Orientation Problem of SLAR Imagery

By

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(Received March 31, 1979)

Abstract

This report treats the orientation and restitution problem of SLAR imagery theoretically. The orientation problem is discussed for single SLAR configuration and also for stereo SLAR configuration. For the former, this paper proposed an analytical orientation method constructed on the geometrical basis of SLAR imagery already studied. For the latter, the author developed an orientation technique to calculate the exterior orientation parameters of the antenna for stereo SLAR imageries simultaneously. With this method the analysis of SLAR imagery may be performed three-dimensionally and more accurately than before. In both cases, some functional form, such as polynomials and Fourier's series, is used to model the behaviors of the exterior orientation elements of the antenna along the flight path, as in the analysis of MSS imagery.

Linearizing the determination equations for the orientation problem of single SLAR imagery, one obtains error equations for the restitution problem of SLAR imagery. This report introduced simple restitution methods of SLAR imagery for a flat terrain and also for a hilly ground surface, and further, clarified some characteristics.

1. Introduction

SLAR records its images in an entirely different manner from a conventional camera. The geometry of SLAR imagery is much more complicated than that of an optical photograph. Many studies about SLAR mapping have been made, and a fundamental analysis of SLAR imagery has already been established. Little, however, has been done in the way of investigating the orientation problem of SLAR imagery. Therefore, this report treats the analytical orientation and restitution problem precisely.

On account of the great difficulties to obtain a SLAR photograph in Japan, no practical research was done in this paper to determine the actual accuracies attainable with the methods proposed. Hence, no findings in the practical analysis of SLAR imagery were shown here. To test SLAR imagery is our next program.

2. Squint Angle

All SLAR systems operate with a particular squint angle. The precise discussion

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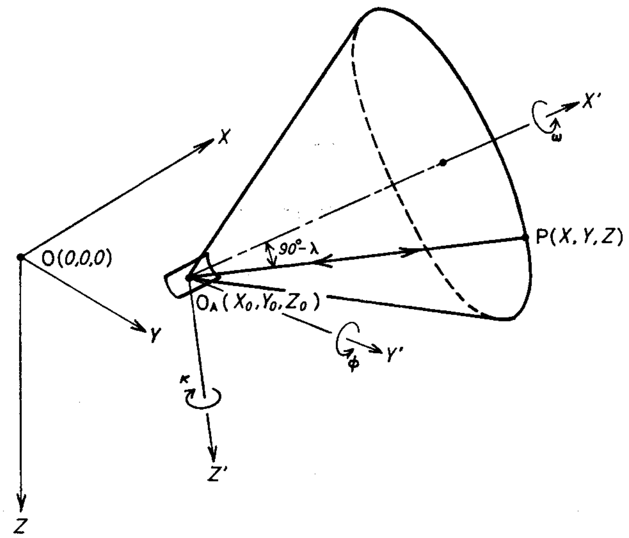

Fig. -1 SLAR geometry

about this angle is to be seen in reference 1. Here, the squint angle is briefly described, as in Fig. -1. For this purpose, the ground coordinate system (X, Y, Z) is taken as a right-handed, rectangular-cartesian system with its origin at an arbitrary point over the ground surface. The antenna coordinate system (X' Y' Z') is selected as a right-handed, rectangular-cartesian system with its origin at the electrical center of the antenna. The $Y'Z'$-plane of the antenna coordinate system is considered to be parallel to the YZ-plane of the ground coordinate system, if the antenna has an ideal position. A pulse of microwave energy transmitted from the electrical center of the antenna travels along a conical surface with an extremely small beam width. The squint angle is an angle of this conical surface relative to the $Y'Z'$-plane.

In reality, the squint angle will be zero or almost zero[1]. If the squint angle is zero, the propagation surface of a pulse of microwave energy is a plane, and then the geometry of SLAR imagery becomes remarkably simple.

3. Projective Characteristics of SLAR Imagery

SLAR imagery is taken with a range projection principle in the across-track direction, and with an orthogonal projection principle in the along-track direction. Consequently, the across-track coordinate (y-coordinate) of an image point is proportional to the time required for a pulse of microwave energy to be transmitted and to be reflected back to the antenna. The along-track coordinate (x-coordinate) is proportional to the ground track distance, if the aircraft travels at a constant speed. Geomet-

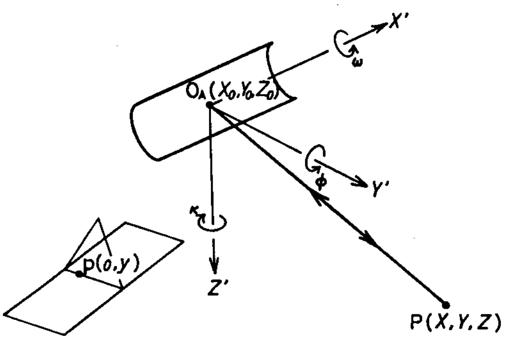

Fig. -2 range projection

rical characteristics of SLAR imagery are discussed in detail in reference 2. Therefore, we consider here the projective characteristics based on a range projection, which are useful for studying the orientation problem of SLAR imagery (see Fig. -2).

If the ground point P(X, Y, Z) and the electrical center $O_A(X_0, Y_0, Z_0)$ of the antenna are given, the slant range between P and O_A can be calculated; and then the image point p(0, y) corresponding to P can be determined from the slant range regardless of the rotation of the antenna. On the contrary, even if the image point p(0, y) and the electrical center $O_A(X_0, Y_0, Z_0)$ of the antenna are given, the ground point P(X, Y, Z) can not be defined uniquely. All points on a spherical surface with its origin at $O_A(X_0, Y_0, Z_0)$ and with its radius $m_0 y$ (=the slant range) fulfill the projective conditions based on a range projection.

A pulse of microwave energy, however, propagates not spherically but along a conical surface. Using this fact, the projective relationship of SLAR imagery can be described as follows: If the ground point P(X, Y, Z) and the electrical center O_A (X_0, Y_0, Z_0) of the antenna are given, the slant range from P to O_A can be calculated; and then the image point p(0, y) can be determined regardless of the rotation of the antenna. On the other hand, even if the image point p(0, y) and the exterior orientation elements of the antenna are given, one can not define the ground point P(X, Y, Z) uniquely. The desired ground point is considered to lie on a circle determined by the sphere with its center at the electrical center of the antenna and its radius $m_0 y$, and by the conical surface along which the pulse travels.

4. Analytical Orientation Problem of SLAR Imagery

In the SLAR system, the range measurement generates a sphere and the squint angle generates a conical surface. The fundamental equations for the orientation problem of SLAR imagery are constructed on this basis[3]. Suppose that exterior orientation parameters of the antenna are constant in the y-direction (on a scan line), and that changes of the exterior orientation elements along the flight path can be modeled with some functional form, such as polynomials and Fourier's series. Then, consider the analytical orientation problem of SLAR imagery in a single SLAR configuration and also in a stereo SLAR configuration.

4-1. analytical orientation problem of single SLAR imagery

4-1-1 in the case of the zero-squint angle

A pulse of microwave energy transmitted from the electrical center of the antenna travels along the $Y'Z'$-plane of the antenna coordinate system, if the squint angle is zero. One must, at first, determine the equation of the $Y'Z'$-plane observed in the ground coordinate system (X, Y, Z). This equation can be constructed in the following way (see Fig. -3). As is well-known, the relationship between the ground coordinate system (X, Y, Z) and the antenna coordinate system (X', Y', Z') is

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \boldsymbol{D}_\varphi \boldsymbol{D}_\omega \boldsymbol{D}_\kappa \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \qquad \cdots\cdots(1)$$

where $\boldsymbol{D}_\varphi$, $\boldsymbol{D}_\omega$ and $\boldsymbol{D}_\kappa$ are the rotation matrices of φ, ω and κ, respectively. By an inverse transformation of equation (1), the next form is obtained:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \boldsymbol{D}_\kappa^t \boldsymbol{D}_\omega^t \boldsymbol{D}_\varphi^t \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix}. \qquad \cdots\cdots(2)$$

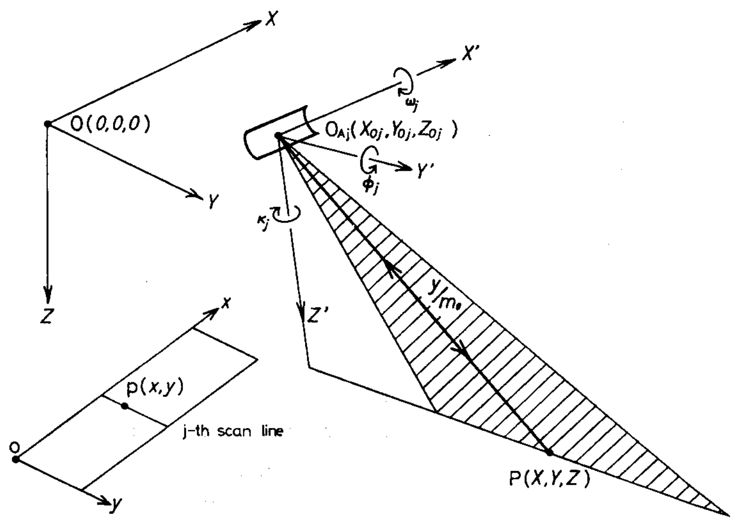

Fig. -3 orientation problem of single SLAR imagery for the 0-squint angle

For simplicity, the following expression is introduced:

$$D_\kappa^t D_\omega^t D_\varphi^t = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}.$$

Then, the equation for the propagation plane of a pulse of microwave energy has the form:[3]

$$d_{11}(X-X_0)+d_{12}(Y-Y_0)+d_{13}(Z-Z_0)=0 \qquad \cdots\cdots(3)$$

because $X'=0$ indicates the propagation plane. It is easily understood that only two of the three coefficients d_{11}, d_{12} and d_{13} in equation (3) are independent with each other. This fact shows that the two rotation parameters of the antenna have a geometrical influence on SLAR imagery. What parameter is not required for the orientation problem of SLAR imagery? The propagation plane $X'=0$ of a pulse is not changed by rotation around the X'-axis (see Fig. -3). From this fact, one can see that ω is the unrequired rotation element of the antenna[4]. Expressing equation (3) in terms of φ and κ, one has

$$\cos\varphi\cos\kappa(X-X_0)+\sin\kappa(Y-Y_0)-\sin\varphi\cos\kappa(Z-Z_0)=0. \qquad \cdots\cdots(4)$$

According to the range projection principle, the y-coordinate of an image point of SLAR imagery is proportional to the slant range from the electrical center $O_A(X_0, Y_0, Z_0)$ of the antenna to the ground point $P(X, Y, Z)$. Therefore, one gets[3]

$$y=m_0\sqrt{(X-X_0)^2+(Y-Y_0)^2+(Z-Z_0)^2} \qquad \cdots\cdots(5)$$

where m_0 is a scale factor and a calibrated value.

Equations (4) and (5) are the fundamental equations for the analytical orientation problem of single SLAR imagery. Rigorously, these two equations are valid at each image point of SLAR imagery independently, which means that the exterior orientation elements (φ, κ, X_0, Y_0, Z_0) of the antenna vary from image point to image point. It is mathematically impossible to determine 5 exterior orientation parameters of the antenna for each image point independently, because there are only two equations for an image point. One assumes, accordingly, that:

(a) the exterior orientation elements of the antenna are constant along a scan line.

Under this assumption, equations (4) and (5) can be set up along each scan line in the form:

$$\cos\varphi_j\cos\kappa_j(X-X_{0j})+\sin\kappa_j(Y-Y_{0j})-\sin\varphi_j\cos\kappa_j(Z-Z_{0j})=0 \qquad \cdots\cdots(6)$$

$$y=m_0\sqrt{(X-X_{0j})^2+(Y-Y_{0j})^2+(Z-Z_{0j})^2}. \qquad \cdots\cdots(7)$$

From equations (6) and (7), one can derive the determination equations for the analytical orientation problem of single SLAR imagery,

$$\left.\begin{aligned} X&=X_{0j}+\frac{\sin\varphi_j\cos\varphi_j\cos^2\kappa_j(Z-Z_{0j})-\sin\kappa_j\sqrt{(1-\sin^2\varphi_j\cos^2\kappa_j)(y/m_0)^2-(Z-Z_{0j})^2}}{1-\sin^2\varphi_j\cos^2\kappa_j}\\ Y&=Y_{0j}+\frac{\sin\varphi_j\sin\kappa_j\cos\kappa_j(Z-Z_{0j})+\cos\varphi_j\cos\kappa_j\sqrt{(1-\sin^2\varphi_j\cos^2\kappa_j)(y/m_0)^2-(Z-Z_{0j})^2}}{1-\sin^2\varphi_j\cos^2\kappa_j}\end{aligned}\right\}\cdots(8)$$

Can the exterior orientation elements of the antenna be calculated with the determination equations (8), if ground control points are given? For a flat ground surface, a desired set of these parameters can not be determined, because one is not able to define only one propagation plane of a pulse of microwave energy, even if more than three ground control points lie on the scan line (see Fig. -4a). On the other hand, when a photographed terrain has relief, the analytical orientation problem of SLAR

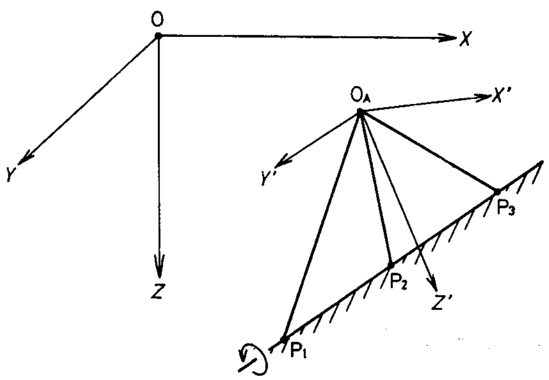

Fig. -4a for a flat terrain

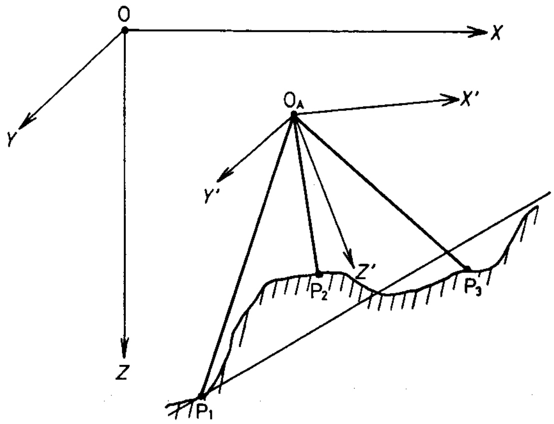

Fig. -4b for a hilly terrain

imagery can be mathematically solved, because only one propagation plane of a pulse of microwave energy through three ground control points can be defined (see Fig. -4b). It is, however, not practical to have over three ground control points on a scan line of SLAR imagery. Therefore, one assumes that:

(b) the changes of exterior orientation elements of the antenna along the flight path can be modeled with some functional form, such as polynomials and Fourier's series.

Using these two assumptions (a) and (b), the orientation of SLAR imagery is able to be performed in the following way. One divides, at first, the exterior orientation elements of the antenna into two parts: the first is the approximation of the elements, and the second is their variation along the flight path. Then, exterior orientation elements of the antenna are described as follows:

$$\xi_{ij}(x)=\xi_{ij0}(x)+\xi_{ij}'(x), \quad (i=1\sim5) \qquad \cdots\cdots(9)$$

where ξ_{ij} represent five orientation parameters, ξ_{ij0} their approximations and ξ_{ij}' the variations along track. As for the approximations, one supposes

$$\left.\begin{array}{ll}\varphi_{j0}=\kappa_{j0}=0, & Y_{0j0}=Z_{0j0}=0,\\ X_{0j0}=x(1-\rho)/m_0 & \end{array}\right\} \qquad \cdots\cdots(10)$$

where x is the image coordinate of SLAR imagery along the flight path and ρ a ratio of the forward overlappings between the scan lines (see Fig. -5). Applying Fourier's series to the along-track variations of the orientation elements, the next form may be written[5]:

$$\xi_{ij}'(x)=\mu_i+\sum_{k=1}^{n}\alpha_{ki}\cos\nu_{ki}x+\sum_{k=1}^{n}\beta_{ki}\sin\nu_{ki}x. \qquad \cdots\cdots(11)$$

Substituting (9), (10) and (11) into (8), the final form of the determination equations is obtained. The unknowns μ_i, α_{ki} and β_{ki} are calculated with the least squares adjustment, if ground control points are given. The obtained μ_i, α_{ki} and β_{ki} determine the changes of the orientation parameters along the flight path.

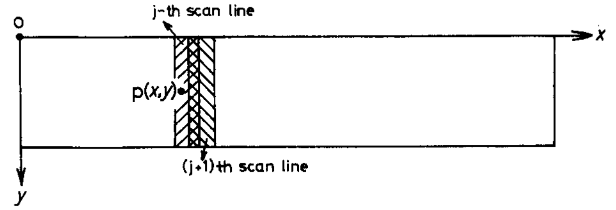

Fig. -5 SLAR imagery

4-1-2 in the case of the general squint angle

If the squint angle is not zero, a pulse of microwave energy travels along a conical surface with its apex at the electrical center of the antenna, and with its central axis on the X'-axis of the antenna coordinate system (X', Y', Z'). Suppose that the squint angle is λ, then the propagation surface of the pulse observed in the ground coordinate system (X, Y, Z) is expressed in the form:

$$-\frac{\{d_{11j}(X-X_{0j})+d_{12j}(Y-Y_{0j})+d_{13j}(Z-Z_{0j})\}^2}{\sin^2\lambda}$$
$$+\frac{\{d_{21j}(X-X_{0j})+d_{22j}(Y-Y_{0j})+d_{23j}(Z-Z_{0j})\}^2}{\cos^2\lambda}$$
$$+\frac{\{d_{31j}(X-X_{0j})+d_{32j}(Y-Y_{0j})+d_{33j}(Z-Z_{0j})\}^2}{\cos^2\lambda}=0 \qquad \cdots\cdots(12)$$

which is valid on the j-th scan line. Also in the case of the general squint angle, ω_j is the geometrically unrequired element. This is easily understood from the fact that the rotation around the X'-axis does not change the attitude of the cone (see Fig. -6). Then, one has the form for the propagation surface of a pulse of microwave energy,

$$-\frac{\{\cos\varphi_j\cos\kappa_j(X-X_{0j})+\sin\kappa_j(Y-Y_{0j})-\sin\varphi_j\cos\kappa_j(Z-Z_{0j})\}^2}{\sin^2\lambda}$$
$$+\frac{\{-\cos\varphi_j\sin\kappa_j(X-X_{0j})+\cos\kappa_j(Y-Y_{0j})+\sin\varphi_j\sin\kappa_j(Z-Z_{0j})\}^2}{\cos^2\lambda}$$
$$+\frac{\{\sin\varphi_j(X-X_{0j})+\cos\varphi_j(Z-Z_{0j})\}^2}{\cos^2\lambda}=0. \qquad \cdots\cdots(13)$$

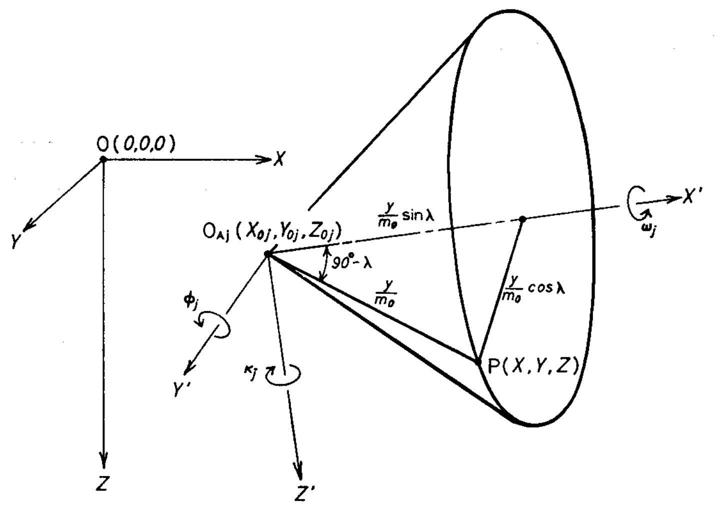

Fig. -6 orientation problem of single SLAR imagery for the general squint angle

Equations (7) and (13) are the fundamental equations for the analytical orientation problem of single SLAR imagery. The analysis is performed in the same way as described in 4-1-1.

4-2. analytical orientation problem of stereo SLAR imageries

It is not possible to recover all three dimensions from single SLAR imagery, since single SLAR imagery is a two-dimensional representation of a generally three-dimensional space. In order to reconstruct a photographed terrain three-dimensionally, stereo SLAR coverage is commonly required.

The most practical method used for the orientation of stereo photographs consists of two main phases: one is relative orientation and the other is absolute orientation. However, this orientation technique can not easily be applied to stereo SLAR imageries, because SLAR imagery is not a central perspective one. We have another orientation method to determine the 12 exterior orientation elements for stereo photographs simultaneously. This technique is easily applicable to the orientation of stereo SLAR imageries. Here, the orientation procedure will be precisely outlined under the suppositions that the squint angle is zero and that the opposite side stereo is taken.

The ground coordinate system (X, Y, Z) is selected as described in 4-1. As for the antenna coordinate system, (X_1', Y_1', Z_1') is for the left SLAR strip, and (X_2', Y_2', Z_2') for the right one. Further, $(\varphi_1, \kappa_1, X_{01}, Y_{01}, Z_{01})$ and $(\varphi_2, \kappa_2, X_{02}, Y_{02}, Z_{02})$ denote the exterior orientation parameters of the antenna in the left- and right SLAR strips, respectively. In addition, image coordinates are expressed by (x_1, y_1) and (x_2, y_2) (see Fig. -7).

As for points required for the analytical orientation problem of stereo SLAR imageries, we have generally the following 4 types: 1) points whose space coordinates (X, Y, Z) are given, 2) points whose planimetric coordinates (X, Y) are known, 3) points whose Z-coordinates are given, 4) points whose space coordinates (X, Y, Z) are not known. We have different types of determination equations for each case. Now, we will derive these determination equations. For this purpose, the fundamental equations for SLAR geometry will, at first, be written down as follows:

$$\cos\varphi_{1j}\cos\kappa_{1j}(X-X_{01j})+\sin\kappa_{1j}(Y-Y_{01j})-\sin\varphi_{1j}\cos\kappa_{1j}(Z-Z_{01j})=0 \quad \cdots\cdots(14a)$$

$$y_1=m_0\sqrt{(X-X_{01j})^2+(Y-Y_{01j})^2+(Z-Z_{01j})^2} \quad \cdots\cdots(14b)$$

for the left SLAR imagery, and

$$\cos\varphi_{2k}\cos\kappa_{2k}(X-X_{02k})+\sin\kappa_{2k}(Y-Y_{02k})-\sin\varphi_{2k}\cos\kappa_{2k}(Z-Z_{02k})=0 \quad \cdots\cdots(14c)$$

$$y_2=m_0\sqrt{(X-X_{02k})^2+(Y-Y_{02k})^2+(Z-Z_{02k})^2} \quad \cdots\cdots(14d)$$

for the right SLAR one. Equations (14a), (14b) are given for the j-th scan line of the left SLAR strip and (14c), (14d) for the k-th scan line of the right one.

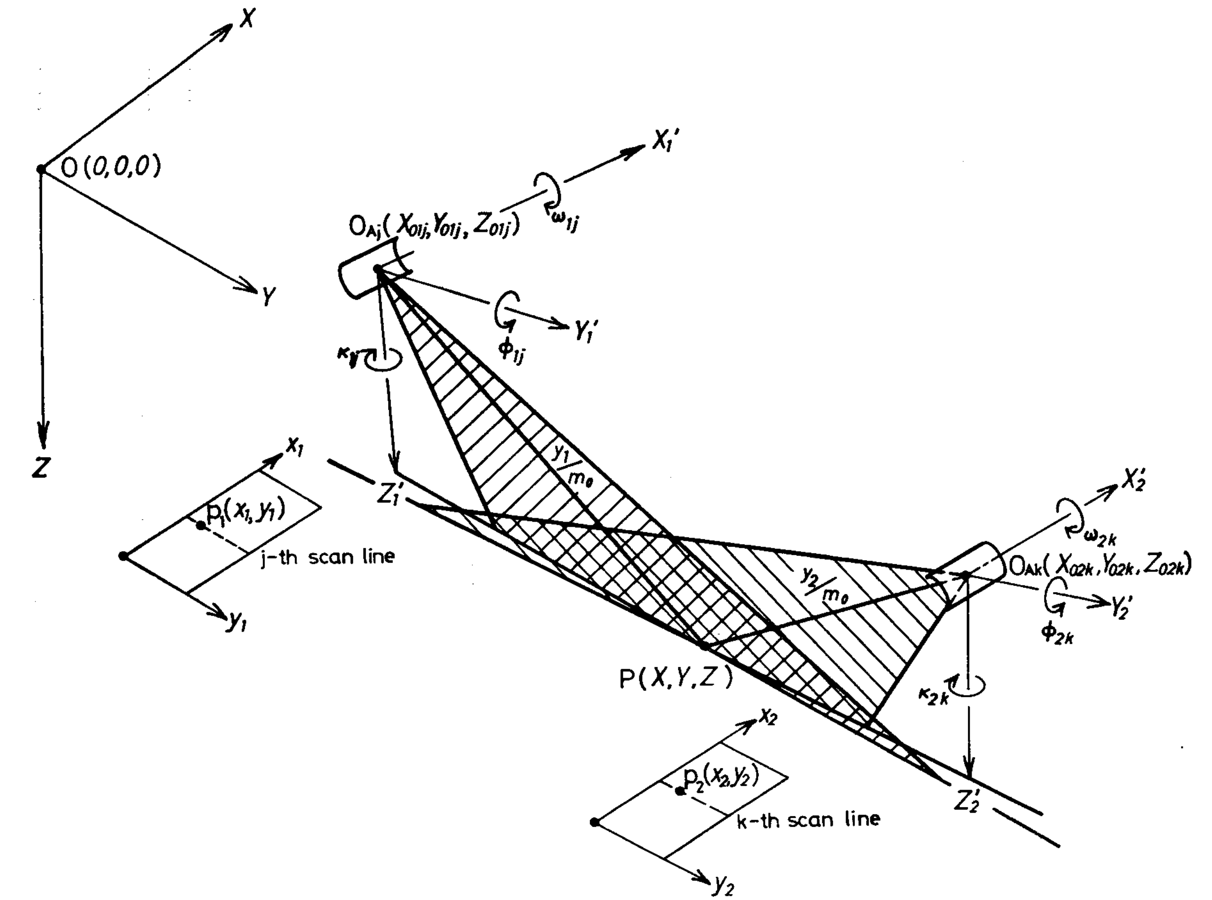

Fig. -7 orientation problem of stereo SLAR imageries

1) for the 1st type of points

If the space coordinates (X, Y, Z) of the points (=ground control points) are given, we can use directly the determination equations (8). They are

$$\left.\begin{aligned}
X&=X_{01j}\\
&+\frac{\sin\varphi_{1j}\cos\varphi_{1j}\cos^2\kappa_{1j}(Z-Z_{01j})-\sin\kappa_{1j}\sqrt{(1-\sin^2\varphi_{1j}\cos^2\kappa_{1j})(y_1/m_0)^2-(Z-Z_{01j})^2}}{1-\sin^2\varphi_{1j}\cos^2\kappa_{1j}}\\
Y&=Y_{01j}\\
&+\frac{\sin\varphi_{1j}\sin\kappa_{1j}\cos\kappa_{1j}(Z-Z_{01j})+\cos\varphi_{1j}\cos\kappa_{1j}\sqrt{(1-\sin^2\varphi_{1j}\cos^2\kappa_{1j})(y_1/m_0)^2-(Z-Z_{01j})^2}}{1-\sin^2\varphi_{1j}\cos^2\kappa_{1j}}
\end{aligned}\right\}\cdots(15)$$

for the left SLAR imagery, and

$$\left.\begin{aligned}
X&=X_{02k}\\
&+\frac{\sin\varphi_{2k}\cos\varphi_{2k}\cos^2\kappa_{2k}(Z-Z_{02k})+\sin\kappa_{2k}\sqrt{(1-\sin^2\varphi_{2k}\cos^2\kappa_{2k})(y_2/m_0)^2-(Z-Z_{02k})^2}}{1-\sin^2\varphi_{2k}\cos^2\kappa_{2k}}\\
Y&=Y_{02k}\\
&+\frac{\sin\varphi_{2k}\sin\kappa_{2k}\cos\kappa_{2k}(Z-Z_{02k})-\cos\varphi_{2k}\cos\kappa_{2k}\sqrt{(1-\sin^2\varphi_{2k}\cos^2\kappa_{2k})(y_2/m_0)^2-(Z-Z_{02k})^2}}{1-\sin^2\varphi_{2k}\cos^2\kappa_{2k}}
\end{aligned}\right\}\cdots(16)$$

for the right one, respectively.

2) for the 2nd type of points

If the planimetric coordinates (X, Y) of the points (=ground control points) are known, we have mathematically three determination equations for one point. These equations can be constructed as follows. Substituting the planimetric coordinates (X, Y) of a given point into (14b), we can calculate the Z-coordinate of this point. This Z-coordinate is named Z_1, since it is obtained from the left sphere expressed by (14b). The given planimetric coordinates (X, Y) and the calculated Z-coordinate must satisfy (14a). From this condition, the first determination equation can be derived in the form:

$$\begin{aligned}\cos\varphi_{1j}\cos\kappa_{1j}(X_{given}-X_{01j})&+\sin\kappa_{1j}(Y_{given}-Y_{01j})\\
&-\sin\varphi_{1j}\cos\kappa_{1j}(Z_{1(calculated)}-Z_{01j})=0.\end{aligned}\qquad\cdots\cdots(17)$$

The second determination equation is constructed in the same manner and has the form:

$$\begin{aligned}\cos\varphi_{2k}\cos\kappa_{2k}(X_{given}-X_{02k})&+\sin\kappa_{2k}(Y_{given}-Y_{02k})\\
&-\sin\varphi_{2k}\cos\kappa_{2k}(Z_{2(calculated)}-Z_{02k})=0.\end{aligned}\qquad\cdots\cdots(18)$$

Further, the third determination equation can be derived from the condition that Z_1 must coincide with Z_2 calculated from the right sphere expressed by (14d). It may be expressed as:

$$Z_1(X_{01j}, Y_{01j}, Z_{01j})=Z_2(X_{02k}, Y_{02k}, Z_{02k}).\qquad\cdots\cdots(19)$$

3) for the 3rd type of points

If only the Z-coordinates of the points (=ground control points) are given, there are mathematically two determination equations for one point. They are derived in the following way. At first, we substitute the Z-coordinate of a given point into (14b) and (14d). Then, the planimetric coordinates (X, Y) of this point can be calculated. The obtained results (X, Y) and the given Z-coordinate must satisfy (14a) and (14c), respectively. Therefore, the determination equations in this case can be expressed in the form:

$$\cos\varphi_{1j}\cos\kappa_{1j}(X_{calculated}-X_{01j})+\sin\kappa_{1j}(Y_{calculated}-Y_{01j}) \\ -\sin\varphi_{1j}\cos\kappa_{1j}(Z_{given}-Z_{01j})=0 \qquad \cdots\cdots(20)$$

$$\cos\varphi_{2k}\cos\kappa_{2k}(X_{calculated}-X_{02k})+\sin\kappa_{2k}(Y_{calculated}-Y_{02k}) \\ -\sin\varphi_{2k}\cos\kappa_{2k}(Z_{given}-Z_{02k})=0. \qquad \cdots\cdots(21)$$

4) for the 4th type of points

Equations (14b) and (14d) express spheres, and these two spheres intersect on a circle in space. This circle, further, intersects the propagation plane of a pulse given by (14a) at two points. The lower point is the desired ground point. Using this fact, we can express the space coordinates (X, Y, Z) of a ground point with the exterior orientation parameters of the left- and right antenna, and these calculated space coordinates must satisfy equation (14c) (=the propagation plane of a pulse of microwave energy in the right SLAR system). The determination equation (only one equation in this case) can be obtained from this condition and has the form:

$$\cos\varphi_{2k}\cos\kappa_{2k}(X_{calculated}-X_{02k})+\sin\kappa_{2k}(Y_{calculated}-Y_{02k}) \\ -\sin\varphi_{2k}\cos\kappa_{2k}(Z_{calculated}-Z_{02k})=0. \qquad \cdots\cdots(22)$$

Equation (22) is effectively used for the simultaneous determination of the exterior orientation elements for stereo SLAR imageries, because it can be applied to arbitrary points recorded in common on the left- and right SLAR imageries. However, we must be careful to use this equation (22), since it plays no roll for the determination of the absolute orientation of stereo SLAR imageries.

Substituting exterior orientation parameters given by (9) into (15), (16), (17), (18), (19), (20), (21), and (22), and setting them up for each type of points, respectively, we have the observation equations for the analytical orientation problem of stereo SLAR imageries. Solving them with respect to the coefficients of Fourier's series by the least squares adjustment, we obtain the exterior orientation parameters of the antenna in the left and right SLAR strips. The calculated Fourier's series indicate the changes in the exterior orientation along the flight path. If the orientation parameters of the antenna are known for the individual scan line, we can calculate $P_1(X_1, Y_1, Z_1)$ from (14a), (14b), and (14d), and $P_2(X_2, Y_2, Z_2)$ from (14b), (14c), and (14d), respectively. The desired ground coordinates (X, Y, Z) of a point are

$$X=(X_1+X_2)/2, \qquad Y=(Y_1+Y_2)/2, \qquad Z=(Z_1+Z_2)/2. \qquad \cdots\cdots(23)$$

5. Approximate Restitution of SLAR Imagery

A simple restitution of single SLAR imagery is possible, if the squint angle is zero and exterior orientation elements of the antenna vary only slightly. Suppose, at first, that the changes in the exterior orientation can be modeled with the following polynomials[6]:

$$\left.\begin{aligned}\Delta\varphi_j &= a_{10}+a_{11}x+a_{12}x^2\\ \Delta\kappa_j &= a_{30}+a_{31}x+a_{32}x^2\\ \Delta x_{0j} &= a_{40}+a_{41}x+a_{42}x^2\\ \Delta y_{0j} &= a_{50}+a_{51}x+a_{52}x^2\\ \Delta z_{0j} &= a_{60}+a_{61}x+a_{62}x^2\end{aligned}\right\} \qquad \cdots\cdots(24)$$

where $\Delta(\ \ )$ denote small quantities of $(\ \ )$. Then, one linearizes equations (8) and expresses displacements in the X- and Y direction in a simple form. By substituting (24) into the displacements, as will soon be seen, one gets simple error equations for the restitution problem of single SLAR imagery. This chapter treats the restitution problem in the next two cases: one is a case where a photographed ground surface is flat, and the other is a case where a terrain has relief.

5-1. the case of a flat terrain

Diaplacements along- and across-track may be expressed in the following form:

$$\left.\begin{aligned}\Delta X &= H\Delta\varphi_j-\sqrt{(y/m_0)^2-H^2}\ \Delta\kappa_j+\Delta X_{0j}\\ \Delta Y &= \Delta Y_{0j}+\frac{H}{\sqrt{(y/m_0)^2-H^2}}\Delta Z_{0j}\end{aligned}\right\} \qquad \cdots\cdots(25)$$

where H is the flight height. In terms of ΔX_{0j}, ΔY_{0j} and ΔZ_{0j} in an image scale, equations (25) are

$$\left.\begin{aligned}\Delta X &= H\Delta\varphi_j-\sqrt{(y/m_0)^2-H^2}\ \Delta\kappa_j+\frac{1}{m_0}\Delta x_{0j}\\ \Delta Y &= \frac{1}{m_0}\Delta y_{0j}+\frac{H}{\sqrt{(y/m_0)^2-H^2}}\ \frac{1}{m_0}\Delta z_{0j}\end{aligned}\right\}. \qquad \cdots\cdots(26)$$

By substituting (24) into (26) and combining terms with equivalent effects, the final form is obtained:

$$\left.\begin{aligned}\Delta X &= A_0+A_1x+A_2\sqrt{y^2-h^2}+A_3x^2+A_4x\sqrt{y^2-h^2}+A_5x^2\sqrt{y^2-h^2}\\ \Delta Y &= B_0+B_1x+B_2\frac{h}{\sqrt{y^2-h^2}}+B_3x^2+B_4x\frac{h}{\sqrt{y^2-h^2}}+B_5x^2\frac{h}{\sqrt{y^2-h^2}}\end{aligned}\right\} \qquad \cdots\cdots(27)$$

where $h=m_0H$.

If ground control points are given, the coefficients of final error equations (27) are determined with the least squares adjustment. Then, one can calculate the displacements of all points photographed and carry out the restitution of SLAR imagery. The changes in the exterior orientation parameters of the antenna, however, can not be obtained from the determined coefficients in (27), because $\Delta\varphi_j$ and Δx_{0j} in (26) have equivalent effects.

5-2. the case of a hilly terrain

If a photographed terrain is hilly, relief displacements occur also in the radar records. These, however, can be corrected as in the analysis of MSS imagery, if the elevations of all photographed ground points are known. Consider the restitution problem of single SLAR imagery for a hilly terrain. Displacements in X and Y corresponding to (26) have the form:

$$\left.\begin{aligned}\Delta X&=(H-Z_H)\Delta\varphi_j-\sqrt{(y/m_0)^2-(H-Z_H)^2}\ \Delta\kappa_j+\frac{1}{m_0}\Delta x_{0j}\\ \Delta Y&=\frac{1}{m_0}\Delta y_{0j}+\frac{H-Z_H}{\sqrt{(y/m_0)^2-(H-Z_H)^2}}\ \frac{1}{m_0}\Delta z_{0j}\end{aligned}\right\}\cdots\cdots(28)$$

where Z_H is the elevation difference of a ground point relative to the average ground height. Substituting (24) into (28), one obtains

$$\left.\begin{aligned}\Delta X=&A_0+A_1x+A_2\sqrt{y^2-(h-z_h)^2}+A_3(h-z_h)+A_4x\sqrt{y^2-(h-z_h)^2}+A_5x^2\\ &+A_6x(h-z_h)+A_7x^2\sqrt{y^2-(h-z_h)^2}+A_8x^2(h-z_h)\\ \Delta Y=&B_0+B_1x+B_2\frac{h-z_h}{\sqrt{y^2-(h-z_h)^2}}+B_3x^2+B_4x\frac{h-z_h}{\sqrt{y^2-(h-z_h)^2}}\\ &+B_5x^2\frac{h-z_h}{\sqrt{y^2-(h-z_h)^2}}\end{aligned}\right\}\cdots\cdots(29)$$

The restitution is carried out in the same way as that in 5-1. Unlike a flat terrain, in this case the changes in the exterior orientation can be determined, because the elevation differences play an effective roll.

6. Conclusion

In the discussion given here, only the theoretical aspects of the orientation problem of SLAR imagery have been considered. For single SLAR configuration, this report proposed an analytical orientation method, which can be easily derived from the geometrical basis already studied. It is, however, quite complicated to consider an orientation for stereo SLAR configuration. The author developed an orientation method for stereo SLAR imageries based on the simultaneous determanation of the exterior orientation parameters of stereo aerial photographs. In both cases, orientation elements of the antenna are modeled with some functional form.

Various difficulties may further occur in the practical analysis of SLAR imagery. To clarify these difficulties and to develop practical effective methods is our next program.

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