Transient Stability Analysis of Electric Power Systems via Lur'e type Lyapunov Function: Part I

By

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Abstract

In this paper, a Lur'e type Lyapunov function derived from the generalized Popov's theorem is applied to the transient stability problem of electric power systems. Also described is the method of evaluating the critical fault clearing time using the Lyapunov function V. On the occasion of the estimation of the critical fault clearing time or the assessment of the transient stability region via the Lyapunov direct method, the determination of the critical value of the Lyapunov function is as important as the construction of the function. In this paper, it is examined what the transient stability region of electric power systems is practically like. It is shown that the degree of the conservativeness of the estimation can be reduced, if V^{uc}, the value of V at the unstable equilibrium point corresponding to the first swing, is used as the critical value instead of the minimum of the values of V at all of the unstable equilibrium points. Two methods of approximately calculating V^{uc} are proposed, and applied to the model systems as numerical examples. Large estimation errors were seen in some cases due to the defect of the Lyapunov function used, which will be overcome in the companion paper.

1. Introduction

The object of electric power systems is to supply customers with electric power of good quality uninterruptedly and sufficiently. This object is attained by a well designed system and its operation. One of the important items which should be investigated at this juncture is the transient stability. Recently, large generating stations are constructed in places remote from the load center due to environmental problems etc. Also, transmission lines are getting to be of large capacity. Accordingly, the transient stability analysis of power systems gets to be more important. The present day transient stability analyses are mainly performed by simulations.

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This method needs to solve multivariable nonlinear differential equations many times, and very much computing time is required for systems with many generators. To substitute for the method of simulation, Lyapunov's direct method was proposed and many kinds of Lyapunov functions have been investigated until now.[1)-5)]

In the Lyapunov method, a critical value is sought which determines the transiently stable region. The system is regarded as stable if the value of the Lyapunov function at the final change of the circuit is less than this critical value. Therefore, the critical fault clearing time is determined by solving the system's differential equations under the sustained fault condition only once. Hence, the total computing time is shortened by a large degree making the on-line estimation of the transient stability possible, if the time for the calculation of the critical value is short.

In this paper, we investigate the method for effectively determining the critical value of the Lur'e type Lyapunov function which is derived from the generalized Popov theorem[6)]. The method for constructing the Lyapunov function from Popov theorem has already been applied to the transient stability problem of electric power systems; and several kinds of Lyapunov functions have been constructed[7)-11)]. However, it can safely be said that the constructed Lyapunov functions have seldom been applied to practical problems, and the feasibility of the function has not been confirmed until now. In this paper, we first investigate the characteristics of the Lur'e type Lyapunov function and examine how to determine the parameters included in the function. The method for determining the critical value, as well as the constuction of the Lyapunov function, has a great effect on the performance of Lyapunov's direct method. So far, the minimum of the values of the Lyapunov function at all unstable equilibrium points has been used as the critical value. The estimation of the stable region, however, is considerably conservative with this value. In bad cases, the estimated value of the critical clearing time is so much smaller than the actual value as to make it meaningless. Besides, it takes a long time to calculate the usual critical value when the number of the generators is large. This is because the number of the equilibrium points is very large, and the usual methods need to calculate all of them.

Recently, a few studies have been made in order to solve this problem[12),13)], the results of which still seem unsatisfactory from the viewpoint of accuracy. From the consideration on the transient stability region, we show that it is possible to reduce the conservativeness of estimation by use of the critical value V^{uc} instead of the usual critical value. V^{uc} is the value of the Lyapunov function at the unstable equilibrium point corresponding to the first swing; and we propose two methods to determine the value V^{uc} approximately. One of the methods supposes that one machine goes out of step, and determines the critical value by a linear combination of the values of the Lyapunov function at the unstable equilibrium points which correspond to each one-machine step-out. The other method approximates the unstable equilibrium point which corresponds to the first swing, and takes the value of the Lyapunov function at this point for the critical value. The latter has the merit that it does not need to calculate the unstable equilibrium points correctly. Therefore, it does not suffer from the difficulties which accompany the calculation of the unstable equilibrium points. In the last part of this paper, we apply the above mentioned methods to 4-machine and 10-machine systems as numerical examples, estimate the critical fault clearing time, and have a discussion on the method of determining the critical value and the Lyapunov function.

2. Lur'e type Lyapunov Function

When the transient stability of electric power systems is analyzed by Lyapunov's direct method, the following assumptions are generally made for multimachine systems although more detailed models are used for one-machine systems.

(1) Each synchronous machine is represented by a constant voltage behind its transient reactance. In other words, the flux linkages are constant during the transient period, and the flux decay and the voltage regulation are not taken into consideration.

(2) Damping power is proportional to slip velocity, and is thus assumed to be mainly due to the mechanical friction and the asynchronous torques.

(3) The mechanical power input is constant, and the governor action is not taken into account.

(4) Each synchronous machine is a round-rotor machine.

(5) The inertia coefficient of each generator is constant.

(6) Loads are represented by constant impedances.

Under the above assumptions, the motion of the i th machine is described by the following differential equation;

$$m_i\frac{d^2\delta_i}{dt^2}+d_i\frac{d\delta_i}{dt}=P_{mi}-\sum_{j=1}^{n}E_iE_jY_{ij}\sin(\delta_{ij}+\theta_{ij}) \quad (i=1,\ 2,\ \cdots,\ n) \cdots\cdots\cdots(1)$$

where

t : time

δ_i : the angle between the rotor shaft of the i th machine and the shaft rotating at the synchronous speed (in electrical degrees)

m_i : the inertia constant of the i th machine

d_i : the damping coefficient of the i th machine

P_{mi} : the mechanical power input for the i th machine

$Y_{ij}\angle\phi_{ij}=Y_{ji}\angle\phi_{ji}$: the post fault short-circuit transfer admittance between i th and j th generator nodes (obtained after reduction of the network retaining only the generator nodes)

$\theta_{ij}=\pi/2-\phi_{ij}$

$\delta_{ij}=\delta_i-\delta_j$.

For the above system, Gudaru[11] has derived the following Lyapunov function by applying the generalized Popov theorem ;

$$V(\delta,\ \omega)=\frac{1}{\sum_{i=1}^{n} m_i}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} m_i m_j(\omega_i-\omega_j)^2+\frac{\alpha}{\sum_{i=1}^{n} m_i}\left(\sum_{i=1}^{n} m_i\omega_i\right)^2$$
$$+\frac{1}{q}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sigma_i D_{ij}^*(2\omega_j^*+\sigma_j)$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n}E_iE_jY_{ij}\{\cos(\delta_{ij}^s+\theta_{ij})-\cos(\delta_{ij}+\theta_{ij})-(\delta_{ij}-\delta_{ij}^s)\sin(\delta_{ij}^s+\theta_{ij})\}$$
$$=V_k+\alpha V_{fk}+\frac{1}{q}V_d+V_p, \qquad \cdots\cdots\cdots\cdots(2)$$

where

$$V_k=\frac{1}{\sum_{i=1}^{n} m_i}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} m_i m_j(\omega_i-\omega_j)^2$$

$$V_{fk}=\frac{1}{\sum_{i=1}^{n} m_i}\left(\sum_{i=1}^{n} m_i\omega_i\right)^2$$

$$V_d=\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\sigma_i D_{ij}^*(2\omega_j^*+\sigma_j)$$

$$V_p=\sum_{i=1}^{n}\sum_{j=1}^{n}E_iE_jY_{ij}\{\cos(\delta_{ij}^s+\theta_{ij})-\cos(\delta_{ij}+\theta_{ij})-(\delta_{ij}-\delta_{ij}^s)\sin(\delta_{ij}^s+\theta_{ij})\}$$

and

$$\sigma_i=\delta_1-\delta_{i+1}$$

$$\omega_i^*=\frac{m_1\omega_1}{d_1}-\frac{m_{i+1}\omega_{i+1}}{d_{i+1}}$$

$$D_{ij}^*=\begin{cases} d_{i+1}-\dfrac{d_{i+1}^2}{\sum_{k=1}^{n} d_k} & \text{for } i=j,\\[2ex] -\dfrac{d_{i+1}d_{j+1}}{\sum_{k=1}^{n} d_k} & \text{for } i\neq j\end{cases}$$

$$\alpha=\frac{\mu_0-\mu}{\mu_0}\geq 0$$

$$\mu_0 = -\frac{1}{\sum_{i=1}^{n} m_i}$$

$$\mu_{min} \leq \mu \leq \mu_{max}$$

μ_{min} and μ_{max} are given by the following equations:

$$\mu_{min} = \nu_{min} - \frac{1}{q\sum_{i=1}^{n} d_i}, \quad \mu_{max} = \nu_{max} - \frac{1}{q\sum_{i=1}^{n} d_i},$$

where ν_{min} and ν_{max} are the solutions of the second order equation,

$$\nu^2 \left\{ \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(d_i m_j - d_j m_i)^2}{4\left(d_i - \frac{m_i}{q}\right)\left(d_j - \frac{m_j}{q}\right)} \right\} - \nu \sum_{i=1}^{n} \frac{d_i m_i}{d_i - \frac{m_i}{q}} - 1 = 0.$$

In order to assure the absolute stability of the system, q must satisfy the condition

$$q > \max_{1 \leq i \leq n} \left(\frac{m_i}{d_i}\right).$$

V_k represents the kinetic energy due to the relative angular velocity, and V_{fk} represents the kinetic energy due to the average angular velocity $\sum_{i=1}^{n} m_i \omega_i / \sum_{i=1}^{n} m_i$. V_{fk} does not appear in the function in the case of uniform damping, because α equals zero. V_d includes damping coefficients but also vanishes as $q \to \infty$. V_p represents the potential energy which is stored by the deviation of the angles from their values at the stable equilibrium point. Hereafter, we call V_k, V_{fk} and V_p kinetic energy, kinetic energy of the inertia center and potential energy, respectively.

3. The Effect of V_d on the Estimation of the Stability Region

We investigate the effect of V_d on the estimation of the transient stability region by the example of a single machine connected to an infinite bus. The swing equation of the machine can be written as

$$m\frac{d^2\delta}{dt^2} + d\frac{d\delta}{dt} = P_m - P_e \sin\delta \qquad \cdots\cdots\cdots\cdots(3)$$

and the Lur'e type Lyapunov function is written as follows;

$$V(\delta, \omega) = m\omega^2 + \frac{m}{q}\left(2\delta\omega + \frac{d}{m}\delta^2\right) + 2P_e(\cos\delta^s - \cos\delta) - 2P_m(\delta - \delta^s), \quad \cdots(4)$$

where

$$q > d/m.$$

The estimations of the stability region in the case of putting $q \to \infty$ and $q = m/d$ are

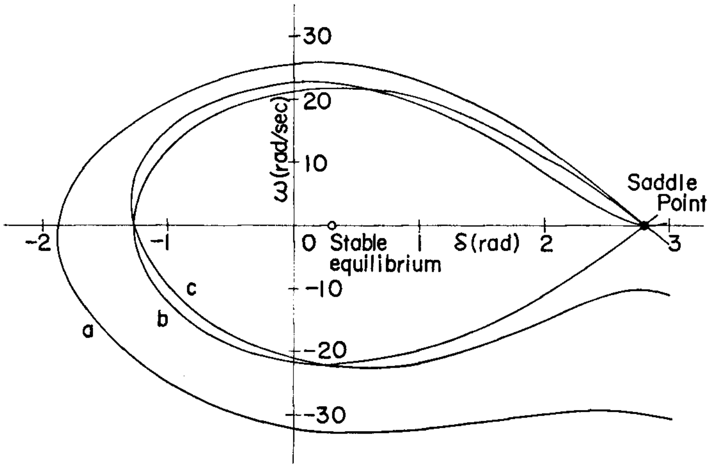


Fig. 1. Estimations of the stable region.
a : actual boundary
b : estimated boundary with q→∞
c : estimated boundary with q=d/m

shown in Fig. 1, where the critical value V^u is given by the value of the Lyapunov function at the unstable equilibrium point $(\delta, \omega)^u = (\pi - \delta^s, 0)$. The system parameters are as follows;

$m = 0.0138$ $\quad$ $d = 0.0285$

$P_m = 0.91$ $\quad$ $P_e = 3.02$

The curve (a) represents the boundary of the actual stability region, and the curve (b) and the curve (c) represent the stability boundaries estimated by the Lyapunov function with $q \to \infty$ and $q = m/d$ respectively. The estimation is better with $q \to \infty$ than with $q = m/d$ at the first and the third quadrants, and conversely at the second and the fourth quadrants. Since the first quadrant is mainly important for the transient stability, we choose q to be infinite in this paper.

4. The Conditions for Transient Stability

When an electric power system is going to lose its synchronism owing to some fault, it is separated into two groups of machines at the first instant. The aspect of this loss of synchronism is called the mode of the step-out. The maximum number of the modes is $2^{n-1}-1$ for an n-machine system, and each mode corresponds to an unstable equilibrium point δ^u, which is a solution of the equations[14],

$$\ddot{\delta}_i = \frac{1}{m_i}\left\{P_{mi} - \sum_{j=1}^{n} E_i E_j Y_{ij} \sin(\delta_{ij} + \theta_{ij})\right\} = \text{constant}, \quad (i = 1, 2, \cdots, n) \quad \cdots\cdots(5)$$

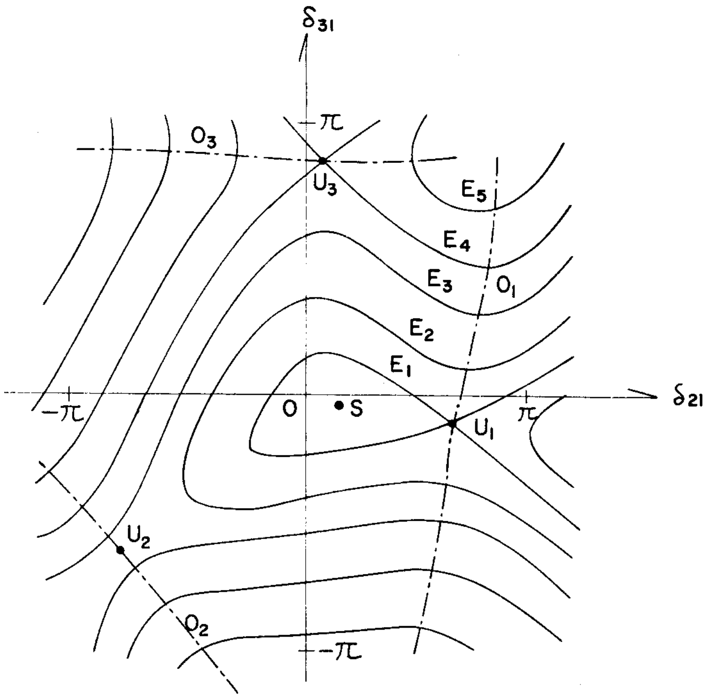


Fig. 2. Equipotential curves for a 3-machine system.

Fig. 2 shows the potential energy of a three-machine system in the plane of the angular differences δ_{21} and δ_{31}. The potential energy takes its minimum value at the point s. The points u_1, u_2 and u_3 correspond to the saddle points. The equi potentials are yielded by the following equation

$$V_p(\delta) = \text{constant}, \qquad \cdots\cdots\cdots\cdots (6)$$

where $V_p(\delta)$ denotes the potential energy as described before. O_1, O_2 and O_3 are curves which are orthogonal to the equipotentials and go through the points u_1, u_2 and u_3, respectively.

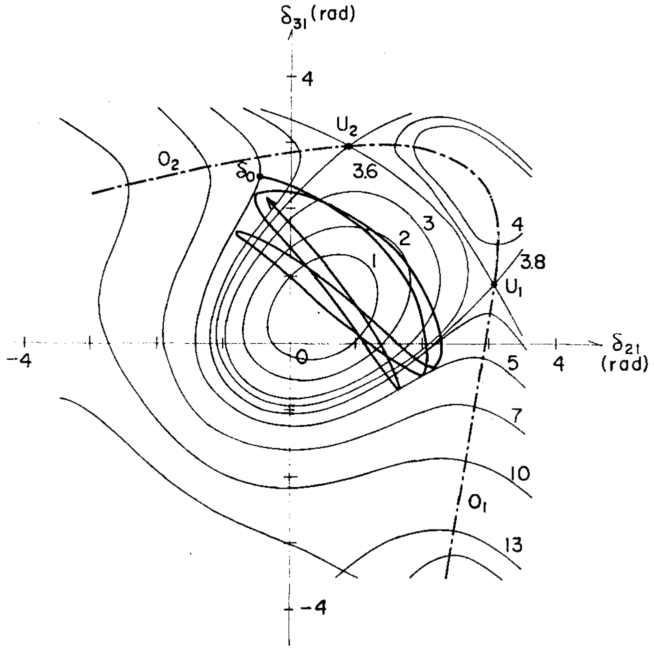
The total torque applied to the system is represented as follows:

$$T = -\frac{\partial V_p}{\partial \delta_r} \qquad \cdots\cdots\cdots\cdots (7)$$

(The proof is given in Appendix 1.)
Accordingly, the direction of the torque is always orthogonal to the equipotential curves, and synchronism will be lost at the moment when the system crosses one of the curves O_x, since afterward the torque T is such that it will separate the system from the curve O_x.

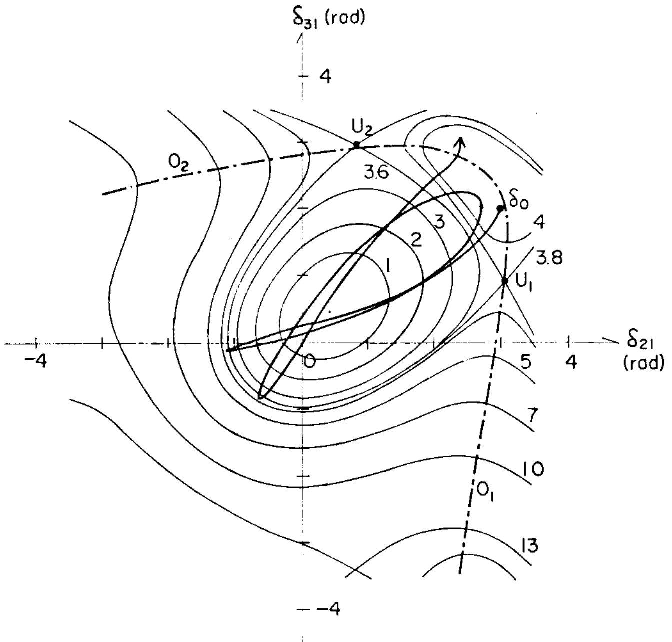
If the system has a total energy equal to E_3, the synchronism can be lost by crossing O_1 or O_2. In other words, machine 2 or machine 1 can be separated from the rest of the system. However, which one will actually be separated depends upon the direction in which the system swings.

Fig. 3 shows some examples of the trajectory of the 3-machine system projected onto the angular space. The angular velocities are initially assumed to be zero, and the initial state is put on the $\delta_{21}-\delta_{31}$ plane. The initial point in Fig. 3 (a) is $(\delta_{21}, \delta_{31})=(-0.5, 2.5)$ rad., and the total energy is 5.03, which is enough to cross the curve O_1 or O_2. The system, however, stays in the stable region after three oscillations for two seconds. In Fig. 3 (b), starting from the point $(\delta_{21}, \delta_{31})=(3.0, 2.0)$ rad., the system crosses the curve O_2 after two oscillations. The total energy is 4.27, and is also enough to cross the curve O_1 or O_2. In Fig. 3 (c), starting from the point $(\delta_{21}, \delta_{31})=(-1.0, -1.0)$ rad., the system crosses the curve O_2 without oscillation. The total energy is 5.61 in this case. As seen in these examples, the system can stay in the stable region for a fairly long time, even if it has enough energy to become unstable. In these cases, it is possible to keep the system in synchronism by changing the system's construction or parameters, or by decreasing the total energy via the damping effect.

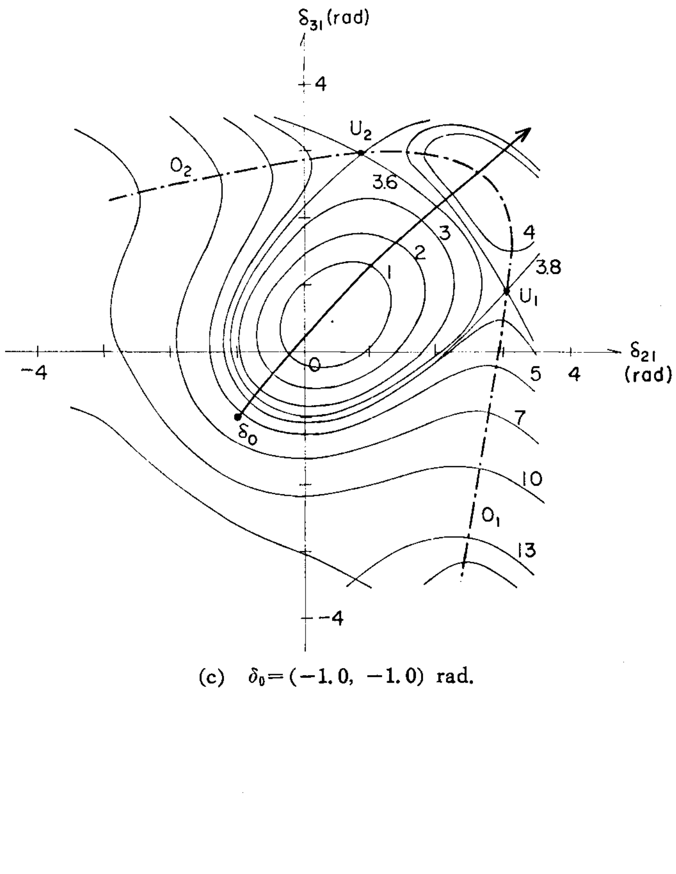


(a) $\delta_0=(-0.5, 2.5)$ rad.

Fig. 3. Trajectory projected onto the angular space for the 3-machine system.



(b) $\delta_0=(3.0,\ 2.0)$ rad.



(c) $\delta_0=(-1.0,\ -1.0)$ rad.

Now we can establish the conditions for the stability of power systems. First consider the loss of synchronism around an unstable equilibrium point δ^{ux}. Since δ^{ux} is the point which makes the potential energy minimum on the curve O_x, the system must have more energy V than V^{ux} in order to cross the curve O_x, where V^{ux} is the potential energy at the point δ^{ux}. Hence, a sufficient condition for stability can be written as follows;

Stability Condition 1

If the system satisfies the condition

$$V<V^{ux} \qquad \cdots\cdots\cdots\cdots(8)$$

around the unstable equilibrium point δ^{ux}, then the system is stable around the point δ^{ux}.

Stability Condition 1 is related to the stability around a certain unstable equilibrium point, and in order to be stable for all of the unstable equilibrium points, the system must satisfy the following condition.

Stability Condition 2

Let V^{um} designate the minimum of the values of V at all of the unstable equilibrium points. Then, if it is

$$V<V^{um}, \qquad \cdots\cdots\cdots\cdots(9)$$

the system is stable with respect to all the unstable equilibrium points.

Stability Condition 2 is the condition which is generally used in Lyapunov's direct method. However, it is well known that by using V^{um} as the critical value of V, the estimation of the critical clearing time is considerably conservative compared with the actual value. In many cases, the system loses synchronism during the first swing, and if the system stays in step after the first swing, the system will be stable for the succeeding swings. Hence, in such cases, Stability Condition 2 is too strict; and Stability Condition 1 is suitable for checking the stability for the first swing. Therefore, Condition 1 is rewritten as follows;

Stability Condition 3

Designating δ^{uc} as the unstable equilibrium point which corresponds to the first swing, and V^{uc} as the value of the Lyapunov function at δ^{uc}, the system is stable for the first swing if

$$V<V^{uc}. \qquad \cdots\cdots\cdots\cdots(10)$$

It is expected that the conservativeness which exists in the usual Lyapunov direct method can be reduced by use of V^{uc} as the critical value.

5. Methods of Determining the Critical Value

In this paper, we pay attention only to the stability for the first swing. By using the critical value V^{uc}, we try to reduce the conservativeness which is inevitable to the usual method which uses V^{um}. In order to determine the critical value V^{uc} described in the last section, it is necessary to know the unstable equilibrium point which corresponds to the first swing. Two methods are proposed in the following.

Method 1 (V_{est1})

For an n-machine system, (n−1) unstable equilibrium points at the most correspond to a one-machine step-out with respect to the No. 1 generator. Each of them can be represented by the (n−1) dimensional vector y_i, $i=2, 3, \cdots, n$, and they are independent each other, where the suffix i denotes the number of the generator which goes out of step. In the same manner, the angles of the machines during transient swings can also be represented by the (n−1) dimensional vector as follows;

$$x=(\delta_2-\delta_1,\ \delta_3-\delta_1,\ \cdots\cdots,\ \delta_n-\delta_1)\,. \qquad \cdots\cdots\cdots\cdots(11)$$

Then x can be represented by the linear combination of the unstable equilibrium vectors y_i's as

$$x=\sum_{i=2}^{n} a_i y_i. \qquad \cdots\cdots\cdots\cdots(12)$$

The coefficients a_i's can be obtained by solving the simultaneous linear algebraic equations,

$$(x,\ y_j)=\sum_{i=2}^{n} a_i(y_i,\ y_j) \quad (j=2,\ 3,\ \cdots\cdots,\ n) \qquad \cdots\cdots\cdots\cdots(13)$$

and indicates the magnitude of the i th unstable swing which is an element included in x. Method 1 supposes that the i th one-machine step-out is going to occur with the magnitude of $|a_i|$, and determines the critical value V_{est1} by the following equation;

$$V_{est1}=\frac{\sum_{i=2}^{n}|a_i|\,V_i^u}{\sum_{i=2}^{n}|a_i|}, \qquad \cdots\cdots\cdots\cdots(14)$$

where V_i^u is the value of the Lyapunov function at the unstable equilibrium point y_i. The critical value V_{est1} is interpreted as a weighted average of V_i^u's. If some $|a_i|$ is much greater than the other $|a_j|$'s $(j\neq i)$, then

$$V_{est1}\doteqdot V_i^u, \qquad \cdots\cdots\cdots\cdots(15)$$

i. e., V_{est1} is equal to the critical value for the i th one-machine step-out. V_{est1} is a function of time, and the time when V_{est1} crosses the Lyapunov function for the sustained fault is determined as the critical clearing time. In case of a step-out of two or more machines, however, accurate results can not be expected, since V_{est1} is derived by assuming only a one-machine step-out.

Method 2 (V_{est2})

As in Method 1, the angles during transient swings are represented by the (n−1) dimensional vector x. Using x, we approximate the unstable equilibrium point δ^{uc} which corresponds to the actual step-out by the equation

$$y=\alpha x, \qquad \cdots\cdots\cdots\cdots(16)$$

where α is a positive constant given by

$$\alpha=\frac{\pi}{\max\limits_{2\leq i\leq n}(|\delta_i-\delta_1|)}.$$

The critical value V_{est2} is defined by the following equation:

$$V_{est2}=V_p(y), \qquad \cdots\cdots\cdots\cdots(17)$$

where V_p represents the potential energy. The meaning of this method can be considered as follows: It is supposed in this method that the system loses its synchronism when the absolute value of some component of x becomes greater than π. Then the unstable equilibrium point is approximated by y, and the potential energy at y is taken as the critical value. Since the angles of the machines which go out of step are nearly equal to π at the corresponding unstable equilibrium point, and the Lur'e type Lyapunov function is not very sensitive to the small change of δ near the equilibrium point[12], the critical value suggested above is considered to be a good approximation of the value V^{uc} which corresponds to the first swing. Moreover, this method does not need to calculate the unstable points correctly, so the computing time is very short. Since V_{est2} is also a function of time, the estimation of the critical clearing time is made in the same way as by V_{est1}.

6. Examples

The Lur'e type Lyapunov function and the methods of determining the critical value are applied to the transient stability analysis of 4-and 10-machine systems.

6-1 4-machine system

The construction of the system is shown in Fig. 4. This system was used by El-Abiad[3] for the first time, and since then it has often been used in the studies of

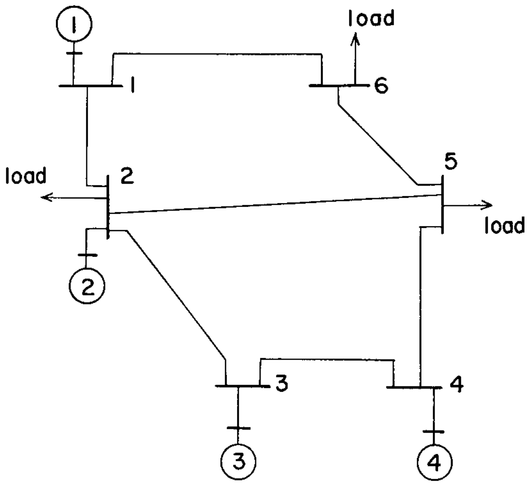


Fig. 4. Construction of the 4-machine system.

the transient stability via Lyapunov's direct method. The assumed disturbance $x-y$ is the 3-phase short circuit which occurs at the terminal x of the transmission line $x-y$, and which is cleared by opening the line at both terminals after a certain lapse of time.

Table 1 shows the estimated critical clearing times for seven faults together with the critical values V^{um}, V_{est1} and V_{est2}. The estimation by V^{um} is conservative for all faults compared with the actual values. As mentioned in Section 4, the estimation error is large when the unstable equilibrium point which gives V^{um} does not coincide with the actual mode of the step-out. Table 2 shows which machine (or machines) swings largely, which machine gives V^{um}, and the results of the estimation by V^{uc}. It can be seen from this table that the error for fault 2-3 which was the greatest by V^{um} is greatly reduced by V^{uc}.

Table. 1. Estimations of the critical clearing time (sec.) for the 4-machine system.
Tcr: actual critical clearing time.

Fault	V^{um}	Vest 1	Vest 2	Tcr
2-1	0.56	0.58	0.57	0.61
2-3	0.41	0.50	0.55	0.54
2-5	0.50	0.53	0.59	0.53
3-2	0.37	0.38	0.37	0.42
3-4	0.40	0.40	0.40	0.42
4-3	0.40	0.44	0.46	0.43
4-5	0.43	0.48	0.43	0.47

Table. 2. Estimations with the critical value V^{uc} for the 4-machine system.
δ^{um}: number of the generator corresponding to V^{um}
δ^{uc} : number(s) of the generator(s) losing synchronism

Fault	δ^{um}	δ^{uc}	V^{uc}	Tcr
2-1	1	2	0.58	0.61
2-3	3	2	0.52	0.54
2-5	2	2	0.50	0.53
3-2	3	3	0.37	0.42
3-4	3	3	0.40	0.42
4-3	3	4	0.45	0.43
4-5	4	4	0.43	0.47

The estimation with V_{est1} gives conservative results for all faults, but its degree of conservativeness is smaller than the results with V^{um}. Also, those results are very similar to those with V^{uc}. This is because only a one-machine step-out occurs in this example, and V_{est1} approximates the value of V at unstable equilibrium point for the first swing.

Fig. 5 shows an example of a variation of V_{est1} for the sustained fault 3-4. V_{est1} decreases gradually as time proceeds, and is almost constant after the time when the potential energy V_p reaches its maximum value. This is because the mode of the step-out becomes apparent with the increase of the potential energy, i. e., the increase of the angular differences of the machines. The mode is almost determined at the time when the potential energy reaches its peak, and does not vary afterwards. This situation is the same for the other faults.

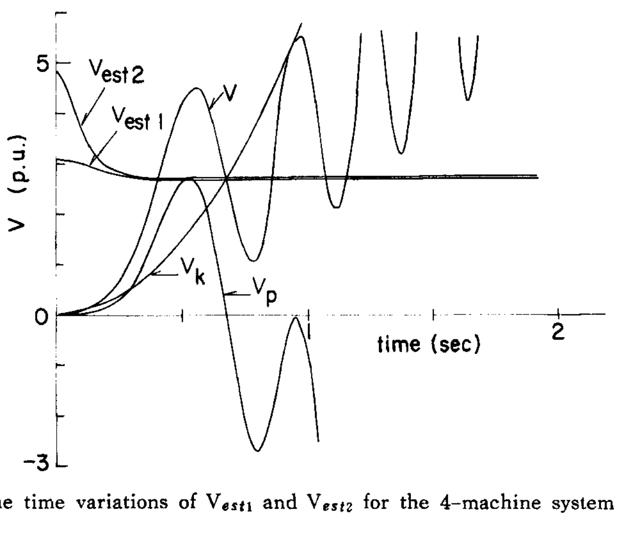


Fig. 5. The time variations of V_{est1} and V_{est2} for the 4-machine system; fault 3-4.

The estimation with V_{est2} gives greater values of critical clearing time than the actual values for some faults, but has the same accuracy as with V_{est1} on the average. Fig. 5 also shows the time variation of V_{est2} for the sustained fault 3-4. It has similar characteristics to the time variation of V_{est1}. It should be noted that the estimation with V_{est2} gives more accurate results than with the usual V^{um}, though V_{est2} can be obtained quite easily without a calculation of the saddle points.

6-2 10-machine system

Fig. 6 shows the construction of the 10-machine system which is called the New England test system[15]. For this system, two cases of load conditions are considered, which are designated as the light load condition and the heavy load condition. The heavy load is taken twice as large as the light load. The faults are assumed to be the 3-phase short circuit, and their expression is the same as for the 4-machine system.

(a) Light load

Table 3 shows the results of the estimation of the critical clearing time under the light load condition. The error is relatively large in the case of the estimation with V^{um}. Table 4 shows the number of the machine which corresponds to the unstable equilibrium point δ^{um} and the number of the machine(s) which actually step out. Both coincide only for the fault 11-12, for which the error of estimation is small. When the critical value V^{uc} corresponding to the first swing is used, the

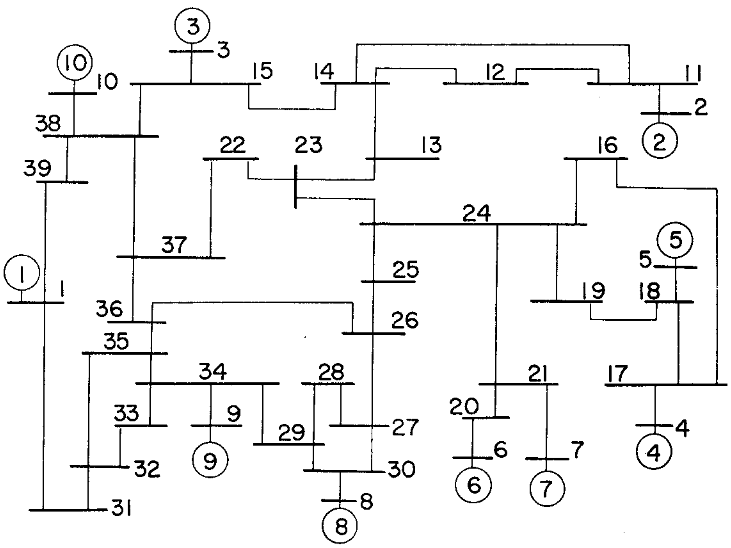


Fig. 6. Construction of the 10-machine system.

Table. 3. Estimations of the critical clearing time (sec.) for the 10-machine system; light loads.

Fault	V	Vest 1	Vest 2	Tcr
11-12	0.33	0.37	0.36	0.30
15-14	0.27	0.36	0.42	0.41
17-18	0.34	0.42	0.47	0.47
18-17	0.33	0.42	0.49	0.50
24-16	0.28	0.35	0.46	0.45
30-27	0.35	0.45	0.48	0.47
34-29	0.33	0.42	0.47	0.46
38-15	0.43	0.54	0.63	0.66

Table. 4. Estimations with the critical value V^{uc} for the 10-machine system; light loads.

Fault	δ^{um}	δ^{uc}	V^{uc}	Tcr
11-12	2	2	0.33	0.30
15-14	6	3	0.34	0.41
17-18	6	4	0.42	0.47
18-17	6	5	0.44	0.50
24-16	6	1	0.44	0.45
30-27	6	8	0.43	0.47
34-29	6	9	0.39	0.46
38-15	6	1	0.67	0.65

estimation is improved to a great extent, compared with the estimation with V^{um}. Also by comparing the results between the two model systems, it can be seen that the improvement of estimation by using V^{uc} instead of V^{um} as the critical value is salient in the case of a system with many machines. V^{uc} is here selected from the values of V at the saddle points corresponding to a one-machine step-out by inspecting the swing curves. Hence, this V^{uc} is not adequate when two or more machines go out step. The relatively large errors in Table 4 are considered to be generated from this cause.

In the case of an estimation using V_{est1} as the critical value, conservative results are given except for the fault 11-12; and the errors are smaller than those with V^{um}. The errors for the faults 24-16 and 38-15 are greater than those with V^{uc}. This is because all the machines except No. 1 go out of step, and V_{est1} is effective only for a one-machine step-out as seen from its definition. The reason why the estimation for the fault 11-12 is larger than with V^{uc} is that the mode of step-out is not apparent at the time when V_{est1} crosses the Lyapunov function. The estimation can be improved by using the value of V_{est1} at the time when the mode is established. For this fault, even the estimation with V^{uc} is larger than the actual value. This is due to the characteristics of the Lyapunov function, about which we will later consider.

In the case of an estimation with V_{est2}, very good results are given except for the fault 11-12; and the average error is smaller than with V^{uc}. This is because V_{est2} can select the unstable equilibrium point which just corresponds to the actual mode of step-out. Hence, there are good estimations for the step-out of two or more machines as well as for a one-machine step-out. The errors are very small, especially for the faults 24-16 and 38-15 in comparison with V_{est1}. The reason why the result for the fault 11-12 is worse than the one with V^{uc} is the same as for V_{est1}.

(b) Heavy load

Table 5 shows the results of the estimation under the heavy load condition. By using V^{um} as the critical value, conservative results are obtained except for the faults 11-12 and 38-15. For the fault 11-12, No. 2 generator loses synchronism, and the critical clearing time estimated by using V^{um} is still larger than the actual value.

Table. 5. Estimations of the critical clearing time (sec.) for the 10-machine system; heavy loads.

Fault	V^{um}	Vest 1	Vest 2	Tcr
11-12	0.10	0.13	0.18	0.06
15-14	0.15	0.19	0.25	0.20
17-18	0.17	0.22	0.26	0.22
18-17	0.17	0.21	0.27	0.22
24-16	0.13	0.15	0.23	0.18
30-27	0.18	0.22	0.27	0.23
34-29	0.18	0.22	0.26	0.22
38-15	0.21	0.21	0.27	0.17

Table. 6. Estimations with the critical value V^{uc} for the 10-machine system; heavy loads.

Fault	δ^{um}	δ^{uc}	V^{uc}	Tcr
11-12	2	2	0.10	0.06
15-14	2	1	0.24	0.20
17-18	2	1	0.26	0.22
18-17	2	1	0.26	0.22
24-16	2	1	0.21	0.18
30-27	2	1	0.27	0.23
34-29	2	1	0.25	0.22
38-15	2	1	0.28	0.17

Table 6 shows the results of estimation with V^{uc}. The estimated values are larger than the actual values, and its average error is large compared with V^{um}. This is again due to the Lyapunov function used.

The estimation with V_{est1} and V_{est2} gives similar results to the estimation with V^{uc}. In the next section 6-3, we investigate the cause of the fact that the estimation is not good by using V^{uc}, V_{est1} and V_{est2} under the heavy load condition.

Under the heavy load condition, the kinetic energy rapidly increases and the potential energy slowly increases during a fault. Hence, in many cases the value of V exceeds the critical value early, while the angular differences are small. In these cases, the establishment of the step-out mode is delayed compared with the increase of the Lyapunov function; and V_{est1} and V_{est2} do not settle down to the value which corresponds to the mode of the step-out at the time when they intersect the Lyapunov function. Therefore, it is preferable to adopt the value to which V_{est1} and V_{est2} settle down as the critical value.

6-3 Consideration of the cause of the large error for the fault 11-12

The time variations of the Lur'e type Lyapunov function and its components when the fault is cleared at the critical time are shown in Figs. 7 and 8. In the case of a 10-machine system (unlike the 4-machine system), the value of V increases after the clearance of the fault, reaches its peak, and decreases to the level at the clearing time. Also, V takes its peak value at nearly the same time as V_p. The above tendency is remarkable for the heavy load condition. The increase of V_p means the increase of the angles of the machines which go out of step. Fig. 9 shows the swing curves for the fault 11-12. In this case, the No. 2 generator suffers a large disturbance, and its angle and the value of V equal about 115 degrees and 3.66, respectively, at the peak. On the other hand, at the unstable equilibrium point

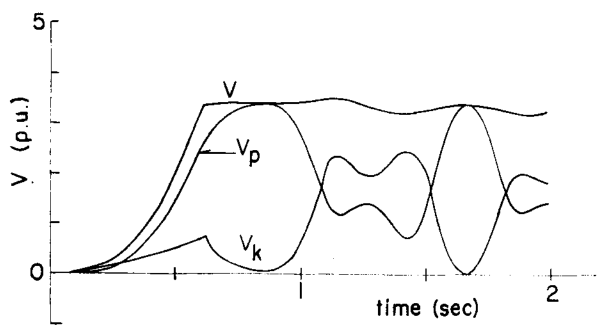
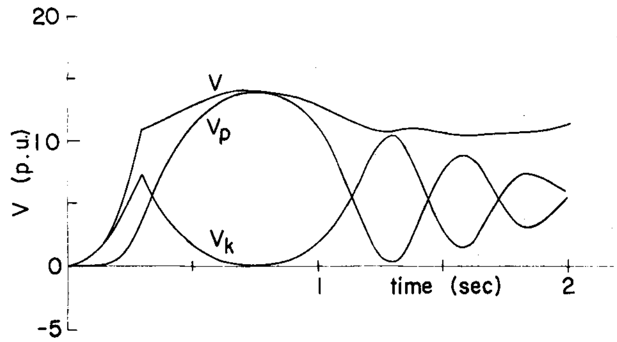
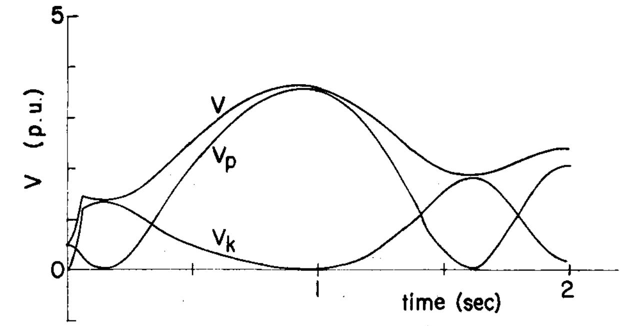


Fig. 7. The time variations of the Lyapunov function and its components for the 4-machine system; fault 2-1.



(a) light loads



(b) heavy loads

Fig. 8. The time variations of the Lyapunov function and its components for the 10-machine system; fault 11-12.

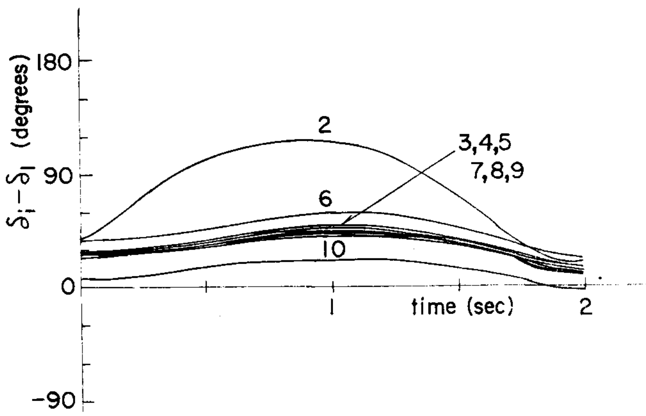


Fig. 9. The swing curves for the 10-machine system; heavy loads, fault 11-12 cleared at 0.06 sec.

for this fault, the angle of the No. 2 generator is 124 degrees, and V^{uc} is 4.09. Since a step-out occurs when the angles exceeds a certain limit, the value of V at the peak instead of at the clearing time must be compared with the critical value.

The above explanation is the reason why the estimation with V^{uc} does not work well for a 10-machine system under the heavy load condition. Considering the fact that the Lyapunov function which correctly takes the transfer conductances into account has not been obtained in spite of the efforts of many people, it is worth studying to appropriately modify the function used in this paper and make a more accurate estimation. One method of modification is proposed in the companion paper.

7. Conclusion

In this paper, we considered a method of determining a critical value which is one of the most important problems in Lyapunov's direct method applied to the transient stability problem. First, we investigated the condition under which the system is transiently stable. We showed that it is preferable in actual problems to use V^{uc} as the critical value rather than V^{um}. We devised two approximations of V^{uc}, i. e. V_{est1} and V_{est2}, from the angles of each machine during fault. As numerical examples, we estimated the critical clearing time for 4-and 10-machine systems by using V^{um}, V^{uc}, V_{est1} and V_{est2} as the critical values. The following results were obtained.

(1) As expected, the estimation with V^{uc} gives much better results than the estimation with V^{um}.

(2) Since V_{est1} supposes that one machine goes out of step, it gives results as accurate as V^{uc} in cases where the fault leads to a one-machine step-out.

(3) V_{est2} gives considerably good results even when more than one machine go out of step, in spite of the ease in its calculation which omits the computation of the unstable equilibrium points.

(4) The Lyapunov function used in this paper has the property that its time derivative can be positive when transfer conductances are large, and V can increase after the clearance of the fault to its peak value. Estimation errors are large in such cases.

(5) The peak value of V corresponds to V^{uc}.

In the companion paper we propose the modification of the Lyapunov function to improve the accuracy of estimation.

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Appendix

Kinetic energy V_k is written as

$$V_k=\frac{1}{\sum_{i=1}^{n} m_i}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} m_i m_j(\omega_i-\omega_j)^2, \qquad \cdots\cdots\cdots\cdots(A\text{-}1)$$

and its partial derivative with respect to $(\omega_i-\omega_1)$ can be written as

$$\frac{\partial V_k}{\partial(\omega_i-\omega_1)}=\frac{2m_i}{\sum_{j=1}^{n} m_j}\{(\sum_{j=1}^{n} m_j)(\omega_i-\omega_1)-\sum_{j=2}^{n} m_j(\omega_j-\omega_1)\}. \qquad \cdots\cdots\cdots\cdots(A\text{-}2)$$

Now suppose that angular momentum constant m_1 of the No. 1 machine is exceedingly larger than that of the other machine, then eq. (A-2) can be rewritten as

$$\frac{\partial V_k}{\partial(\omega_i-\omega_1)}=2m_i(\omega_i-\omega_1), \qquad \cdots\cdots\cdots\cdots(\text{A-3})$$

and in vector form

$$\frac{\partial V_k}{\partial \omega_r}=M\omega_r, \qquad \cdots\cdots\cdots\cdots(\text{A-4})$$

where $\omega_r=(\omega_2-\omega_1,\ \omega_3-\omega_1,\ \cdots\cdots,\ \omega_n-\omega_1)$, $M=diag(2m_2,\ 2m_3,\ \cdots\cdots,\ 2m_n)$. $\dot{V}$ can be written as

$$\dot{V}=\frac{\partial V_k}{\partial \omega_r}\cdot\dot{\omega}_r+\frac{\partial V_p}{\partial \delta_r}\cdot\omega_r, \qquad \cdots\cdots\cdots\cdots(\text{A-5})$$

where $\delta_r=(\delta_2-\delta_1,\ \delta_3-\delta_1,\ \cdots\cdots,\ \delta_n-\delta_1)$. Substituting eq. (A-4) into eq. (A-5). we get

$$\dot{V}=M\omega_r\cdot\dot{\omega}_r+\frac{\partial V_p}{\partial \delta_r}\cdot\omega_r,$$

$$=\left(M\dot{\omega}_r+\frac{\partial V_p}{\partial \delta_r}\right)\cdot\omega_r. \qquad \cdots\cdots\cdots\cdots(\text{A-6})$$

Since $\dot{V}$ always equals zero in the case of neglected transfer conductances, the following relation can be obtained

$$M\dot{\omega}_r+\frac{\partial V_p}{\partial \delta_r}=0,$$

that is,

$$M\dot{\omega}_r=-\frac{\partial V_p}{\partial \delta_r}. \qquad \cdots\cdots\cdots\cdots(\text{A-7})$$