Transient Stability Equivalents Based on Lyapunov Function

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(Received December 18, 1979)

Abstract

Owing to the increasing size and complexity in power systems, the study of the stability equivalent is receiving a great deal of attention. This paper describes a systematic method for the recognition of coherent machines by means of the Lyapunov function which is used for the transient stability analysis. A group of generators, whose partial Lyapunov function has a small value compared with that for the whole system, is aggregated into one equivalent generator. This method does not require a long time simulation of the entire system. The parameters of the simplified system are also determined by using the Lyapunov function. The method is applied to 10-machine and 50-machine sample systems and the results are shown.

1. Introduction

Power systems today continue to increase in size and complexity because of the increasing demand for electric power, and because the power supplies are located far from the demands. Furthermore, as the inter-connection among the power systems tend to be strengthened, aiming for improved economy and reliability, it has become impossible in the stability analysis to neglect the effect of the adjacent power systems. In the analysis of such large-scale power systems, it is impossible, or even if possible, it is not economical to represent the whole system in uniform detail. Hence, the study of the stability equivalents has been receiving a great deal of attention. The methods are classified as the following three categories :

i) the use of network equivalents and the allocation of inertia by distribution factors,[1]

ii) replacing coherent generators-generators which swing in a group under transient conditions-by one equivalent generator,[2]

iii) modal analysis of the linearized system equation and order reduction.

Although method ii) is advantageous in the point that the existing transient stability program can be used without any significant modification, it requires a coherency

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recognition which is made using the observed results of the stability study of the entire system.

In this paper, we propose a systematic method for coherency recognition, using the Lyapunov function which is used for the transient stability criterion. A group of generators, whose partial Lyapunov function has a small value compared with that for the whole system, is aggregated into one equivalent generator. This method does not require a long time simulation of the whole system. The parameters of the equivalent generator and the equivalent circuit are determined in such a way that the value of the Lyapunov function is changed as little as possible by the aggregation. The method is applied to 10-machine and 50-machine sample systems and its validity is examined.

2. Coherency Recognition Using Lyapunov Function

2.1 Principle of the Method

As is well known, the transient stability criterion by Lyapunov's direct method is made through the following procedure. First of all, an appropriate Lyapunov function (V-function) is constructed and V_{cr}, the critical value of V, is obtained which gives the stability boundary. The system differential equation is solved under a faulted condition using a numerical method until the value of V exceeds V_{cr}. The critical clearing time t_c is given as the time when the value of V equals V_{cr}. If the actual clearing time is less than t_c, the system is judged to be stable.

From the above procedure of the transient stability criterion, it can be seen that, if the value of the Lyapunov function, not for the entire system but for a group of generators, is sufficiently smaller than that of the entire system, the value of V is not changed very much by aggregating the group of generators. Accordingly, the stability criterion using the V function can be made with almost the same accuracy even after the aggregation. Furthermore, as the generators, whose partial V function is of small value, have similar time variations in their phase angles and angular velocities, and also tend to swing together, it is thought that the simplified system well retains the performance of the original system.

2.2 Swing Equations and Lyapunov Function

The movement of the generator rotor is assumed to be represented by the following classical model, which is used in the simplest analysis of transient stability.

$$\left.\begin{aligned} &\frac{d\delta_k}{dt}=\omega_k \\ &M_k\frac{d\omega_k}{dt}=P_{mk}-P_{ek}-D_k\omega_k \end{aligned}\right\} \quad (1)$$

$$P_{ek}=G_{kk}E_k^2+\sum_{i=1,\neq k}^{n} E_iE_k[G_{ki}\cos(\delta_k-\delta_i)+B_{ki}\sin(\delta_k-\delta_i)]$$

where E_k : induced voltage behind transient reactance x_d' of the k-th generator

δ_k, ω_k : phase angle and angular velocity of the k-th generator, respectively

$G_{ij}+jB_{ij}$: transfer admittance between the induced voltages of the i-th generator and that of the j-th generator

M_k, D_k : inertia constant and damping coefficient of the k-th generator, respectively

P_{mk}, P_{ek} : mechanical power input and electrical power output concerned with the k-th generator, respectively

In this paper, we use the following Lyapunov function obtained from the first integration (also called the energy integration) of the differential equation (1):

$$\begin{aligned} V(\delta,\omega)=&\sum_{k=1}^{n-1}\sum_{j=k+1}^{n} M_jM_k\omega_{jk}^2/2M_T \\ &+\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}(-M_kP_{mj}+M_jP_{mk}+M_kG_{jj}E_j^2-M_jG_{kk}E_k^2)\cdot(\delta_{jk}-\delta_{jk}^s)/M_T \\ &-\sum_{k=1}^{n-1}\sum_{j=k+1}^{n} B_{jk}E_jE_k(\cos\delta_{jk}-\cos\delta_{jk}^s) \end{aligned} \tag{2}$$

where

$\omega_{jk}=\omega_j-\omega_k$

$\delta_{jk}=\delta_j-\delta_k$

$M_T=\sum_{i=1}^{n} M_i$

δ_{jk}^s : the position of the stable equilibrium point after the clearance of the fault

2.3 Algorithm of Coherency Recognition

The Lyapunov function (2) is rewritten without using the Σ mark as follows, for example, for a four-machine system.

$$\begin{aligned} V=&(M_1M_2\omega_{12}^2+M_1M_3\omega_{13}^2+M_1M_4\omega_{14}^2+M_2M_3\omega_{23}^2+M_2M_4\omega_{24}^2+M_3M_4\omega_{34}^2)/2M_T \\ &+[\{-M_1(P_{m2}-G_{22}E_2^2)+M_2(P_{m1}-G_{11}E_1^2)\}(\delta_{21}-\delta_{21}^s) \\ &+\{-M_1(P_{m3}-G_{33}E_3^2)+M_3(P_{m1}-G_{11}E_1^2)\}(\delta_{31}-\delta_{31}^s) \\ &+\{-M_1(P_{m4}-G_{44}E_4^2)+M_4(P_{m1}-G_{11}E_1^2)\}(\delta_{41}-\delta_{41}^s) \\ &+\{-M_2(P_{m3}-G_{33}E_3^2)+M_3(P_{m2}-G_{22}E_2^2)\}(\delta_{32}-\delta_{32}^s) \\ &+\{-M_2(P_{m4}-G_{44}E_4^2)+M_4(P_{m2}-G_{22}E_2^2)\}(\delta_{42}-\delta_{42}^s) \\ &+\{-M_3(P_{m4}-G_{44}E_4^2)+M_4(P_{m3}-G_{33}E_3^2)\}(\delta_{43}-\delta_{43}^s)]/M_T \end{aligned}$$

$$-B_{21}E_2E_1(\cos\delta_{21}-\cos\delta_{21}^s)-B_{31}E_3E_1(\cos\delta_{31}-\cos\delta_{31}^s)-B_{41}E_4E_1(\cos\delta_{41}-\cos\delta_{41}^s)$$
$$-B_{32}E_3E_2(\cos\delta_{32}-\cos\delta_{32}^s)-B_{42}E_4E_2(\cos\delta_{42}-\cos\delta_{42}^s)-B_{43}E_4E_3(\cos\delta_{43}-\text{ces}\,\delta_{43}^s) \quad (3)$$

Now, if we assume that generators No. 2, 3 and 4 behave in completely same way, that is, the swing curves being parallel with each other, then that part of the above V function encircled by the dotted line becomes zero. This part is the Lyapunov function just for the three machines No. 2, 3 and 4, except that the denominator is M_T instead of $M_2+M_3+M_4$. Hence, it is seen that the value of the partial Lyapunov function for a group of coherent generators is small. On the other hand, it cannot always be said that a group of generators whose partial Lyapunov function is of small value is coherent, because it is possible that the value of the partial V function is accidentally small. The coherency, however, can be recognized very accurately via the algorithm described below.[5)]

Step 1. Classify the whole system, using some method suitable for the given system, into the part which is retained in detail and the part to be equivalized.

Step 2. Pick up the pair of generators with the maximum transfer admittance, which belongs to the part to be equivalized, and has not yet been assigned to any coherent group nor been calculated in the value of the partial V function. If no such pair is found, the coherency recognition is completed. Even if one is found, if the value of the transfer admittance is less than a certain definite value Y_{th}, the recognition is completed.

Step 3. Calculate the value of V for the pair of generators chosen in Step 2. If the value is less than the threshold value V_{th}, proceed to Step 4. If it is greater than V_{th}, then go to Step 2.

Step 4. Select the generator which does not yet belong to any coherent group and which has the largest admittance from the group under consideration*. If the admittance value is less than Y_{th}, make the group one coherent group and go to Step 2. If the admittance value is greater than Y_{th}, then add the generator to the group and calculate the value of the partial V function.

Step 5. If the value of V is still less than V_{th}, leave the last generator added to the group and go to Step 4. If the value of V exceeds V_{th}, then remove the last added generator and make one coherent group. Go to Step 2.

In the above algorithm, the values of the partial Lyapunov functions are calculated at an appropriate time after the fault occurrence. Hence, the coherency can be recognized systematically by performing the simulation of the whole system under the

* The admittance between a group of generators and one generator is defined by the maximum value of admittances between that one generator and each generator belonging to the group.

faulted condition for only a short time period. Furthermore, if the angular acceleration can be assumed to be constant, for a while after the fault occurrence, the angular velocity and the phase angle can be approximately calculated for the short faulted period. In such a case, the coherency can be recognized without the simulation of the entire system. (See chapter 4.)

3. Determination of the Equivalent System Parameters

The construction of the stability equivalents using this coherency consists of two phases, i. e., the recognition of the coherency and the determination of the parameters of the equivalents. While the coherency recognition has not been studied very much, several methods for determining the parameters of the equivalent circuits and generators have been reported. They are quite similar to each other in many points. In this paper, we determine the parameters of the equivalents so as to make the change in the value of the V function as small as possible. That is to say that the parameters are determined in such a way whereby the value of V is not changed, provided that the following three conditions are satisfied : (a) the relative angular velocity between the aggregated generators is zero ($\omega_{ij}=0$, for $i, j\in S$, where S is a set of the aggregated generators), (b) the angular difference is equal to that at the stable equilibrium point of the post-fault system ($\delta_{ij}=\delta^s_{ij}$, for $i, j\in S$), and (c) the voltages behind the transient reactance are equal to each other ($E_i=E_j$, for $i, j\in S$). Equivalent parameters (subscript e denotes the quantities for the equivalent generator)

from the part corresponding to the kinetic energy of the Lyapunov function :

$$M_e=\sum_{i\in S} M_i \tag{4}$$

from the first term of the part corresponding to potential energy :

$$P_{me}=\sum_{i\in S} P_{mi} \tag{5}$$

$$Y_{ee}=\sum_{i,j\in S} Y_{ij} \tag{6}$$

from the second term of the part corresponding to the potential energy :

$$Y_{ej}=\sum_{i\in S} Y_{ij} \tag{7}$$

As the effect of the damping is not considered in the Lyapunov function used in this paper, the damping coefficient of the equivalent generator is chosen to be the average value of D_i/M_i weighted by M_i ;

$$D_e=\sum_{i\in S} D_i \tag{8}$$

The equivalent internal voltage is taken from the average value of the induced voltages of the generators weighted by the driving point admittance value;

$$E_s = \sum_{i \in S} |Y_{ii}| E_i / \sum_{i \in S} |Y_{ii}| \tag{9}$$

The stable equilibrium state is obtained for the equivalent system with the above parameters, and transient performance is calculated by taking the equilibrium as the initial state. As regards the initial values in the calculation of the equilibrium state, if we use the values of the original system for the retained generators and the average values of the original system (weighted by the inertia constant) for the aggregated generators, the number of repetitions is only one or two.

4. Numerical Examples and Discussions

4.1 10-machine System

The method described so far is first applied to the 10-machine system shown in Fig. 1. The fault assumed is a three-phase short circuit at one of the x-marks in the figure, and is denoted as Fault A or Fault B. Reclosing is not considered. The first step of the coherency recognition algorithm, i. e., the classification of the whole system into the part retained in detail and the part which has a possibility of simplification, was performed using the absolute value of the admittance from the faulted point (admittance distance[4], see Appendix). In practical problems it may be possible to classify intuitively from the regional characteristics and the boundaries between utilities.

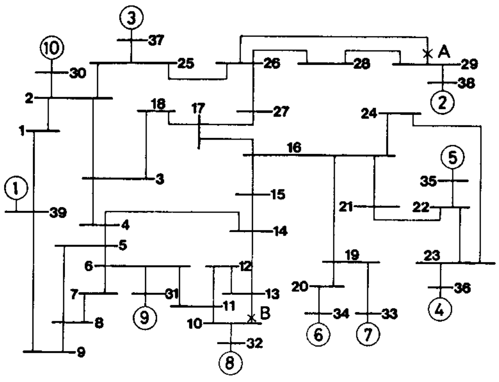


Fig. 1. Sample 10-machine system.

Table 1 shows the admittance matrix (absolute value) including the generator transient reactances. In the case of Fault A, if we take the generators which have a mutual admittance less than 1 p. u. from the very close generator No. 2 to the fault point, generators Nos. 4-9 are chosen as eligible for aggregation. The coherency recognition algorithm previously described is applied to these six generators, and the

Table 1. Transfer admittance between generators.

1	2	3	4	5	6	7	8	9	10	No. of generator
21.08	1.39	2.35	1.14	1.36	0.52	1.52	2.82	2.60	4.62	1
	7.95	1.18	0.57	0.68	0.26	0.76	0.53	0.40	1.44	2
		10.14	0.52	0.62	0.24	0.70	0.63	0.49	2.79	3
			9.70	3.13	0.49	1.42	0.70	0.49	0.86	4
				11.28	0.58	1.69	0.83	0.58	1.02	5
					5.53	1.96	0.32	0.23	0.39	6
						12.20	0.93	0.65	1.14	7
							11.00	1.76	1.15	8
								9.20	0.90	9
									15.20	10

Table 2. Coherency recognition for Fault A.

	group of generators	value of partial V fn.	result*
1	4, 5	0.88×10^{-4}	○
2	4, 5, 7	0.13×10^{-3}	○
3	4, 5, 7, 6	0.67×10^{-2}	×
4	8, 9	0.24×10^{-4}	○

* The value of V fn. for the entire system is 4.465.
○ : coherent, × : not coherent

results are shown in Table 2. First, the pair of generators Nos. 4 and 5, which have the largest transfer admittance between them, are considered. We take 1 p.u. for Y_{th}, which is the same value as that used in Step 1. The admittance value now is 3.13 p.u. and greater than Y_{th}. Hence, the partial V function is calculted for this pair. The value of the V function is computed at 0.31 sec after the fault occurrence. The critical clearing time for Fault A was 0.32 sec. We use for V_{th} 1/1000 of the V function value for the entire system (=4.465). As the value of V for generators Nos. 4 and 5 is less than V_{th}, the third generator No. 7 which has the largest admittance from Nos. 4 and 5 (=1.69 p.u.$>Y_{th}$) is added to this pair and the value of V for the three generators is computed. Since its value is still less than V_{th}, we add generator No. 6 which has the largest admtitance ($1.96>Y_{th}$) from the group Nos. 4, 5 and 7 and calculate the value of V. As the value now exceeds V_{th}, we remove No. 6 and make one coherent group by Nos. 4, 5 and 7. Among the remaining generators Nos. 6, 8 and 9, we consider the pair of No. 8 and No. 9 which have the largest transfer admittance ($1.76 > Y_{th}$) and calculate the V function. Since the value is less than V_{th}, we take generator No. 6 which has the largest admittance from the pair under consideration. (In fact, No. 6 is the only remaining generator.) However, the admittance value 0.32 is less than Y_{th}, hence No. 8 and No. 9 make one coherent group. There is only the one generator No. 6 remaining and the coherency recognition is completed. Consequently, the ten generators are aggregated into seven.

Fig. 2 shows the swing curves for both the original and the simplified system with Fault A cleared at 0.31 sec. after its occurrence. Although the magnitude of the oscillation is a little exaggerated by the simplification, the coincidence of the two systems is fine. Table 3 shows the results of the stability criteria via the Lyapunov function. The correct critical clearing time 0.32 sec. was obtained for this fault using the Lyapunov function. The criterion is unchanged by the simplification of the system.

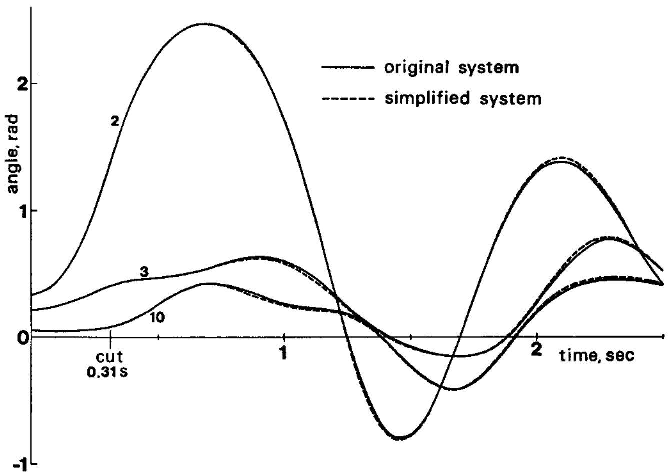


Fig. 2. Swing curves of 10-machine system (Fault A).

Table 3. Stability criterion via Lyapunov function (Fault A).

	original system	simplified system
V_{cr}	5.015	5.117
0.31sec	4.465	4.561
0.32sec	4.867	4.969
0.33sec	5.292	5.398

Table 4. Coherency recognition for Fault B.

	group of generators	value of partial V fn.	result*
1	4.5	0.24×10^{-3}	○
2	4, 5, 7	0.12×10^{-2}	○
3	4, 5, 7, 6	0.16×10^{-1}	×
4	2, 3	0.22×10^{-1}	×

* The value of V fn. for the entire system is 10.63.
○: coherent, × : not coherent

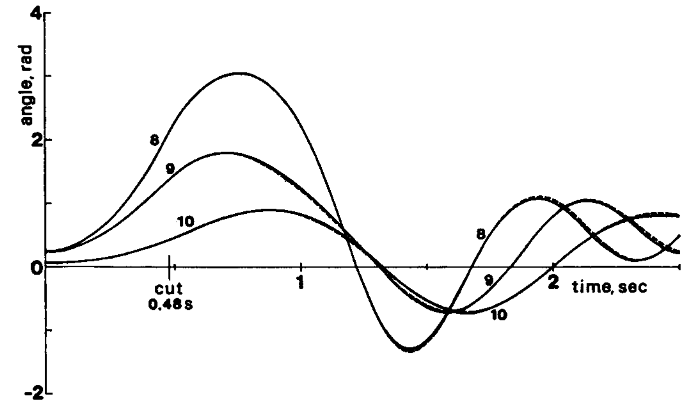


Fig. 3. Swing curves of 10-machine system (Fault B).

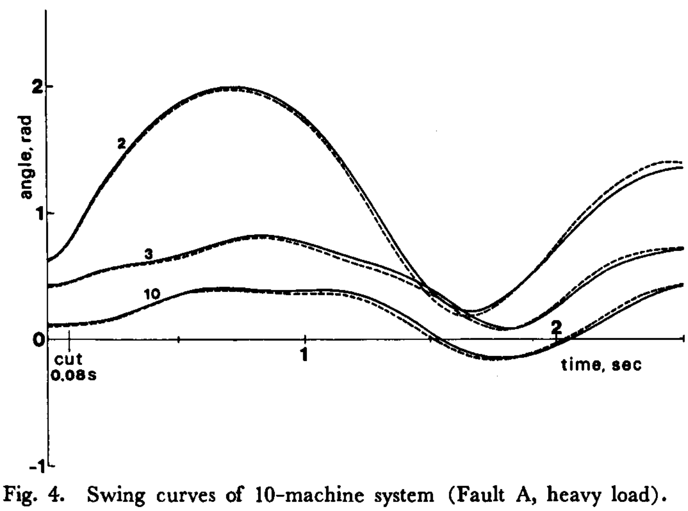


Fig. 4. Swing curves of 10-machine system (Fault A, heavy load).

The coherency recognition was also made for Fault B and generators Nos. 4, 5 and 7 from the eligible ones Nos. 2-7 were found to be aggregated (Table 4). The effects of the simplification are shown in Fig. 3.

Fig. 4 shows the swing curves for Fault A with all of the generator outputs and the load powers doubled. In the case of the simplified system, the coherent groups used are the same as those found in Table 2. Although the effects of the system simplification are a little greater than Fig. 2, they are not serious and the critical clearing time 0.08 sec. was not changed by the simplification. Therefore, it can be said that the coherency recognized under one operating condition can be used under other conditions without a great loss of accuracy, because the operating condition does not affect the coherency very much.

4.2 50-machine System

The simplification method is next applied to the 50-machine system shown in Fig. 5. The fault assumed is a three-phase short-circuit at the point marked by x in the figure. The 35 generators which have a transfer admittance less than 50 p.u. from the faulted point are taken as eligible for aggregation. The values of the partial Lyapunov functions are calculated at 0.27 sec. which is the critical clearing time for the assumed fault. V_{th} is 1/1000 of the Lyapunov function for the entire system, the same as in the case of the 10-machine system, and Y_{th} is 50 p.u. These values can be determined in accordance with the degree of the requirement for the simplification. That is to say, when it is required to simplify the system and reduce the number of the generators very much, V_{th} should be large and Y_{th} should be small. The results of the coherency recognition are shown in Table 5. Consequently, the eligible

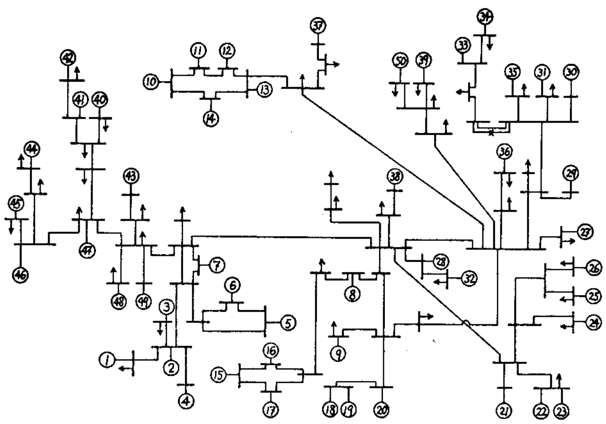


Fig. 5. Sample 50-machine system.

Table 5. Coherency recognition for 50-machine system (I).

non-aggregated group	
21, 23, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 49, 50	
coherent groups (recognized at 0.27 sec)	
1	40, 41
2	44, 45
3	1, 2
4	5, 6, 4
5	10, 12, 11
6	18, 20, 19
7	47, 48
8	22, 25
9	15, 17, 16

35 generators are aggregated into 22. (The total of 50 is reduced to 37.)

Fig. 6 shows the swing curves of some generators in the original system and the simplified system when the fault is cleared at the critical clearing time. Fig. 6 (a) shows the performance of the generators represented in detail. The magnitude of the first swing becomes a little greater by the simplification and, accordingly, the

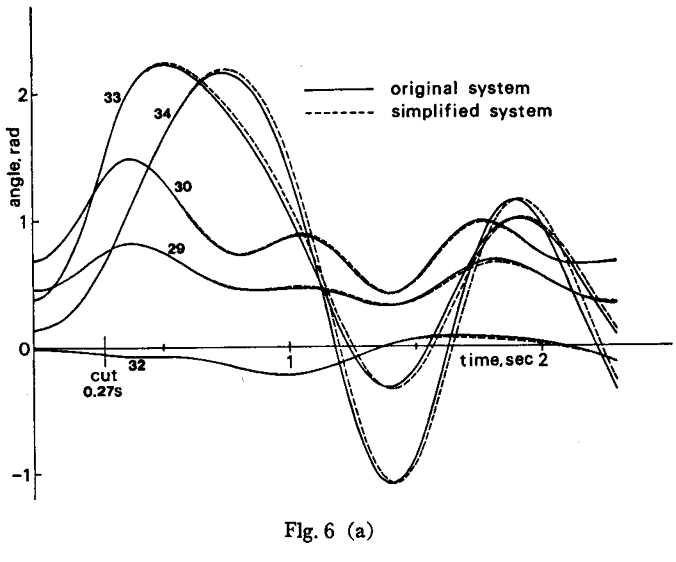
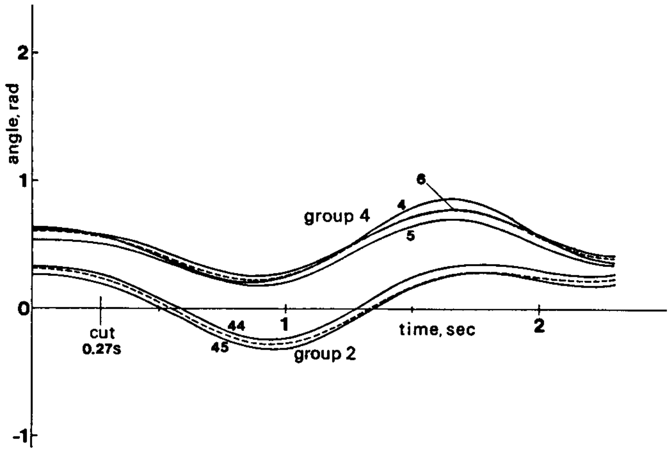
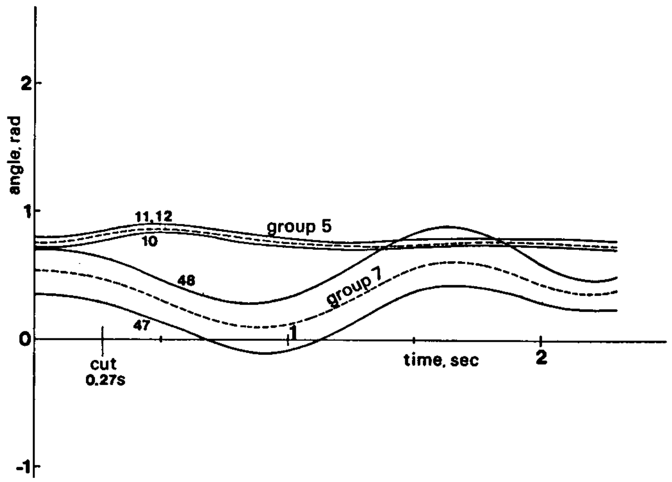


Fig. 6 (a)



Flg. 6 (b)



Flg. 6 (c)

Fig. 6 Swing curves of 50-machine system.

subsequent swings are delayed. The magnitude and frequency of the oscillations, however, are almost unchanged, and it can be seen that the effect of the simplification is small. Fig. 6 (b) and (c) show the performance of the generators which belong to the coherent groups. It is seen from these figures that the swing curves of the generators which belong to a coherent group are almost parallel to each other and the coherency recognition algorithm works well. It is also noted that the performance of the equivalent generator is nearly the average of those of the aggregated generators.

As described before, the fault clearing time 0.27 sec. is the critical clearing time

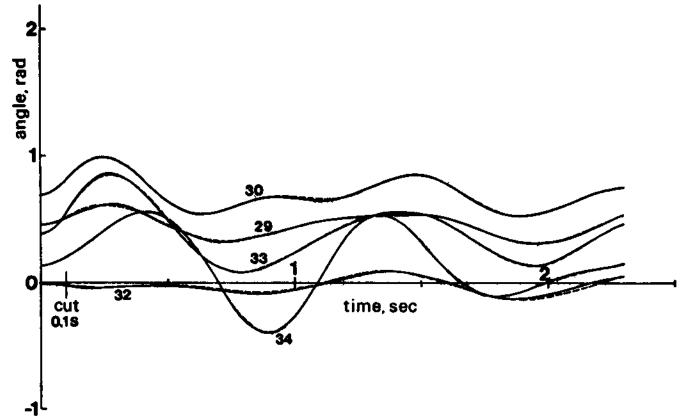


Fig. 7. Swing curves of 50-machine system (fault cleared at 0. 1 sec).

for this fault. Hence, the swing of the generators is large and it is the severest case from the viewpoint of the system simplification. Fig. 7 shows the performance of some non-aggregated generators, when the fault is cleared at 0. 1 sec. As is clear from the figure, the performance of the original system is very well retained with the fault cleared at 0. 1 sec., which is a less severe condition for the simplification.

In the above calculation, the critical clearing time was obtained in advance, and the coherency was recognized using the values of the Lyapunov functions at that time. In reality, the critical clearing time can not be known beforehand. Therefore, it is necessary that the coherency can be recognized using the value of the V functions computed at the actual fault clearing time. Table 6 shows the results of the cohere-

Table 6. Coherency recognition for 50-machine system (II).

coherent groups (recognized at 0. 1 sec)	
1	40, 41
2	44, 45
3	1, 2
4	5, 6, 4
5	10, 12, 11
6	18, 20, 19
7	47, 48
8	15, 17, 16
9	24, 25, 22

ncy recognition using the value of V at 0.1 sec. The result is the same as Table 5 which recognized at 0.27 sec, except that generator No. 24. is included in the last group. Therefore, it is seen that the coherency can be recognized at the actual fault clearing time by the proposed algorithm.

In the above coherency recognitions, the original system was simulated for 0.1 or 0.27 sec. with the faulted condition to obtain the values of δ and ω, and the partial Lyapunov functions were computed. As was referred to in Section 2.3, if the angular accelerations can be considered constant during a short time after the fault occurrence, we can obtain the values of δ and ω after the fault from the prefault values δ_0, ω_0 and the angular acceleration at the moment of the fault occurrence a, as follows:

$$\left.\begin{aligned}\delta&=\delta_0+at^2/2\\ \omega&=\omega_0+at\end{aligned}\right\} \tag{10}$$

As a result, the calculation of the original system becomes unnecessary. Table 7 shows the comparison of the approximate values of δ and ω, obtained by using eq. (10) with the accurate values for the arbitrarily chosen four generators. Although a considerable difference is recognized for generator No. 30, which is located near the faulted point, the approximate values of the other generators are very close to their accurate values. Table 8 shows the results of the coherency recognition using these approximate values of δ and ω. Comparing this table with Tables 5 and 6, it is seen that for 0.1 sec. recognition, the results are completely the same. For 0.27 sec. recognition, the only difference is that generator No. 19 is not included in the sixth group in Table 8. Therefore, it was made clear that the coherency can be recognized without the simulation of the entire system by applying the algorithm

Table 7. Approximate values of δ and ω.

	no. of generator	accurate value		approximate value	
		δ(rad)	ω(rad/sec)	δ(rad)	ω(rad/sec)
0.1 sec	10	0.730	0.187	0.731	0.198
	20	0.453	0.203	0.454	0.223
	30	0.812	2.306	0.823	2.688
	40	0.331	−0.192	0.333	−0.194
0.27 sec	10	0.783	0.412	0.793	0.535
	20	0.506	0.368	0.524	0.602
	30	1.330	3.152	1.668	7.257
	40	0.270	−0.555	0.272	−0.525

Table 8. Coherency recognition for 50-machine system (III).

	coherent groups	
	approximate recognition at 0.27 sec	approximate recognition at 0.1 sec
1	40, 41	40, 41
2	44, 45	44, 45
3	1, 2	1, 2
4	5, 6, 4	5, 6, 4
5	10, 12, 11	10, 12, 11
6	18, 20	18, 20, 19
7	47, 48	47, 48
8	22, 25	15, 17, 16
9	15, 17, 16	24, 25, 22

proposed in this paper to the approximately calculated during-fault condition.

5. Conclusions

We proposed a method for short-term dynamic equivalents based on the Lyapunov function. It was shown from the results of the numerical examples that the coherent groups of generators can be known through a systematic procedure without the transient calculation of the entire system. Furthermore, using the Lyapunov function, we could derive a method for determining the parameters of the equivalent circuits and generators, which method is similar to those so far reported.

It is expected that the method will be extended so that the effects of the control devices (AVRs and governors) can be taken into account.

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Appendix. Admittance Distance

The bus voltages are determined from the load-flow calculation, and the loads can be represented by the constant impedances between the node and the ground as follows,

$$I=YV \tag{A-1}$$

If we separate I and V into I_G, V_G for the generator nodes, I_F, V_F for the faulted node and I_L, V_L for load nodes, then

$$\begin{bmatrix} I_G \\ I_F \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GF} & Y_{GL} \\ Y_{FG} & Y_{FF} & Y_{FL} \\ Y_{LG} & Y_{LF} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_F \\ V_L \end{bmatrix} \tag{A-2}$$

Because the loads are contained in Y as constant impedances, $I_L=0$, and V_L can be eliminated from eq. (A-2) to get

$$\begin{bmatrix} I_G \\ I_F \end{bmatrix} = \left(\begin{bmatrix} Y_{GG} & Y_{GF} \\ Y_{FG} & Y_{FF} \end{bmatrix} - \begin{bmatrix} Y_{GL} \\ Y_{FL} \end{bmatrix} \cdot Y_{LL}^{-1} \cdot [Y_{LG} Y_{LF}] \right) \cdot \begin{bmatrix} V_G \\ V_F \end{bmatrix} = Y' \begin{bmatrix} V_G \\ V_F \end{bmatrix} \tag{A-3}$$

The following relation holds between the induced voltage E_G behind the transient reactance x_d' and the terminal voltage V_G

$$E_G = V_G + X_d \cdot I_G \tag{A-4}$$
$$X_d = \mathrm{diag}\{jx_d'\}$$

Eliminating V_G from eqs. (A-3) and (A-4), we get

$$\begin{bmatrix} I_G \\ I_F \end{bmatrix} = \left(U + Y' \cdot \begin{bmatrix} X_d & O \\ O & O \end{bmatrix}^{-1} \right) \cdot Y' \cdot \begin{bmatrix} E_G \\ V_F \end{bmatrix} = Y'' \cdot \begin{bmatrix} E_G \\ V_F \end{bmatrix} \tag{A-5}$$

$|y''_{iF}|$ $(=|y''_{Fi}|)$, the absolute value of the element of the matrix Y'', is the admittance distance from the faulted point to the i-th generator. At the opposite of the usual distance, the large value of the admittance distance is interpreted as the electrical proximity.