Stability of the Welding Arc in a Constant Feeding Speed System

By

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This paper deals with some problems concerning the stability of the D. C. arc welding system in which the electrode wire is fed with a constant speed. The arc welding system may produce a self-sustained oscillation where the current and length of the arc vary slowly and periodically in the steady state. The welding system is described by nonlinear differential equations. The phase-plane analysis is applied to the study of the phenomena. A steady state having a constant arc length is correlated with a singular point. A self-sustained oscillation is then represented by a limit cycle in the phase-plane. From the analytical results, it may be inferred that the self-sustained oscillations occur due to the self-regulation peculiar to the arc in the case where the power source has a rising characteristic.

1. Introduction

When the D. C. power source is applied to the arc welding system with a constant feeding speed, the steady state response of the system may usually, but not necessarily, be stable. There are also certain special cases in which the response of the welding system is unstable even when subjected to the D. C. power source. This paper deals with the so-called 'self-sustained oscillation,' where the current and length of the arc vary slowly and periodically in the steady state. Andoh has reported an experimental investigation concerning the stability of the welding arc and has clarified its physical mechanism.[1] The analysis of the arc welding system has not been so far performed from theoretical view points, although some experimental works have been done.[1)~5] This kind of oscillation also occurs in mechanical systems, and in various systems with nonlinear elements.[6]

The behavior of the arc welding system is described by two first-order nonlinear differential equations. The phase-plane analysis is used for the present investigation. The current and voltage of the arc constitute the coordinates of a representative point in

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the phase-plane. The steady solutions which are correlated with singular points in the phase-plane are first sought for various combinations of the system parameters. The stability of steady solutions is discussed by considering the behavior of variations from the steady solutions. The transient state of the arc is investigated by illustrating the geometrical configuration of integral curves in the phase-plane. Consequently, a steady arc having a constant length is correlated with a stable singular point, and an oscillating arc is correlated with a limit cycle in the phase-plane. When the current and length of the arc keep on varying periodically in the steady state, the representative point does not tend to a singular point but moves along the limit cycle repeatedly.

The relationship between the initial conditions and the resulting steady state response is also investigated by making use of the phase-plane analysis. It is a distinctive feature of nonlinear systems that various types of steady state responses may take place even in the same system depending upon the different values of initial conditions. The region of initial conditions leading to the arc response is shown on the phase-plane for the particular sets of system parameters.

2. Fundamental Equations

The schematic diagram in Fig. 1 shows an arc welding system in which the oscillating arc take place due to the self-regulation peculiar to the arc under the impression of D. C. voltage. In the figure, one of the rollers is revolved by a motor, another roller revolves freely, and the electrode wire is fed with a constant speed 'v_0'. 'L' is the internal inductance of the welder having the no load voltage 'E', and 'R' is a constant characterizing the welder.* The arc voltage being 'V' in the case of arc current 'i', the following equation is established.

$$\frac{di}{dt}=\frac{1}{L}(E-Ri-V)\equiv X(i, l, V). \tag{1}$$

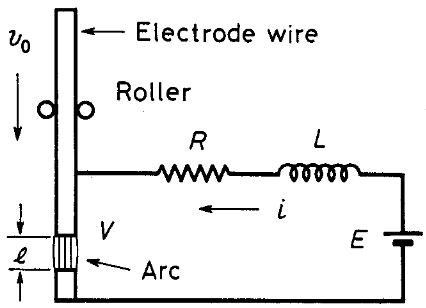

Fig. 1. Welding system in which the electrode wire is fed with constant speed.

* The voltage induced in a DC generator is directly proportional to the flux across which the armature coils cut. Hence, the performance characteristics of a generator can be controlled by the ways the field coils are connected to the armature and the load.

The characteristic of a series generator with a separately excited field coil may be approximated by Eq. (1) in which the coefficient R is negative. A field coil is connected in series between the output of the armature and the load. This means that all the load current flows through the field coil. When there is a heavy load, there may be more field current and more flux, tending to compensate excessively for the voltage drop in the armature and field coil. In this case, the generator has the so-called rising characteristic, i.e., the larger the arc current, the higher the voltage.

The melting speed of the electrode wire is proportional to the arc current when its density is not very large. If the current is held constant in an arc of short length, the melting speed of the wire increases or decreases as the arc length decreases or increases.[1] Thus, the welding arc has a self-regulation characteristic by which the arc tends to keep its length constant itself. We have the following equation with regard to the arc length:

$$\frac{dl}{dt}=\alpha i+\beta i/(l+\gamma)-v_0\equiv Y(i, l, V), \tag{2}$$

where α, β, and γ are constants. The first and second terms represent the melting speed of the electrode wire. It is noted that the effect of the welding speed is neglected in Eq. (2).

The voltage-ampere characteristic of the welding arc may be approximated by the form:

$$0=V-[al+b+(cl+d)i+(el+f)/i]\equiv Z(i, l, V), \tag{3}$$

where $a, b, \cdots, f$ are constants* characterizing the welding arc. The characteristic of Eq. (3) shows a fairly good approximation to the arc in steady states.[7] An long as we deal with cases where the arc current and length vary slowly, Eq. (3) may be considered to be legitimate.[1]

Equations (1), (2) and (3) play a significant role in the following investigation, since they serve as the fundamental equations in studying the transient state as well as the steady state of the welding arc.

3. Steady Solutions and Their Stability

In order to discuss the relationship between the initial conditions and the resulting responses, we have to first investigate the type of steady solutions under the various combinations of system parameters. Therefore, a welding arc having a constant length in the steady state will be studied in this sections.

(a) Steady Solutions

We consider the steady state in which current $i(t)$, arc length $l(t)$ and voltage $V(t)$

* These coefficients should be determined by experiments.

in Eqs. (1), (2) and (3) are constant, so that

$$\frac{di}{dt}=0,\ \frac{dl}{dt}=0 \text{ and } \frac{dV}{dt}=0. \tag{4}$$

Substituting these conditions in Eqs. (1), (2) and (3), the steady solutions are determined by

$$X(i_0, l_0, V_0)=0, \quad Y(i_0, l_0, V_0)=0 \text{ and } Z(i_0, l_0, V_0)=0, \tag{5}$$

where the subscript 0 is used to designate the values of i, l and V for the steady solution.

(b) Stability Investigation

The equilibrium states determined by Eqs. (5) are not always realized, but are actually able to exist only so long as the equilibrium states are stable. In order to investigate the stability of steady solutions as given by Eqs. (5), we consider sufficiently small variations ξ_1, ξ_2 and ξ_3 from the equilibrium state defined by

$$\xi_1=i-i_0, \quad \xi_2=l-l_0, \quad \xi_3=V-V_0. \tag{6}$$

Then, if these variations ξ_1, ξ_2 and ξ_3 tend to zero with increasing t, the solutions are stable. Substituting Eqs. (6) in (1), (2) and (3) gives

$$\left.\begin{aligned}\frac{d\xi_1}{dt}&=a_{11}\xi_1+a_{12}\xi_2+a_{13}\xi_3\\ \frac{d\xi_2}{dt}&=a_{21}\xi_1+a_{22}\xi_2+a_{23}\xi_3\\ 0&=a_{31}\xi_1+a_{32}\xi_2+a_{33}\xi_3,\end{aligned}\right\} \tag{7}$$

where

$$\begin{aligned}a_{11}&=\left(\frac{\partial X}{\partial i}\right)_0=-\frac{R}{L}\\ a_{12}&=\left(\frac{\partial X}{\partial l}\right)_0=0\\ a_{13}&=\left(\frac{\partial X}{\partial V}\right)_0=-\frac{1}{L}\\ a_{21}&=\left(\frac{\partial Y}{\partial i}\right)_0=\alpha+\beta/(l_0+\gamma)\\ a_{22}&=\left(\frac{\partial Y}{\partial l}\right)_0=-\beta i_0/(l_0+\gamma)^2\\ a_{23}&=\left(\frac{\partial Y}{\partial V}\right)_0=0\\ a_{31}&=\left(\frac{\partial Z}{\partial i}\right)_0=-(cl_0+d)+(el_0+f)/i_0^2\\ a_{32}&=\left(\frac{\partial Z}{\partial l}\right)_0=-(a+ci_0+e/i_0)\\ a_{33}&=\left(\frac{\partial Z}{\partial V}\right)_0=1,\end{aligned} \tag{7a}$$

where $(\partial X/\partial i)_0, \cdots, (\partial Z/\partial V)_0$ stand for $(\partial X/\partial i), \cdots, (\partial Z/\partial V)$ at $i=i_0$, $l=l_0$ and $V=V_0$ respectively. Letting

$$\xi_i = c_i e^{\lambda t} \quad (i=1\sim3), \tag{8}$$

we substitute Eqs. (8) into (7) to obtain

$$\begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = 0. \tag{9}$$

In order for c_i in Eq. (9) to take a non-zero value, λ should satisfy the following characteristic equation:

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0. \tag{10}$$

We assume that the two solutions λ_j $(j=1, 2)$ for Eq. (10) are different from each other. Denoting c_i for $\lambda=\lambda_j$ by c_{ij}, we can express two sets of solution for Eqs. (7) as $\xi_{ij}=c_{ij}e^{\lambda_j t}$ $(i=1\sim3, j=1, 2)$. Therefore, the general solutions of Eqs. (7) may be written as

$$\xi_i = c_{i1}e^{\lambda_1 t} + c_{i2}e^{\lambda_2 t} \qquad (i=1\sim3). \tag{11}$$

Since the ratio $c_{1j}:c_{2j}:c_{3j}$ can be determined from Eq. (9), it is possible to select two constants in Eqs. (11) arbitrarily.

If the real parts of λ_j $(j=1,2)$ are negative, the variational components ξ_i $(i=1\sim3)$ will converge to zero with time, and the corresponding steady solution will be stable.

(c) Numerical Examples

In order to present a more concrete description of the arc, a numerical analysis of the system Eqs. (1), (2) and (3) was carried out for the parameters, as given by Table. 1

By varying the welder parameters E and R, the current and arc length are calculated from Eqs. (5); and the isolength curves are plotted on the $E-R$ plane in Fig. 2. Following an analysis of the preceding section, the stability of steady solutions is investigated, and the stable region is hatched in Fig. 2. There are certain cases in which the steady solution becomes unstable when the welder has a rising characteristic.

Table 1. System parameters in Eqs. (1), (2) and (3).

L	0.02 H	α	0.05 mm/A·s	β	0.0308 mm²/A·s
γ	4.92 mm	v_0	20.0 mm/s	a	3.25 V/mm
b	17.0 V	c	0.400 V/m·A	d	0.00350 V/A
e	30.0 V·A/mm	f	10.0 V·A		

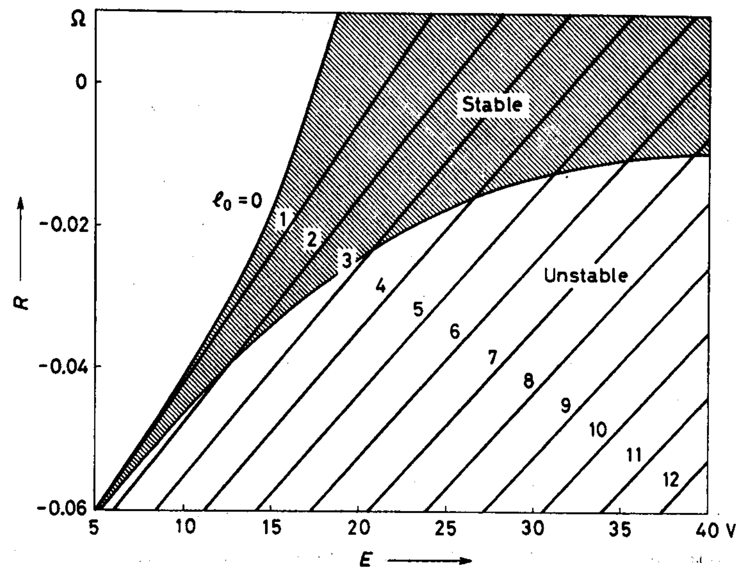

Fig. 2. Iso-length curves and region in which the welding arc is sustained in the stable state.

4. Limit Cycle Correlated with Self-sustained Oscillations

(a) Phase-plane Analysis[8)~11)]

Our object is to study the self-regulation and the stability of the welding arc depending on the welder characteristic. These problems are investigated by studying the solution of Eqs. (1), (2) and (3) in the transient state, which, with the lapse of time, ultimately yields the steady solution. For this purpose, it is particularly useful to investigate the integral curves of the following equations derived from Eqs. (1), (2) and (3):

$$\left.\begin{aligned} &\frac{dl}{di}=\frac{Y(i,l,V)}{X(i,l,V)} \\ \text{with} \quad & \\ &Z(i,l,V)=0. \end{aligned}\right\} \qquad (12)$$

One readily sees from Eq. (3) that V is uniquely determined once the values of i and l are known. Since the time t does not appear explicitly in Eqs. (12), we can draw the integral curves in the $i-V$ plane. Thus, the behavior of system may be described by the movement of the representative point $(i(t),\ V(t))$ along the integral curves of Eqs. (12). The steady solutions satisfy the conditions (4) and are, therefore, expressed by the singular points of Eqs. (12), i.e., by the points at which $X(i, l, V)$ and $Y(i, l, V)$ both vanish.

Now suppose that the initial condition is prescribed by a point whose coordinates are given by $i(0)$ and $V(0)$ in the $i-V$ plane. Then, the representative point $(i(t),\ V(t))$

moves, with increasing t, along the integral curve which starts from the point ($i(0)$, $V(0)$), and tends ultimately to a stable singular point or to a limit cycle. Thus, the transient-state solutions are correlated with the integral curves of Eqs. (12). If the integral curve leads to a limit cycle with increasing t, the representative point ($i(t)$, $V(t)$) moves along the limit cycle repeatedly, so that the current and length of the arc keep on varying periodically, resulting in a self-sustained oscillation. The period of oscillation is equal to that required for the representative point to complete one round along the limit cycle. We show in Section 4(b) examples of the integral curves which tend to a singular point or to a limit cycle.

The character of the singular point reveals the behavior of the arc in the vicinity of the equilibrium state, and consequently determines the stability of steady solution.

The types of singular points are classified according to the character of the integral curves near the singular points, that is, according to the roots (λ's) of characteristic equation (10). The singular points of Eqs. (12) are classified as follows:[10]

(1) If the characteristic roots λ are both real and of the same sign, the singularity is a node.

(2) If the characteristic roots λ are both real but of an opposite sign, the singularity is a saddle which is intrinsically unstable.

(3) If the characteristic roots λ are complex conjugates, the singularity is a focus.

(4) If the characteristic roots λ are imaginary, the singularity is either a center or a focus.[8]

A singularity is stable or unstable according as a point ($i(t)$, $V(t)$) on the neighboring integral curves tends to it or not with increasing t, that is, according as the real part of λ is negative or positive.

(b) Numerical Example

The numerical analysis used for the above is as follows. Let us consider an example in which system parameters are given by Table 1. The current i_0 and arc length l_0 are first calculated from Eqs. (5) for various values of E and R. We now distinguish these equilibrium states according to the types of singularity. By varying the values of E and R, we determine the regions for the different types of singularity. The analytical results are shown in Fig. 3.

Since the transient state of the arc is correlated with the integral curve of Eqs. (12), it will be useful to show the geometrical configuration of integral curves for representative cases.

First, let us consider an example where the welder parameters are given by

$$E=13.0\,V \text{ and } R=-0.03\,\Omega. \qquad (13)$$

A point having these parameters is indicated by 'A' in Fig. 3. The integral curves for

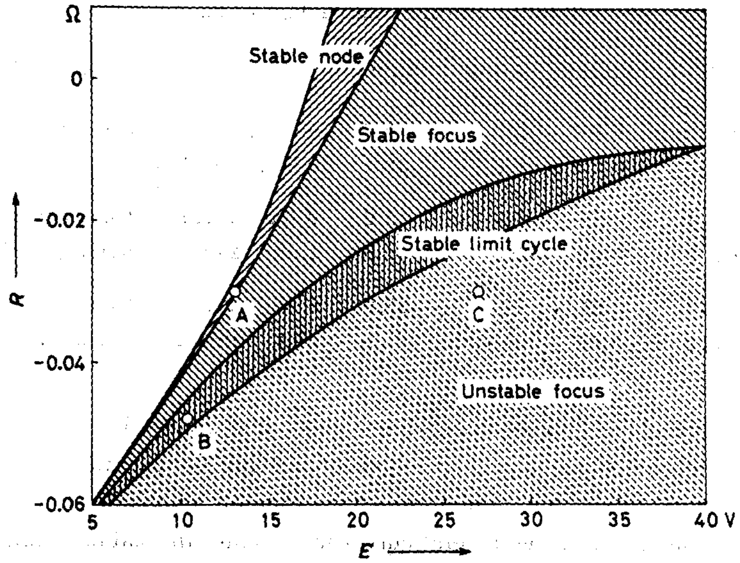

Fig. 3. Regions in which different types of singularity and the limit cycle exist.

this particular case are plotted in Fig. 4. The singular points are determined by Eqs. (4); the details are listed in Table 2. By Eqs. (1), (2) and (3), the representative point ($i(t)$, $V(t)$) moves, with increasing t, along the integral curve in the direction of the arrows. In Fig. 4 there are two singularities. Point 1 is stable, and the corresponding steady state is realized. The remaining singularity, point 2, is a saddle point, which is

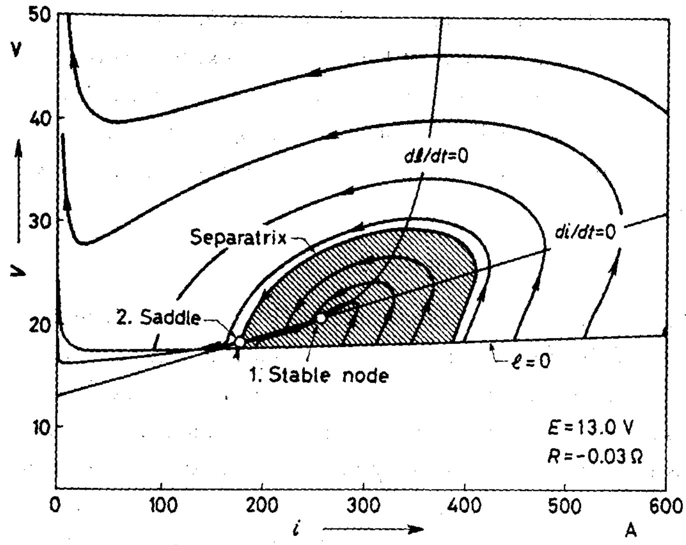

Fig. 4. Integral curves of Eqs. (12) in the i-V plane, the system parameters being E=13.0 V and R=−0.03 Ω.

Table 2. Singular points in Figs. 4, 5 and 6.

Singular point	l_0 [mm]	i_0 [A]	V_0 [V]	λ_1, λ_2	Classification
Fig. 4, 1	0.800	257	20.7	−1.27, −3.85	Stable node
Fig. 4, 2	0.179	176	18.3	0.503, −22.1	Saddle (unstable)
Fig. 5, 1	2.15	320	25.6	$0.290 \pm 2.68i$	Unstable focus
Fig. 5, 2	0.103	160	18.0	1.53, −28.5	Saddle (unstable)
Fig. 6, 1	5.63	362	37.9	$0.482 \pm 2.99i$	Unstable focus
Fig. 6, 2	−0.308	0.070	27.0	7804, −185100	Saddle (unstable)

intrinsically unstable. The corresponding steady state cannot be sustained, because any slight deviation from point 2 will lead the welding arc to either the stable node represented by point 1 or the short-circuit state expressed by $l=0$.

The separatrices, i.e., the integral curves which enter the saddle point, divide the whole plane into two regions. An arc discharge started with any initial conditions in the shaded region tends to singularity 1, resulting in a stable welding arc. However, an arc discharge started from the unshaded region tends either to the short-circuit state where the arc length is zero, or to the V axes on which the current vanishes, resulting in no arc discharge. Hence the relationship existing between the initial condition ($i(0)$, $V(0)$) and the resulting response will be made clear. It is also seen that singular point 2 is situated on the boundary curve between these two regions.

We now turn to our present investigations. When an oscillating arc occurs in the system described by Eqs. (1), (2) and (3), the current $i(t)$ and the arc length $l(t)$ vary periodically. The stable limit cycle of Eqs. (12), if it exists, is correlated with an oscillating arc. It depends upon the system parameters whether Eqs. (12) have a limit cycle or not. The region in which a limit cycle exists is sought by varying values of E and R. The analytical result is shown in Fig. 3. Let us consider a case in which the welder parameters are given by

$$E=10.3\,V \text{ and } R=-0.048\,\Omega. \tag{14}$$

A point having these parameters is indicated by 'B' in Fig. 3. The integral curves for the above values of the welder parameters are plotted in Fig. 5. By substituting Eqs. (14) into Eqs. (5), the singular points are located in the $i-V$ plane. There are, as in the first case, two singularities, 1 and 2, the details of which are listed in Table 2. Contrary to the case illustrated in Fig. 4, singularity 1 becomes unstable. We see in Fig. 5 a stable limit cycle which encircles the unstable focus 1. One of integral curves that contains the saddle point 2, that is, the separatrix, divides the whole plane into two regions. In one of them, all integral curves tend to the limit cycle, and in the other, all integral curves

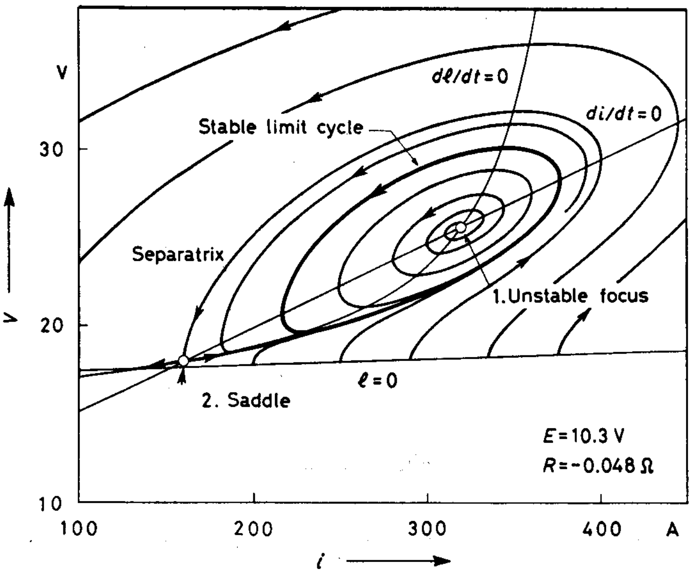

Fig. 5. Integral curves of Eqs. (12) in the i–V plane, the system parameters being $E=10.3\,\mathrm{V}$ and $R=-0.048\,\Omega$.

tend to either the 'V' axes or the curve on which 'l' vanishes. Thus, the occurrence of a limit cycle will depend on the initial condition. The period required for the representative point to complete one round along the limit cycle, that is, the period of current

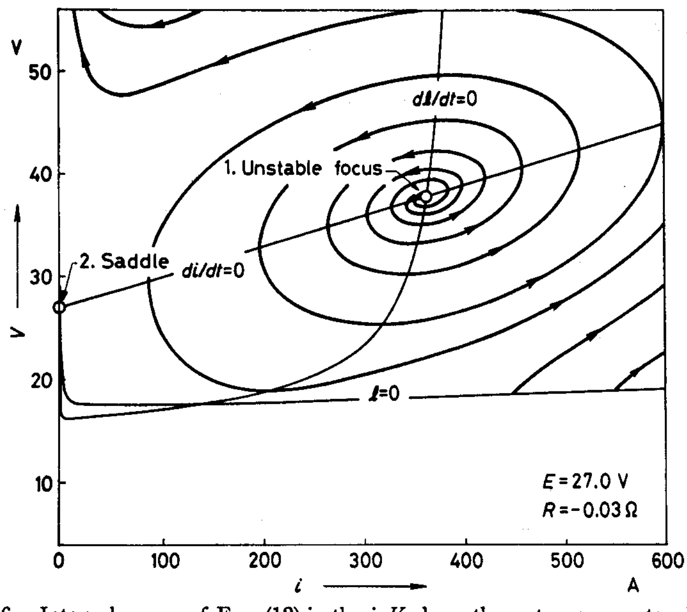

Fig. 6. Integral curves of Eqs. (12) in the i–V plane, the system parameters being $E=27.0\,\mathrm{V}$ and $R=-0.03\,\Omega$.

variation of the oscillating arc, is 2.75··· second.

An arresting feature previously reported by Nobuhara and the auther[2] is that the self-regulation of the arc of the rising characteristic welder may not necessarily be stronger than that in the constant voltage characteristic. However, no theoretical consideration was given at that time. From the present analysis, however, it will be deduced that when the welder has a rising characteristic, the steady arc might become unstable and change into an oscillating arc.

Finally, we consider the third example where the welder parameters are given by

$$E=27.0\,V \text{ and } R=-0.03\,\Omega. \tag{15}$$

A point having these parameters is indicated by 'C' in Fig. 3. The integral curves of this particular case are plotted in Fig. 6. There are two singularities, 1 and 2, the details of which are listed in Table 2. As in the preceding case illustrated in Fig. 5, singularity 1 is an unstable focus. However, in the present case, Eqs. (12) have no limit cycle. Therefore, we have no welding arc for any of the initial conditions prescribed.

5. Concluding Remarks

The welding arc has a self-regulation by which the arc tends to keep its length constant itself. The self-regulation and the stability of the aıc depend on the welder characteristic. These problems are discussed by considering the behavior of variation from the steady state. The phase-plane method is useful for studying the stability of the arc.

The welding system under consideration is described by two first-order nonlinear differential equations. The arc current and voltage constitute the coordinates of a representative point in the phase-plane. The representative point moves, with the increase of time, along the integral curve and leads ultimately to a stable singular point or to a stable limit cycle. Hence the transient solutions are correlated with the integral curves and the steady solutions having constant currents are correlated with the singular points. If the integral curve leads to a limit cycle with the increase of time, the representative point travels repeatedly along the limit cycle, so that the arc current and voltage keep on varying periodically. Thus an oscillating arc is represented by a limit cycle. Typical examples of the phase-plane diagram are illustrated for particular sets of welder parameters. The illustrations provide a general view of welding arcs in both transient and steady state conditions. Proceding in this way, we may discuss the relationship between initial conditions and resulting steady states in addition to the stability of the arc.

From the present analysis, it will be deduced that when the welder has a rising characteristic, the steady arc might become unstable owing to the buildup of self-sustained oscillations and change into an oscillating arc.

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