac{\partial \alpha_g}{\partial \hat{t}} = -D\alpha_g \quad \left(D = \frac{\partial \cdot}{\partial \hat{t}}\right) \tag{3}$$

The effective angle of attack $(\alpha + \alpha_g)$ and the effective pitch-rate $(\hat{q}+\hat{q}_g)$ are used to calculate the aerodynamic forces and moments. Then, in the case of the short-period approximation, the equations of an airplane's motion induced by the gust are given as follows:

$$\begin{aligned} (2\mu D + C_{z\alpha})\alpha \qquad\qquad -2\mu\hat{q} &= C_{z\alpha}\alpha_g + C_{z\delta_f}\delta_f + C_z\delta_e\delta_e \\ -(C_{m\dot{\alpha}}D + C_{m\alpha})\alpha + (i_B D - C_{m_q})\hat{q} &= (C_{m\dot{\alpha}}D - C_{m_q}D + C_{m\alpha})\alpha_g \\ &\quad + C_{m\delta_f}\delta_f + C_{m\delta_e}\delta_e \end{aligned} \tag{4}$$

where $C_{z\dot{\alpha}}$ and C_{z_q} are assumed to be negligible.

Next, since the gust is assumed to be one-dimensional, the power spectrum of gust given by Dryden is available. However in this analysis, for simplicity the following approximate equation is used instead of the original Dryden model[1].

$$\Phi_g(k) = \frac{H^2}{I^2 + k^2} \tag{5}$$

where

$$H^2 = 3\sigma_g^2 \bar{c} / 4\pi L$$
$$I = 0.725 \bar{c} / L$$

and σ_g is the intensity of gust and L is the scale of turbulence.

In order to advance the analysis in time-domain and apply the stochastic control theory to this problem, "shaping filter" should be introduced (Fig. 1). The gust velocity w_g with assigned power spectrum, Eq. (5), can be generated by a white noise through this filter. The transfer function of such a filter is expre-

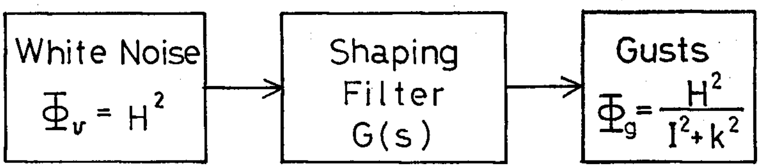

Fig. 1. Shaping filter.

ssed by the following form, if the power-spectral density of the input white noise is H^2.

$$G(s) = \frac{1}{I+s} \tag{6}$$

Therefore, the differential equation which α_g satisfies is given as follows:

$$D\alpha_g = -I\alpha_g + v \tag{7}$$

where v is a white noise with the co-variance $E\{v(t)v(\tau)\} = (H^2/u_0^2)\delta(t-\tau)$.

Putting the above equations in matrix form, the state equation of the gust-alleviation system is expressed as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{C}v \tag{8}$$

where $\boldsymbol{x} = [\alpha, \theta, \hat{q}, \alpha_g]^T$ is a state vector and $\boldsymbol{u} = [\delta_f, \delta_e]^T$ is a control vector. The matrices $\boldsymbol{F}$, $\boldsymbol{G}$ and $\boldsymbol{C}$ are given by

$$\boldsymbol{F} = \begin{pmatrix} C_{z\alpha}/2\mu & 0 & 1 & C_{z\alpha}/2\mu \\ 0 & 0 & 1 & 0 \\ \dfrac{C_{m\dot{\alpha}}C_{z\alpha}/2\mu + C_{m\alpha}}{i_B} & 0 & \dfrac{C_{m\dot{\alpha}} + C_{m_q}}{i_B} & \dfrac{C_{m\dot{\alpha}}C_{z\alpha}/2\mu + C_m - I(C_{m\dot{\alpha}} - C_{m_q})}{i_B} \\ 0 & 0 & 0 & -I \end{pmatrix},$$

$$\boldsymbol{G} = \begin{pmatrix} C_{z\delta_f}/2\mu & C_{z\delta_e}/2\mu \\ 0 & 0 \\ \dfrac{C_{m\dot{\alpha}}C_{z\delta_e}/2\mu + C_{m\delta_f}}{i_B} & \dfrac{C_{m\dot{\alpha}}C_{z\delta_e}/2\mu + C_{m\delta_e}}{i_B} \\ 0 & 0 \end{pmatrix}, \quad \boldsymbol{C} = \begin{pmatrix} 0 \\ 0 \\ \dfrac{C_{m\dot{\alpha}} - C_{m_q}}{i_B} \\ 0 \end{pmatrix}$$

The normal acceleration and the pitch-rate at the center of gravity of the airplane are adopted to assess the performance of gust-alleviation systems. The reason is because these quantities are the principal factors which affect passenger comfort or structual fatigue. In order to non-dimensionalize these variables, the normal acceleration factor n_z is introduced as follows:

$$\begin{aligned} n_z &= \frac{\text{normal acceleration at } C.G.}{g} \\ &= -\frac{2u_0^2}{g\bar{c}}\left(\frac{C_{z\alpha}}{2\mu}\alpha + \frac{C_{z\alpha}}{2\mu}\alpha_g + \frac{C_{z\delta_f}}{2\mu}\delta_f + \frac{C_{z\delta_e}}{2\mu}\delta_e\right) \end{aligned} \tag{9}$$

According to the random process theory for a linear system, the power-spectra of a normal acceleration factor and a non-dimensional pitch-rate, i.e. n_z and $\hat{q}$

can be related with the input, or the power spectrum of gust as follows:

$$\Phi_{n_z}(k) = \left| \frac{n_z}{w_g}(jk) \right|^2 \Phi_g(k) \tag{10}$$

$$\Phi_{\hat{q}}(k) = \left| \frac{q}{w_g}(jk) \right|^2 \Phi_g(k) \tag{11}$$

The mean square values of response can be obtained by integrating the power-spectra with respect to k throughout frequency range.

III. Gust-Alleviation by Stochastic Control Theory

Fig. 2 shows the block diagram of the optimal stochastic control system. Since the gust is random, the state variable $\boldsymbol{X}$ is also a random one. Therefore, the state of an airplane should be estimated by several observed quantities. However, in order to foucs our attention on the optimal control problem, it will be assumed that the instantaneous and exact measurements of the state are possible.

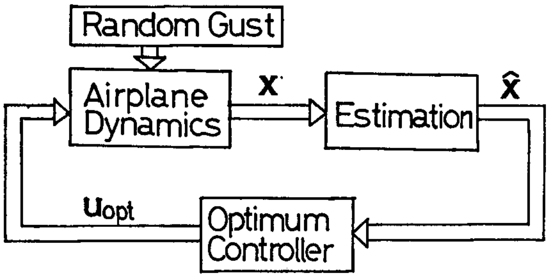

Fig. 2, Brock diagram of a stochastic optimal linear control system.

Then, let the performance index be given by

$$J = E\left\{ \lim_{\hat{T}\to\infty} \frac{1}{\hat{T}} \int_0^{\hat{T}} (\boldsymbol{d}\boldsymbol{W}\boldsymbol{d} + \boldsymbol{u}\boldsymbol{Z}\boldsymbol{u})\, d\hat{t} \right\} \tag{12}$$

$$\boldsymbol{W} = \begin{bmatrix} w_1^2 & 0 \\ 0 & w_2^2 \end{bmatrix}, \quad \boldsymbol{Z} = \begin{bmatrix} r_1^2 & 0 \\ 0 & r_2^2 \end{bmatrix}$$

where $E\{\cdot\}$ means expected values and $\boldsymbol{W}$ and $\boldsymbol{Z}$ are positive definite weighting matrices. Furthermore, putting $\boldsymbol{d}=[n_z, \hat{q}]^T$, this is the response vector given by

$$\boldsymbol{d} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{13}$$

where
$$\boldsymbol{A} = \begin{pmatrix} -\left(\frac{2u_0^2}{g\bar{c}}\right)\frac{C_{z\alpha}}{2\mu} & 0 & 0 & -\left(\frac{2\mu_0^2}{g\bar{c}}\right)\frac{C_{z\alpha}}{2\mu} \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\boldsymbol{B}=\begin{pmatrix} -\left(\frac{2u_0^2}{g\bar{c}}\right)\frac{C_{z\delta_f}}{2\mu} & -\left(\frac{2u_0^2}{g\bar{c}}\right)\frac{C_{z\delta_e}}{2\mu} \\ 0 & 0 \end{pmatrix}.$$

Accordingly, the optimal control problem can be treated as follows:

"Given the state equation of the system expressed by Eq. (8), and assuming that the states may be observed exactly, determine the optimal linear feed-back control law $\boldsymbol{u}=\boldsymbol{Kx}$ which minimizes the criterion functional J given by Eq. (12)."

By introducing the state second-moment matrix $\boldsymbol{X}$; $\boldsymbol{X}=E\{\boldsymbol{x}\cdot\boldsymbol{x}^T\}$, the covariance equation of the state equation, Eq.(8), can be derived as follows:

$$\dot{\boldsymbol{X}}=(\boldsymbol{F}+\boldsymbol{GK})\boldsymbol{X}+\boldsymbol{X}(\boldsymbol{F}+\boldsymbol{GK})^T+(H^2/u_0^2)\boldsymbol{CC}^T \tag{14}$$

Exchanging the order of integration and expectation, the performance index J may be transformed into the following form after some manipulations.

$$J=\text{tr.}[\{(\boldsymbol{A}+\boldsymbol{BK})\boldsymbol{W}(\boldsymbol{A}+\boldsymbol{BK})^T+\boldsymbol{KZK}^T\}\boldsymbol{X}] \tag{15}$$

In order to solve this problem, *the Matrix Minimum Principle* by Athans is applied as follows:[6]

The Hamiltonian function H is introduced into this problem as follows:

$$H=\text{tr.}[(\boldsymbol{A}+\boldsymbol{BK})\boldsymbol{W}(\boldsymbol{A}+\boldsymbol{BK})^T\boldsymbol{X}+\boldsymbol{KZK}^T\boldsymbol{X}]+\text{tr.}[\dot{\boldsymbol{X}}\boldsymbol{P}] \tag{16}$$

where $\boldsymbol{P}$ is the co-state matrix associated with the state second-moment matrix $\boldsymbol{X}$. If $\boldsymbol{K}^*$ denotes the optimal feed-back gain matrix which minimizes J, and if $\boldsymbol{X}^*$ is the state matrix which corresponds to $\boldsymbol{K}^*$, there exists a co-state matrix $\boldsymbol{P}^*$ that satisfies the following canonical equations and minimizes the Hamiltonian function H.

Canonical equations are given as follows:

$$\left.\frac{\partial H}{\partial \boldsymbol{P}}\right|_*=\dot{\boldsymbol{X}}^*=(\boldsymbol{F}+\boldsymbol{GK}^*)\boldsymbol{X}^*+\boldsymbol{X}^*(\boldsymbol{F}+\boldsymbol{GK}^*)^T+(H^2/u_0^2)\boldsymbol{CC}^T=\boldsymbol{0} \tag{17}$$

$$\left.\frac{\partial H}{\partial \boldsymbol{X}}\right|_*=-\dot{\boldsymbol{P}}^*=(\boldsymbol{A}+\boldsymbol{BK}^*)^T\boldsymbol{W}(\boldsymbol{A}+\boldsymbol{BK}^*)+\boldsymbol{K}^{*T}\boldsymbol{P}^*\boldsymbol{K}^* \\ +(\boldsymbol{F}+\boldsymbol{GK}^*)^T\boldsymbol{P}^*+\boldsymbol{P}^*(\boldsymbol{F}+\boldsymbol{GK}^*)=\boldsymbol{0} \tag{18}$$

If it is assumed that $\boldsymbol{K}$ is not constrained, then the minimization of the Hamiltonian function H implies the following necessary condition.

$$\left.\frac{\partial H}{\partial \boldsymbol{K}}\right|_*=[\boldsymbol{B}^T\boldsymbol{WA}+\boldsymbol{G}^T\boldsymbol{P}^*+(\boldsymbol{B}^T\boldsymbol{WB}+\boldsymbol{Z})\boldsymbol{K}^*]\boldsymbol{X}^*=\boldsymbol{0} \tag{19}$$

From this equation, $\boldsymbol{K}^*$ can be solved as follows:

$$\boldsymbol{K}^* = -(\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{B}+\boldsymbol{Z})^{-1}(\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{A}+\boldsymbol{G}^T\boldsymbol{P}^*) \tag{20}$$

In the above equation, the co-state matrix $\boldsymbol{P}^*$ is the solution of the following Riccati equation of matrix form.

$$\begin{aligned}&[\boldsymbol{F}-\boldsymbol{G}(\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{B}+\boldsymbol{Z})^{-1}\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{A}]^T\boldsymbol{P}^*+\boldsymbol{P}^*[\boldsymbol{F}-\boldsymbol{G}(\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{B}+\boldsymbol{Z})^{-1}\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{A}]\\&-\boldsymbol{P}^*\boldsymbol{G}(\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{B}+\boldsymbol{Z})^{-1}\boldsymbol{G}^T\boldsymbol{P}^*+\boldsymbol{A}^T\boldsymbol{W}[\boldsymbol{I}-\boldsymbol{B}(\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{B}+\boldsymbol{Z})^{-1}\boldsymbol{B}^T\boldsymbol{W}]\boldsymbol{A}=\boldsymbol{0}\end{aligned} \tag{21}$$

where $\boldsymbol{I}$ is a unit matrix.

IV. Examples of Numerical Calculations

In order to illustrate the performance of the optimal gust-alleviation system, let us calculate a numerical example of a conventional transport. Flight conditions and stability derivatives of this transport are shown in Table 1. It is assumed that the scale of turbulence L is 300 m and the intensity of gust σ_g is 2.7 m/sec.

Table 1. Aerodynamic data of the model transport

Dimensions of the Model Transport		
W	Airplane weight	21,800 Kg
S	Wing area	94.8 m^2
$\bar{c}$	Mean aerodynamic chord	3.2 m
I_B	Moment of inertia of airplane about y-axis	50,300 Kgm·sec^2
l	Total length	26.3 m
u	Flight velocity	459 Km/h (123 m/sec)
	Cruising altitude	6,100 m

Aerodynamic Derivatives		
$C_{z\alpha} = -5.82$	$C_{m_q} = -34.8$	$C_{m\dot{\alpha}} = -1.83$
$C_{m\alpha} = -9.50$	$C_{z\delta_f} = -1.65$	$C_{z\delta_e} = -0.40$
$C_{m\delta_f} = -0.30$	$C_{m\delta_e} = -1.74$	

In this paper, we calculated two types of control systems. The first one is the linkage-control system (Fig. 3(a)), and in this system it is assumed that the elevator and the wing-flaps are linked with a fixed operation ratio r. The value r should be determined to satisfy the static equilibrium condition of the airplane as follows:[1)]

$$r = \frac{C_{m\alpha}C_{z\delta_f}-C_{z\alpha}C_{m\delta_f}}{C_{z\alpha}C_{m\delta_e}-C_{m\alpha}C_{z\delta_e}} \tag{22}$$

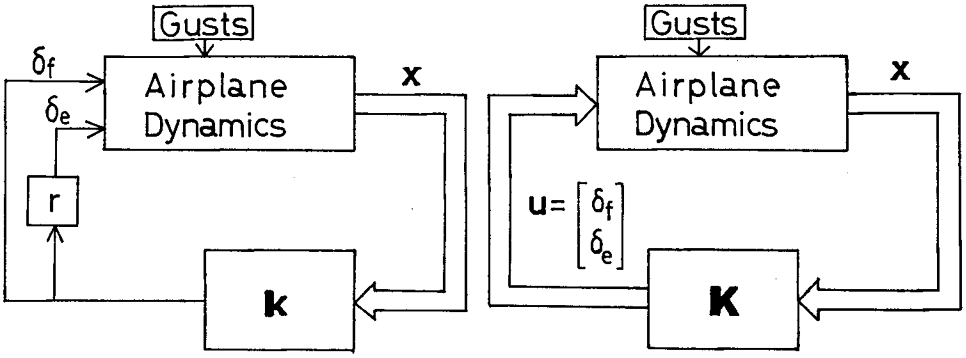

Fig. 3 (a). The Linkage-control system. Fig. 3 (b). The Independent-control system.

The next one is called the independent-control system (Fig. 3(b)), in which two kinds of controllers, that is, elevator and flap, can be operated independently. The latter is the "exact optimal" system, but the former has the advantage of simplicity in calculation and realization.

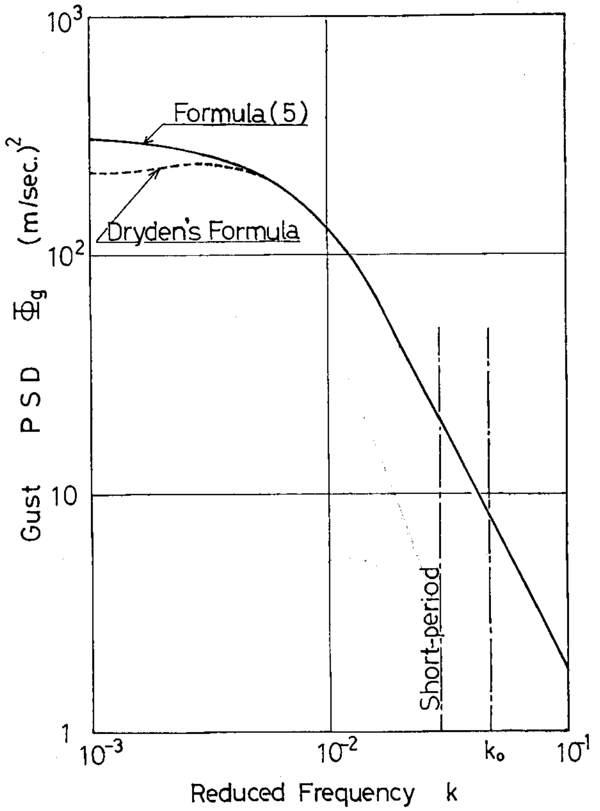

Fig. 4. Power spectral density of gusts; $L=300$ m, $\sigma_g=2.7$ m/sec.

Fig. 4 shows the power-spectral density of the gust. In this figure it is shown the approximate equation previously mentioned agrees well with Dryden's spectral density for the frequency range of a short-period mode. It should be noted that k_0 inidicates the limit point of point approximation.

Applying the analyses given above to this example, the optimal control laws can be determined, and the opwer-spectral densities as well as the R.M.S. values of the responses can be calculated for the controlled airplane.

First, in the case of the linkage-control system, the power-spectral densities of the normal acceleration factor and those of pitch-rate are illustrated in Figs. 5(a) and 5(b) respectively. From these figures it is found that the optimal gust-alleviation system is remarkably effective for this transport. It is also obvious that the power-spectral densities of the responses are decreased with the reduction of γ, which is the weighting factor for the control term in the performance index. Fig. 6 shows the relation between the R.M.S. values of gust responses and those of controller (i.e. elevator and flap) deflections.

Next, Figs. 7(a) and 7(b) shows the R.M.S. responses of the independent-control system. Compared with the linkage-control system, the effectiveness of this system seems fairly better, which is indicated in the figures.

In order to examine the sensitivity of the controlled systems to the change of

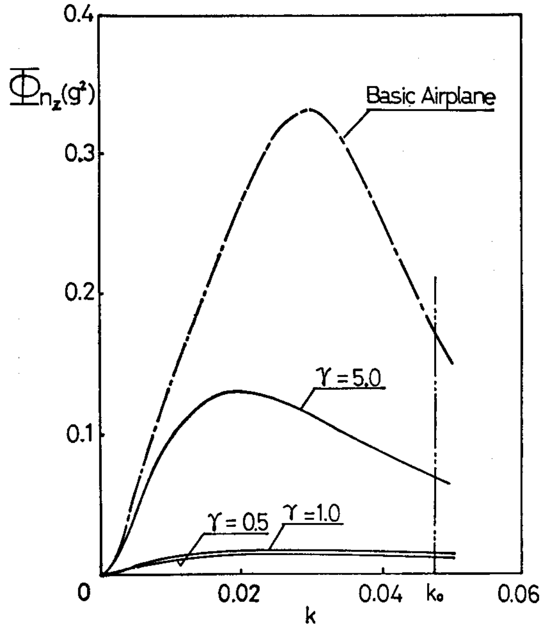

Fig. 5 (a). Power spectral density of the normal acceleration factor (the Linkage-control system).

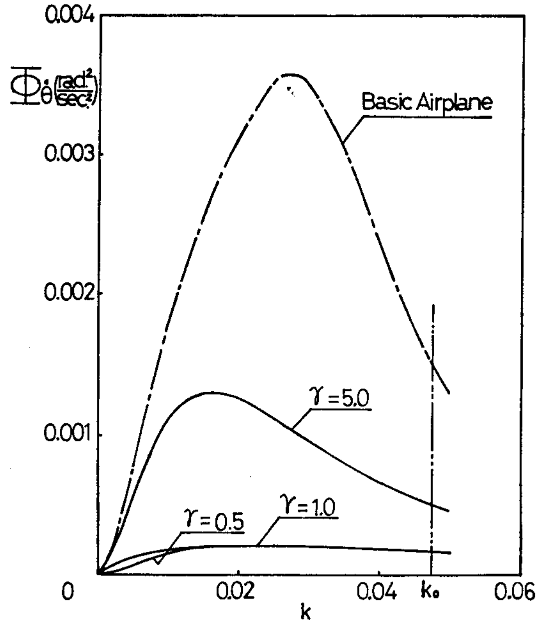

Fig. 5 (b). Power spectral density of the pitch rate (the Linkage-control system).

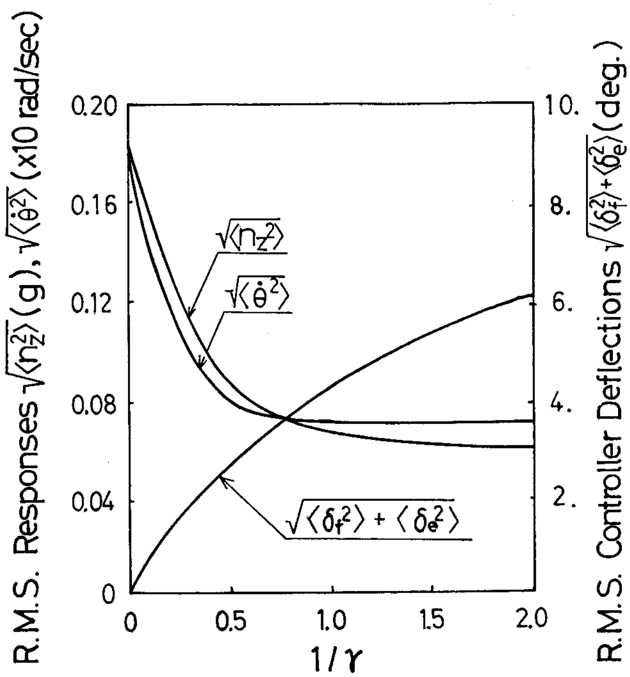

Fig. 6. R.M.S. values of the responses and those of the controller deflections (the Linkage-control system).

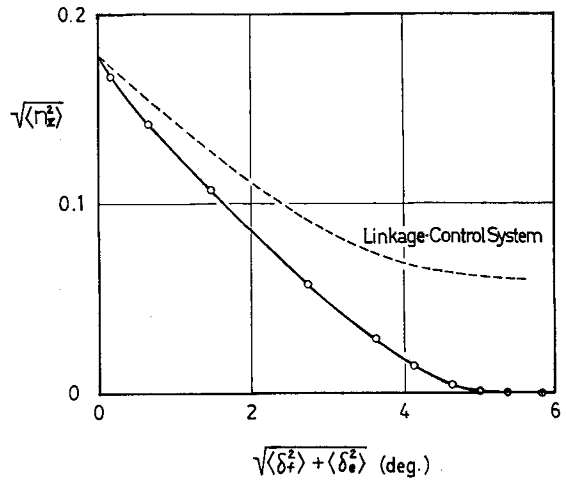

Fig. 7 (a). R.M.S. values of the normal acceleration factor (the Independent-control system).

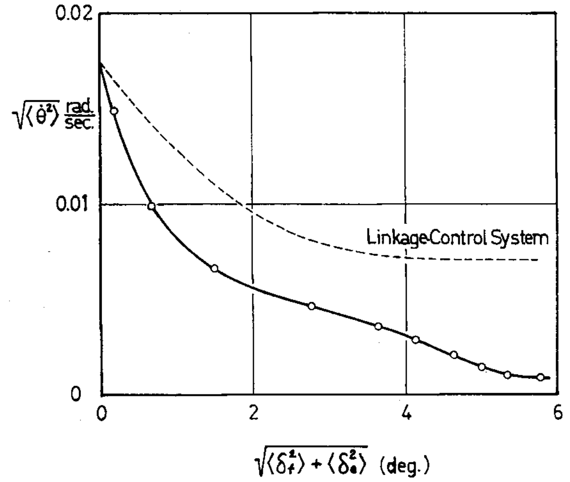

Fig. 7 (b). R.M.S. values of the pitch rate (the Independentcontrol system).

gust wave-length, the effects of scale of turbulence L to the responses are indicated in Figs. 8(a) and 8(b). It is found that the responses of optmally controlled systems have not only lower levels at the nominal value of L (i.e. $L_0=300$ m), but also have a lower sensitivity for the change of the scale of turbulence.

V. Conculsion

Applying the stochastic control theory, we obtained the optimal control law that minimizes gust responses of an airplane under some assumptions and approximations.

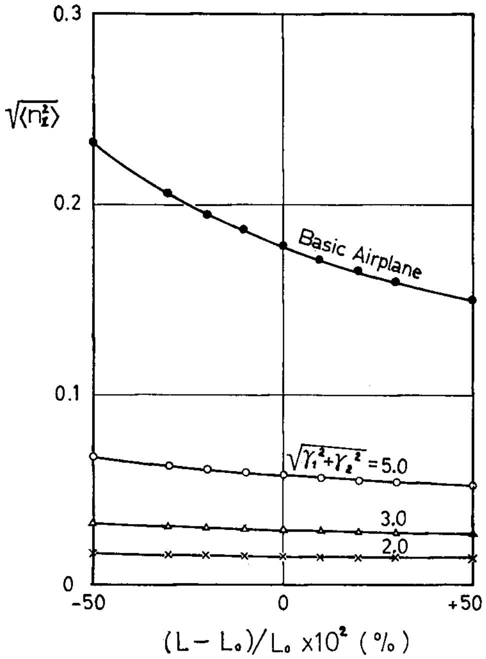

Fig. 8 (a). Sensitivity of the gust-alleviation systems to the change of the scale of turbulence; normal acceleration factor.

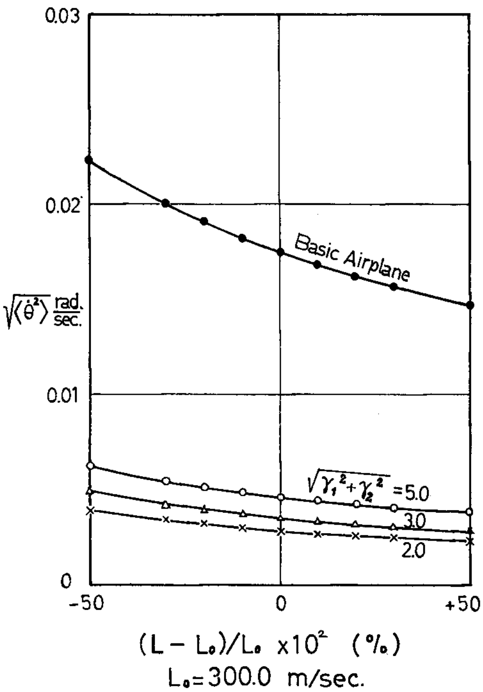

Fig. 8 (b). Sensitivity of the gust-alleviation systems to the change of the scale of turbulence; pitch rate

Two kinds of control systems are investigated, i.e. one is the linkage-control system, and the other is the independent-control system. In the case of the conventional transport used in the numerical calculation, the former system shows about 70 % reduction of the normal acceleration factor in the R.M.S. value as compared with an unalleviated airplane. This performance of alleviation has the same order as those investigated by Nakagawa.[1)]

On the other hand, utilizing the latter type of system (i.e. the independent-control system), more than 92 % reduction in the normal acceleration factor is possible, even if the flap deflection is restricted within 4 degrees in the R.M.S. value.

However, the practical realization of this system seems to be difficult, because the on-line measurement of gust velocity and the exact determination of the power-spectral density of gust are necessary. To overcome the first difficulty, an alternative sub-optimal system was suggested by Hess. In this system, instead of a direct physical measurement of gust, Eq. 9 is utilized to construct α_g indirectly by measuring n_z, α, δ_e and δ_f.

The scale of turbulence, L, is also a quantity difficult to measure. However, in this paper it is shown that the change of L scarcely influences the performance of gqet-alleviation for the present optimal system.

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