Transient Stability Analysis of Electric Power Systems via Lur'e type Lyapunov Function: Part II

By

Naoto KAKIMOTO,* Yasuharu OHSAWA,* and Muneaki HAYASHI*

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Abstract

It was found in the previous paper that the time derivative of the Lyapunov function used for the transient stability analysis can take a positive value in the case of a system with large transfer conductances; and it makes the estimation error of the critical fault clearing time large. In this paper, the Lyapunov function is modified in order to remove the defect and fit it to the analysis of the transient stability. Moreover, a new method of determining the critical value of the Lyapunov function is proposed. which is derived from the consideration of the modification. The method uses the value of the potential energy as the critical value when the system gets out of the stability domain. The modified Lyapunov function and the new critical value are applied to a 10-machine system as a numerical example.

1. Introduction

Recently, Lyapunov's direct method has been examined as a method of on-line assessment of transient stability[1)-8)]. In the previous paper[10)], we investigated the estimation of the critical fault time via the Lur'e type Lyapunov function which is derived from the generalized Popov's theorem. From the results of the paper, it was shown that a more accurate estimation than by the usual method of Lyapunov can be made by paying attention to the stability for the first swing, which the system suffers by a fault. However, it was also ascertained that the time derivative of the used Lyapunov function becomes positive for the system with large transfer conductances, and it makes the error of the estimation large. In this paper, we make its cause clear, and appropriately modify the Lyapunov function so as to remove its defect and fit it to the analysis of transient stability. So far, no Lyapunov function correctly

* Department of Electrical Engineering.

takes the effect of transfer conductances into account. The method via the modified Lyapunov function proposed in this paper is one which can correctly evaluate the transient stability when transfer conductances can not be neglected.

Moreover, we have devised a new method of determining the critical value of the Lyapunov function from a consideration about the modification of the function. This method makes use of the fact that the time derivative of kinetic energy becomes zero at the instant when the system gets out of the stable region, and uses the value of the potential energy at that instant as the critical value. The idea about the critical value in the previous paper is refined to this method to give a more accurate critical value, and to make the calculation easier. Hence, it can be said that this method is very efficient in comparison with the usual method of Lyapunov.

In the last part of this paper, the modified Lyapunov function and the new critical value are applied to a 10-machine system; and the critical fault clearing time is estimated.

2. Modification of Lur'e type Lyapunov Function

The Lyapunov function used in the previous paper is of the Lur'e type, which is derived from the generalized Popov's theorem by Gudaru[7]. It takes the following form, when the damping is not considered:

$$
\begin{aligned}
V(\delta,\ \omega) &= \frac{1}{\sum_{i=1}^{n} m_i} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} m_i m_j (\omega_i - \omega_j)^2 \\
&+ \sum_{i=1}^{n} \sum_{j=1}^{n} E_i E_j Y_{ij} \{\cos(\delta_{ij}^s + \theta_{ij}) - \cos(\delta_{ij} + \theta_{ij}) - (\delta_{ij} - \delta_{ij}^s)\sin(\delta_{ij}^s + \theta_{ij})\} \\
&= V_k(\omega) + V_p(\delta) \qquad \cdots\cdots\cdots\cdots (1)
\end{aligned}
$$

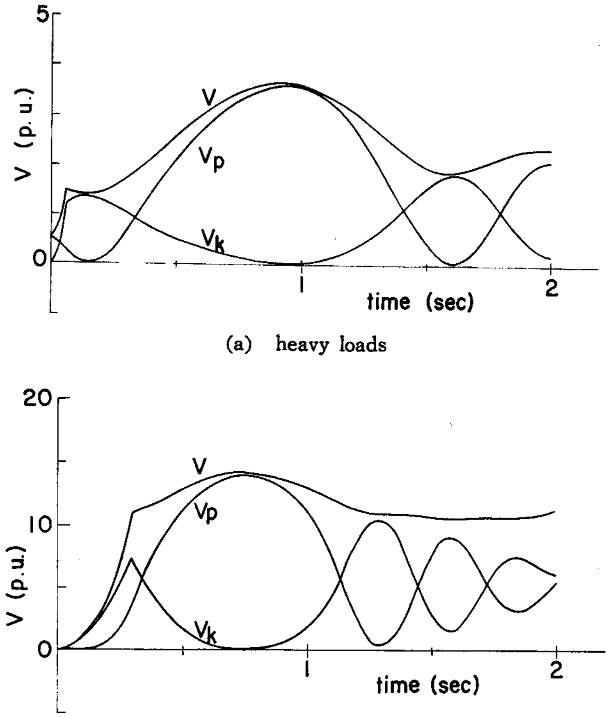
where V_k is called kinetic energy, and V_p is called potential energy. By investigating the time variation of V when the fault is cleared at the critical time (Fig. 1), it becomes clear that V increases after the clearance of the fault to its peak value at the time when V_p becomes maximum; and that this tendency is more remarkable as the loads get heavier. The time derivative of V is calculated as follows:

$$
\begin{aligned}
\dot{V}(\delta,\ \omega) &= \sum_{i=1}^{n} \sum_{j=1}^{n} E_i E_j Y_{ij} \{\sin(\delta_{ij}^s + \theta_{ij}) - \sin(\delta_{ij} + \theta_{ij})\} (\Delta\omega_i + \Delta\omega_j) \\
&= \dot{V}_k(\delta,\ \omega) + \dot{V}_p(\delta,\ \omega), \qquad \cdots\cdots\cdots\cdots (2)
\end{aligned}
$$

where

$$
\dot{V}_k = \sum_{i=1}^{n} \sum_{j=1}^{n} E_i E_j Y_{ij} \{\sin(\delta_{ij}^s + \theta_{ij}) - \sin(\delta_{ij} + \theta_{ij})\} (2\omega_i - 2\overline{\omega})
$$

$$
\dot{V}_p = -\sum_{i=1}^{n} \sum_{j=1}^{n} E_i E_j Y_{ij} \{\sin(\delta_{ij}^s + \theta_{ij}) - \sin(\delta_{ij} + \theta_{ij})\} (\omega_i - \omega_j)
$$



(b) light loads

Fig. 1. The time variations of the Lyapunov function and its components for the 10-machine system; critically cleared fault 11-12.

$$\bar{\omega}=\sum_{i=1}^{*} m_i\omega_i/\sum_{i=1}^{*} m_i$$

$$\Delta\omega_i=\omega_i-\bar{\omega}.$$

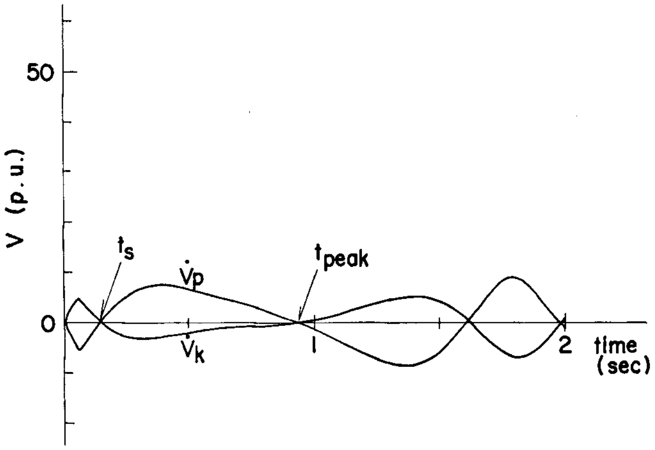
From eq. (2) it is seen that $\dot{V}$ is not generally negative definite, when $\theta_{ij}\fallingdotseq 0$, i. e., transfer conductances exist, although they equal zero and V=*constant* in case of $\theta_{ij}=0$. Consequently, V does not satisfy the conditions for the Lyapunov function (there was a mistake found in the process of induction in Gudaru's paper). Very accurate estimations, however, can be made via this function in the case of small θ_{ij}'s, as was shown by the numerical examples in the previous paper. It is difficult to get a Lyapunov function which correctly takes the transfer conductances into account. Therefore, we modify the V function in this paper so that an accurate estimation is possible even if the loads are heavy and θ_{ij}'s are large.

Since the peak value of V corresponds to the critical value for the first swing of the system, we consider such a modification of V that the value of V is kept constant at its peak value after the fault clearance. We first consider the increment

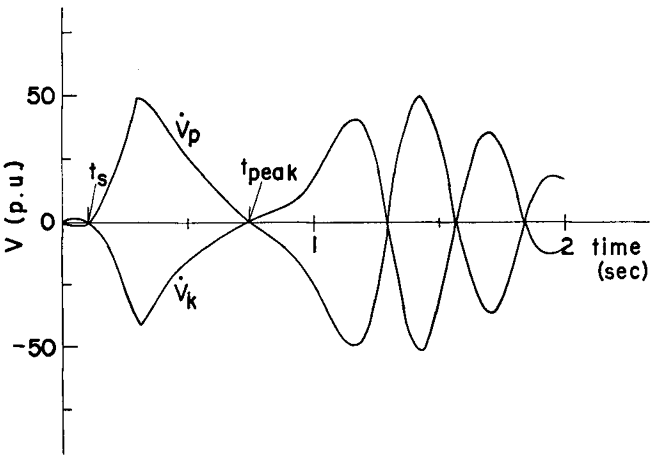
ΔV of V after the fault clearance. ΔV can be written as

$$\begin{aligned}\Delta V &= \int_{t_{cut}}^{t_{peak}} \dot{V}\, dt \\ &= \int_{t_{cut}}^{t_{peak}} (\dot{V}_p + \dot{V}_k)\, dt \qquad \cdots\cdots\cdots\cdots(3) \\ &= \Delta V_p + \Delta V_k,\end{aligned}$$

where t_{peak} is the time when V reaches its peak, and t_{cut} is the time of fault clearance. If ΔV is added to V at the clearing time, V equals its peak value. Performing the same operation for t, $t \geq t_{cut}$, we get



(a) heavy loads



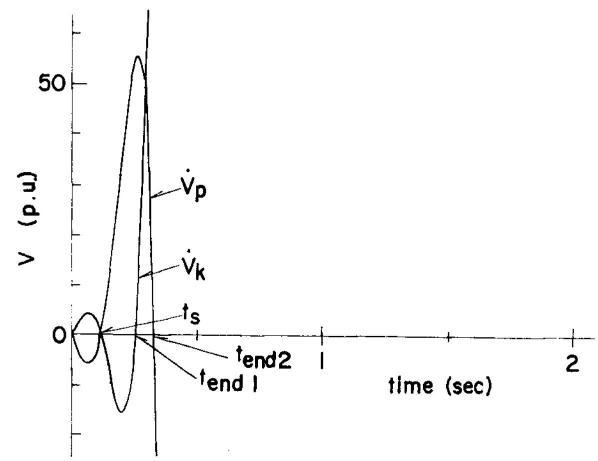
(b) light loads

Fig. 2. The time variations of $\dot{V}_p$ and $\dot{V}_k$ for the 10-machine system; critically cleared fault 11-12.

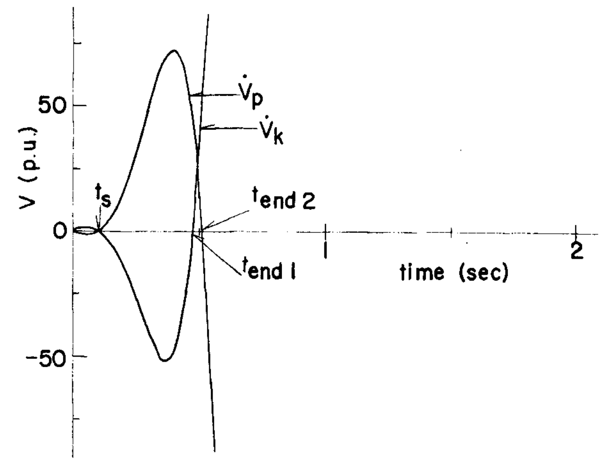
$$V_{peak} = V(t) + \Delta V(t), \qquad \cdots\cdots\cdots\cdots (4)$$

and the value of V equals V_{peak} after the time of fault clearance. Since the critical clearing time is unknown at the time of the estimation, it is necessary to perform this modification without knowing the variation of $\dot{V}_p$ and $\dot{V}_k$ after the fault clearance.

Fig. 2 shows an example of the time variations of $\dot{V}_p$ and $\dot{V}_k$ when the fault is cleared at the critical clearing time. $\dot{V}_p$ and $\dot{V}_k$ vanish at the time t_s, because the angles of the machines at this time are similar to those at the stable equilibrium point. After t_s, $\dot{V}_p$ becomes positive and $\dot{V}_k$ becomes negative, i. e., V_p increases



(a) heavy loads



(b) light loads

Fig. 3. The time variations of $\dot{V}_p$ and $\dot{V}_k$ for the 10-machine system ; sustained fault 11-12.

and V_k decreases, and V increases because $|\dot{V}_p|>|\dot{V}_k|$. $\dot{V}_p$ and $\dot{V}_k$ again vanish at time t_{peak}, when V takes its peak value.

Fig. 3 shows the time variations of $\dot{V}_p$ and $\dot{V}_k$ for a sustained fault. Both $\dot{V}_p$ and $\dot{V}_k$ vanish at time t_s by the same reason as in Fig. 2. After t_s, $\dot{V}_p$ is positive, $\dot{V}_k$ is negative, and $\dot{V}_p$ and $\dot{V}_k$ again equal zero at time t_{end1} and t_{end2}, respectively. t_{end1} is generally smaller than t_{end2}, and its discrepancy becomes larger as loads get heavier, as is seen by a comparison with Fig. 3 (a) and (b).

As will be shown in Section 3, the system loses its synchronism after the time t_{end1}, when $\dot{V}_k$ becomes zero. Therefore, the value of V_p at t_{end1} is to be the critical value for stability. Because the peak value of V after the fault-clearing corresponds to this critical value, ΔV in eq. (3) is approximated as follows:

$$\begin{aligned}\Delta V &= \int_t^{t_{end1}} \dot{V}\, dt \\ &= \int_t^{t_{end1}} (\dot{V}_p+\dot{V}_k)\, dt \\ &= \Delta V_p + \Delta V_k,\end{aligned} \qquad \cdots\cdots\cdots\cdots (5)$$

where $\dot{V}$, $\dot{V}_p$ and $\dot{V}_k$ are expressed by eq. (2), and with parameters for the post -fault condition.

Eq. (5) is interpreted as follows: if the system reaches the boundary of the stable region after the clearance of the fault at time t, the increment of V is approximately given by eq. (5).

Table 1. ΔV_p, ΔV_k and β for the 10-machine system; fault 11-12.

(a) heavy loads

Critically cleared fault	$\Delta V_p = 3.597$ $\Delta V_k = -1.325$ $\beta = 2.715$
Sustained fault	$\Delta V_p = 3.574$ $\Delta V_k = -1.326$ $\beta = 2.695$

(b) light loads

Critically cleared fault	$\Delta V_p = 13.894$ $\Delta V_k = -10.466$ $\beta = 1.328$
Sustained fault	$\Delta V_p = 14.475$ $\Delta V_k = -10.347$ $\beta = 1.399$

Table 1 shows an example of the values of ΔV_p and ΔV_k, from t_s to t_{end1} for the sustained fault, and from t_s to t_{peak} for the critically cleared fault. It shows that the approximation is very reasonable. Subsequently, we consider the reason why the values of ΔV_p and ΔV_k are almost the same in both the cases of a sustained fault and a critically cleared fault. From eq. (1),

$$\dot{V}_k=\frac{\partial V_k}{\partial \omega_r}\cdot\dot{\omega}_r, \qquad \dot{V}_p=\frac{\partial V_p}{\partial \delta_r}\cdot\omega_r . \qquad \cdots\cdots\cdots\cdots (6)$$

Accordingly, ΔV_p can be written as

$$\begin{aligned} \Delta V_p &= \int \frac{\partial V_p}{\partial \delta_r}\,\omega_r\,dt \\ &= \int \frac{\partial V_p}{\partial \delta_r}\,d\delta_r \\ &= V_p(\delta_e) - V_p(\delta_s), \end{aligned} \qquad \cdots\cdots\cdots\cdots(7)$$

and it depends upon only δ_e and δ_s, which are the values of δ_r at the upper and lower boundary of the integral. Suppose that δ_s is the stable equilibrium point, then $V_p(\delta_s)=0$, and ΔV_p can be determined only by δ_e. Hence, if δ_e changes a little between the cases of a sustained fault and a critically cleared fault, then ΔV_p is almost the same between them. On the other hand, ΔV_k can be written as follows;

$$\begin{aligned} \Delta V_k &= \int \frac{\partial V_k}{\partial \omega_r} \cdot \dot{\omega}_r\,dt \\ &= \int \dot{\omega}_r(\delta_r) \cdot L(d\delta_r), \end{aligned} \qquad \cdots\cdots\cdots\cdots(8)$$

where $L(d\delta_r)$ is a linear function of $d\delta_r$.
ΔV_k depends upon the integral path. Therefore, if the projections of the trajectories onto the angular space are almost the same for both the cases of a sustained fault and a critically cleared fault, then ΔV_k is almost the same (see Fig. 4).

Accordingly, it is permitted to substitute eq. (5) for eq. (3), if the mode of the swing is not varied very much by the clearance of the fault. On that occasion, the value of t must be large enough to satisfy the assumption that the system reaches

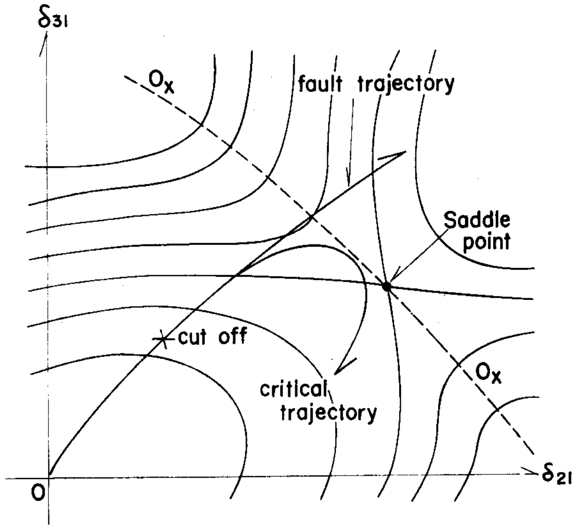


Fig. 4. Trajectories projected onto the angular space.

the boundary of the stable region after the fault-clearance. We can use eq. (9) in order to modify the Lyapunov function in the same way as eq. (4), namely,

$$V_a(t) = V(t) + \Delta V(t). \qquad \cdots\cdots\cdots\cdots(9)$$

As $\Delta V(t)$ is a function of time t, and should be computed with time t varied, the calculations become a little complex. To avoid it, we consider another modification,

$$V_\beta(t) = V_p(t) + \beta V_k(t), \qquad \cdots\cdots\cdots\cdots(10)$$

where

$$\beta = -\int_{t_s}^{t_{end1}} \dot{V}_p dt \Big/ \int_{t_s}^{t_{end1}} \dot{V}_k dt$$

In this modification, β is a constant which represents the average efficiency of transformation from kinetic energy V_k to potential energy V_p when the system moves from the stable equilibrium point to the boundary of the stable region. That is, after the clearance of the fault, kinetic energy V_k decreases and is transformed into potential energy V_p, and its efficiency is β on the average. Therefore, the value of V after transformation can be obtained by multiplying V_k by β. This latter modification has the advantage that the Lyapunov function is represented analytically unlike the former one. It is inevitable, however, that V will oscillate a little after fault-clearance because β is an average value.

3. Determination of the Critical Value of the Lyapunov Function

Fig. 5 shows an example of equipotential curves of the 3-machine system. The three stability conditions shown in the previous paper were based on the unstable equilibrium point. According to the previous paper, however, the system becomes unstable when it crosses the curve O_x. Hence, it is more suitable to use the value of V_p at the time when the system crosses the curve O_x as the critical value rather than the value at the unstable equilibrium.

The time when the system crosses O_x can be known in the following manner. Fig. 6 shows the projection of the trajectory at the instant when the system crosses the curve O_x, where ω_r is orthogonal to the curve O_x. $\dot{V}_p$ can be written as

$$\dot{V}_p = \frac{\partial V_p}{\partial \delta_r} \omega_r.$$

Accordingly, when ω_r is orthogonal to O_x as in Fig. 6, $\dot{V}_p$ equals zero because $\partial V_p / \partial \delta_r$ is parallel with O_x. On the other hand, $\dot{V}_k$ can be written as

$$\dot{V}_k = \frac{\partial V_k}{\partial \omega_r} \dot{\omega}_r = -\frac{\partial V_p}{\partial \delta_r} \cdot \omega_r,$$

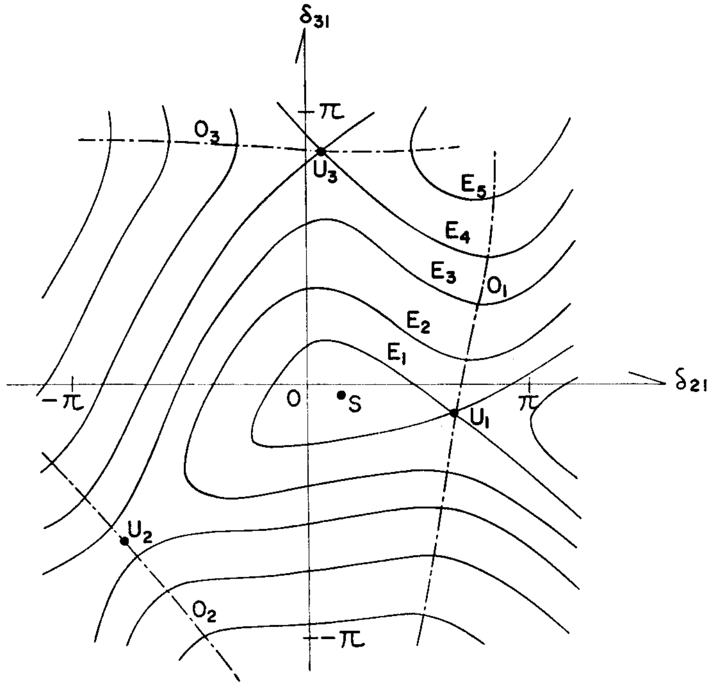


Fig. 5. Equipotential curves for a 3-machine system.

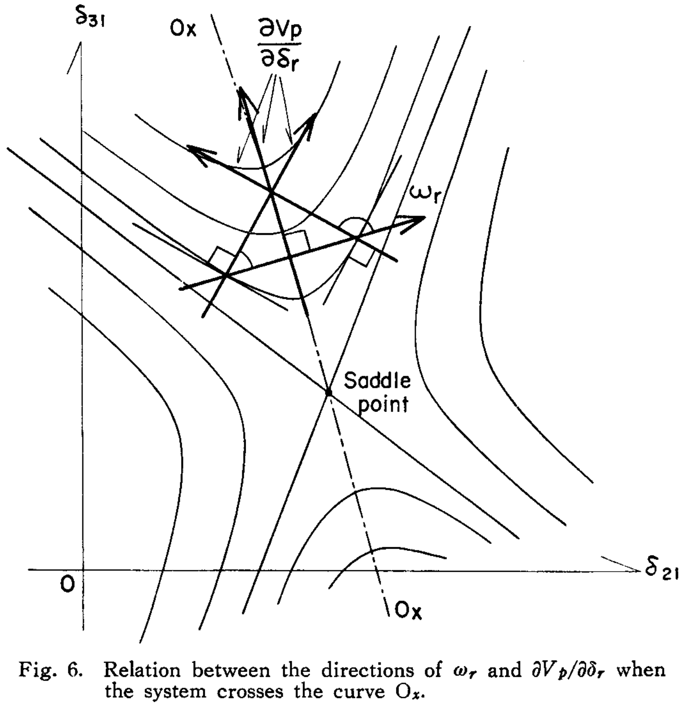


Fig. 6. Relation between the directions of ω_r and $\partial V_p/\partial\delta_r$ when the system crosses the curve O_x.

and hence

$$\dot{V}_k=0.$$

Therefore, $\dot{V}_p$ and $\dot{V}_k$ vanish at the instant when the system crosses the curve O_x orthogonally. The point where $\dot{V}_p$ and $\dot{V}_k$ vanish does not lie on O_x, when ω_r is not just orthogonal to O_x. This discrepancy, however, is small, because the direction of $\partial V_p/\partial\delta_r$ is greatly changed near O_x.

Next, the signs of $\dot{V}_p$ and $\dot{V}_k$ are considered. When the system is inside the curve O_x, i. e., in the stable region, $\dot{V}_p$, which equals the inner product of $\partial V_p/\partial\delta_r$ and ω_r, is positive, and $\dot{V}_k$ is negative because $\partial V_p/\partial\delta_r$ and ω_r make an acute angle. On the other hand, when the system is outside O_x, $\partial V_p/\partial\delta_r$ and ω_r make an obtuse angle, and accordingly, $\dot{V}_p$ is negative and $\dot{V}_k$ is positive. Table 2 summarizes this situation. That is, in the stable region, the potential energy V_p increases and the kinetic energy V_k decreases. The decrease of V_k means the decrease of $|\omega_r|$, and prevents the system from getting out of O_x. The situation is the contrary outside O_x. Consequently, it is possible to regard the time when $\dot{V}_p$ changes its sign from positive to negative, or when $\dot{V}_k$ changes from negative to positive as the time when the system crosses O_x.

Table 2. Sign of $\dot{V}_p$ and $\dot{V}_k$ when the system crosses the curve O_x.

	Inside of O_x	Outside of O_x
$\dot{V}_p$	positive	negative
$\dot{V}_k$	negative	positive

When transfer conductances are not neglected, $\dot{V}$ is not always zero, and accordingly, the times when $\dot{V}_p$ and $\dot{V}_k$ vanish do not coincide. In this case, the instant when $\dot{V}_k$ becomes zero is considered as the time of the crossing of the stability boundary. This is because V_k is expressed by the same equation as the case when transfer conductances are neglected; and the distinction between the inside and outside of O_x was made by the decrease or increase of $|\omega_r|$.

The instant when the system crosses the boundary of the stable region can be known through the process mentioned above; and V_{cr}, the value of V_p at that instant, shows the boundary value for stability on the trajectory. Therefore, if the total energy of the system V satisfies the condition

$$V<V_{cr}, \qquad \cdots\cdots\cdots\cdots(11)$$

then the system is stable. Hence, we can use V_{cr} as the critical value. Actually, the trajectory changes, when the fault is cleared so as to satisfy the above condition.

The change must not be large for the above condition to be valid. When the change of the trajectory is large, we can modify the value of V_{cr} as follows. First calculate the V_{cr} from the sustained fault, and call it V_{cr}^1. V_{cr}^1 is generally greater than the true critical value. Hence, the system crosses the curve O_x if the fault is cleared at $V = V_{cr}^1$. We call the value of V_{cr} of this case V_{cr}^2. V_{cr}^2 is closer to the true critical value than V_{cr}^1. By repeating the above operation, the more accurate critical values V_{cr}^3, V_{cr}^4,... can be obtained. The number of repetitions, however, should be limited at most to 2 or 3 times, in order that the advantage of the Lyapunov's method may not be cancelled.

4. Examples

Following the method described so far, we have estimated the critical fault clearing time of the 10-machine system[9], which was used in the previous paper.

Table 3 shows the estimated values of the critical clearing time and the actual values which are obtained by simulations. Although the values estimated by V are

Table 3. Estimations of the critical clearing time (sec.) for the 10-machine system.

(a) heavy loads

Fault	V	V_α	V_β	T_{cr}
11-12	0.10	0.06	0.06	0.06
15-14	0.23	0.21	0.21	0.20
17-18	0.26	0.20	0.21	0.22
18-17	0.27	0.21	0.21	0.22
24-16	0.23	0.18	0.18	0.18
30-27	0.26	0.21	0.22	0.23
34-29	0.25	0.20	0.21	0.22
38-15	0.28	0.22	0.22	0.17

(b) light loads

Fault	V	V_α	V_β	T_{cr}
11-12	0.33	0.29	0.30	0.30
15-14	0.42	0.40	0.41	0.41
17-18	0.51	0.47	0.49	0.47
18-17	0.53	0.48	0.50	0.50
24-16	0.49	0.44	0.46	0.45
30-27	0.51	0.47	0.49	0.47
34-29	0.50	0.46	0.48	0.46
38-15	0.71	0.67	0.69	0.65

greater than the actual values, the values by V_α and V_β are exceedingly near the actual values. In general, V_α gives smaller estimated values than V_β, but the differences between them are small. For the fault 38-15, however, the estimated value is exceptionally greater than the actual value, and its error is great compared with those for other faults.

Table 4 shows the values of V, V_α and V_β at the critical clearing time and the

Table 4. V, V_α and V_β at the critical clearing time, V_{peak} and V_{cr} for the 10-machine system.

(a) heavy loads

Fault	V	V_α	V_β	V_{peak}	V_{cr}
11-12	1.52	3.69	3.65	3.66	3.90
15-14	22.04	29.25	26.73	30.94	32.52
17-18	22.62	37.58	37.35	36.24	34.38
18-17	21.64	40.89	39.53	39.14	39.19
24-16	23.52	40.71	38.84	35.45	41.93
30-27	22.21	33.69	34.02	32.70	31.04
34-29	22.53	34.78	35.20	30.72	32.35
38-15	8.96	21.46	16.03	14.00	28.70

(b) light loads

Fault	V	V_α	V_β	V_{peak}	V_{cr}
11-12	11.10	14.83	14.06	14.00	14.75
15-14	45.99	48.53	47.08	50.41	47.11
17-18	40.25	49.96	45.32	47.65	50.17
18-17	50.37	60.90	56.45	61.42	58.61
24-16	54.07	66.79	62.08	64.06	66.41
30-27	36.54	44.36	40.55	40.50	45.89
34-29	39.61	48.25	44.31	44.16	49.17
38-15	45.99	52.82	49.23	49.61	56.13

peak value of V. The value of V is small in comparison with V_{peak}, and this trend is remarkable in the case of heavy loads. The values of V_α and V_β obtained by modifying V are almost the same as with V_{peak}, so it is ascertained that the modification of V is successfully performed as was aimed. In Table 4, the critical value V_{cr} is also shown. It is proved from the comparison between V_{cr} and V_{peak} that the critical value proposed in this paper is quite proper. For the fault 38-15, especially in case of the heavy loads, there are large differences between V_{cr} and V_{peak}, while V_β is almost equal to V_{peak}. The cause of this discrepancy will be investigated later.

Fig. 7 shows the time variations of V, V_α and V_β when the fault 11-12 is susta-

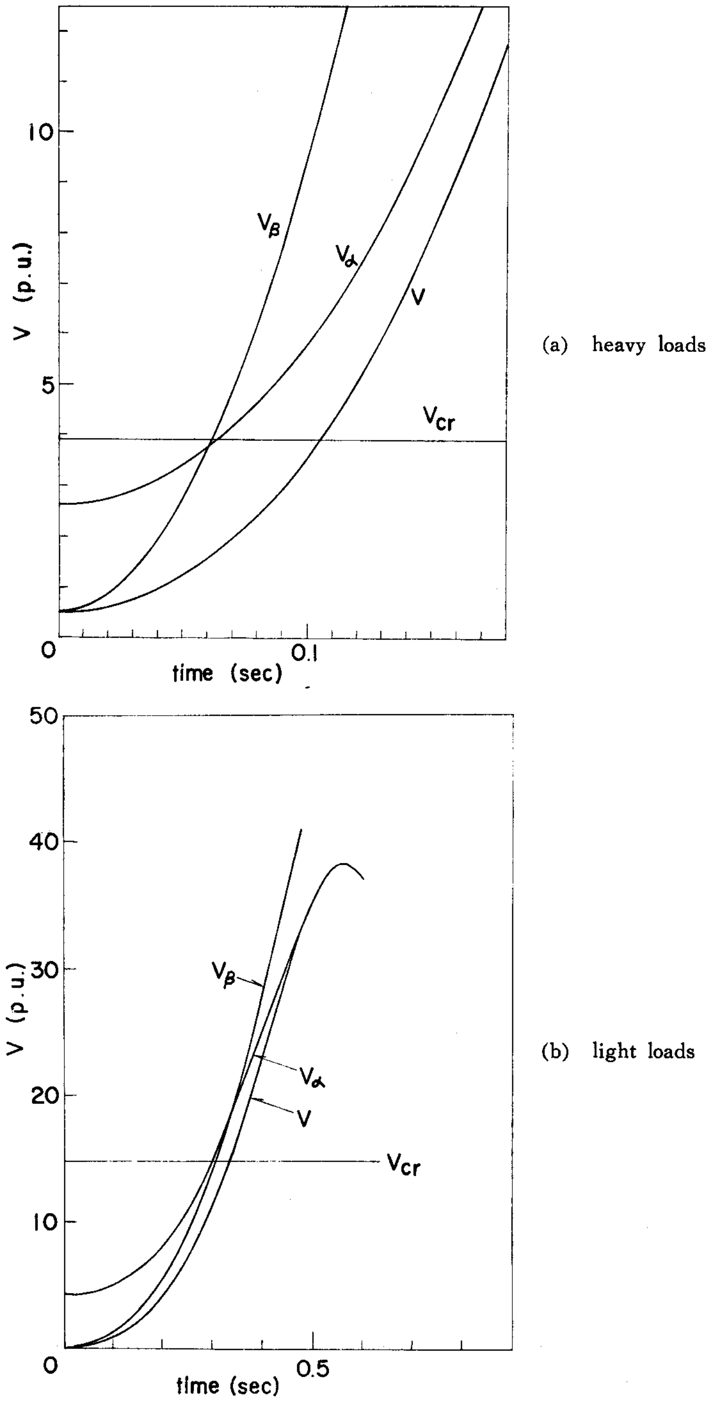
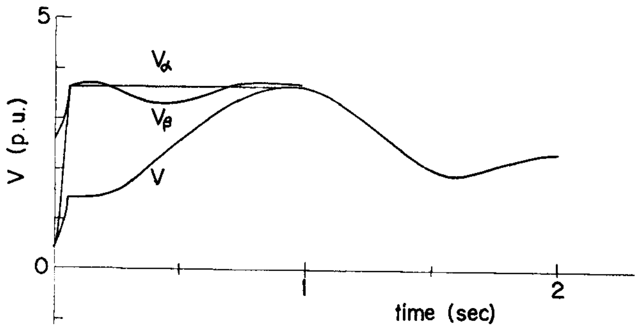


Fig. 7. The time variations of V, V_α and V_β for the 10-machine system; sustained fault 11-12.

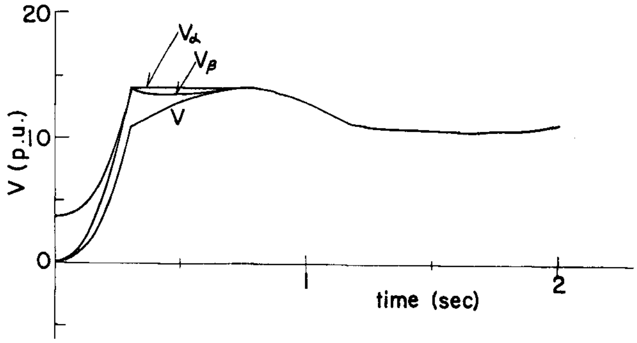
ined. V_α is the function which can be obtained by adding ΔV, the increment of V from the instant of the fault clearing to the time of crossing the boundary of the stable region, to V. Accordingly, the difference with V is large when the time t is small, but it approaches V assymptotically as time proceeds. On the other hand, V_β is given by multiplying V_k by β in V, so its difference with V gets larger as V_k gets larger.

Fig. 8 shows the time variations of V, V_α and V_β when the fault 11-12 is critically cleared. The function V_α follows eqs. (3), (9), when the fault is cleared. Hence, it is kept at the constant value of V_{peak} after the clearance of the fault. V_α, however, does not always coincide with V_{peak} in the case of the estimation, because eq. (5) is used. V_β is almost equal to V_{peak} at the clearing time, and oscillates a little afterwards, where the value of β is that used in the estimation.

From the above results, it is verified that the the method for modifying the Lyapunov function and determining the critical value proposed in this paper are proper except for the fault 38-15. In the following, we consider the case of the



(a) heavy loads

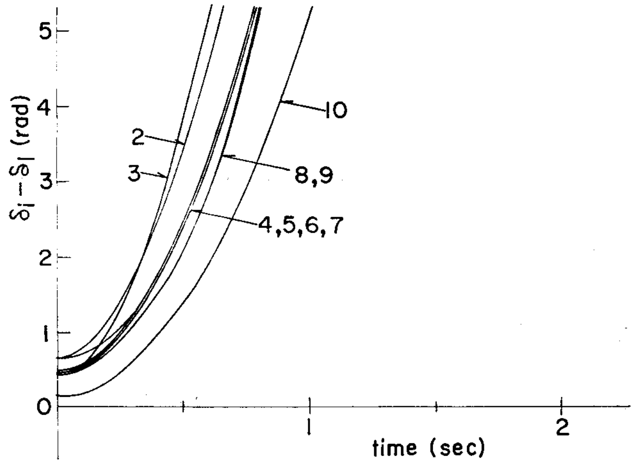


(b) light loads

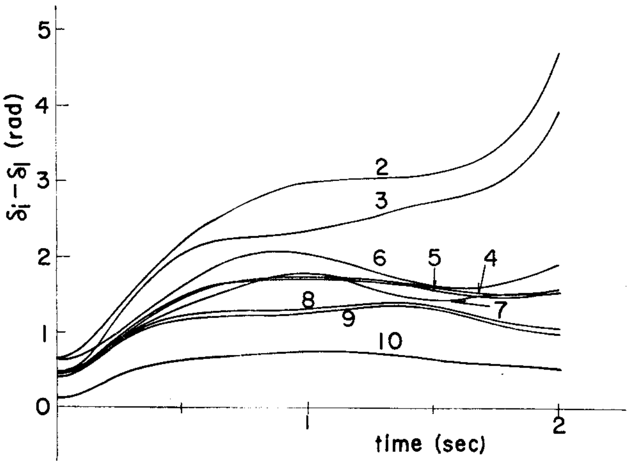
Fig. 8. The time variations of V, V_α and V_β for the 10-machine system; critically cleared fault 11-12.

fault 38-15.

From Table 2 it is shown that the assumption in Section 3 is not satisfied, because the critical value V_{cr} largely differs from V_{peak}. The swing curves, when the fault is sustained and when the fault is cleared at a time a little later than the critical value, are shown in Fig. 9. The modes of the swings are very different, and so there is a great difference between V_{cr} and the actual critical value. One method to improve the estimation in such cases, as described in the last section, is to repeat the calculation of V_{cr} by clearing the fault at the time which is obtained in the previous estimation. Table 5 shows the result. An estimated value adequately near



(a) sustained fault 38-15.



(b) fault 38-15 cleared at 0.18 sec.

Fig. 9. The swing curves for the 10-machine system; heavy loads.

Table 5. Improvement of the estimation by repetition in case of the fault 38-15.

(a) heavy loads

No.	V_{cr}	V_α	V_β	β
1	28.70	0.22	0.22	1.831
2	21.92	0.19	0.19	1.890
3	19.73	0.18	0.18	1.891

(b) light loads

No.	V_{cr}	V_α	V_β	β
1	56.12	0.67	0.69	1.189
2	53.06	0.65	0.67	1.176

the true value can be obtained after three iterations in the case of heavy loads, and after two iterations in the case of light loads. The change of V_{cr} is particularly large for the heavy loads, and its value gets smaller with repetition. The value of β, however, does not change so much, and does not affect the estimation. From the above results, an adequately accurate estimation can be obtained by repeating the calculations 2 or 3 times even for the fault 38-15.

5. Conclusion

In this paper, we considered a modification of the Lyapunov function V used in the previous paper, so as to make the estimation of the critical clearing time better, when transfer conductances are taken into account. From the fact that the increment of V after the fault clearance can be approximated by the data for the sustained fault, under the assumption that the mode of the swings does not change so much when the fault is cleared, the function V was modified so as to get the functions V_α and V_β.

On the other hand, we advanced the considerations made in the previous paper about the condition under which the system is transiently stable, and showed that as the critical value we can use V_{cr}, the value of V_p at the instant when the time derivative of the kinetic energy vanishes for the sustained fault.

As an example, we applied the functions V_α, V_β and the critical value V_{cr} to the 10-machine system and estimated the critical fault clearing time. Consequently, the following results were obtained.

1) The estimation via V_α or V_β gives better results compared with the estimation

via V.

2) The values of V_α and V_β at the critical clearing time are almost the same as the peak value of V.

3) The critical value V_{cr} correctly indicates the boundary of the stable region, and its calculation is easy.

4) When the mode of swings changes largely between the sustained fault and the cleared fault as in case of the fault 38-15, a good estimated value can be obtained by repeating the estimation two or three times at most.

Accordingly, we can correctly estimate the critical fault clearing time by using the functions V_α, V_β and the critical value V_{cr}, even if the transfer conductances are large. Besides, we can avoid the difficulties accompanying the calculation of the unstable equilibrium points, which is a great obstacle to the application of the Lyapunov direct method.

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