Finite Element Analysis of Foundation on Inhomogeneous Rock Mass

By

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Abstract

The finite element procedure that can allow for the nonlinear behavior of inhomogeneous rock masses and nonlinear interaction effects was described. The use of the interface and joint elements was presented. Inclusion of these elements very often change the results of the deformation behavior and the stability of foundations on rock masses. As for the typical examples, practical problems were solved and the influence of discontinuities in the rock mass was investigated.

It is found that the results are significantly influenced by material properties, which should carefully be determined from the in-situ and laboratory tests.

1. Introduction

The stability of a foundation on the rock mass has long been examined by the Terzaghi formula, which is based on the theory of the limit equilibrium. The Terzaghi formula is known to be very simple to use, but is not applied to an inhomogeneous medium, and the accuracy of the solution is sometimes in doubt. Recently, the extended theory of plasticity which is applicable to the inhomogeneous medium has been used by several researchers.[1)2)] However, such a theory has the assumption that the collapse load is unique and not dependent on the load path to collapse, and also it can not solve complex problems.

On the contrary, the use of a finite element method has become very popular in the stability analysis with the development of the high speed computer. The finite element method may be the only feasible approach to some problems, particularly practical ones involving complicated geometry and boundary conditions. And it can follow the complete load deformation behavior prior to collapse which may be of as much practical interest as the collapse load.

Application of the finite element method to the elasto-plastic problems of foundations is not uncommon. However, most of the previous studies have considered

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only the behavior of homogeneous rock masses and have not included interface or joint effects. Much has been written in recent years about the significance of planes of weakness on the stability of structures in hard rock mass. However, usually in the analysis, the discontinuous rock mass has been treated as an anisotropic mass by averaging the whole rock mass properties. Such an approach may overlook an important behavior of rock mass subjected to complex loads from foundations. Often, a large relative movement occurs between structures and rocks. Such movements and transfer of shear stresses across the interfaces must be taken into account in the stability analysis of foundations.

The purpose of this study is to develop a finite element procedure that can allow for nonlinear behavior of rock masses, nonlinear interaction effects and to investigate the behavior of foundation on the inhomogeneous rock masses.

2. Finite Element Procedure

The whole process of the construction of a foundation is a three-dimensional problem. Because of the prohibitive cost of such a three-dimensional analysis, an approximate two-dimensional idealization assuming plane strain conditions for foundations has been used.

Since the general procedure to formulate the two-dimensional continuum elements is already given in literature[4], the finite element formulation for a joint or interface element will be described.

2.1 Joint or Intreface Element

The joint element is intended to represent the rock joints, faults, interfaces and similar discontinuities in continuum systems. The joint element has the capability of representing the main characteristics of the deformation behavior of the rock joints such as debonding and slip.

Previous attempts have been made to develop such elements to represent the joint behavior[4][5]. However, in the methods previously developed, it was found that numerical difficulties may arise from a poor conditioning of the stiffness matrix due to very large off-diagonal terms or very small diagonal terms which are generated by these elements in certain cases.

In order to avoid such numerical problems, a new joint element has recently been developed, which uses relative displacements as the independent degrees of freedom[6]. For example, in a two-dimensional problem the joint element will have four degrees of freedom as shown in Fig. 1. The relative normal and tangential displacements, Δu_n and Δu_s, are assumed to vary linearly along the element as follows:

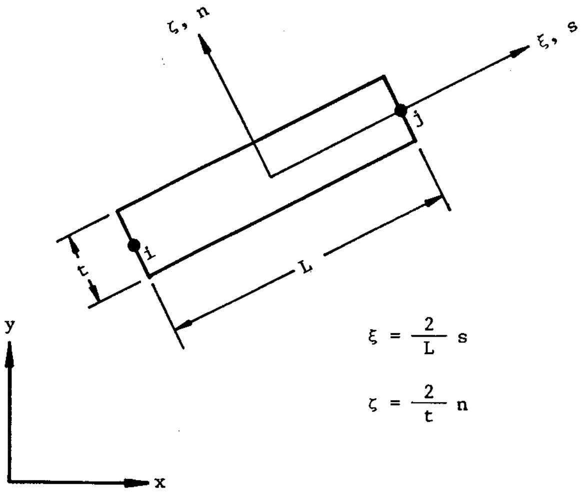

Fig. 1. Coordinate Systems for Joint Element.

$$\left.\begin{aligned} \Delta u_n &= h_i \Delta u_{ni} + h_j \Delta u_{nj} \\ \Delta u_s &= h_i \Delta u_{si} + h_j \Delta u_{sj} \end{aligned}\right\} \tag{1}$$

where the h_i and h_j are the linear interpolation functions and Δu_{ni}, Δu_{nj}, Δu_{si} and Δu_{sj} are the nodal point values of the relative displacement. The joint "strain" vector, $\{\varepsilon\}_J$, is defined by

$$\{\varepsilon\}_J = \begin{Bmatrix} \varepsilon_n \\ \varepsilon_s \end{Bmatrix}_J = [B]_J \{\Delta u\}_J \tag{2}$$

where the subscript symbol J denotes the joint element, $[B]_J$ is the strain-displacement transformation matrix which is derived from Eq. (1) and the transposition of $\{\Delta u\}_J = \{\Delta u_{ni}, \Delta u_{si}, \Delta u_{nj}, \Delta u_{sj}\}$. Simlarly, the joint "stress" vector, $\{\sigma\}_J$, is also obtained. The joint "stresses" and "strains" are linked through the following material property matrix $[C]$.

$$\begin{Bmatrix} \sigma_n \\ \sigma_j \end{Bmatrix}_J = \begin{bmatrix} C_{nn} & C_{ns} \\ C_{sn} & C_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_n \\ \varepsilon_s \end{Bmatrix}_J \tag{3}$$

In general the stress-strain relationship for rock joints is nonlinear. The stiffness matrix for the joint element is formed in the n–s coordinate system

$$[K]_{ns} = \int_V \{B\}_J^T [C] \{B\}_J dV \tag{4}$$

2.2 Constitutive Laws

The elasto-plastic constitutive law was adopted for the two-dimensional plane strain element. The Drucker-Prage model[7] extended from the Mohr-Coulomb yield condition for ideal plasticity was implemented in the computer program[8]. It is written in the form:

$$F = \alpha J_1 + \sqrt{J_2} - k = 0 \tag{5}$$

where J_1 and J_2 are stress invariants, and[3]

$$\alpha = \frac{2 \sin \phi}{\sqrt{3}\,(3-\sin \phi)}, \quad k = \frac{6c \cos \phi}{\sqrt{3}\,(3-\sin \phi)} \tag{6}$$

where c is the cohesion, ϕ is the friction angle for the Mohr-Coulomb yield criterion. At the time of the plane strain condition, Eq. (6) may be changed to:

$$\alpha = \frac{\tan \phi}{(9+12 \tan^2 \phi)^{1/2}}, \quad k = \frac{3c}{(9+12 \tan^2 \phi)^{1/2}} \tag{7}$$

The stress-strain relationship for rock joint is usually complex and is difficult to model mathematically. Fig. 2 is an example of stress-deformation relationship which is determined from laboratory tests. Dilatancy or contractancy of rock joints are often ignored. The terms C_{ns} and C_{sn} in Eq. (3) are, therefore, set to be zero.

In an idealized stress-deformation relationship in normal direction (Fig. 3), three distinct stages can be recognized;

a. Separation, $C_{nn}=C_{ss}=0$ when $\varepsilon_n \geq 0$.

b. Crushing of the surface asperities or the compression of the material in the joint, if any $C_{nn}=E_c$ when $\varepsilon_n^c < \varepsilon_n < 0$. For smooth surfaces or intefaces this case does not exist, therefore $\varepsilon_n^c=0$.

c. Contact, $C_{nn}=E_f$, which is normally a very large value. $(\varepsilon_n < \varepsilon_n^c)$

The stress-deformation relationship in shear direction (Fig. 3) is assumed to be elastic-perfectly plastic using a Mohr-Coulomb yield criterion:

$$\begin{aligned} C_{ss} &= G & \sigma_s &< c+\sigma_n \tan \phi \\ C_{ss} &= 0 & \sigma_s &= c+\sigma_n \tan \phi \end{aligned} \tag{8}$$

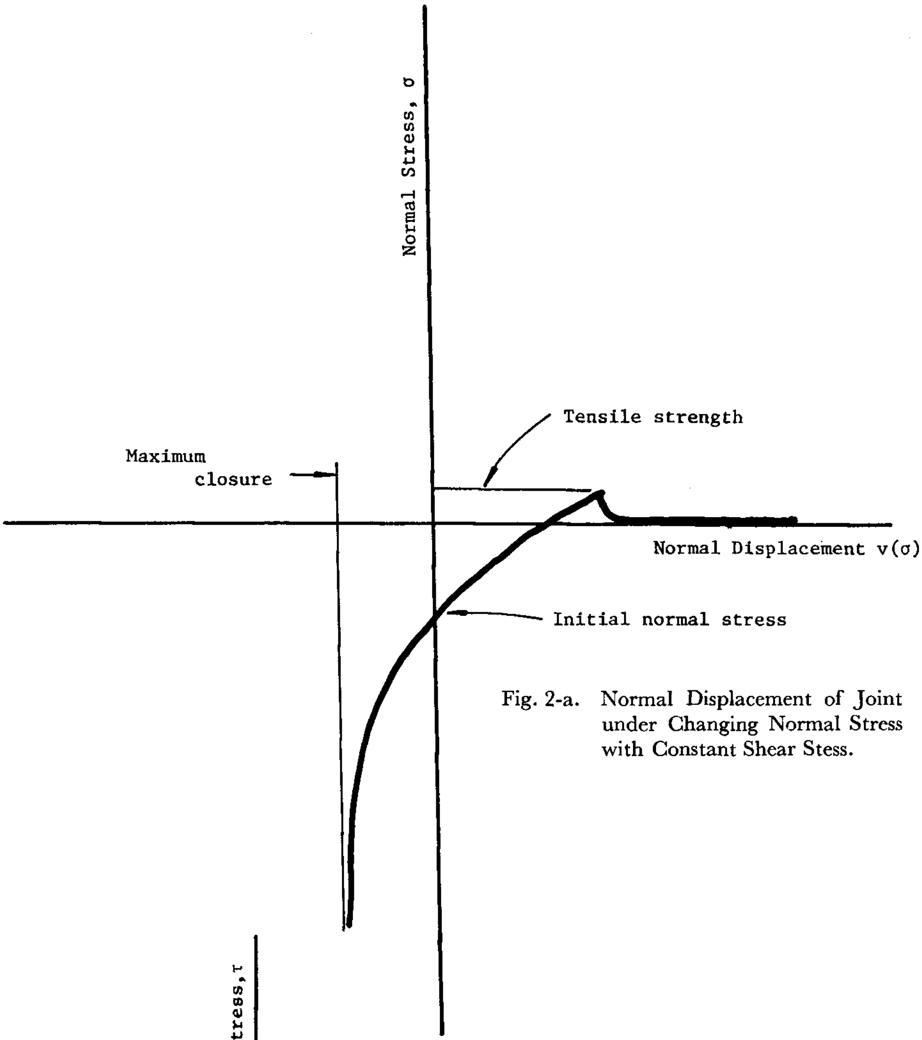

Fig. 2-a. Normal Displacement of Joint under Changing Normal Stress with Constant Shear Stess.

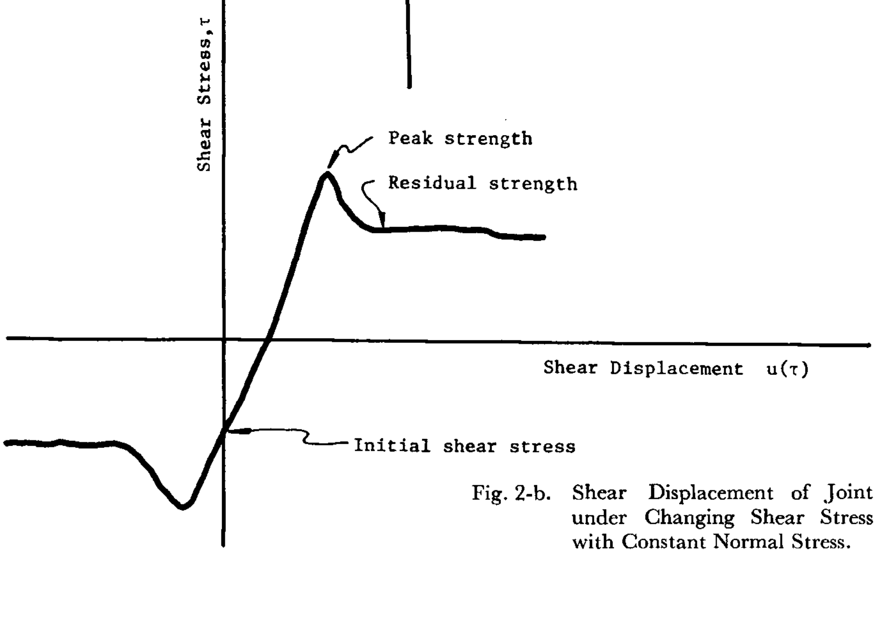

Fig. 2-b. Shear Displacement of Joint under Changing Shear Stress with Constant Normal Stress.

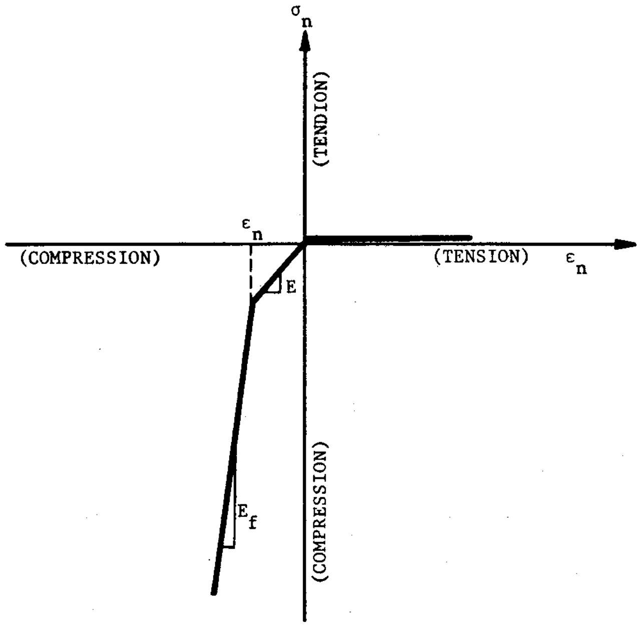

Fig. 3–a. Normal Stress-Displacement Relation for Joint.

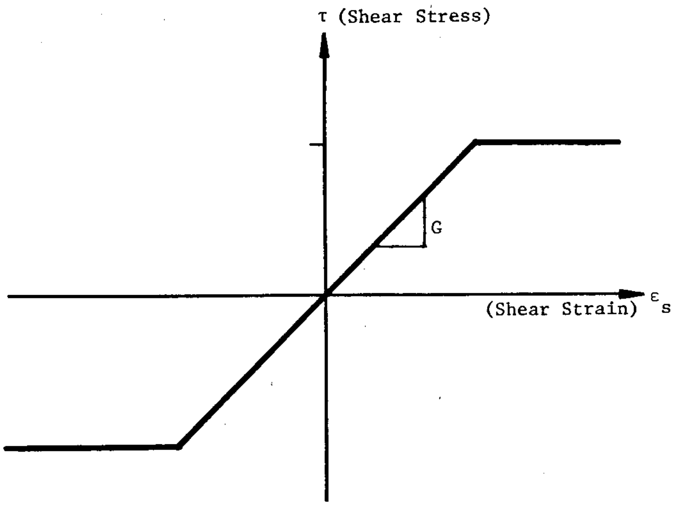

Fig. 3-b. Shear Stress-Displacement Relation for Joint.

3. Modeling of Geologic Site

The geological conditions of the sites where the foundations of a bridge are to be constructed were determined from the site investigations, geophysical methods and drill holes. The actual geologic condition of the ground is usually so complex that the idealization or modeling of the actual site has to be done in order that the analysis may be performed. The first step to model the complex rock mass is to investigate the boring core carefully, classify the rock and determine the fracture frequency.

Secondly, with these results the pressuremeter tests are performed and the deformability of the rock mass at the desired point can be estimated. At the same time, in-situ plate bearing tests, pressuremeter tests in bore holes and laboratory triaxial tests are recommended to know the material properties of rock and rock mass. Finally, the data obtained from all kinds of investigation are averaged with a certain weight depending upon the importance of the results.

Fig. 4 is a cross section of a pier type foundation and rock mass, which is an example of modeling of the site with six zones (ranks of rock mass) determined by the proecdure described above. The rock mass is mainly the multiple layers of

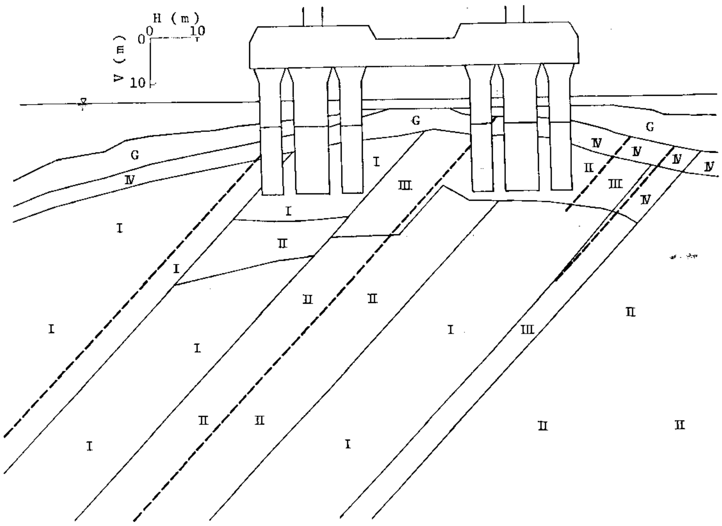

Fig. 4. Modeling of Geologic Site at Pier Type Foundation.

sandstone and shale dipping at about 45 degrees. Fig. 5 show the geologic site model of the rock mass where the anchorage is to be seated. The rock mass also consists of multiple layers of sandstone and shale with a fault and weak zones. The material properties of six ranks (zones) of the rock mass in Figs. 4 and 5 are shown in Table 1, and they were used as the input data in the finite element analysis.

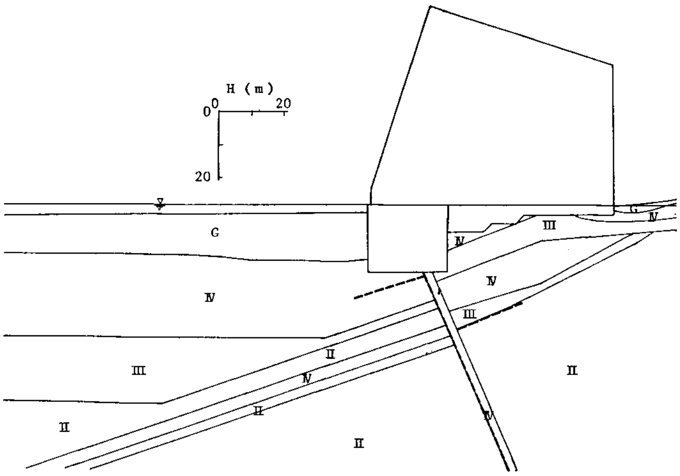

Fig. 5. Modeling of Geologic Site at Anchorage.

Table 1. Material Properties of Rock Masses.

Mat. No.	Rank	E	ν	c	ϕ	density
		kg/cm²	—	kg/cm²	(°)	t/m³
1	I	20,000	0.35	6.0	45.0	2.5
2	II	12,000	0.35	3.0	40.0	2.5
3	III	6,000	0.38	2.0	37.5	2.4
4	IV	2,000	0.38	1.0	32.5	2.2
5	V	1,000	0.40	0.5	25.0	2.0
6	G	1,000	0.40	0.1	30.0	1.8
7	C	267,000	0.17	—	—	2.5

G: gravel C; Concrete

4. Finite Element Analysis

Fig. 6 and Fig. 7 are the finite element meshes of Fig. 4 and Fig. 5 respectively. The material constants are assigned to the element to represent the modeling of the ground site as much as possible. In the finite element analysis of each problem, two meshes were used: One included interface or joint elements and the other did not. No adequate laboratory tests were available to define the behavior of interfaces between rocks and concrete specimens. Hence, the required moduli were obtained on the bais of experience with similar geologic materials in pre-previous studies.[10),11)]

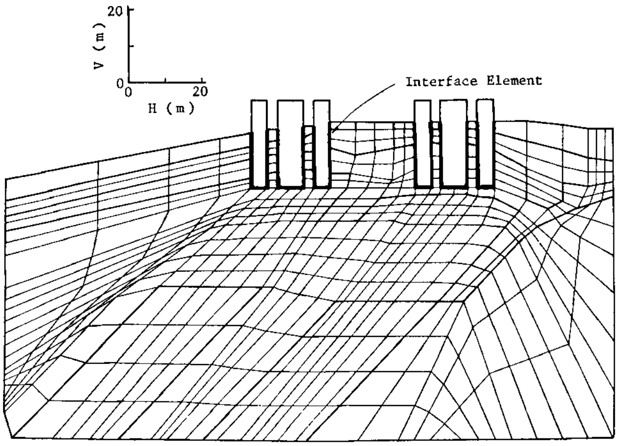

Fig. 6. Finite Element Mesh.

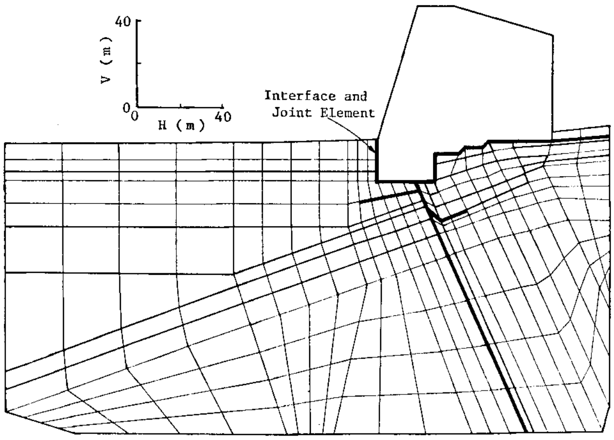

Fig. 7. Finite Element Mesh.

The shear stiffnes C_{ss} for the interfaces was chosen as 2280 kg/cm^3 and the normal stiffness C_{nn} was adopted as 10^8 kg/cm^3. The shear stiffness for rock joints was 360 kg/cm^3, which was estimated from laboratory test results on rock materials[10]. The strength parameters for the interfaces and rock joints were taken as $c=0.0$ and $\phi=30°$.

4.1 Pier type foundation

Fig. 8–a shows the σ_1 (major principal stress) contour without the interface elements and Fig. 8–b with the interface elements, when the maximum design load was applied on the foundation. In Fig. 8–a it is seen that the columns move together with the base rock mass, and the distribution of the stress in the rock

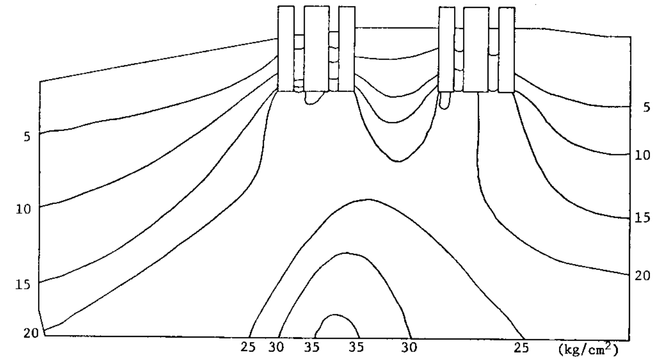

Fig. - 8-a. Stress Contour without Interface Elements.

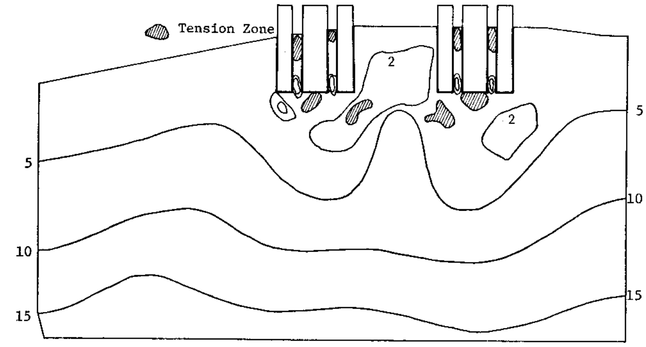

Fig. 8-b. Stress Contour with Interface Elements.

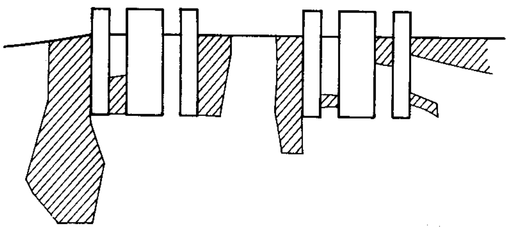

Fig. 9-a. Yield Zone without Interface Elements.

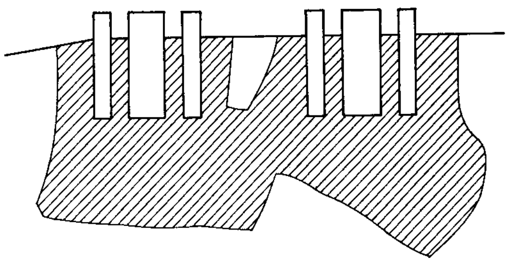

Fig. 9-b. Yield Zone with Interface Elements.

mass is fairly uniform. The total settlement of the foundation is less than 3 cm and no unusual behavior was expected. On the contrary, with the interface elements in Fig. 8–b, each column behaves independently and uneven stress distribution was predicted. It was found that there existed tension zones at several regions and the high intensity of shear stresses was induced. Although the total settlement of the foundation with interfaces was small, the yield zone spread to a wide range as shown in Fig. 9–b, whereas the only local failure was expected without interfaces as shown in Fig. 9–a.

The results of these analyses call attention to construct the foundation very carefully, particularly to bond the concrete and rock mass intefaces with grouting or with some other methods.

4.2 Anchorage

Figs. 10–a and 10–b show the σ_1 contour and yield zones when the maximum design load was applied. It is found that the left front of the anchorage is heavily damaged and high stress concentration occurs. The anchorage settles about 10 cm

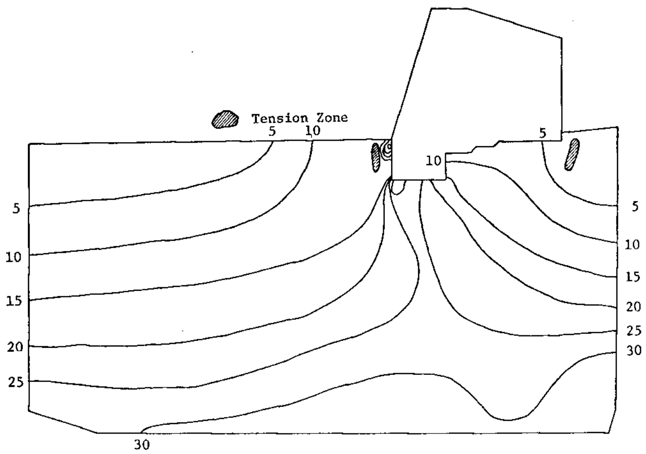

Fig. 10-a. Stress Contour without Joint Elements.

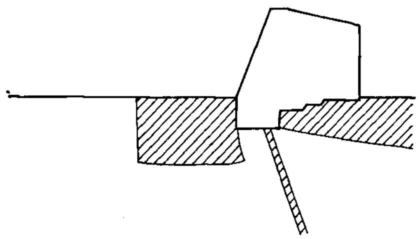

Fig. 10-b. Yield Zone without Joint Elements.

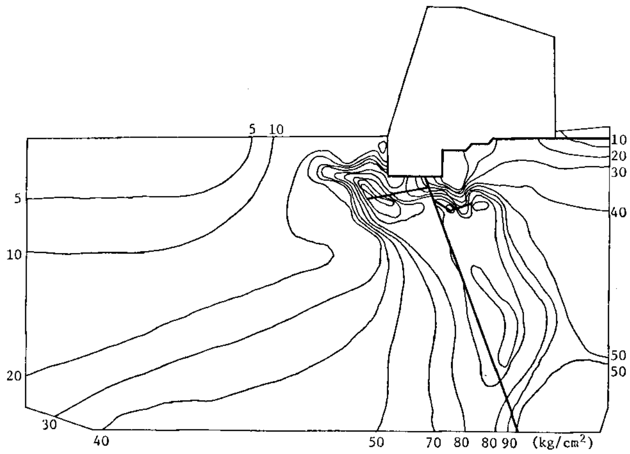

Fig. 10–c Stress Contour with Joint Elements.

and rotates to the left a little. As a result, the tension zone appears at the right front. Along the fault zone, stresses are concentrated and the yield zone is developed.

With interface and joint elements, extreme stress concentration along the joints are developed, as shown in Fig. 10–c. The anchorage slides to the left and some parts of the rock-concrete interfaces separate. Yield zones are everywhere in the rock mass and the settlement of the ancAorage is very large. There is a possibility that the stability of the anchorage might be lost unless the rock mass is strengthened by rock anchors or grouting.

Although the results of those with and without joint elements are different, both represent the certain aspects of behavior of the anchorage. In order to evaluate the stability of the anchorage more realistically, the material properties (particularly the properties of rock joints) have to be intensively investigated since the results show that the stability of the foundation is highly dependent upon the behavior of the joints.

5. Conculsions

The finite element procedure presented in this paper is well suited for a nonlinear analysis of practical problems. It is found that the results are significantly influenced by material properties adopted and the material properties should be determined very carefully from the in-situ and laboratory tests.

The use of the interface and joint elements was presented. Inclusion of these elements very often changes the results of the deformation behavior and the stability of foundations on rock masses. As for the typical example, two kinds of practical problems were solved and the behaviors of the foundations on the inhomogeneous rock masses were throughly investigated.

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