# Optimum Feedback Control of AC-DC Parallel Transmission System

By

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**Abstract**

This paper describes a method of damping a system oscillation on an ac-dc parallel transmission system by applying the optimum feedback law. The optimum feedback control can be obtained by solving the Riccati matrix equation for the linearized system equation. The reference value of the constant current control system of the dc system is chosen as the control variable. The method is applied to a generator feeding an infinite-bus through an ac-dc parallel transmission system, and its effectiveness is ascertained.

## 1. Introduction

The application of a dc power transmission has been considered as a method for solving the stability problem of long distance, large capacity electric power transmissions. However, it has demerits whereby the construction of a dc system is greatly restricted because the multi-terminal dc transmission technique and a dc circuit breaker are still under development. Therefore, an ac-dc parallel transmission system is thought advantageous for long distance, large capacity transmission. It has been investigated and found that the above demerit of dc power transmission can be supplemented by ac power transmission. It has been reported that the stability of the ac system in an ac-dc parallel transmission system is considerably improved by controlling the dc transmitted power. As an ac-dc parallel transmission system is described by a set of nonlinear differential equations, a nonlinear time optimal control is most effective for damping system oscillations. However, it is almost impossible to realize such a control, because it depends upon the kind, the location and the clearance time of the fault. A linear time optimal control for the linearized system with the dc transmitted power as a control variable is also investigated. As the optimal switching curve for this method is obtained in a second-order space, the effects of an AVR and a governor of the generator and

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the dynamic performance of the dc system including the phase angle control cannot be considered.

In this paper, we use a linear optimum control with a performance index of quadratic form. Using this method we can represent the system in detail as required. Furthermore, as the optimum control law is obtained as a feedback form of the state variables, once the feedback coefficients are calculated, effects of the control can be expected for any kind of faults. We investigate in this paper the effects of a linear optimum feedback control for an ac-dc parallel system feeding an infinite-bus, with the reference value of the constant current control of the dc system as a control variable.

## 2. Power Flow Calculation for AC-DC Interconnected Systems

Before the transient stability analysis is made for an ac-dc interconnected power system, the initial value of each state variable must be known by performing the power flow calculation of the ac-dc system. AC-DC connecting points can be dealt as $P$-$Q$ specified nodes by taking note of the rectifying effective power $P$ and the required reactive power $Q$. Hence, it is necessary that the values of $P$ and $Q$ can be calculated from the specified value for the dc system and the ac voltage at the rectifier node.

### 2.1 Equations for DC System

The steady-state equations for the dc system shown in Fig. 1 are considered. The rectifier and the inverter are assumed to be the type of three-phase bridge connection as shown in Fig. 2 and Fig. 3 respectively.

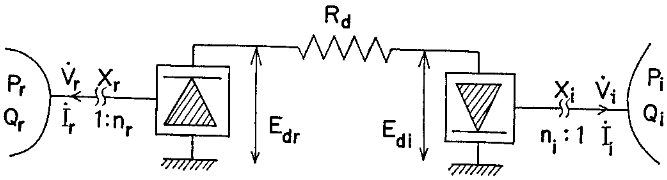
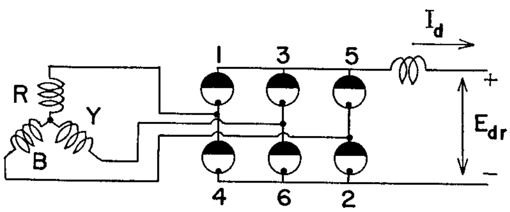
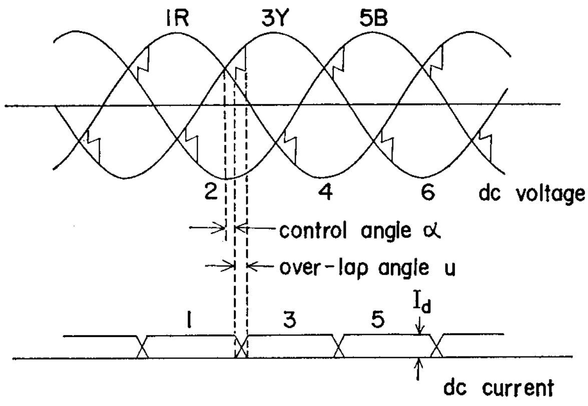



Fig. 1. DC power transmission system.

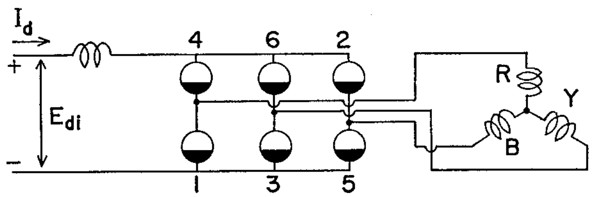



(a)

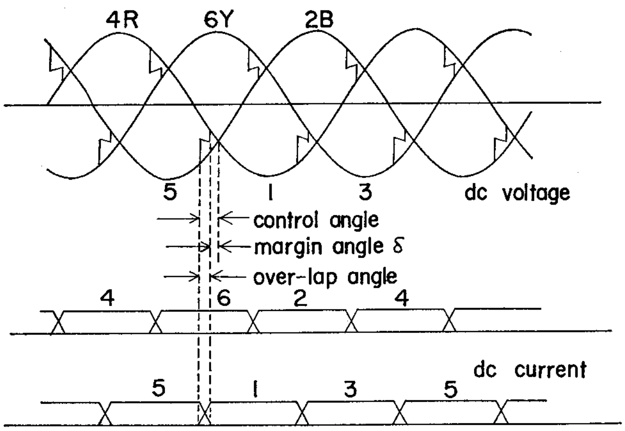



(b)

Fig. 2. Circuit and commutation of rectifier.
(a) three-phase bridge circuit
(b) commutating action




(a)




(b)

Fig. 3. Circuit and commutation of inverter.
(a) three-phase bridge circuit
(b) commutating action

### (a) Equations of rectifier

DC voltage $E_{dr}$ Neglecting the voltage drop inside the rectifier, the dc voltage $E_{dr}$ can be represented as a function of the ac voltage at the ac-dc connecting point $|\dot{V}_r|$, the control angle of the rectifier $\alpha$, dc current $I_d$ and the commutation reactance $X_r$.

$$E_{dr} = \frac{3\sqrt{2}}{\pi} n_r |\dot{V}_r| \cos\alpha - \frac{3X_r}{\pi} I_d \tag{2.1}$$

where $n_r$ denotes the tap-ratio of the main transformer. $n_r$ can be changed in order to adjust the secondary voltage.

AC current If we assume that the harmonic currents are eliminated by an ac filter, the absolute value of ac current $|\dot{I}_r|$ can be expressed as follows, using the dc current $I_d$.

$$|\dot{I}_r| = \frac{\sqrt{6}}{\pi} n_r I_d \tag{2.2}$$

Power factor angle $\phi_r$ The lagging angle $\phi_r$ of ac current $\dot{I}_r$ with regard to ac voltage $\dot{V}_r$ can be calculated from the control angle $\alpha$ and the over-lap angle $u_r$. As the over-lap angle is represented as follows, using dc current $I_d$ and the commutation reactance $X_r$

$$\cos\alpha - \cos(\alpha + u_r) = \frac{\sqrt{2} X_r}{n_r |\dot{V}_r|} I_d \tag{2.3}$$

the power factor angle $\phi_r$ is expressed as follows

$$\begin{aligned} \cos\phi_r &= \frac{1}{2}[\cos\alpha + \cos(\alpha + u_r)] \\ &= \cos\alpha - \frac{X_r}{\sqrt{2}\, n_r |\dot{V}_r|} I_d \\ &= \frac{E_{dr}}{\dfrac{3\sqrt{2}}{\pi} n_r |\dot{V}_r|} \end{aligned} \tag{2.4}$$

### (b) Equations of inverter

The equations of the inverter can be obtained by changing the control angle $\alpha$ into the margin angle $\delta$ in eqs.(2.1)–(2.4). The dc voltage of the inverter $E_{di}$ expressed by the control angle of the inverter $\beta$ is as follows:

$$E_{di} = \frac{3\sqrt{2}}{\pi} n_i |\dot{V}_i| \cos\beta + \frac{3X_i}{\pi} I_d \tag{2.5}$$

where $\beta=u_i+\delta$, $n_i$, $V_i$ and $X_i$ correspond to $n_r$, $V_r$ and $X_r$ of the rectifier. (The subscript $i$ denotes the quantities for the inverter.)

### (c) Equations of dc line

Considering only the voltage drop due to the dc line resistance at the steady-state condition, the dc voltages $E_{dr}$, $E_{di}$ and the dc current $I_d$ are related by the following equation.

$$E_{dr}-E_{di} = R_d I_d \tag{2.6}$$

where $R_d$ denotes the total line resistance.

### (d) Quantities which dc system receives from ac system

Using eq.(2.2), the effective power $P_r$ and the reactive power $Q_r$ which flow from the ac system to the dc system at the rectifier are represented by the following equations.

$$P_r = \sqrt{3}\,|\dot{V}_r|\frac{\sqrt{6}}{\pi}n_r I_d \cos\phi_r \tag{2.7}$$

$$Q_r = \sqrt{3}\,|\dot{V}_r|\frac{\sqrt{6}}{\pi}n_r I_d \sin\phi_r \tag{2.8}$$

Analogous equations are obtained for the inverter, the only difference being that the sign of $P_i$ is reversed. Furthermore, the dc system is regarded as the equivalent driving point admittances $Y_{rr}$ and $Y_{ii}$ at the converter points.

$$\dot{Y}_{rr} = \sqrt{3}\frac{\sqrt{6}}{\pi}n_r I_d(\cos\phi_r - j\sin\phi_r)/|\dot{V}_r| \tag{2.9}$$

$$\dot{Y}_{ii} = \sqrt{3}\frac{\sqrt{6}}{\pi}n_i I_d(-\cos\phi_i - j\sin\phi_i)/|\dot{V}_i| \tag{2.10}$$

## 2.2 Calculating Method of DC System

When the rectifier is controlled by a constant power control system and the inverter by a constant margin angle control, the procedure to calculate $P_r$-$Q_r$ and $P_i$-$Q_i$ from the ac voltages $\dot{V}_r$ and $\dot{V}_i$ is as follows.

(a) Obtaining $I_d$ from the power setting $P_{dro}$ and the margin angle $\delta_0$. The dc voltage at the inverter $E_{di}$ is

$$E_{di} = \frac{3\sqrt{2}}{\pi}n_i|\dot{V}_i|\cos\delta_0 - \frac{3X_i}{\pi}I_d \tag{2.11}$$

From eq.(2.11) and (2.6),

$$P_{dro} = E_{dr}I_d$$

$$=\left(\frac{3\sqrt{2}}{\pi}n_i|\dot{V}_i|\cos\delta_0-\frac{3X_i}{\pi}I_d+R_dI_d\right)I_d \tag{2.12}$$

Eq. (2.12) is a quadratic equation for $I_d$, and $I_d$ is determined as the more reasonable solution of the two (usually the smaller one).

(b) Substituting the obtained $I_d$ into eq. (2.11), we can get $E_{di}$. $E_{dr}$ is calculated from eq. (2.6). From the values of $E_{dr}$ and $E_{di}$ with eq. (2.4), the power factors $\cos\phi_r$ and $\cos\phi_i$ are obtained.

(c) $P$ and $Q$ are calculated from eq. (2.7) and (2.8), and they become the specified value for the ac node.

The above calculations are performed every time when the values of $\dot{V}_r$ and $\dot{V}_i$ change. Therefore, the following process is necessary. First, (a)-(c) are performed using the initially assumed value $\dot{V}_{r0}$ and $\dot{V}_{i0}$, and the ac flow calculation program is executed once regarding the ac-dc connecting nodes as $P$-$Q$ specified nodes. (a)-(c) are performed again using the modified values of $\dot{V}_{r1}$ and $\dot{V}_{i1}$, and the ac flow calculation is performed. The above is repeated until convergence is obtained. Fig. 4 shows the flow chart of the above process. During the calculation of (a)-(c) above, the tap ratios $n_r$ and $n_i$ must be adjusted so that the secondary voltage of the inverter transformer equals unity, and that the control angle $\alpha$ takes a value between 15° and 20°. The flow diagram for the dc system

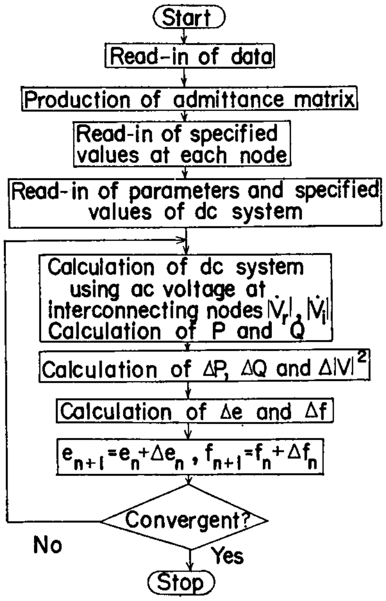



Fig. 4. Flow diagram for power flow calculation of ac-dc interconnected system.

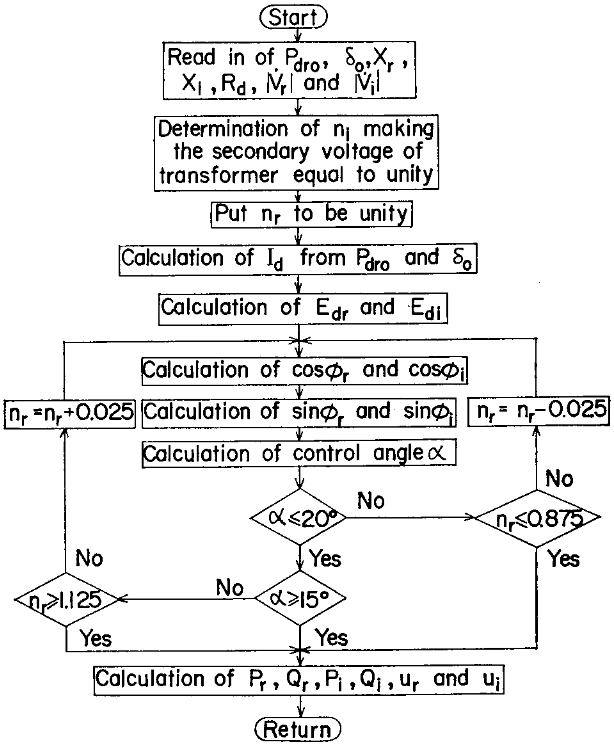



Fig. 5. Flow diagram for dc system calculation.

calculation is shown in Fig. 5, where the step of the tap changer is assumed to be 0.025 p.u. and the upper and lower limit of the tap is 1.125 p.u. and 0.875 p.u., respectively.

## 3. Transient Performance Calculation of AC-DC Interconnected System

The transient performance of an ac-dc interconnected system can be calculated in the same way as the load flow calculation. This is done by calculating the dynamic performance of the dc system using the ac voltages at the ac-dc connecting points, and representing the dc system by an equivalent driving-point admittance at the converter points. The dynamic performance of the dc system is calculated from the equations describing the steady-state characteristics.

### 3.1 Modification of the Admittance Matrix

We represent the quantities for load nodes, generator nodes and ac-dc connecting nodes by using the subscripts $L$, $G$ and $D$, respectively. The admittance matrix is expressed as follows.

$$\begin{pmatrix} \boldsymbol{i}_L \\ \boldsymbol{i}_G \\ \boldsymbol{i}_D \end{pmatrix} = \begin{pmatrix} A & D & E \\ D^T & B & F \\ E^T & F^T & C \end{pmatrix} \begin{pmatrix} \boldsymbol{v}_L \\ \boldsymbol{v}_G \\ \boldsymbol{v}_D \end{pmatrix} \tag{3.1}$$

As the loads are represented by the constant impedances,

$$\boldsymbol{i}_L = -Y_L \boldsymbol{v}_L \tag{3.2}$$

$$\boldsymbol{i}_D = -Y_D \boldsymbol{v}_D \tag{3.3}$$

Substituting eq. (3.2) and (3.3) into eq. (3.1), and eliminating $\boldsymbol{v}_L$, we get

$$\boldsymbol{i}_G = [-D^T(A+Y_L)^{-1}D+B]\boldsymbol{v}_G+[-D^T(A+Y_L)^{-1}E+F]\boldsymbol{v}_D \tag{3.4}$$

$$\boldsymbol{v}_D = [-E^T(A+Y_L)^{-1}E+C+Y_D]^{-1}\cdot[-E^T(A+Y_L)^{-1}D+F^T]\boldsymbol{v}_G \tag{3.5}$$

Therefore, the relation $\boldsymbol{i}_G=Y\boldsymbol{v}_G$ results in obtaining $Y_D$ from the calculation of the dc system dynamic performance and by substituting eq. (3.5) into eq. (3.4). Only $Y_D$ is changed during the transient calculation, and all other matrices are unchanged.

### 3.2 Dynamic Performance of DC System

The ac voltage at the connecting points $\boldsymbol{v}_D$ is changed when a fault occurs or the internal phase angle of the generator changes. $Y_D$ is recalculated from the new value of $\boldsymbol{v}_D$, considering the dynamic performance of the dc system containing control systems.

#### (a) Equations for dc transmission line

Assuming that the dc transmission line is composed of an overhead line - cable - overhead line and its equivalent circuit is represented as shown in Fig. 6, the dynamic performance of the dc system is described by the following differential equations.

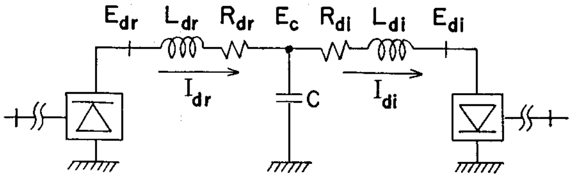



Fig. 6. Model of dc system.

$$L_{dr}\frac{dI_{dr}}{dt}+R_{dr}I_{dr} = E_{dr}-E_c \tag{3.6}$$

$$L_{di}\frac{dI_{di}}{dt}+R_{di}I_{di} = E_c-E_{di} \tag{3.7}$$

where $L_d$ contains the reactance of the dc reactor.

### (b) Control systems for converter

The basic control systems for ac-dc converters are a constant current control, a constant voltage control and a constant margin angle control. As we are interested in only a short time transient performance, it is assumed that the constant current and the constant voltage control can be represented by a time lag of the first order, and that the constant margin angle control system is an open-loop type. Hence, denoting the control angle of the rectifier and inverter by $\alpha_1$ and $\alpha_2$ respectively, the characteristic of each control system is represented as shown below.

Constant current control system

$$\begin{aligned} \alpha_1 &= \frac{K_c}{1+sT_c}(I_{dr}-I_{d0}) \\ \alpha_2 &= \frac{K_c}{1+sT_c}(I_{di}-I_{d0}+I_m) \end{aligned} \tag{3.8}$$

where $T_c$ and $K_c$ denote the time constant and the gain of the control system. $I_{d0}$ is the specified value of the rectifier current, and $I_m$ is the margin of current.

Constant voltage control system

$$\begin{aligned} \alpha_1 &= \frac{K_v}{1+sT_v}(E_{dr}-E_{d0}) \\ \alpha_2 &= \frac{K_v}{1+sT_v}(E_{di}-E_{d0}) \end{aligned} \tag{3.9}$$

where $E_{d0}$ is the specified value of the voltage, and $T_v$ and $K_v$ represent the time constant and the gain of the control system.

Constant margin angle control system Denoting the margin angle by $\delta_0$, this control system is described by the following equations.

$$\begin{aligned} \alpha_1 &= \pi-\cos^{-1}\left(\cos\delta_0-\frac{\sqrt{2}X_r}{n_r|\dot{V}_r|}I_{dr}\right) \\ \alpha_2 &= \pi-\cos^{-1}\left(\cos\delta_0-\frac{\sqrt{2}X_i}{n_i|\dot{V}_i|}I_{di}\right) \end{aligned} \tag{3.10}$$

$$\alpha_{\min}\leqslant\alpha_1,\ \alpha_2\leqslant\alpha_{\max} \tag{3.11}$$

Inequality (3.11) represents the range of the control angle determined by the capacity of the phase control system.

The dynamic performance of the dc system can be calculated by the repetition of solving the differential equations (3.7)-(3.9) for one time step, and obtaining the dc voltages at the converters from the new control angles determined by eqs. (3.8)-(3.11). $Y_D$, the information transmitted to the ac system, is obtained from eq. (2.9) and (2.10) using the values of the control angles and the dc current after the dc system is solved for one step of the ac system. (The time step length for a dc system is usually one fifth or one tenth of that for an ac system.)

**(c) Process of transient performance calculation of ac-dc interconnected system**

The process of transient performance calculation is summarized as follows.

(i) Calculate the generator voltage behind the transient reactance and the initial phase angle from the initial state of the ac system.

(ii) Calculate $Y_D$ from the initial state of the dc system and obtain the admittance matrix corresponding to the generator nodes only, using eq. (3.4) and (3.5).

(iii) Calculate the transient performance of the ac system for one time step using the admittance matrix given in (ii).

(iv) Obtain the generator voltage $\boldsymbol{v}_G$ from the new value of the generator phase angle given in (iii), and further, obtain the new value of $\boldsymbol{v}_D$ using eq. (3.5).

(v) Calculate the dynamic performance of the dc system for the same time interval as for the ac system using the above $\boldsymbol{v}_D$ through the process given in (b), and obtain the new value of $Y_D$.

(vi) Modify the admittance matrix corresponding to generator nodes only, using eqs. (3.4) and (3.5) with the above $Y_D$, and go to (iii).

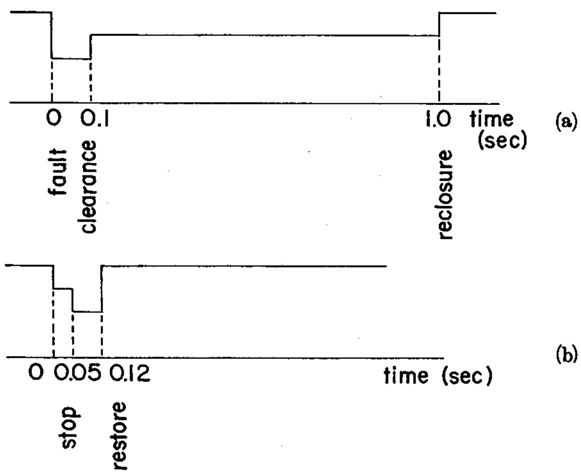



Fig. 7. Time schedule of system protection.
(a) clearance of faults on ac line
(b) protection of dc system in case of voltage drop below 0.5 p.u. at the interconnecting point

### (d) Protecting method of dc system

When the voltage at the ac-dc connecting point drops to a great degree, a commutation failure occurs and the dc system suffers a short-circuit fault. In such a case an overcurrent flows through the converter. The control angle is rapidly increased to 90° and the dc voltage is decreased in order to prevent the overcurrent, when the ac voltage comes below some threshold value. The dc system is stopped when the dc current becomes zero. After the ac voltage is recovered, the dc system is started again. The time schedule of the above protective action is shown in Fig. 7 (b).

## 4. Application of Linear Optimum Control to AC-DC Interconnected Transmission System

We apply the linear optimum control theory to the ac-dc interconnected system shown in Fig. 8, and try to improve the stability by controlling the dc transmitted power. In this section, the modeling of the system and the linear optimum control theory are described.

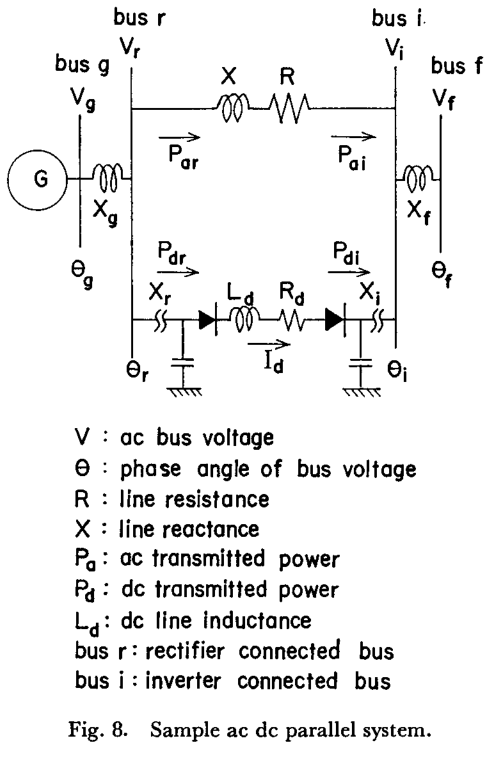



Fig. 8. Sample ac dc parallel system.

### 4.1 Modeling of the System

#### (a) Equation of the motion of the generator

$$M\frac{d^2\theta_g}{dt^2} = P_m-(P_{ar}+P_{dr}) \tag{4.1}$$

where $M$=inertia constant

$P_m$=power input to the generator

$P_{ar}, P_{dr}$=transmitted ac and dc power,

and the damping is neglected.

#### (b) Equation for dc system

If the rectifier is controlled by a constant current control system the response of which is represented by a time delay of the first order, and if the inverter is controlled by an open-loop constant margin angle control system, then the dc system can be described by the following equations. (By means of the open-loop constant margin angle control, the control angle $\beta$ is controlled so that the margin angle $\delta$ might be kept fixed all the time. Hence, the dc voltage at the inverter side $E_{di}$ is independent of the control angle $\beta$.) Here we assume that the dc transmission line does not include a cable, and that the capacitance to the ground is negligible.

$$\begin{aligned}
&L_d\frac{dI_d}{dt}+R_dI_d = E_{dr}-E_{di}\\
&E_{dr} = \frac{3\sqrt{2}}{\pi}n_r|\dot{V}_r|\cos\alpha-\frac{3X_r}{\pi}I_d\\
&E_{di} = \frac{3\sqrt{2}}{\pi}n_i|\dot{V}_i|\cos\delta-\frac{3X_i}{\pi}I_d\\
&T_c\frac{d\alpha}{dt}+\alpha = K_c(I_d-I_{ds})
\end{aligned} \tag{4.2}$$

where $I_{ds}$ denotes the specified value of the dc current.

#### (c) State variables and state equations

If we choose the state variables and the control variable as follows,

$$x_1 = \theta_g,\ x_2 = \frac{d\theta_g}{dt} = \dot{\theta}_g,\ x_3 = I_d,\ x_4 = \alpha,\ u = I_{ds} \tag{4.3}$$

then the state equations are established as follows from eqs. (4.1) and (4.2)

$$\begin{aligned}
&\dot{x}_1 = x_2\\
&\dot{x}_2 = \frac{1}{M}\left[P_m-P_{ar}-\left(\frac{3\sqrt{2}}{\pi}n_r|\dot{V}_r|\cos x_4-\frac{3X_r}{\pi}x_3\right)\cdot x_3\right]
\end{aligned} \tag{4.4}$$

$$\dot{x}_3=\frac{1}{L_d}\left[\left(-R_d-\frac{3X_r}{\pi}+\frac{3X_i}{\pi}\right)\cdot x_3+\frac{3\sqrt{2}}{\pi}n_r|\dot{V}_r|\cos x_4-\frac{3\sqrt{2}}{\pi}n_i|\dot{V}_i|\cos\delta\right]$$

$$\dot{x}_4=\frac{1}{T_c}(-x_4+K_cx_3-K_cu)$$

$|\dot{V}_r|$ and $|\dot{V}_i|$ are the functions of the state variables as clearly seen from eq. (3.5). Besides, $P_{ar}$ can also be expressed in terms of the state variables using the following equation.

$$\begin{aligned}\frac{V_gV_r}{X_g}\sin(\theta_g-\theta_r)&=P_{ar}+P_{dr}\\ \frac{V_iV_f}{X_f}\sin(\theta_i-\theta_f)&=P_{ai}+P_{di}\end{aligned}\tag{4.5}$$

Therefore, eq.(4.4) becomes the equation which includes the state variables and the control variable only.

### (d) Linearization of the state equations

Linearizing eqs. (4.4) and (4.5) around the equilibrium point, we get the following equations, where $\Delta$ denotes a small deviation from the value at the equilibrium point.

$$\begin{aligned}\Delta\dot{x}_1&=\Delta x_2\\ \Delta\dot{x}_2&=\frac{1}{M}\Big[\{-GV_rV_i\sin(\theta_{r0}-\theta_{i0})+BV_rV_i\cos(\theta_{r0}-\theta_{i0})\}(\Delta\theta_r-\Delta\theta_i)\\ &-\left(\frac{3\sqrt{2}}{\pi}n_rV_r\cos x_{40}-\frac{6X_r}{\pi}x_{30}\right)\Delta x_3+\frac{3\sqrt{2}}{\pi}n_rV_rx_{30}\sin x_{40}\Delta x_4\Big]\\ \Delta\dot{x}_3&=\frac{1}{L_d}\left[\left(-R_d-\frac{3X_r}{\pi}+\frac{3X_i}{\pi}\right)\Delta x_3-\frac{3\sqrt{2}}{\pi}n_rV_r\sin x_{40}\Delta x_4\right]\\ \Delta\dot{x}_4&=\frac{1}{T_c}(K_c\Delta x_3-\Delta x_4-K_c\Delta u)\end{aligned}\tag{4.6}$$

$$\begin{aligned}a_{11}\Delta\theta_r+a_{12}\Delta\theta_i&=b_{11}\Delta x_1+b_{12}\Delta x_3+b_{13}\Delta x_4\\ a_{21}\Delta\theta_r+a_{22}\Delta\theta_i&=b_{21}\Delta x_1+b_{22}\Delta x_3+b_{23}\Delta x_4\end{aligned}\tag{4.7}$$

where

$$\begin{aligned}a_{11}&=GV_rV_i\sin(\theta_{r0}-\theta_{i0})-BV_rV_i\cos(\theta_{r0}-\theta_{i0})+\frac{V_gV_r}{X_g}\cos(x_{10}-\theta_{r0})\\ a_{12}&=-GV_rV_i\sin(\theta_{r0}-\theta_{i0})+BV_rV_i\cos(\theta_{r0}-\theta_{i0})\\ a_{21}&=-GV_rV_i\sin(\theta_{r0}-\theta_{i0})-BV_rV_i\cos(\theta_{r0}-\theta_{i0})\\ a_{22}&=GV_rV_i\sin(\theta_{r0}-\theta_{i0})+BV_rV_i\cos(\theta_{r0}-\theta_{i0})-\frac{V_iV_f}{X_f}\cos(\theta_{i0}-\theta_{f0})\end{aligned}$$

$$
\begin{aligned}
b_{11} &= \frac{V_g V_r}{X_g}\cos(x_{10}-\theta_{r0}) \\
b_{12} &= -\left(\frac{3\sqrt{2}}{\pi} n_r V_r \cos x_{40} - \frac{6X_r}{\pi} x_{30}\right) \\
b_{13} &= \frac{3\sqrt{2}}{\pi} n_r V_r x_{30} \sin x_{40} \\
b_{22} &= -\left(\frac{3\sqrt{2}}{\pi} n_i V_i \cos\delta - \frac{6X_i}{\pi} x_{30}\right) \\
b_{21} &= b_{23} = 0
\end{aligned}
\tag{4.8}
$$

Subscript $o$ denotes a value at the equilibrium point. Choosing the phase angle of the infinite-bus $f$ as the reference angle, $\theta_{f0}$ becomes zero. Representing $\Delta\theta_r$ and $\Delta\theta_i$ in terms of $\Delta x_1$, $\Delta x_3$ and $\Delta x_4$ from eq. (4.7) and substituting them into eq. (4.6), we get the following state equation:

$$
\begin{pmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \\ \Delta\dot{x}_3 \\ \Delta\dot{x}_4 \end{pmatrix}
=
\begin{pmatrix}
0 & 1 & 0 & 0 \\
c_{21} & 0 & c_{23} & c_{24} \\
0 & 0 & \frac{1}{L_d}\left(-R_d-\frac{3X_r}{\pi}+\frac{3X_i}{\pi}\right) & -\frac{3\sqrt{2}}{\pi L_d} n_r V_r \sin x_{40} \\
0 & 0 & \frac{K_c}{T_c} & -\frac{1}{T_c}
\end{pmatrix}
\begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{pmatrix}
+
\begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{K_c}{T_c} \end{pmatrix} \Delta u
\tag{4.9}
$$

where

$$
\begin{aligned}
C_{21} &= \frac{1}{M}[-GV_r V_i \sin(\theta_{r0}-\theta_{i0}) + BV_r V_i \cos(\theta_{r0}-\theta_{i0})]\frac{(a_{21}+a_{22})\cdot b_{11}}{a_{11}a_{22}-a_{12}a_{21}} \\
C_{23} &= \frac{1}{M}\Big[\{-GV_r V_i \sin(\theta_{r0}-\theta_{i0}) + BV_r V_i \cos(\theta_{r0}-\theta_{i0})\} \\
&\quad \times \frac{(a_{21}+a_{22})b_{12}-(a_{11}+a_{12})b_{22}}{a_{11}a_{22}-a_{12}a_{21}} - \left(\frac{3\sqrt{2}}{\pi} n_r V_r \cos x_{40} - \frac{6X_r}{\pi} x_{30}\right)\Big] \\
C_{24} &= \frac{1}{M}\Big[\{-GV_r V_i \sin(\theta_{r0}-\theta_{i0}) + BV_r V_i \cos(\theta_{r0}-\theta_{i0})\} \\
&\quad \times \frac{(a_{21}+a_{22})\cdot b_{13}}{a_{11}a_{22}-a_{12}a_{21}} + \frac{3\sqrt{2}}{\pi} n_r V_r x_{30} \sin x_{40}\Big]
\end{aligned}
\tag{4.10}
$$

### 3.2 Theory of Optimum Control

We consider a linear system

$$
\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} \tag{4.11}
$$

with a performance index ($PI$)

$$PI = \int_0^T (\boldsymbol{x}^T Q \boldsymbol{x} + \boldsymbol{u}^T P \boldsymbol{u}) dt \tag{4.12}$$

where $\boldsymbol{x}$ is an $n$-dimensional state variable vector, $\boldsymbol{u}$ is an $m$-dimensional control variable vector, and $A$, $B$, $Q$ and $P$ are $n \times n$, $n \times m$, $n \times n$ and $m \times m$ matrices, respectively. $P$ is a positive definite and symmetric matrix, and $Q$ is at least positive semi-definite and symmetric.

If there are no constraints on the control variables, the control which makes $PI$ minimum can be obtained by solving the following matrix Riccati equation

$$\dot{R}(t) + Q - R(t)BP^{-1}B^T R(t) + R(t)A + A^T R(t) = 0 \tag{4.13}$$

under the condition that $R(T)=0$, and we have

$$\boldsymbol{u}(\boldsymbol{x}, t) = -P^{-1}BR(t)\boldsymbol{x} \tag{4.14}$$

The optimum control given by eq. (4.14) depends upon the time. However, it can be represented as a time-independent linear feedback control of the state variables by making the upper limit of the integration in eq. (4.12) infinite. As the solution $R(t)$ of eq. (4.13) depends upon the upper and lower limits of the integration in eq. (4.12), we denote it as $R(t, 0, T)$. Then the steady solution obtained by making the upper limit infinite becomes

$$R_0 = \lim_{T \to \infty} R(0, 0, T) \tag{4.15}$$

$R_0$ can be calculated either by solving the degenerated Riccati equation

$$Q - RBP^{-1}B^T R + RA + A^T R = 0 \tag{4.16}$$

or by integrating eq. (4.13) backward from the terminal condition $R(\infty)=0$ until it can be regarded as constant. In this paper we use the latter method.

### 4.3 Optimum Feedback Control of AC-DC Parallel Transmission System

We apply the optimum feedback control law given in the foregoing section to the system described by eq. (4.9). The Riccati equation corresponding to eq. (4.13) is as follows,

$$\begin{aligned}
\dot{r}_{11} &= -kr_{14}^2 + 2(a_{11}r_{11} + a_{21}r_{12} + a_{31}r_{13} + a_{41}r_{14}) + q_{11} \\
\dot{r}_{22} &= -kr_{24}^2 + 2(a_{12}r_{12} + a_{22}r_{22} + a_{32}r_{23} + a_{42}r_{24}) + q_{22} \\
\dot{r}_{33} &= -kr_{34}^2 + 2(a_{13}r_{13} + a_{23}r_{23} + a_{33}r_{33} + a_{43}r_{34}) + q_{33} \\
\dot{r}_{44} &= -kr_{44}^2 + 2(a_{14}r_{14} + a_{24}r_{24} + a_{34}r_{34} + a_{44}r_{44}) + q_{44} \\
\dot{r}_{12} &= -kr_{14}r_{24} + a_{12}r_{11} + (a_{11} + a_{22})r_{12} + a_{32}r_{13} + a_{42}r_{14} + a_{21}r_{22} + a_{31}r_{23} + a_{41}r_{24} + q_{12} \\
\dot{r}_{13} &= -kr_{14}r_{34} + a_{13}r_{11} + a_{23}r_{12} + (a_{11} + a_{33})r_{13} + a_{43}r_{14} + a_{21}r_{23} + a_{31}r_{33} + a_{41}r_{34} + q_{13} \\
\dot{r}_{14} &= -kr_{14}r_{44} + a_{14}r_{11} + a_{24}r_{12} + a_{34}r_{13} + (a_{11} + a_{44})r_{14} + a_{21}r_{24} + a_{31}r_{34} + a_{41}r_{44} + q_{14}
\end{aligned} \tag{4.17}$$

$$\dot{r}_{23} = -kr_{24}r_{34}+a_{13}r_{12}+a_{23}r_{22}+(a_{22}+a_{33})r_{23}+a_{43}r_{24}+a_{12}r_{13}+a_{32}r_{33}+a_{42}r_{34}+q_{23}$$
$$\dot{r}_{24} = -kr_{24}r_{44}+a_{14}r_{12}+a_{24}r_{22}+a_{34}r_{23}+(a_{22}+a_{44})r_{24}+a_{12}r_{14}+a_{32}r_{34}+a_{42}r_{44}+q_{24}$$
$$\dot{r}_{34} = -kr_{34}r_{44}+a_{14}r_{13}+a_{13}r_{14}+a_{24}r_{23}+a_{23}r_{24}+a_{34}r_{33}+(a_{33}+a_{44})r_{34}+a_{43}r_{44}+q_{34}$$

where $k=(K_c/T_c)^2$, and $r$, $a$ and $q$ are the elements of the matrices $R$, $A$ and $Q$. The Riccati matrix $R$ is obtained by integrating eq. (4.17) from the initial condition $r_{11}=r_{12}=\cdots=r_{44}=0$ until all $r$s converge.

The performance index in eq. (4.12) was chosen as $P=1$ and $Q=\mathrm{diag}(10,1,0,0)$. Although the optimum feedback control is calculated under the assumption that the control variable is not constrained, from the practical point of view we put an upper and lower limit on the control variable as follows,

$$\begin{aligned} &u_{\min} \leqslant u \leqslant u_{\max} \\ &u_{\min} = \mathrm{cof}_{\min} \cdot u_0, \quad u_{\max} = \mathrm{cof}_{\max} \cdot u_0 \end{aligned} \tag{4.18}$$

## 5. Numerical Example

### 5.1 Model AC-DC Parallel Transmission System and Assumed Fault

#### (a) Model system

Fig. 9 shows a model ac-dc parallel system adopted for computations. The generator $G$ represents several power stations the total capacity of which amounts to 10,000 MW. The transmission voltage is 1,000 kV for the ac lines and $\pm 500$ kV for the dc line, and the transmission distance is assumed to be about 600 km. The receiving end can be regarded as an infinite-bus. As for the generators, the damping effect is neglected and control systems such as an AVR or a governor are not considered. Furthermore, it is assumed that the generator can be represented by a constant voltage behind the transient reactance. There are no transient stability improving schemes such as series capacitors or intermediate switching stations installed on the ac transmission line. Moreover, the ac power and dc power are assumed to be equal to one another at the steady-state operating condition.

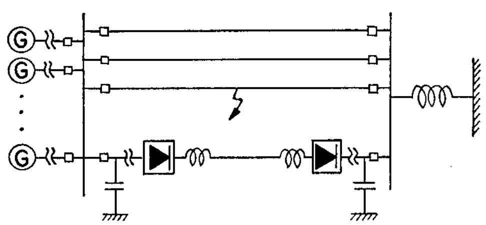



Fig. 9. Sample ac-dc parallel system.

### (b) Controlling method for the dc system

It is assumed that both the rectifier and the inverter are installed with a constant current control system and an open-loop constant margin angle control system. When the voltage at the interconnecting point falls below 0.5 p.u., the protective system comes into operation. The dc system stops its operation 0.05 sec. later and begins to operate again 0.02 sec. after the voltage is recovered. The diagram of the protection was shown in Fig. 7. The current margin (cf. eq. (3.8)) is taken to be 18 % of the specified current value of the constant current control at the rectifier side. The optimum control comes into operation 0.02 sec. after the clearance of the fault. The range of the control variable is constrained by eq.(4.18) and $cof_{\max}$ and $cof_{\min}$ are chosen to be 1.5 and 0.5, respectively.

### (c) Values of the parameters used for the calculations

The parameters of the ac and dc systems are shown in Table 1 and Table 2. The capacity of the capacitor to supply reactive power at the converter points is 0.25 p.u. The steady-state voltage at the rectifier side interconnecting point is 1.05 p.u., and the control angle of the rectifier is controlled to range between 15° and

Table 1. Parameters for model ac system.

| | constants | constants in computer |
|---|---|---|
| generator | capacity=12 000 MVA<br>output =10 000 MW<br>$x_d'$ (transient reactance)=35 %<br>$M$ (inertia constant) =7 sec | $x_d'$=0.291<br>$M$=0.0223<br>damping coefficient=0.0<br>(without AVR and Gov) |
| transformer | capacity= 6 000 MVA<br>12 000 MVA<br>$x_{t1}$ =23 % | $x_{t1}$=0.387 ( 6 000 MVA)<br>0.192 (12 000 MVA) |
| ac transmission line | voltage 1 000 kV<br>positive sequence impedance (Ω/km/cct)=0.012+$j$0.276<br>distance=600 km | $R+jX$ (p.u./cct)=0.072+$j$1.656<br>reactance between inverter connected bus and infinite bus $x_s=j0.142$ |

Table 2. Parameters for model dc system.

| | | | |
|---|---|---|---|
| dc transmission line | | constant current control | |
| inductance (including dc reactor) | 0.026 | gain | 100 |
| line resistance | 0.1 | time constant (sec) | 0.01 |
| capacitance to ground | 0.0 | time constant for varying reference value of current (sec) | 0.01 |
| reactance of main transformer | 0.4 | | |
| capacity of reactive power supplying condenser | 0.25 | constant margin angle control (open-loop type) | 15° |

20° by adjusting the tap of the rectifier transformer.

### (d) Assumed fault

When the ac power and dc power are equal to one another, faults on the ac line are usually more severe than those on the dc line because the former cause a voltage drop at the interconnecting points. Therefore, we assumed in this paper a three-phase short circuit at the center of one of the ac transmission lines as a fault. The fault is cleared 0.1 sec. later and the faulted line is reclosed 1.0 sec. after the occurrence of the faults.

## 5.2 Effect of Optimum Feedback Control

Transient performances of the above described ac-dc parallel system were calculated for the following two cases; (i) constant current control, (ii) optimum beedback control. Fig. 10 shows the results; the phase difference between the generator and the infinite-bus, the power output of the generator and the dc power at the rectifier, and also the control variable (the reference value of the current) for the case (ii). The figure shows that in the case of a constant current control, the phase difference angle and the generator output power continue to oscillate as in the case with an ac system only, because the dc transmitted power is kept almost constant owing to the constant current control. In the case of the optimum control, on the other hand, the dc transmitted power increases after the clearance of the fault because of the optimum control, and the acceleration of the generator is suppressed. When the generator begins to decelerate the dc power decreases, and the oscillation is dampened very rapidly. Since the reference value of the current is limited by the constraints shown in eq. (4.18), it varies from the upper limit to the lower limit as if the system was controlled by a bang-bang control,

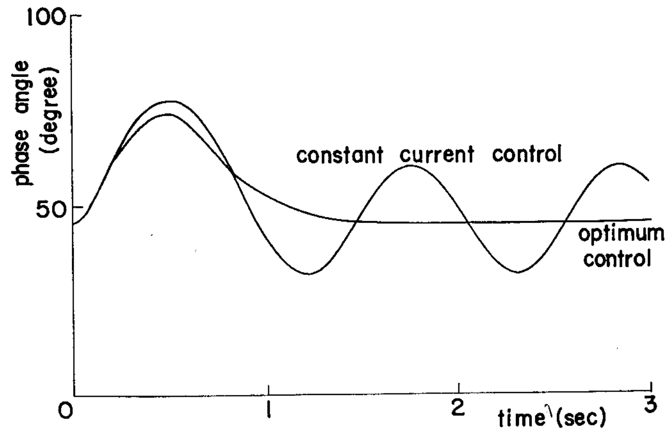



Fig. 10(a). Comparison of phase angle oscillations.

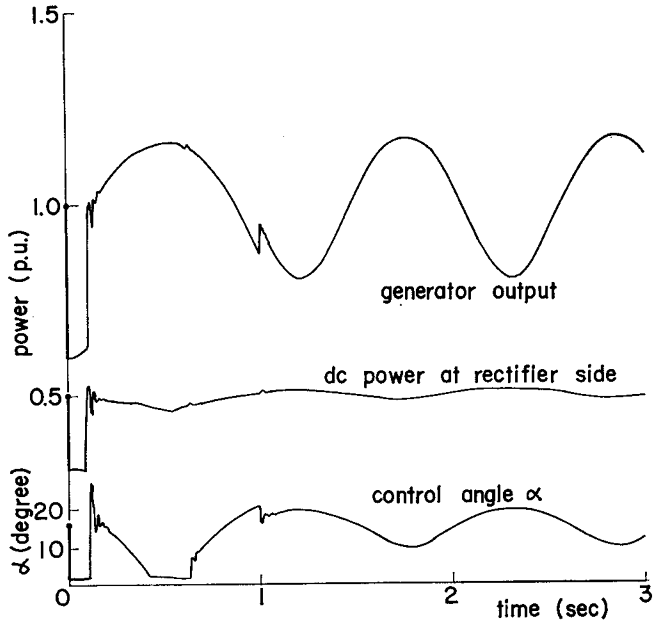



Fig. 10(b). Transient performance under constant current control.

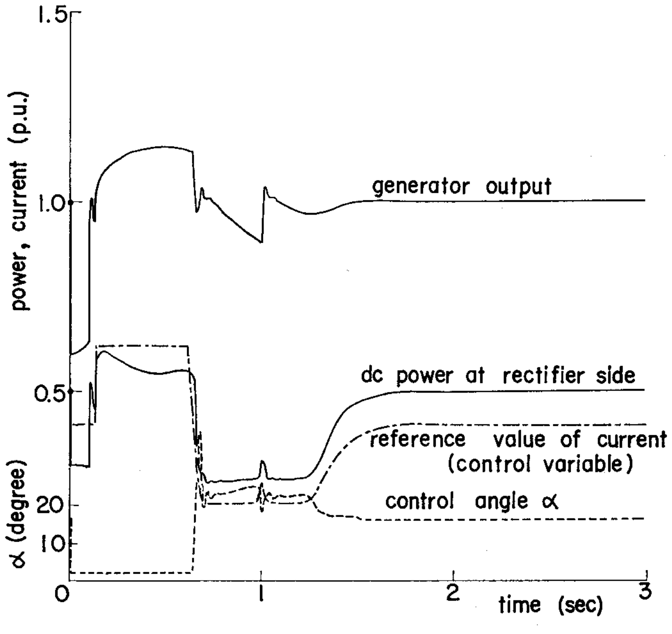



Fig. 10(c). Transient performance under optimum feedback control.

and the dc power is controlled accordingly. The eigenvalues of the linearized system for cases (i) and (ii) are as follows:

(i) $0.0 \pm j6.410, \ -51.92 \pm j390.5$

(ii) $-3.903, \ -21.51, \ -79.78 \pm j395.5$

As the damping of the generator is neglected, the constant current controlled system has a pair of eigenvalues with zero real part. On the other hand, the optimumly controlled system has a considerable amount of damping.

## 6. Conclusion

We tried in this paper to control the dc current of an ac-dc parallel transmission system by varying the reference value of the constant current control system, and examined how the transient characteristics of the ac system in the case of a fault is improved. As a method of control, we used the optimum feedback control for the linearized system. Hence, a transient performance of the system in the case of a large disturbance is not always improved. However, for the fault assumed in this paper, -a three-phase short circuit for 0.1 sec. at the center of an ac line,- it was made clear that the oscillations of the phase angle and the generator output power are dampened rapidly owing to the optimum control.

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### Reference

1) S. Koyama: J.I.E.E.J., **89-4**, 759 (1969) (in Japanese).
2) C. Adamson and N.G. Hingorani: "HVDC Power Transmission," Garaway, §3 (1960).
3) T. Hayashi, T. Machida and I. Masumo: Report of CRIEPI Japan, No. 73068 (1974) (in Japanses).
4) S. Koyama: J.I.E.E.J., **89-11**, 2203 (1969) (in Japanese).
5) Y. Sekine: "Analizing Theories of Electric Power Systems," Denki-Shoin Co., Ltd., Tokyo, §5 (1971) (in Japanese).
6) T. Hayashi: Report of CRIEPI Japan, No. 175003 (1976) (in Japanese).
7) Expert Commitee of DC Transmission: Technical Report of I.E.E.J. II, No. 53 (1977) (in Japanese).
8) D.G. Schultz and J.L. Melsa: "State Functions and Linear Control Systems," McGraw-Hill, New York, §4, §7 (1967).
9) Y. Yoshida: Report of CRIEPI Japan, No. 74023 (1974) (in Japanese).
10) T. Machida: "Direct Current Transmission," Tokyo Denki University Press, Tokyo, §2, §4, §6 (1971) (in Japanese).