A Renewal Process Model for Use in Seismic Risk Analysis

By

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Abstract

A model of random occurrence of earthquakes using a renewal process is proposed on the basis of the catalogue of historical earthquakes for Kyoto, Japan. The model accounts for the variation of annual occurrence rate depending on the intervals between successive earthquakes. It can incorporate widely recognized nonstationary features of earthquake occurrences that can not be explained with the conventional simple Poisson process models. By using the renewal process model, called herein the double Poisson process model, one can estimate the probability of future earthquake occurrences in terms of the conditional probability based on the time since the last earthquake. Numerical results are presented for the Kyoto area.

1. Introduction

Seismic risk assessment is a major problem in the earthquake resistant design of modern engineering structures. Since earthquakes take place randomly in space and time, probabilistic methods are required in the risk assessment of future earthquakes. Seismic risk analysis involves a hazard prediction which is presented in terms of the *seismic intensity* to be felt at a site in a *future period*, corresponding to a given level of *probability of exceedence* (or return period). When any two of them are specified, the rest is determined.

A number of works have been published for Japan[6,9,11,13,14,15], the United States[2,3,4], and other countries[5,12] using various probabilistic models and various forms for presenting the above three risk parameters. In all these studies it has been assumed, at least implicitly, that earthquakes occur in a stationary random sequence, and consecutive earthquakes take place independently. This means that the occurrence of earthquakes is treated as a Poisson process. A Poisson process would be appropriate if the period being discussed is long enough for several destructive earthquakes to be expected with a high probability. However,

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destructive earthquakes are such rare events that they may or may not be experienced by the people living in a site throughout their lives. Many engineering structures are also usually designed for useful lives of the same order, say, several tens of years. On the other hand, it is widely recognized that after a major earthquake has occurred in a seismic region, a certain period must elapse for accumulation of strain energy sufflcient to cause the next earthquake. It is also recognized that the longer the quiescent period is, the larger the next earthquake will be. The same idea applies when earthquakes felt at a site are discussed; i.e., the longer the time since the last earthquake experienced at the site, the closer in time and the stronger the next occurrence. Such statements are supported by recent developments in techtonics and seismology as well as reflecting widely recognized experiences.

JMA	0	I	II	III	IV	V	VI	VII			
MM	I	II	III	IV	V	VI	VII	VIII	IX	X	

Fig. 1. Comparison of JMA and MM Intensities

Thus, when we descuss destructive earthquakes to be felt at a site within a period of time significant in an engineering or social sense, the occurrence rate of earthquakes is obviously nonstationary. Engineering seismic risk analysis should incorporate this nonstationarity and provide information at least in terms of a conditional probability of earthquake occurrence on the basis of the times of previous earthquakes. The conventional simple Poisson process models cannot meet this requirement, since the seismic risk obtained from them is independent of the time of the last earthquake: the data of past earthquakes are reflected only in the mean occurrence rate.

The objective of this study is to provide a solution for the problem mentioned above. The present paper is its first report. First, the historical data of major earthquakes are re-examined in detail, and evidence of an interrelation between succeeding earthquakes is identified. The catalogue of major earthquakes felt at Kyoto, Japan, within the period of 827~1936 A.D. is reviewed. The felt intensity is represented in terms of the JMA* (Japan Meteorological Agency) scale I. On this basis, a renewal process model called a double Poisson process is proposed. Following the basic formulation of the model, the recurrence time distribution for major earthquakes and the conditional probability of future earthquakes based on the time of the last earthquake are obtained. The numerical

* JMA and MM intensity scales are compared in Fig. 1.

results demonstrate that the double Poisson process is particularly useful for engineering purposes in the sense that it can provide information on the future seismic risk viewed from the time the risk is evaluated. This has been impossible with time-invariant simple Poisson processes.

2. Historical Earthquake Data for Kyoto

2.1 Dataset

In finding a correlation in the sequence of earthquakes, it is useful to examine the probability distribution of recurrence times between successive earthquakes. This requires a catalogue of past earthquakes to include those which occurred far in the past, in spite of the relatively low reliability of information for such times.

Fortunately, Japanese engineers can make use of a catalogue[16,18] of historical earthquakes dating back as far as 599 A.D., compiled after the Nobi earthquake of 1891 by Tayama, Omori and later by Musha, as well as those registered after modern instrumental observations began. Work to improve information on historical earthquakes is being continued through searching old documents that would supplement the catalogue or provide information useful for a more accurate estimation of the felt intensity and location of the epicenter.

It is very likely that the reliability of the catalogue of historical earthquakes varies with time. In this study, the catalogue of earthquakes which damaged the city of Kyoto (JMA $I \geq \text{V}$) is used. The capital of Japan was established in Kyoto in 794 A.D., and it has been a political or cultural center of Japan. Therefore, the catalogue for this area should be reliable compared to other districts which were developed centuries later. This aspect has been discussed by Katayama[10] and Kameda[8]. The catalogue used herein is primarily based on those earthquakes with a felt intensity V and over in Kyoto, as listed by Usami and Hisamoto[17] and Usami[18]. In this study, the earthquakes which occurred after 800 A.D. are used. Some events which are clearly aftershocks of the Nobi earthquake of 1891 are excluded. Besides, a further review was made as to the felt intensity of the registered earthquakes*. The catalogue adopted herein is listed in Table 1. It contains 37 earthquakes which consist of 27 earthquakes with a felt intensity of V, 7 with VI, and 3 with VII.

2.2 Subgrouping of the dataset

The catalogue of historical earthquakes described in **2.1** is further rearranged

* Comments offered by Professor T. Usami of the University of Tokyo were very helpful for this purpose.

Table 1. Catalogue of Major Earthquakes in Kyoto (selected from Usami and Hisamoto[17])

No.	Date year-month-day	Epicenter*	Magintude M	JMA intensity in Kyoto	Earthquake set E_A	Earthquake set E_B	Earthquake set E_G
1	827 VIII 11	135.75, 34.9	6.7	V	○		○
2	856 — —	Kyoto	6.4	V	○		○
3	881 [illegible]I 13	Kyoto	6.4	V	○		⎫
4	887 VIII 26	135.3 , 33.0	8.6	VI	○	○	⎬ ○
5	890 VII 10	Kyoto	6.2	V	○		⎭
6	938 V 22	135.8 , 34.8	6.9	VI	○	○	○
7	976 VII 22	135.8 , 34.9	6.7	VII	○	○	○
8	1041 VIII 25	Kyoto	6.4	V	○		○
9	1093 III 19	Kyoto	6.4	V	○		⎫ ○
10	1096 XII 17	137.3 , 34.2	8.4	V	○		⎭
11	1185 VIII 13	136.1 , 35.3	7.4	VII	○	○	○
12	1245 VIII 27	Kyoto	6.2	V	○		○
13	1317 II 24	135.8 , 35.1	6.7	VI	○	○	○
14	1350 VII 6	Kyoto	6.2	V	○		⎫
15	1361 VIII 3	135.0 , 33.0	8.4	V	○		⎬ ○
16	1369 IX 7	Kyoto	6.1	V	○		⎭
17	1425 XII 23	Kyoto	—	V	○		○
18	1449 V 13	135.75, 35.0	6.4	VI	○	○	⎫ ○
19	1466 V 29	Kyoto	—	V	○		⎭
20	1498 IX 20	138.2 , 34.1	8.6	V	○		⎫ ○
21	1510 IX 21	135.7 , 34.6	6.7	V	○		⎭
22	1586 I 18	136.8 , 36.0	7.9	VI	○	○	⎫ ○
23	1596 IX 5	135.75, 34.85	7.0	VII	○	○	⎭
24	1662 VI 16	136.0, 35.3	7.6	VI	○	○	⎫
25	1664 I 4	Kyoto Yamashiro	5.9	V	○		⎬ ○
26	1665 VI 25	Kyoto	6.1	V	○		⎭
27	1694 XII 12	Tango	6.1	V	○		⎫ ○
28	1707 X 28	135.9 , 33.2	8.4	V	○		⎭
29	1751 III 26	Kyoto	6.4	V	○		○
30	1830 VIII 19	135.7 , 35.0	6.4	VI	○	○	○
31	1854 XII 24	135.0 , 33.2	8.6	V	○		○
32	1891 X 28	136.6 , 35.6	7.9	V	○		⎫
33	1896 V 7	135.7 , 35.1	5.1	V	○		⎬ ○
34	1899 III 7	136.0 , 34.2	7.6	V	○		⎭
35	1927 III 7	135.1 , 35.6	7.5	V	○		⎫
36	1936 I 8	135,8 , 35.1	4.4	V	○		⎬ ○
37	1936 II 21	135,7 , 34.5	6.4	V	○		⎭

* Numerical values represent longitude(East) and latitude (North) in degrees.

into the following three sets.

(i) Set E_A: All earthquakes listed in Table 1: i.e., 37 events with a felt intensity of V and over in Kyoto.

(ii) Set E_B: 10 events with an intensity VI and over in Kyoto.

(iii) Set E_G: Earthquakes with an intensity V and over that occurred within 20 years are treated as a single event of an earthquake group. Table 1 shows the results of 22 earthquake groups, 9 of which contain at least one earthquake with an intensity VI and over. The occurrence time of group is determined as the centroid on the time axis with weights 1,2, and 3 for the intensities V, VI, and VII, respectively.

The earthquakes included in each set are identified on the right-hand side of Table 1. The sets E_A and E_B are used for obtaining the probability of earthquakes with their corresponding intensities. Set E_G has been prepared in order to avoid the effects of a clustering sequence of earthquakes occuring within a relatively small time interval, 20 years in this study, and to maintain the premise that major earthquakes are rare events. Set E_G has the possibility of eliminating the effect of earthquakes which possibly took place in historical ages but are not registered in the catalogue. This is achieved when such missing earthquakes have actually occurred within 20 years. Comparison between the results obtained from the sets E_A and E_G will be a help for judging the reliability of the catalogue used herein.

3. Earthquake Occurernce Model

3.1 Recurrence time distribution and intensity ratio

The cumulative probability of the recurrence time of earthquakes and earthquake groups has been plotted on exponential probability papers in Figs. 2∼4, corresponding to the earthquake sets E_A, E_B, and E_G, respectively. The felt intensity for each sample point is identified. If the data points lie on a single straight line passing through the origin, the recurrence time follows an exponential distribution and it can be assumed that the events take place in a Poisson process. In these figures, however, the data points are obviously divided into two groups which fit different straight lines: these lines have been obtained from linear regression. If the complete data is fitted to a single straight line, the dashed line is obtained. It is clear that the two straight lines agree with the data much better than the single line.

The inverse of the slope of the straight lines in Figs. 2∼4 represents the mean occurrence rate ν of the events. The values of ν for the corresponding straight lines are also given in the figures. Observe that ν_2 is larger than ν_1 for all cases.

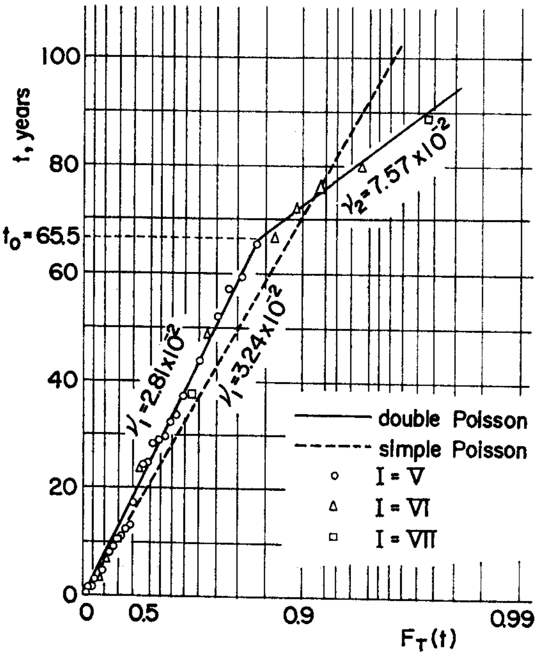

Fig. 2. Recurrence-Time Distribution (earthquake set E_A)

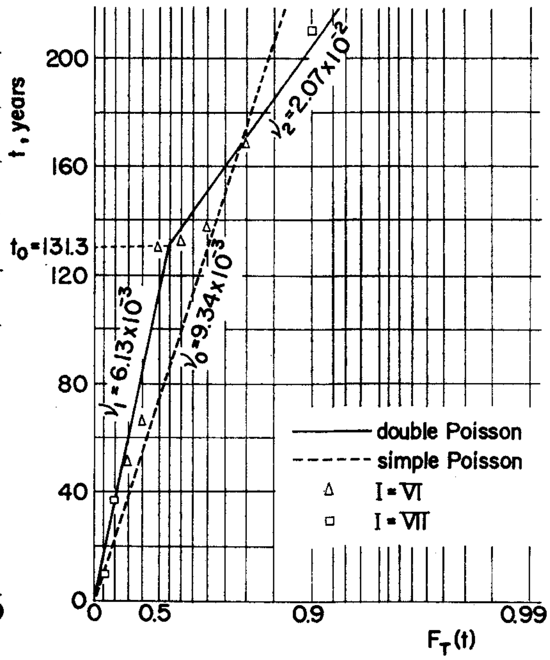

Fig. 3. Recurrence-Time Distribution (earthquake set E_B)

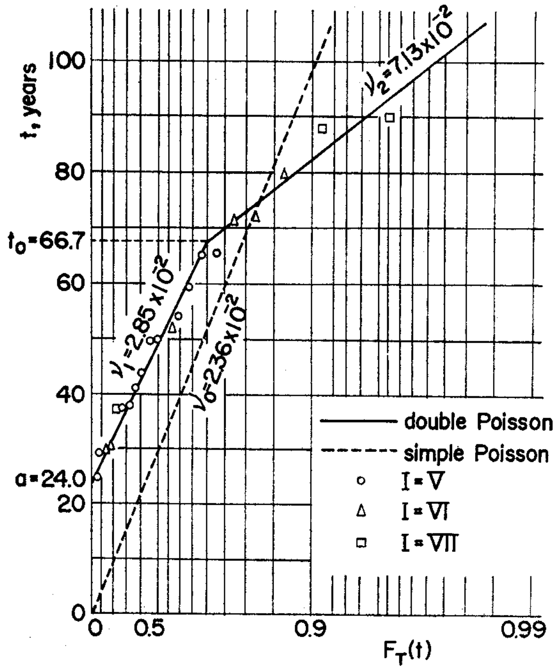

Fig. 4. Recurrence-Time Distribution (earthquake set E_G)

Therefore, when no earthquakes occur for a period of t_0 years, the occurrence rate increases from ν_1 to ν_2. It will be verified later that this feature can be modeled by a double Poisson process proposed herein. The result in Fig. 4 shows that the minimum interval between successive events (for Kyoto) is $a=24$ years. This is a consequence of counting earthquakes occurring within 20 years as a single event in the set E_G.

Another remarkable point in Figs. 2 and 4 is that all events occurring with recurrence times larger than t_0 years involve the felt intensity of $I \geq$ VI, whereas all earthquakes with $I=$ V occur within the interval of t_0 years. This implies that after an elapse of time without damaging earthquakes beyond the limit of t_0 years, the next earthquake will be a severe one.

These points are consistent with the generally recognized features of earthquake occurrences discussed in the introduction **1**. In the following section, they are incorporated in the earthquake occurrence model. The values of parameters ν_1, ν_2, a and t_0 are again listed in Table 2 along with the values of ν_0, the mean occurrence rate for a simple Poisson process. Table 2 also shows the proportion r_{VI} of the events with $I \geq$ VI among those occurring under $\nu=\nu_1$.

3.2 Modeling with double Poisson process

A random sequence of earthquakes, having the recurrence-time distribution as described in the previous section, can be defined in the following manner.

(1) Case with $a=0$.

After an earthquake, subsequent earthquakes occur in a Poisson process with a mean occurrence rate $\nu=\nu_1$ as long as the interval between any adjacent earthquakes does not exceed t_0 years. If an interval exceeds t_0 years, the mean occurrence rate for the next earthquake increases to $\nu=\nu_2$. As soon as an earthquake occurs under $\nu=\nu_2$, the mean occurrence rate reduces again to $\nu=\nu_1$. This model applies to the earthquake sets E_A and E_B.

(2) Case with $a>0$.

It is the same as (1), except that there is a period of a years with a zero occurrence rate after each event; i.e., the minimum interval between any successive events is a years. This applies to the earthquake set E_G.

These models constitute a renewal process, since the earthquake occurrence rate varies, depending only on the time of the last event. The model in (1) is a modification of simple Poisson process models to take account for variation of the mean occurrence rate. The model in (2) is a further modification to incorporate a minimum interval between earthquake groups. For these reasons, the models defined above will be called *double Poisson process* models in this study.

The recurrence-time distribution for the double Poisson process is obtained in the following. Suppose that an earthquake occured at $t=0$, and let random variables X_t, X_{t_1}, X_{t_2} be defined as follows:

X_t: number of earthquakes occurring in a time interval $(0, t)$.

X_{t_1}: number of earthquakes included in X_t and occurring under $\nu=\nu_1$.

X_{t_2}: number of earthquakes included in X_t and occurring under $\nu=\nu_2$.

Then the distribution function $F_T(t)$ of the recurrence time T is represented in the following form:

(i) $0<t\leq a$

$$F_T(t) = 0 \qquad \cdots\cdots(1)$$

(ii) $a<t\leq t_0$

Since there is no possibility that $X_{t_2}>0$,

$$\begin{aligned} F_T(t) &= 1-P(X_t = 0) = 1-P(X_{t_1} = 0,\ X_{t_2} = 0) \\ &= 1-P(X_{t_1} = 0) = 1-e^{-\nu_1(t-a)}\ . \qquad \cdots\cdots(2) \end{aligned}$$

(iii) $t>t_0$

In this case, X_{t_2} can be $X_{t_2}>0$. Therefore,

$$\begin{aligned} F_T(t) &= 1-P(X_{t_1} = 0,\ X_{t_2} = 0) \\ &= 1-e^{-\nu_1(t_0-a)}e^{-\nu_2(t-t_0)} \\ &= 1-e^{-\{\nu_2(t-a)-(\nu_2-\nu_1)(t_0-a)\}} \qquad \cdots\cdots(3) \end{aligned}$$

Eqs. (1)~(3) coincide with the solid lines in Figs. 2~4 for the corresponding values of ν_1, ν_2, t_0 and a listed in Table 2.

Table 2. Parameters Characterizing Earthquake Occurrences.

	Earthquake set		
	E_A	E_B	E_G
ν_1 (10^{-2} year^{-1})	2.81	0.613	2.85
ν_2 (10^{-2} year^{-1})	7.57	2.07	7.13
a (years)	0.0	0.0	24.0
t_0 (years)	65.5	131.3	66.7
r_{VI}	0.194	—	0.25
ν_0 (10^{-2} year^{-1})	3.24	0.934	2.36

Goodness-of-fit tests[1)] were performed on the recurrence time distributions obtained from the double Poisson process model and those from the simple Poisson process model by using the Kolmogorov-Smirnov test for the earthquake sets

Table 3. Goodness-of-Fit Test of Recurtence-Time Distribution.

Test	Earthquake set	Sample value of test variable		Critical value for 5 % significance level*
		double Poisson process model	simple Poisson process model	
Kolmogorov-Smirnov	E_A	0.082	0.120	0.22
	E_B	0.17	0.24	0.41
	E_G	0.12	0.44	0.28
chi-square	E_A	0.89	1.53	7.81, 9.49
	E_G	0.62	7.87	3.84, 5.99

* Two numerical values correspond to double Poisson, and simple Poisson model: degrees of freedom in the chi-square tests differ with the models.

E_A, E_B, and E_G. Chi-square tests were also performed for the earthquake sets E_A and E_G which have enough sample sizes. The test results are shown in Table 3. They tell that both the simple Poisson and the double Poisson processes cannot be rejected with a 5 % significance level. However, the sample values of the test variables for the double Poisson process model are much smaller than those for the simple Poisson process model, thus demonstrating a better fitting of the double Poisson model in the statistical sense as well as in the physical sense described earlier.

The distribution of the earthquake recurrence time for Kyoto has been discussed by Usami and Hisamoto[17]. Fig. 5 shows the density function of the recurrence time. The data plots and the dashed line showing the exponential distribution have

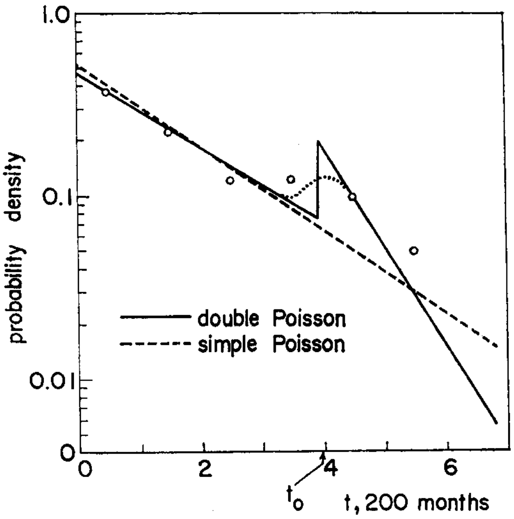

Fig. 5. Density Function of Recurrence Time (Data points and simple Poisson model are after Usami and Hisamoto).

been given by Usami and Hisamoto*. The solid line shows the result obtained from Eqs. (2) and (3) based on the double Poisson process for the earthquake set E_A. It has been concluded by Usami and Hisamoto that the exponential distribution agrees well with the observed data. However, the deviation of the data plots from the exponential distribution in Fig. 5 is systematic, and this leads to the double Poisson process model. The discontinuity of the density function from the double Poisson model in Fig. 5 results from assigning discrete values ν_1 and ν_2 to the mean occurrence rate. The actual case would be a smoother curve such as the one shown by a dotted line. However, this difference has only a minor effect, and the significance of the double Poisson process model remains unchanged.

4. Probability of Future Earthquake Occurrence

4.1 General remarks

On the basis of the double Poisson process model defined in the previous section, the probability of future earthquake occurrence is obtained. As emphasized in **1.**, required information is the conditional probability of an earthquake occurrence within a given future period, based particularly on the time of the last earthquake occurrence. This is achieved in the following, first by using the recurrence-time distribution, and then by using the intensity ratio.

4.2 Analysis using the recurrence-time distribution

Let the event $C(t_l)$ denote that t_l years have passed since the last earthquake occurred. Then the probability that there will be an earthquake within the future τ years is represented by the following conditional probability.

$$P\{N_I(\tau)\geq 1\,|\,C(t_l)\} = \frac{F_T(\tau+t_l)-F_T(t_l)}{1-F_T(t_l)}. \qquad \cdots\cdots\cdots\cdots\cdots(4)$$

Here $N_I(\tau)$ is a random variable denoting the number of earthquakes occurring in a future period $(0, \tau)$, whose intensities are no less than I. The random variable $N_I(\tau)$ differs from the X_t used in **3.2** in that the time origin in this case is located at the present or at the time the seismic risk is evaluated. Then, on substitution from Eqs. (1)~(3), depending on the values of t_l and τ as illustrated in Fig. 6, Eq. (4) is expressed in the following form; i.e., by setting

$$s_0 = t_0 - t_l$$

* They are based on Case C in ref. (17). The data are slightly different from those used in this study, in that they include four earthquakes which took place before 800 A.D.

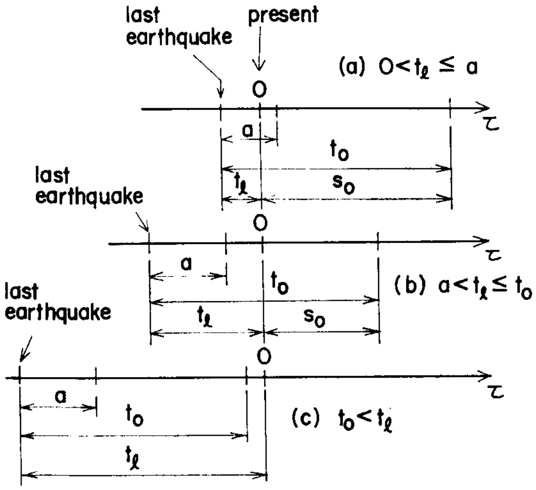

Fig. 6. Arrangement of Time Parameters ($a>0$).

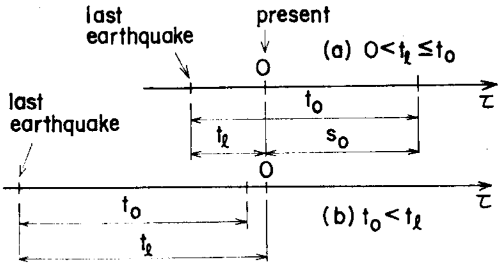

Fig. 7 Arrangement of Time Parameters ($a=0$).

we have

(1) $0<t_l\leq a$.

$$P\{N_l(\tau)\geq 1\,|\,C(t_l)\} = \begin{cases} 0 & ;\ 0<\tau\leq a-t_l \\ 1-e^{-\nu_1(\tau+t_l-a)} & ;\ a-t_l<\tau\leq s_0 \\ 1-e^{-\{\nu_2(\tau-s_0)+\nu_1(t_0-a)\}} & ;\ \tau>s_0 \end{cases} \quad \cdots(5)$$

(2) $a<t_l\leq t_0$.

$$P\{N_l(\tau)\geq 1\,|\,C(t_l)\} = \begin{cases} 1-e^{-\nu_1\tau} & ;\ 0<\tau\leq s_0 \\ 1-e^{-\{\nu_2(\tau-s_0)+\nu_1 s_0\}} & ;\ \tau>s_0 \end{cases} \quad \cdots\cdots\cdots\cdots(6)$$

(3) $t_0<t_l$

$$P\{N_l(\tau)\geq 1\,|\,C(t_l)\} = 1-e^{-\nu_2\tau}\,. \quad \cdots\cdots\cdots\cdots\cdots\cdots(7)$$

When the earthquake sets E_A and E_B are considered, only Eqs. (6) and (7) are involved, since $a=0$ for these cases as illustrated by Fig. 7.

For comparison, the result from a simple Posison process with a mean occurrence rate ν_0 is represented by

$$P\{N_I(\tau)\geq 1 \mid C(t_l)\} = P\{N_I(\tau)\geq 1\} = 1-e^{-\nu_0\tau} \quad \cdots\cdots\cdots\cdots\cdots(8)$$

which does not depend on t_l (or s_0).

The conditional probability of the future earthquake occurrence $P\{N_I(\tau)\geq 1 \mid$

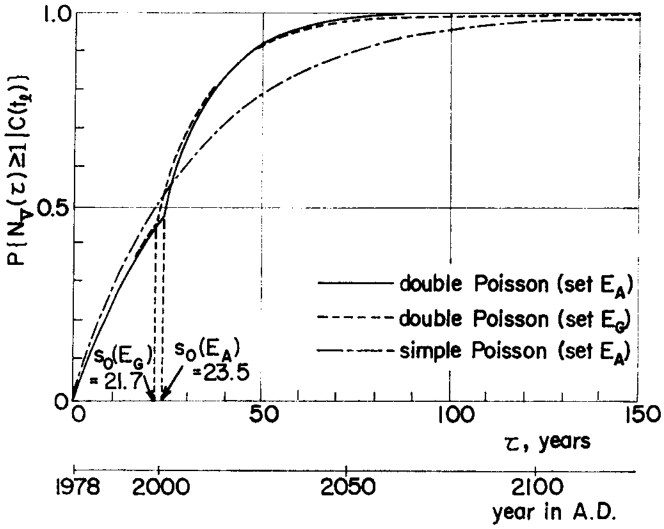

Fig. 8. Conditional Probability of Future Earthquakes Starting from 1978, (based on Eq. (4); intensity $I \geq$ V).

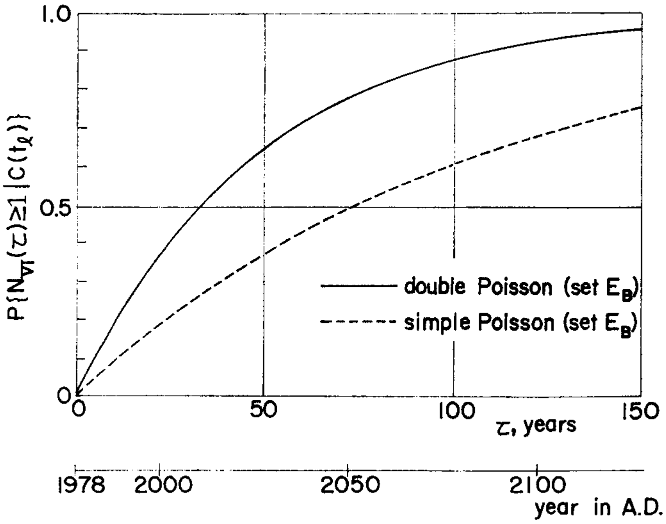

Fig. 9. Conditional Probability of Future Earthquakes Starting from 1978, (based on Eq. (4); intensity $I \geq$ VI).

$C(t_l)\}$ has been computed using these formulae. Figs. 8 and 9 show the values of $P\{N_I(\tau)\geq 1 \mid C(t_l)\}$, starting from 1978. Fig. 8 is for the intensity $I\geq$V based on the earthquake sets E_A and E_G, and Fig. 9 is for $I\geq$VI based on E_B. In Fig. 8, the curves for the double Poisson process have clear breaks at $\tau=s_0$, which correspond to the breaks at $t=t_0$ in Figs. 2 and 4. These are the points beyond which the mean occurrence rate of earthquakes increases from ν_1 to ν_2. Therefore, in the range $\tau\geqq s_0$, the values of $P\{N_V(\tau)\geq 1 \mid C(t_l)\}$ based on the double Poisson process increase more rapidly with τ than those from the simple Poisson assumption. In Fig. 9, based on the earthquake set E_B with the higher intensity $I\geq$VI, the result from the double Poisson process does not have such a break point, since in this case $t_0<t_l$ and Eq. (7) applies. However, because of the large value of ν_2, the value of $P\{N_{VI}(\tau)\geq 1 \mid C(t_l)\}$ from the double Poisson process is much larger than that from the simple Poisson process. In Fig. 8, there is little difference between the results from the double Poisson process for the set E_A and those for the set E_G. This implies that there is enough uniformity in the earthquake set E_A in the sense described in **2.2.** Hence, discussion on earthquakes with intensities $I\geq$V in the following part of this paper will be confined to the results for the set E_A.

It is important to find how the probability of a future earthquake varies after the occurrence of an earthquake. For this purpose, the value of $P\{N_I(\tau)\geq 1 \mid$

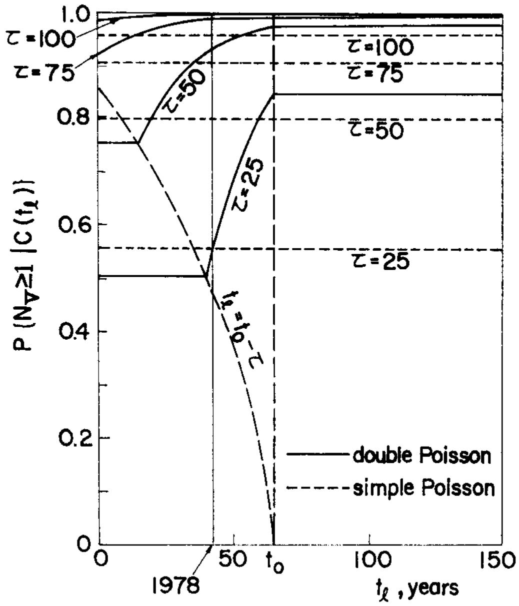

Fig. 10. Conditional Probability of Future Earthquakes as a Function of t_l, (based on Eq. (4); intensity $I\geq$V).

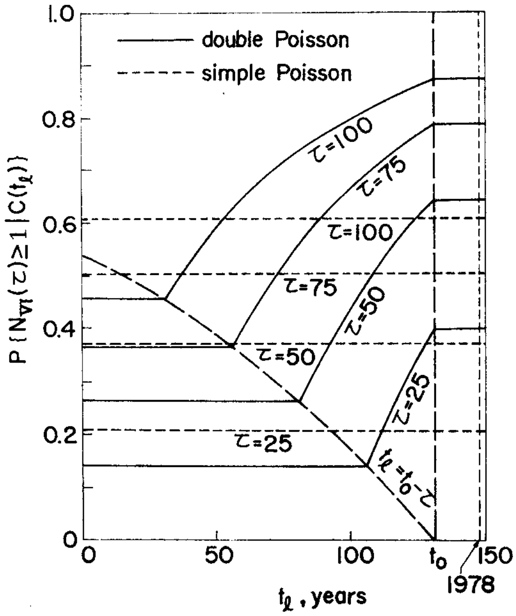

Fig. 11. Conditional Probability of Future Earthquakes as a Function of t_l, (based on Eq. (4); intensity $I \geq$VI).

$C(t_l)\}$ has been plotted against t_l in Figs. 10 and 11 for the various values of the future period τ. In these figures, the probability level based on the double Poisson process for $t_l \leq t_0 - \tau$ and that for $t_l > t_0$ are independent of t_l, the latter assuming a larger value. The values for $t_l \leq t_0 - \tau$ are obtained from the first of Eq. (6) which corresponds to cases where all earthquakes within future τ years will occur in a Poisson process with the mean occurrence rate ν_1. However, the values for $t_l > t_0$ are obtained from Eq. (7), in which case the first earthquake to occur within future τ years will necessarily take place under $\nu = \nu_2$. The probability level in the intermediate region of $t_0 - \tau < t_l \leq t_0$ reflects two possibilities, that all earthquakes occur under $\nu = \nu_1$ and that they contain an earthquake occurring under $\nu = \nu_2$. The results based on the simple Poisson process shown by the dashed lines in Figs. 10 and 11 do not account for such effects of t_l.

In Figs. 10 and 11, the year 1978 is marked on the t_l-axis. In order to compare the probability of future earthquakes on a common time axis using Figs. 10 and 11, it is necessary to shift them horizontally so that these year marks coincide. It should also be noted that Figs. 9 and 11 for the intensity $I >$VI are based on the earthquake set E_B with only 10 samples. The set E_B excludes further information pertaining to earthquakes with an intensity $I \geq$VI contained in the set

EA. For example, the difference in the proportion of such strong earthquakes among the total between those occurring under $\nu=\nu_1$ and those under $\nu=\nu_2$ is useful information contained in the set E_A but not considered in E_B. Therefore, it is desirable to analyze the earthquakes with an intensity $I \geq$ VI also using the set E_A. This requires a more complicated formulation and is dealt with in the next section.

4.3 Analysis of earthquakes with $I \geq$ VI using intensity ratio

From the points raised at the end of the previous section, the probability of a future earthquake occurrence for the intensity $I \geq$ VI is obtained using the earthquake set E_A. It incorporates the probability that an earthquake has an intensity $I \geq$ VI, depending on whether it occurs under $\nu=\nu_1$ or $\nu=\nu_2$. Let random variables $K_{\tau 1}$ and $K_{\tau 2}$ denote the number of earthquakes occuring under $\nu=\nu_1$ and $\nu=\nu_2$, respectively, within a future period $(0, \tau)$. They are different from X_{t_1} and X_{t_2} in that the time origin in this case is taken in the same manner as in $N_I(\tau)$. Then, the probability that an earthquake with an intensity $I \geq$ VI will occur within the future period τ, under the condition that the last earthquake occurred t_l years ago is represented by

$$\begin{aligned} &P\{N_{\mathrm{VI}}(\tau) \geq 1 \mid C(t_l)\} \\ &\quad = P\{N_{\mathrm{VI}}(\tau) \geq 1,\ K_{\tau 2} = 0 \mid C(t_l)\} \\ &\qquad + P\{N_{\mathrm{VI}}(\tau) \geq 1,\ K_{\tau 2} \geq 1 \mid C(t_l)\} \\ &\quad = \sum_{k=0}^{\infty} P\{N_{\mathrm{VI}}(\tau) \geq 1 \mid K_{\tau 1} = k,\ K_{\tau 2} = 0,\ C(t_l)\} \\ &\qquad \cdot P\{K_{\tau 1} = k,\ K_{\tau 2} = 0 \mid C(t_l)\} \\ &\qquad + P\{N_{\mathrm{VI}}(\tau) \geq 1 \mid K_{\tau 2} \geq 1,\ C(t_l)\} P\{K_{\tau 2} \geq 1 \mid C(t_l)\}\ . \qquad \cdots\cdots(9) \end{aligned}$$

The closed form solutions of Eq.(9) are obtained depending on the value of t_l. A detailed analysis is described in Appendix A, whose results are shown below. They have been obtained for the case of $a=0$, Fig. 7, to which the earthquake set E_A applies. Sustitution from Eqs.(A.1)～(A.10) into Eq. (9) yields

(1) $0 < t_l \leq t_0$

a) $0 < \tau \leq s_0$

$$P\{N_{\mathrm{VI}}(\tau) \geq 1 \mid C(t_l)\} = 1 - e^{-r_{\mathrm{VI}} \nu_1 t} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

b) $s_0 < \tau \leq t_0$

$$\begin{aligned} &P\{N_{\mathrm{VI}}(\tau) \geq 1 \mid C(t_l)\} \\ &\quad = \sum_{k=0}^{\infty} \{1-(1-r_{\mathrm{VI}})^k\}\{\tau^k - (\tau - s_0)^k\} \nu_1^k \frac{e^{-\nu_1 \tau}}{k!} + e^{-\nu_1 s_0}\{1 - e^{-\nu_2(\tau - s_0)}\} \\ &\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10') \end{aligned}$$

c) $t_0<\tau\leq t_0+s_0$

$$
\begin{aligned}
&P\{N_{\mathrm{VI}}(\tau)\geq 1\,|\,C(t_l)\}\\
&=\sum_{k=0}^{\infty}\{1-(1-r_{\mathrm{VI}})^k\}\Bigg[\frac{\nu_1^k}{(k-1)!}\,e^{-\{\nu_2\tau-(\nu_2-\nu_1)t_0\}}\\
&\quad\cdot\left\{\sum_{r=0}^{k-1}(-1)^r\frac{(\tau-t_0)^{k-r-1}}{(\nu_2-\nu_1)^{r+1}}e^{(\nu_2-\nu_1)(\tau-t_0)}\frac{(k-1)!}{(k-r-1)!}+(-1)^k\frac{(k-1)!}{(\nu_2-\nu_1)^k}\right\}\\
&\quad+\sum_{j=1}^{k-1}\frac{\nu_1^k e^{-\nu_1\tau}}{(k-j-1)!(j-1)!}\sum_{i=0}^{j-1}\frac{(j-1)!}{\nu_1^{i+1}(j-i-1)!}\Bigg[e^{\nu_1 t_0}\sum_{h=0}^{j-i-1}\binom{j-i-1}{k}(-1)^h\frac{(\tau-t_0)^{k-i-1}}{k-j+h}\\
&\quad-e^{\nu_1 s_0}(\tau-s_0)^{j-i-1}\left\{\frac{(k-j-1)!}{\nu_1^{k-j}}-\sum_{l=0}^{k-j-1}\frac{(k-j-1)!}{\nu_1^{l+1}(k-j-l-1)!}(\tau-t_0)^{k-j-l-1}e^{-\nu_1(\tau-t_0)}\right\}\Bigg]\\
&\quad+\sum_{m=1}^{k}\sum_{p=1}^{m}\frac{(\tau-t_0)^{k-m}(t_0+s_0-\tau)^{m-p+1}(\tau-s_0)^{p-1}}{(k-m)!(m-p+1)!(p-1)!}\nu_1^k e^{-\nu_1\tau}\Bigg]\\
&\quad+e^{-\nu_1 s_0}\{1-e^{-\nu_2(\tau-s_0)}\}+e^{-\nu_1 t_0}\sum_{m=1}^{\infty}\Bigg[1-\left(\frac{-\nu_1}{\nu_2-\nu_1}\right)^m e^{-\nu_2(\tau-t_0)}\\
&\quad-e^{-\nu_1(\tau-t_0)}\sum_{r=0}^{m-1}\frac{\{\nu_1(\tau-t_0)\}^{m-r-1}}{(m-r-1)!}\left\{1-\left(\frac{-\nu_1}{\nu_2-\nu_1}\right)^{r+1}\right\}\Bigg] \qquad \cdots\cdots(10'')
\end{aligned}
$$

(2) $t_l>t_0$

$$P\{N_{\mathrm{VI}}(\tau)\geq 1\,|\,C(t_l)\}=1-e^{-\nu_2\tau}. \qquad \cdots\cdots(11)$$

For $t_l\leq t_0$, the closed form solution has been obtained only for $0<\tau\leq s_0+t_0$. Further analysis requires formulation for separate cases, as τ exceeds $2t_0$, s_0+2t_0, $3t_0$, s_0+3t_0, etc. The complexity of reduction for these cases is formidable. In the case of Kyoto, dealt with in this paper, the result within the range of Eqs. (10), (10′), (10″) is adequate for periods significant in the prediction of the future earthquake probability.

The numerical values obtained using the above results are shown in Figs. 12 and 13. Fig. 12 is a plot of $P\{N_{\mathrm{VI}}(\tau)\geq 1\,|\,C(t_l)\}$ starting from 1978, shown by the heavy solid curve. Since $t_l<t_0$ for the set E_A, this curve has been obtained from Eqs. (10)～(10″). The results based on set E_B shown in Fig. 9 are also plotted for comparison. It is noted from these results that a more detailed variation of the probability has been obtained by using the intensity ratio than that from the recurrence-time distribution. The probability of future earthquakes with $I\geq$VI based on the intensity ratio does not increase with τ in such a simple manner as that from the recurrence-time distribution. It is smaller than even the result of the simple Poisson process until $\tau=s_0$, whereas it grows rapidly thereafter to get close to the result based on the recurrence-time distribution, and exceeds it before $\tau=s_0+t_0$. Since the analysis using the intensity ratio developed in this

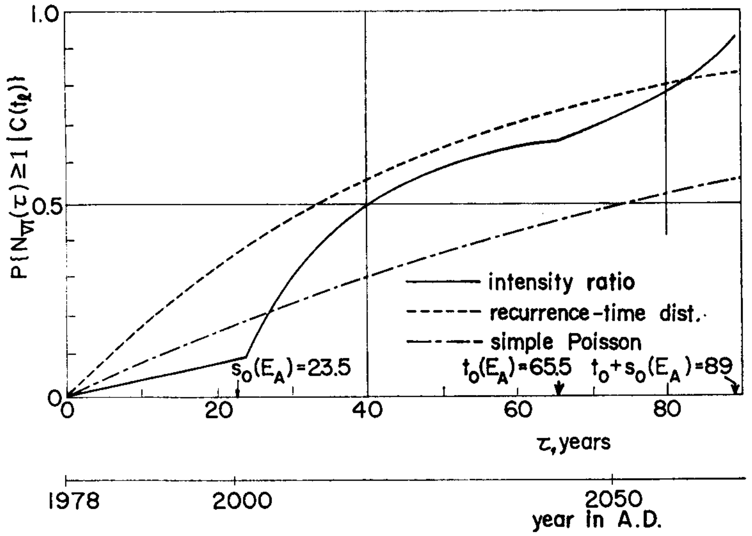

Fig. 12. Conditional Probability of Furture Earthquakes Starting from 1978, (based on Eq. (9); intensity $I \geq$ VI).

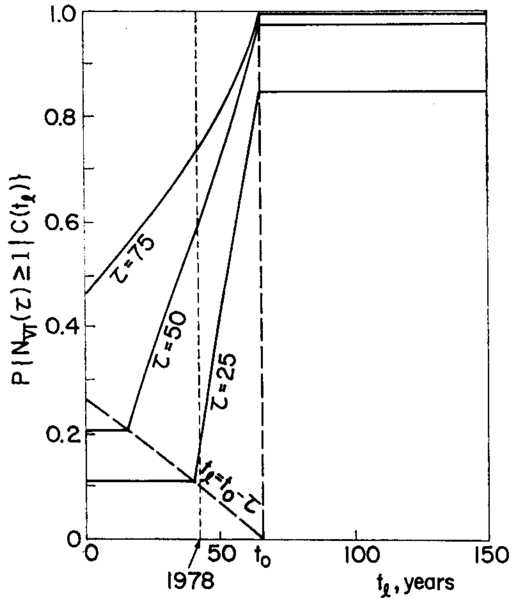

Fig. 13. Conditional Probability of Furture Earthquakes as a Function of t_l, (based on Eq. (9); intensity $I \geq$ VI).

section incorporates more information on earthquakes with $I \geq$ VI than the analysis in the previous section using the set E_B, the solid curve in Fig. 12 should be a more realistic estimate of the future earthquake probability than the others in the same figure.

Fig. 13 is plot of $P\{N_{VI}(\tau)\geq 1 | C(t_l)\}$ against t_l that can be compared with Figs. 10 and 11. Since the analysis in this section is using the set E_A, the value of t_0 and the year mark for 1978 in Fig. 13 coincide with Fig. 10. The constant probability for $t_l \leq t_0 - \tau$ is determined from Eq. (10) which represents the Poisson probability with $\nu = r_{VI}\nu_1$ giving a much lower probability level than in the corresponding region in Fig. 10. For $t_l > t_0$, on the other hand, the probability is obtained from Eq. (11) which is identical with Eq. (7). Therefore, in this region the probability in Fig. 13 coincides with that in Fig. 10. The results for $t_0 - \tau < t_l \leq t_0$ are determined from Eqs. (10′) and (10″).

Comparing Fig. 13 with Fig. 11, both obtained for the intensity $I \geq$ VI, we can observe that the difference between the probability values for $t_l \leq t_0 - \tau$ and $t_l > t_0$ in Fig. 13 is much greater than that in Fig. 11. It should also be noted that after t_l exceeds t_0, the probability of an earthquake with $I \geq$ VI in Fig. 13 is much larger than that obtained from the recurrence-time distribution based on the double Poisson process as well as the simple Poisson probability shown in Fig. 11.

5. Conclusions

The major results of this study may be summarized as follows:

(1) From a detailed survey of the recurrence-time distribution for earthquakes with felt intensity $I \geq$ V, in the JMA scale, it has been found that there is evidence of a correlation in the time series of destructive earthquakes that can be explained qualitatively on a physical basis.

(2) The correlation is characterized by an increase in the mean occurrence rate, if no destructive earthquakes occur for a fixed period of t_0 years. It is also remarkable that all earthquakes occurring after a quiescent period longer than t_0 years have intensities of $I \geq$ VI.

(3) A similar increase in the mean occurrence rate is also observed when only the earthquakes with $I \geq$ VI are considered, although the sample size in this case is not sufficient for definitive results.

(4) A renewal process model of random sequence of earthquakes that can account for the above properties of the earthquake data has been proposed. The model is called herein the *double Poisson process* in the sense that the mean occurrence rate varies from one to the other of two fixed values, depending on the period t_l between the time of last earthquake and the present time.

(5) On the basis of the double Poisson process model, the conditional probability of earthquakes in a future period given the time of the last earthquake has

been obtained. The formulation has been made in two ways: (i) using the recurrence-time distribution, and (ii) using the intensity ratio. The latter formulation is particularly useful when earthquakes with $I \geq$ VI are concerned.

(6) The numerical values of the conditional probability of future earthquakes have been obtained for Kyoto, and their significance has been verified in comparison with conventional simple Poisson process models.

The double Poisson process model developed in this study has been proposed on the basis of the earthquake data felt at a fixed site. It proved to provide useful information for a seismic risk analysis that incorporates a correlation between successive earthquakes within the range of the data for Kyoto. However, Kyoto is rather an exceptional case, where historical earthquake data with a sample size large enough to detect the correlation are available. Therefore, for extending the renewal process model developed herein to less documented districts, it is necessary to deal with earthquake occurrences within seismic sources of a somewhat larger area. Then, the seismic risk analysis for a site will be made with the aid of a suitable attenuation rule. Work in this direction is underway.

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Appendix A. Reduction of Conditional Future Earthquake Probability Using Intensity Ratio

Eqs. (10)∼(11) are reduced in the following. Since the case with $a=0$ is dealt with, the time arrangement in Fig. 7 is involved.

Among the terms appearing in Eq. (9), the following expressions apply to all cases.

$$P\{N_{\mathrm{VI}}(t)\geq 1\,|\,K_{\tau 1}=k,\ K_{\tau 2}=0,\ C(t_l)\} = 1-(1-r_{\mathrm{VI}})^k \quad \cdots\cdots(\mathrm{A.1})$$

$$P\{N_{\mathrm{VI}}(t)\geq 1\,|\,K_{\tau 2}\geq 1,\ C(t_l)\} = 1 \quad \cdots\cdots(\mathrm{A.2})$$

Eq.(A.1) is based on an assumption that the intensities of earthquakes occurring under $\nu=\nu_1$ are statistically independent. Eq. (A.2) result from Fig. 2, in which all earthquakes corresponding to $\nu=\nu_2$ are of intensities $I\geq$ VI. Further reduction of Eq. (9) to Eqs. (10)~(11) is made for the following separate cases.

A.1 Case with $t_l \leq t_0$.

It is required to formulate the problem for $0<\tau\leq s_0$, $s_0<\tau\leq t_0$, $t_0<\tau\leq s_0+t_0$, $s_0+t_0<\tau\leq 2t_0$, $2t_0<\tau\leq s_0+2t_0$, etc. Herein, the analysis is made only for the first three cases; i.e., within the time range of $0<\tau\leq s_0+t_0$.

(1) $0<\tau\leq s_0$

In this case all possible earthquakes occur in a Poisson process with $\nu=\nu_1$. Therefore, we have

$$P\{K_{\tau 1}=k,\ K_{\tau 2}=0\,|\,C(t_l)\} = \frac{(\nu_1\tau)^k}{k!}e^{-\nu_1\tau} \quad \cdots\cdots\cdots\cdots(\text{A.3})$$

$$P\{K_{\tau 2}\geq 1\,|\,C(t_l)\} = 0 \quad \cdots\cdots\cdots\cdots(\text{A.4})$$

(2) $s_0<\tau\leq t_0$

The time parameters for this case are illustrated in Fig. A1. If the first earthquake takes place in the interval $(0, s_0)$, it will occur under $\nu=\nu_1$. In

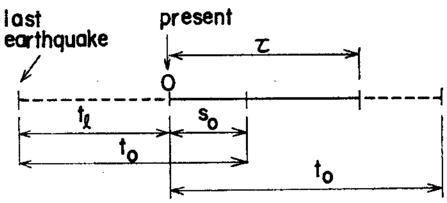

Fig. A1

this case, therefore, all earthquakes, say k events, occurring in $(0, \tau)$ take place under $\nu=\nu_1$ and are counted among $K_{\tau 1}$, since all subsequent events following the first occur within the interval of t_0. Hence we have

$$P\{K_{\tau 1}=k,\ K_{\tau 2}=0\,|\,C(t_l)\} = \int_0^{s_0} e^{-\nu_1\eta}\nu_1 d\eta \frac{\{\nu_1(\tau-\eta)\}^{k-1}}{(k-1)!}e^{-\nu_1(\tau-\eta)}$$

$$= \nu_1^k\{\tau^k-(\tau-s_0)^k\}\frac{e^{-\nu_1\tau}}{k!}. \quad \cdots\cdots\cdots\cdots(\text{A.5})$$

On the other hand, if the first earthquake occurs in (s_0, τ), it will necessarily take place under $\nu=\nu_2$, and therefore is counted as $X_{\tau 2}$. All following earthquakes, if any, take place under $\nu=\nu_1$, and therefore do not contribute to $K_{\tau 2}$. Accordingly, the following relation holds.

$$P\{K_{\tau 2}\geq 1\,|\,C(t_l)\} = P(K_{\tau 2}=1) = e^{-\nu_1 s_0}\{1-e^{-\nu_2(\tau-s_0)}\}. \quad \cdots\cdots\cdots\cdots(\text{A.6})$$

(3) $t_0<\tau\leq t_0+s_0$

First the probability of event $(K_{\tau 1}=k,\ K_{\tau 2}=0)$ is discussed. Divide the future interval $(0,\ \tau)$ into $(0,\ \tau-t_0)$, $(\tau-t_0,\ s_0)$, and $(s_0,\ \tau)$, Fig. A2. Let l_1, l_2, l_3, denote the number of earthquakes occurring in these intervals, respectively. They must satisfy

$$l_1+l_2+l_3=k$$

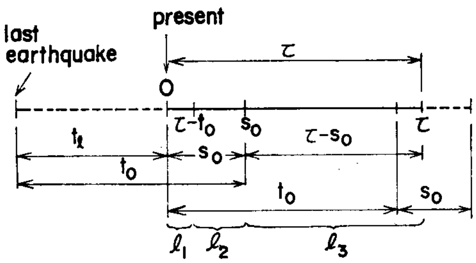

Fig. A2

These k earthquakes are required to take place under $\nu=\nu_1$. This condition is realized in either of the cases listed in Table A.1. Then the probability of each case is obtained as follows:

Table A1. Possible Combination of l_1, l_2, l_3 $(l_1+l_2+l_3=k)$.

Case	l_1	l_2	l_3
A_{00}	k	0	0
A_{01}	$k-1$	0	1
	⋮	⋮	⋮
	$k-j$	0	j
	⋮	⋮	⋮
	1	0	$k-1$
A_1	$k-1$	1	0
	$\begin{cases} k-1 \\ k-2 \end{cases}$	$\begin{cases} 2 \\ 1 \end{cases}$	$\begin{cases} 0 \\ 1 \end{cases}$
	⋮	⋮	⋮
	$\begin{cases} k-m \\ k-m \\ \vdots \\ k-m \end{cases}$	$\begin{cases} m \\ m-1 \\ \vdots \\ 1 \end{cases}$	$\begin{cases} 0 \\ 1 \\ \vdots \\ m-1 \end{cases}$
	⋮	⋮	⋮
	$\begin{cases} 0 \\ \vdots \\ 0 \end{cases}$	$\begin{cases} k \\ \vdots \\ 1 \end{cases}$	$\begin{cases} 0 \\ \vdots \\ k-1 \end{cases}$

$$P(A_{00})=\int_0^{\tau-t_0}\nu_1 d\eta\,\frac{(\nu_1\eta)^{k-1}}{(k-1)!}\,e^{-\nu_1\eta}e^{-\nu_1 t_0}e^{-\nu_2(\tau-(\eta+t_0))} \qquad \cdots\cdots\cdots\cdots(\mathrm{a})$$

$$P(A_{01}) = \sum_{j=1}^{k-1}\int_0^{\tau-t_0} \nu_1 d\eta \frac{(\nu_1\eta)^{k-j-1}}{(k-j-1)!} e^{-\nu_1\eta}\int_{s_0}^{\eta+t_0} \nu_1 d\eta' \frac{\{\nu_1(t-\eta')\}^{j-1}}{(j-1)!} e^{-\nu_1(\tau-\eta')}$$

$$= \sum_{j=1}^{k-1}\int \frac{\nu_1^k e^{-\nu_1\tau}}{(k-j-1)!(j-1)!}\int_0^{\tau-t_0} d\eta \int_{s_0}^{\eta+t_0} \eta^{(k-j-1)}(t-\eta')e^{j-1}e^{-\nu_1\eta}e^{\nu_1\eta'}d\eta' \qquad \cdots\cdots\cdots\cdots(\mathrm{b})$$

$$P(A_1) = \sum_{m=1}^{k}\frac{\{\nu_1(\tau-t_0)\}^{k-m}}{(k-m)!} e^{-\nu_1(\tau-t_0)}\left[\sum_{p=1}^{m}\frac{\{\nu_1(s_0+t_0-\tau)\}^{m-p+1}}{(m-p+1)!} e^{-\nu_1(s_0+t_0-\tau)} \times \frac{\{\nu_1(\tau-s_0)\}^{p-1}}{(p-1)!} e^{-\nu_1(\tau-s_0)}\right] \qquad \cdots\cdots\cdots\cdots(\mathrm{c})$$

Integration of Eq. (b) is particularly a lengthy procedure. Reduction of Eqs. (a), (b), (c) and summing them up lead to the following result:

$$P\{K_{\tau1}=k,\ K_{\tau2}=0\,|\,C(t_l)\}$$
$$= \frac{\nu_1^k}{(k-1)!} e^{-\{\nu_2\tau-(\nu_2-\nu_1)t_0\}}\left\{\sum_{r=0}^{k-1}(-1)^r\frac{(\tau-t_0)^{k-r-1}}{(\nu_2-\nu_1)^{r+1}} e^{(\nu_2-\nu_1)(\tau-t_0)}\frac{(k-1)!}{(k-r-1)!} + (-1)^k\frac{(k-1)!}{(\nu_2-\nu_1)^k}\right\} + \sum_{j=1}^{k-1}\frac{\nu_1^k e^{-\nu_1\tau}}{(k-j-1)!(j-1)!}\sum_{i=0}^{j-1}\frac{(j-1)!}{\nu_1^{i+1}(j-i-1)!}$$
$$\times\left[e^{\nu_1 t_0}\sum_{h=0}^{j-i-1}\binom{j-i-1}{h}(-1)^h\frac{(\tau-t_0)^{k-i-1}}{k-j+h} - e^{\nu_1 s_0}(\tau_0-s)^{j-i-1}\left\{\frac{(k-j-1)!}{\nu^{k-j}} - \sum_{l=0}^{k-j-1}\frac{(k-j-1)!}{\nu_1^{l+1}(k-j-l-1)!}(\tau-t_0)^{k-j-l-1}e^{-\nu_1(\tau-t_0)}\right\}\right]$$
$$+\sum_{m=1}^{k}\sum_{p=1}^{m}\frac{(\tau-t_0)^{k-m}(s_0+t_0-\tau)^{m-p+1}(\tau-s_0)^{p-1}}{(k-m)!\,(m-p+1)!\,(p-1)!}\nu_1^k e^{-\nu_1\tau}. \qquad \cdots\cdots\cdots\cdots(\mathrm{A.7})$$

The first, second and third terms of Eq. (A.7) correspond, repsectively, to Eqs. (a), (b), and (c).

Next, the probability $P(K_{\tau2}\geq-1)$ is dealt with. This even consists of the following two mutually exclusive cases: (i) no earthquakes in $(0, s_0)$, but an earthquake occurs under $\nu=\nu_2$ in (s_0, τ), and (ii) earthquakes occur under $\nu=\nu_1$ in $(0, \tau-t_0)$, after which no earthquakes until t_0, and an earthquake under $\nu=\nu_2$ before the time τ. In these cases also, earthquakes under $\nu=\nu_2$ can occur at most once within $(0, \tau)$. Then, it holds that

$$P\{K_{\tau2}\geq 1\,|\,C(t_l)\} = P(K_{\tau2}=1)$$
$$= e^{-\nu_1 s_0}\{1-e^{-\nu_2(\tau-s_0)}\} + \sum_{m=1}^{\infty}\int_0^{\tau-t_0}\frac{(\nu_1\eta)^{m-1}}{(m-1)!} e^{-\nu_1\eta}\nu_1 d\eta\, e^{-\nu_1 t_0}\{1-e^{-\nu_2(\tau-\eta-t_0)}\}$$
$$= e^{-\nu_1 s_0}\{1-e^{-\nu_2(\tau-s_0)}\} + e^{-\nu_1 t_0}\sum_{m=1}^{\infty}\left[1-\left(\frac{-\nu_1}{\nu_2-\nu_1}\right)^m e^{-\nu_2(\tau-t_0)} - e^{-\nu_1(\tau-t_0)}\sum_{r=1}^{m-1}\frac{\{\nu_1(\tau-t_0)\}^{m-r-1}}{(m-r-1)!}\left\{1-\left(\frac{-\nu_1}{\nu_2-\nu_1}\right)^{r+1}\right\}\right]. \qquad \cdots\cdots\cdots\cdots(\mathrm{A.8})$$

A.2 Case with $t_l < t_0$

In this case, the mean occurrence rate has already increased to ν_2 at the time origin. Therefore, the first earthquake to occur in the future will necessarily take place under $\nu = \nu_2$. Therefore,

$$P\{K_{\tau 1} = k,\ K_{\tau 2} = 0 \,|\, C(t_l)\} = \begin{cases} e^{-\nu_2 \tau}; & k = 0 \\ 0 \quad ; & k \geq 1 \end{cases} \qquad \cdots\cdots\cdots\cdots (A.9)$$

$$P\{K_{\tau 2} \geq 1 \,|\, C(t_l)\} = 1 - e^{-\nu_2 \tau} \qquad \cdots\cdots\cdots\cdots (A.10)$$

for any τ.

Appendix B. Notations

a = period in which $\nu = 0$.
$C(t_l)$ = event that the last earthquake occurred t_l years ago.
E_A = earthquake set with $I \geq V$ for Kyoto.
E_B = earthquake set with $I \geq VI$ for Kyoto.
E_G = set of earthquake groups with $I \geq V$ for Kyoto.
I = earthquake intensity in the JMA scale.
$K_{\tau 1}$ = random variable denoting the number of earthquakes occurring under $\nu = \nu_1$ within future τ years.
$K_{\tau 2}$ = random variable denoting the number of earthquakes occurring under $\nu = \nu_2$ within future τ years.
$N_I(\tau)$ = random variable denoting the number of earthquakes with intensities no less than I within future τ years.
$P(\ )$ = probability
r_{VI} = proportion of earthquakes with intensities $I \geq VI$ among those occurring under $\nu = \nu_1$.
s_0 = $t_0 - t_l$
T = recurrence time of earthquakes.
t = time with origin at the time of last earthquake occurrence.
t_0 = maximum recurrence time for earthquakes occurring under $\nu = \nu_1$.
t_l = time since the last earthquake occurrence.
X_t = random variable denoting the number of earthquakes within t years after an earthquake occurrence.
X_{t_1} = random variable denoting the number of earthquakes included in X_t and occurring under $\nu = \nu_1$.
X_{t_2} = random variable denoting the number of earthquakes included in X_t and occurring under $\nu = \nu_2$.

ν = mean occurrence rate of earthquakes.

ν_1 = mean occurrence rate for $T \leq t_0$.

ν_2 = mean occurrence rate for $T > t_0$.

τ = future period.