A New Approach to Transient Stability Region of Synchronous Generator

By

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Abstract

This paper describes a method of analytically determining the domain of transient stability in state space for a synchronous generator. The motion of a synchronous generator is represented in the state space; and the rate of change of the direction of the motion is expressed by a series expansion of the distance along the motion. The stability boundary is determined by the minimal point of the differential coefficient of a sufficiently high order, based on the facts that every state on the stability boundary approaches the saddle point with time, and also that the direction of the trajectory changes discontinuously at the saddle point. As a numerical example, the method is applied to a simple synchronous generator connected to an infinite-bus. The results show that the most important part of the stability boundary can be obtained very accurately with a reasonable amount of computation.

1. Introduction

The transient stability of electric power systems is assessed by analyzing a set of nonlinear ordinary differential equations called "swing equation." The method which has been used widely for the purpose is that via simulation. This method consists of solving the swing equation numerically, obtaining the performance of the system after the clearance of the fault, and then judging whether the system is stable or not. This method has an advantage that the system can be modelled as minutely as the computer permits. The method also has a weak point in that it requires much computing time, because the differential equations must be solved until the system is judged to have stability.

The other methods, such as the equal-area criterion, do not require the solution of the differential equations, but they are applicable in practice to a 1- or 2-machine system only. As a method which is applicable to a multimachine system, the direct method of Lyapunov has been studied by many researchers.[1-4] This method has some very at-

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tractive features, but there are also some drawbacks. Except in special cases, estimates of the region of stability are conservative. Although there are some methods which can theoretically predict the true region of stability,[5,6] the amount of computation is considerable. Another difficulty is that the method is almost restricted to a simplified model, in which the effects of A.V.R., line conductances etc. are neglected.

This paper describes an analytic method of deriving the boundary of the stability region by using the property whereby the trajectory discontinuously changes its direction at the saddle point. First, the function which represents the partial derivative of the direction of a trajectory with respect to the direction perpendicular to the trajectory is defined. Next, this function is expressed by the series expansion of the distance along the trajectory, and the stability boundary is determined from the fact that the differential coefficient of a sufficiently high order of the series expansion takes its minimal value on the stability boundary. In Section 2, the principle of the method is described. As a numerical example, it is applied to a simple single-machine system in Section 3.

2. Principle of the Method

We consider the autonomous system described by the following set of differential equations:

$$\begin{aligned} \dot{x}_1 &= X_1(x_1, x_2, \ldots, x_n) \\ \dot{x}_2 &= X_2(x_1, x_2, \ldots, x_n) \\ &\vdots \\ \dot{x}_n &= X_n(x_1, x_2, \ldots, x_n) \\ & X_i(0, 0, \ldots, 0) = 0 \qquad i = 1, 2, \ldots, n \end{aligned} \tag{1}$$

or

$$\dot{x} = X(x), \qquad X(0) = 0 \tag{2}$$

The origin is assumed to be a stable equilibrium point, and our problem is to obtain the asymptotically stable region around the origin. First we describe the proposed method for the case of second order system, and then, we generalize it to higher order systems.[7]

Fisrt of all, we define the following function in the state space of the second order.

$$F = \frac{\partial \alpha}{\partial n} = \frac{\partial[arctan\{X_1(x_1, x_2)/X_2(x_1, x_2)\}]}{\partial n} \tag{3}$$

This function is the partial derivative of the direction of the trajectory at a point with respect to the direction perpendicular to the trajectory (Fig. 1). The value of this function is positive when the trajectories approach each other, and negative when they diverge. Since the direction of the trajectory changes discontinuously at the saddle point, the value of this function approaches $-\infty$ along the trajectory which approaches the saddle point.

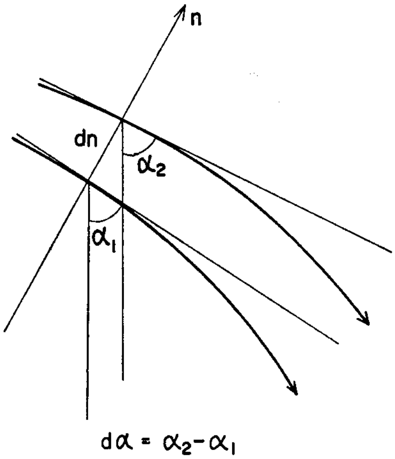

Fig. 1. Definition of the function F.

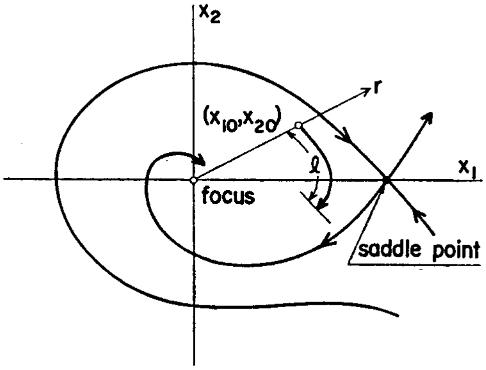

Fig. 2. Definition of the variable l.

This trajectory is called the separatrix. When we take one initial point (x_{10}, x_{20}), a unique trajectory passes through this point, as long as the point is not a singular point. If the distance from the initial point along this trajectory is denoted by l, the function F on the trajectory becomes the function of the initial point and l. Representing the function F by the series expansion with respect to l, we have:

$$F(x_{10}, x_{20}, l) = F(x_{10}, x_{20}, 0) + l \cdot \left.\frac{\partial F}{\partial l}\right|_{(x_{10}, x_{20})} + \frac{l^2}{2!} \cdot \left.\frac{\partial^2 F}{\partial l^2}\right|_{(x_{10}, x_{20})} + \cdots + \frac{l^n}{n!} \cdot \left.\frac{\partial^n F}{\partial l^n}\right|_{(x_{10}, x_{20})} + \cdots \tag{4}$$

where $\partial F/\partial l$, $\partial^2 F/\partial l^2$, $\cdots$ represent the partial derivative of F of each order with respect to the direction of the trajectory l.

If the series expansion (4) is truncated at the finite number of terms m, it yields the error R_m. Since F takes $-\infty$ at the saddle point, R_m is $-\infty$ regardless of the value of m. Since a point on the separatrix approaches the saddle point with time, the partial derivative $\partial^k F/\partial l^k$ takes a larger value on the separatrix than on the other points near it, as long as the order of the derivative k is high enough. Hence, by computing the value of $\partial^k F/\partial l^k$ continuously along some line crossing the separatrix, we can know the point on the separatrix, that is, on the stability boundary, from its minimal value. In connection with the case of the power system transient stability problem, the stability boundary (and at the same time the critical clearing time) can be derived by calculating the value of $\partial^k F/\partial l^k$ for the post-fault system at each during- fault point obtained by the step-by-step method and by identifying its minimal value.

In the case of a general n-dimensional system, the ortho-complement of a vector is of order (n-1). It is, therefore, necessary to impose (n-2) supplementary conditions and to transpose the ortho-complement into a 1-dimensional space in order to define the function F in the same way as eq. (3). For the supplementary conditions, we assume that all coordinates except two are constant, that is, x_i is assumed to be constant for $i=1,2, \cdots, p-1, p+1, \cdots, q-1, q+1, \cdots, n$. Then the plane (x_p, x_q) is defined by this assumption. If we draw the line tangent to the trajectory at the point where the trajectory crosses this plane, then the vector, which is orthogonal to this tangent line, and at the same time lies on the plane, is orthogonal to the projection of the tangent line to the plane. Hence the function F is represented as follows:

$$F_{pq}=\frac{\partial \alpha}{\partial n}=\frac{\partial[\arctan\{X_p(x_1, \cdots, x_n)]/X_q(x_1, \cdots, x_n)\}]}{\partial n}\Bigg|_{x_1=x_{10}, \cdots \overset{p}{\vee} \cdots \overset{q}{\vee} \cdots, x_n=x_{n0}} \tag{5}$$

The possible number of a 2-dimensional space in n-dimensional space is the same as the number of the combination ${}_nC_2$. The function F can, therefore, be defined for an n-dimensional case as follows:

$$F=\sum_{p,q}[F_{pq}] \tag{6}$$

It is not necessary to calculate all of the elements in eq. (6), since each F_{pq} has the same feature that its value is minus infinity at the saddle point.

3. Numerical Example

The method described above is applied to the single-machine infinite-bus system which is represented by the following differential equation[6]:

$$M\frac{d^2\delta}{dt^2}+D\frac{d\delta}{dt}=P_m-P_e \sin\delta \tag{7}$$

$$M=0.0138 \qquad D=0.0285$$

$$P_m=0.91 \qquad P_e=3.02.$$

Introducing a new variable T so that

$$T=\sqrt{(P_e/M)}\cdot t=14.8t,$$

eq. (7) is reduced to

$$\frac{d^2\delta}{dT^2}+0.140\frac{d\delta}{dT}=0.301-\sin\delta. \tag{8}$$

The singularities of eq. (8) are

stable focus $\delta=0.306$ rad.
saddle $\delta=2.836$ rad.

The stable equilibrium is transferred to the origin through the co-ordinate transformation $\delta=x_1+0.306$. Using the state variable representation $\dot{\delta}=\dot{x}_1=x_2$, eq. (8) is rewritten as follows:

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=0.301-\sin(x_1+0.306)-0.140x_2\end{aligned} \tag{9}$$

We consider the asymptotic stability region around the origin. The actual stability region obtained by solving the differential equation (8) using a numerical integration is shown in Fig. 3. $\partial^k F/\partial l^k$ for the various values of k is calculated along the lines radiating from the origin in the first quadrant, where the transient instability usually occurs. (The detailed process of the calculation is included in the Appendix.) Fig. 4 shows the results of taking r as the axis of abscissa and of taking k as the parameter. In Fig. 4(a), the stability boundary is accurately assessed by calculating the derivative of the function F up to only the second order. On the other hand, in Fig. 4(b) and Fig. 4(c), the deri-

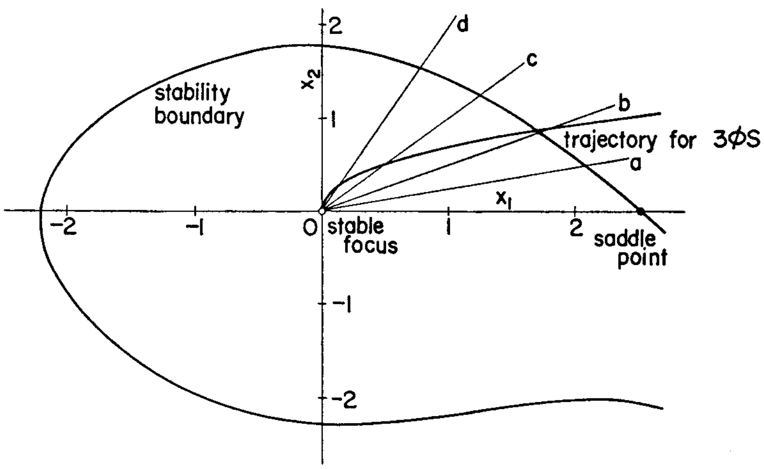

Fig. 3. Stability region of the model system and radial lines for the application of the proposed method.

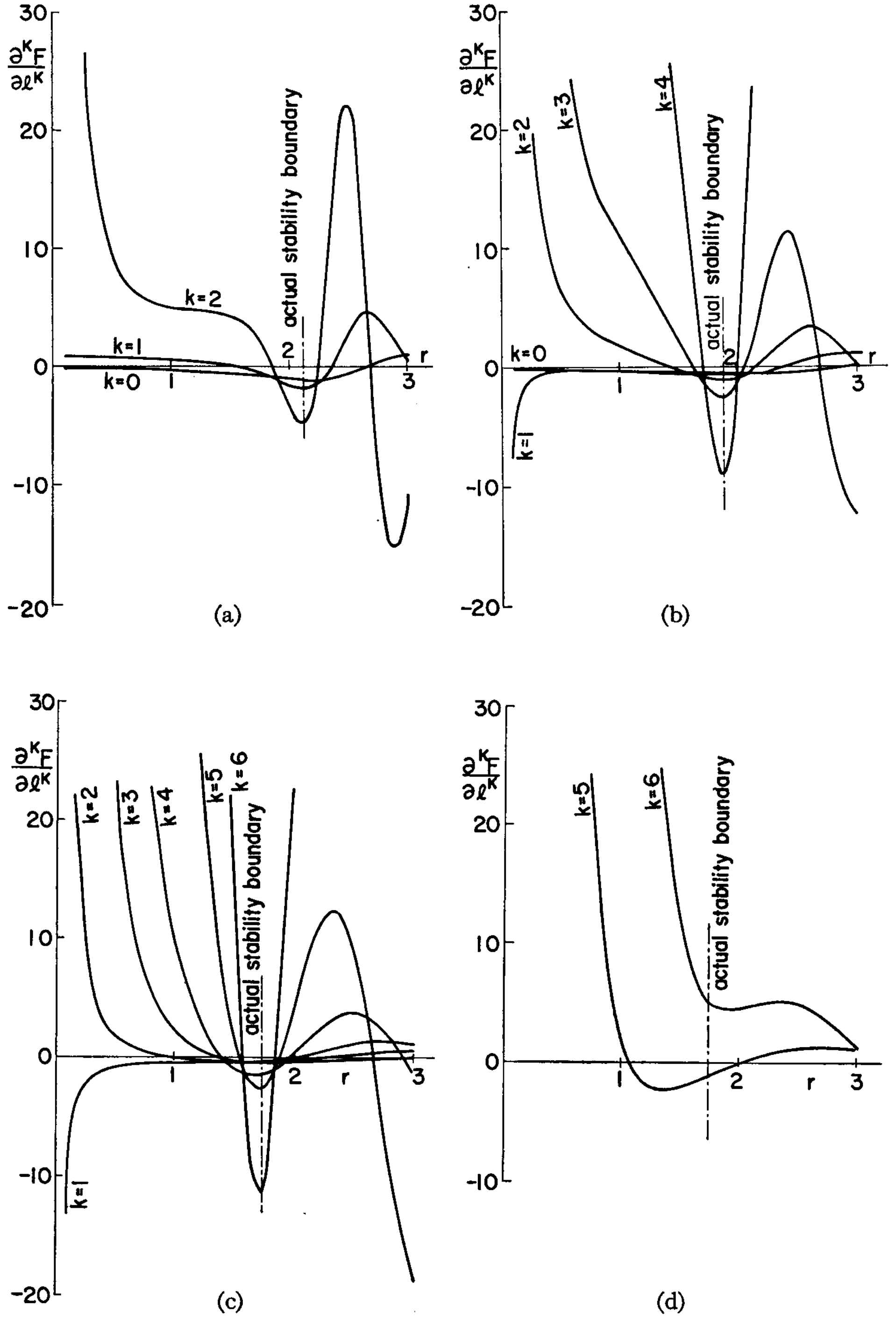

Fig. 4. Assessment of the stability boundary by the proposed method.

vatives must be calculated up to at least the third order and fifth order, respectively. In Fig. 4(d), even the sixth order is still insufficient. This is because of the fact that when the stability boundary point is near the saddle point, the value of l in eq. (3) is small, and hence, the derivative of a low order takes a large negative value at the stability boundary. On the contrary, for the boundary point apart from the saddle point, the value of l is large and high order derivatives must be calculated in order to know the stability boundary. In Fig. 3, the trajectory in the case of the three-phase short circuit on the transmission line is also shown. From this trajectory, it can safely be said that the method proposed here can predict the stability boundary with sufficient accuracy and a reasonable amount of computation at the region which is important for transient stability problems.

4. Conclusions

In this paper, we proposed a method for obtaining the stability boundary by making use of the property of the trajectories at the saddle point, and applied it to a simple power system. The results of the numerical example show that using the method proposed here, a fairly accurate assessment of the stability boundary can be obtained with a reasonable amount of computation at the region where the transient instability of power systems usually occurs.

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Appendix

For a second order system, the function F is calculated as follows:

$$F=\frac{\partial\alpha}{\partial n}=\frac{\partial\alpha}{\partial x_1}\cdot\frac{-X_2}{v}+\frac{\partial\alpha}{\partial x_2}\cdot\frac{X_1}{v}$$

where $v=\sqrt{(X_1^2+X_2^2)}$

$$\alpha=\arctan(X_1/X_2)$$

$$\frac{\partial\alpha}{\partial x_1}=\frac{X_2^2}{v^2}\cdot\frac{1}{X_2^2}\left(\frac{\partial X_1}{\partial x_1}\cdot X_2-\frac{\partial X_2}{\partial x_1}\cdot X_1\right)$$

$$\frac{\partial\alpha}{\partial x_2}=\frac{X_2^2}{v^2}\cdot\frac{1}{X_2^2}\left(\frac{\partial X_1}{\partial x_2}\cdot X_2-\frac{\partial X_2}{\partial x_2}\cdot X_1\right)$$

$$\therefore\ F=\frac{X_2^2}{v^2}\cdot\frac{1}{X_2^2}\left(\frac{\partial X_1}{\partial x_1}\cdot X_2-\frac{\partial X_2}{\partial x_1}\cdot X_1\right)\frac{-X_2}{v}+\frac{X_2^2}{v^2}\cdot\frac{1}{X_2^2}\times\left(\frac{\partial X_1}{\partial x_2}\cdot X_2-\frac{\partial X_2}{\partial x_2}\cdot X_1\right)\frac{X_1}{v}$$

$$=\frac{-X_2}{v^3}\left(\frac{\partial X_1}{\partial x_1}\cdot X_2-\frac{\partial X_2}{\partial x_1}\cdot X_1\right)+\frac{X_1}{v^3}\left(\frac{\partial X_1}{\partial x_2}\cdot X_2-\frac{\partial X_2}{\partial x_2}\cdot X_1\right)$$

The $(k+1)$ th order derivative of F is obtained using the k th order deiivative as follows:

$$\frac{\partial^{k+1}F}{\partial l^{k+1}}=\frac{1}{v}\cdot\frac{d}{dt}\left(\frac{\partial^k F}{\partial l^k}\right)$$

$$=\frac{1}{v}\cdot\left[\frac{\partial}{\partial x_1}\left(\frac{\partial^k F}{\partial l^k}\right)\cdot X_1+\frac{\partial}{\partial x_2}\left(\frac{\partial^k F}{\partial l^k}\right)\cdot X_2\right]$$

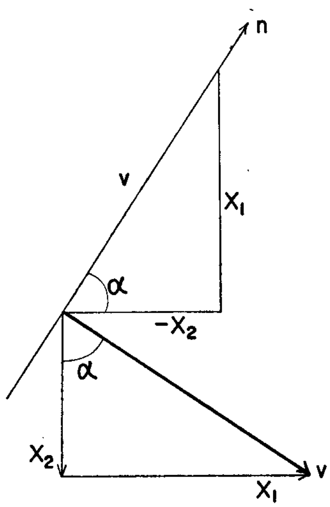

Fig. A-1. Calculation of the function F for second order system.