Fundamental Investigation on Design of Tunnel Steel Arch Supports

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Abstract

For the purpose of contributing to a proper design of tunnel supports with steel arches, it is necessary to clarify their behavior under earth pressure. As a first approach, the authors investigated a method to calculate the deformation of, and the stress in, steel supports subjected to earth pressure. Special attention was paid to the treatment of the passive forces induced, the mechanical characteristics of the ends of these supports, as well as the joints of the members.

As a result of theoretical investigations followed by experiments, the authors obtained a method that may be used in practice.

1. Introduction

Steel arch supports are widely used for rock tunneling, because of their obvious merits such as rapid construction, safety, etc. In spite of the increase in the use of steel supports, no standard for designing steel arch supports has yet been established.

An estimation of the load which acts on the support might be used for a theoretical design of steel arch supports, but such loads are not well understood. In addition to this, the mechanical construction characteristics of steel arch supports are unknown factors, because of the uncertainty involved in supports. This may cause a difficulty in obtaining a rational design.

A load will be estimated by several practical measurements. However, the latter factors may be elucidated experimentally, using considerably simplified construction characteristics. The problem can be divided into two: one is the problem of what kind of passive load is generated under the action of an active load; and the other is the problem of what kind of stress is generated under such conditions.

In this research, theoretical considerations were made on the mechanical properties

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of steel arch supports, based on the experimental results obtained by full scale laboratory tests. Also, the relation between the active load and the passive load and the construction characteristics which are essential for designing a steel arch support have been mainly discussed.

2. Principles of the Calculation Method of Passive Loads

The method adopted by Proctor and White[1] for a design of steel arch supports, which is a graphical solution of the equilibrium of forces, has met a number of difficulties in application because of the lack of consideration for the construction characteristics of the supports. For instance, the distribution of a load is necessarily determined by the shape of the support and the magnitude of the maximum active load. Consequently, as far as the magnitude of the active load which governs the force relationship of the support is concerned, the same force relationship should hold, regardless of whether the active loads are symmetrical or non-symmetrical. Such a procedure might yield results which differ from the accepted results.

Undoubtedly, the occurrence of a passive load is caused by the restriction of the deformation of the support on the rock wall side. Let us assume that the active loads, which are indicated by the black arrows in Fig. 1(a), are caused from the

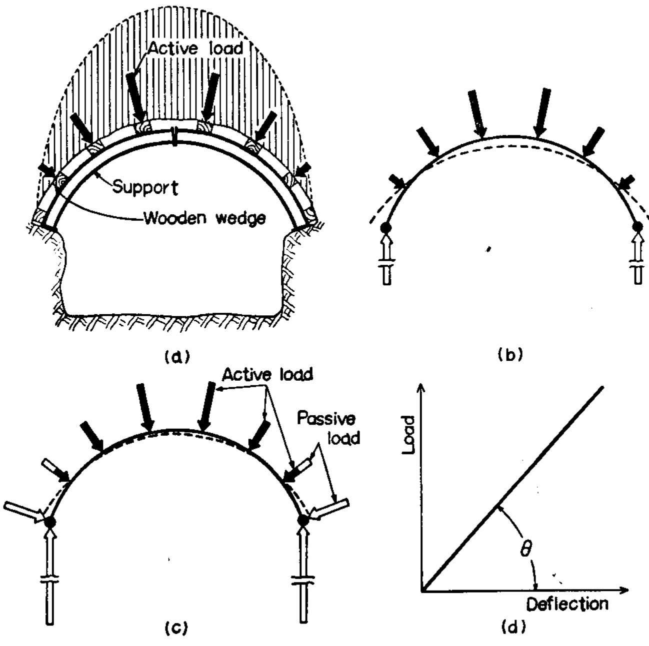

Fig. 1. Relation between active loads and passive loads generated on tunnel arch support.

weight of loose rock, shown by the hatching in the same figure, and that they act on the support fixed to the rock roof through the wedges. However, if there were no rock wall around the support, the support might deform as shown by the broken line in Fig. 1(b). The deformation of the support on the rock wall side is actually limited to the corresponding deformation of the wedge, while the deformation on the open space side is free. Thus, the reactions (passive loads), which are indicated by the blank arrows in Fig. 1(c), may be generated at the block points, which are the contact between the support and the wedges. The magnitude of the passive loads and their distribution must be determined by several factors, such as the distribution of the active loads, the material of the support, the structure and dimension of the support, the conditions of the support ends, the joint load-deformation characteristics of the wedge (Fig. 1(d)), the number of the block points etc., which are related to the deformation of the support.

Now, let us equate the relationship between the passive loads and the items which govern the deformation of the support. For this purpose, it is necessary to decide the conditions of the ends of the steel arch support and its joint structure at the top. For simplicity, it is assumed in this paper, that the loads act perpendicularly to the support at every block point. Also, it is assumed that a linear relationship can be held between the load on the wedge and its deformation.

The relationship between the active load acting on the j-th block point ($j=1$, 2, ……, n, where n is the number of block points), A_j, and the deflection at the i-th block point ($i=1$, 2, ……, n), d_i, which is primarily due to the active loads, can be given by Equation (1),

$$d_i=\sum_{j=1}^{n} a_{ij}A_j, \qquad i=1, 2, \cdots\cdots, n. \tag{1}$$

where a_{ij} denotes the deflection generated at the i-th block point, when a unit load acts on the j-th block point.*

The passive load generated on the j-th block point, P_j, must satisfy the following relations at the i-th block point ($i=1$, 2, ……, n), because the deflections to the rock wall side are limited.

$$\begin{array}{l} (a_{11}+b_1)P_1+a_{12}P_2+\cdots\cdots\cdots\cdots+a_{1n}P_n\geqq-(d_1+b_1A_1) \\ a_{21}P_1+(a_{22}+b_2)P_2+\cdots\cdots\cdots\cdots+a_{2n}P_n\geqq-(d_2+b_2A_2) \\ \vdots \qquad\qquad\qquad\qquad\quad \vdots \qquad\quad \vdots \\ a_{n1}P_1+a_{n2}P_2+\cdots\cdots\cdots+(a_{nn}+b_n)P_n\geqq-(d_n+b_nA_n) \end{array} \tag{2}$$

and

$$P_j\geqq 0 \tag{3}$$

* d_i has a positive sign when the deflection occurs on the open space side.

where b_i is a constant which can be estimated by the load-deflection curve of the wedge, and the following relation holds between b_i and the angle of inclination θ which is shown in Fig. 1(d)

$$b_i = \cot\theta = 1/K_r \qquad (4)$$

where K_r is the coefficient of the reaction plate. The values of the right-hand side of Equation (2) and each coefficient of P_j are known, so that each value of P_j can be obtained by solving Equation (2) with Equation (3).

This is a basic principle of determination of passive loads. However, the actual solution of these unequal equations is carried out as follows: First of all, the block points, where the passive loads are generated, are arbitrarily presumed. Then, at these block points, the equalities of Equation (2) hold. Since the passive loads on the other block points are zero, the number of these equations becomes equal to the number of the unknown factors. This indicates the possibility of a solution. Then, we examine whether each sign of the P_j obtained is positive or not. If each P_j is positive, then we examine the satisfaction of the unequal Equation (2) at the block points, where no passive load is expected. By using such a method of trial and error, each value of P_j can be determined. The uniqueness of the solution has not yet been proved theoretically, but by experience, it is expected that we can determine P_j uniquely.

3. Conditions of the Support Ends and the Joint of a Steel Arch Support

In order to determine a_{ij} which was defined in the previous chapter, it is necessary first to determine the properties of the support ends as well as the joint of the steel arch support. It would seem that one approach to this problem would be to assume that the support ends and the joint are the construction of the hinges. However, it is feared that such an assumption is too far from the actual condition. Therefore, the authors assumed the unique condition of the support ends, considering the practical conditions of the support installation and the convenience of the calculations. These assumptions are based on the presumptions that small movements of the support legs are observed. Now, first calculate the deformation of the support by the method described later, under the assumption that the end of one of the legs of support functions as a roller, whereas the other functions as a hinge, as shown in Fig. 2(a). Then, repeat the calculation under the assumption that the conditions of the support ends are exchanged with each other, as shown in Fig. 2(b). We consider the average of the results of the calculations under these two conditions as the actual deformation of the support. (See Fig. 2(c)). The displacement of the roller may be horizontal or inclined, as shown Fig. 2.

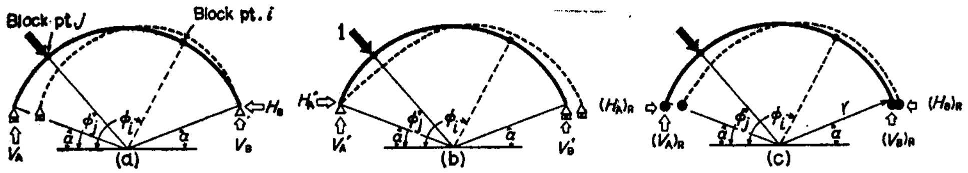

Fig. 2. Condition of support ends.*

The formulas for calculating the deformation of the support can be derived for each shape of the support. As an example, the formula for the arch support shown in Fig. 2 will be derived. When an active load of a unit magnitude, A_j, acts on the block point j of the steel support which has a roller on one end and a hinge on the other as shown in Fig. 2(a), the deflection, δ_{ij}, at the i-th block point is expressed by the following equation according to the principle of virtual work:

$$\delta_{ij}=\int_{\alpha}^{\pi-\alpha}\frac{M\bar{M}}{EI}ds+\int_{\alpha}^{\pi-\alpha}\frac{N\bar{N}}{ES}ds+\int_{\alpha}^{\pi-\alpha}\kappa\frac{Q\bar{Q}}{GS}ds \tag{5}$$

where M, N and Q denote the bending moment, the axial force and the shearing force, respectively. These are generated on each part of the support by the action of a unit load acting on the point j. $\bar{M}$, $\bar{N}$ and $\bar{Q}$ are caused by a unit load acting on the point i. The notations E, I, S, G and κ denote the modulus of the longitudinal elasticity, the geometrical moment of inertia, the sectional area, the modulus of rigidity and the shearing constant, respectively. In Equation (5), the influence of the axial force N and of the shearing force Q on the deflection of the support is very small, compared with that of the bending moment M. This indicates the negligibility of the second term as well as the third term of right-hand of Equation (5). The influence of the axial force, however, appears sometimes marked, depending on the conditions of the loads. In such cases, it is not permitted to neglect the second term on the right hand side of Equation (5). The formula to find the deflection for the case shown in Fig. 2(a), derived by neglecting the second and the third terms of Equation (5), is shown in the following:

$$\delta_{ij}=\frac{r^3}{EI}\Big[V_A\bar{V}_A\Big\{(\phi_j-\alpha)\Big(\cos^2\alpha+\frac{1}{2}\Big)-2\cos\alpha\sin\phi_j+\frac{1}{4}(\sin 2\phi_j+3\sin 2\alpha)\Big\}$$

$$+V_B\bar{V}_A\Big\{(\phi_i-\phi_j)\Big(\cos^2\alpha-\frac{1}{2}\Big)+\frac{1}{4}(\sin 2\phi_j-\sin 2\phi_i)\Big\}$$

$$-H_B\bar{V}_A\Big\{\cos(\phi_j+\alpha)-\cos(\phi_i+\alpha)+\frac{1}{4}(\cos 2\phi_i-\cos 2\phi_j)-\frac{1}{2}(\phi_i-\phi_j)\sin 2\alpha\Big\}$$

$$+V_B\bar{V}_B\Big\{(\pi-\alpha-\phi_i)\Big(\cos^2\alpha+\frac{1}{2}\Big)-2\cos\alpha\sin\phi_i+\frac{1}{4}(3\sin 2\alpha-\sin 2\phi_i)\Big\}$$

* $V'_A=V_A$, $V'_B=V_B$, $H'_A=-H_B$
$(V_A)_R=V_A$, $(V_B)_R=V_B$, $(H_A)_R=H'_A/2$, $(H_B)_R=H_B/2$

$$-(H_B\bar{V}_B+V_B\bar{H}_B)\left\{\cos(\phi_i-\alpha)+\frac{1}{4}(\cos 2\phi_i+3\cos 2\alpha)-\frac{1}{2}(\pi-\alpha-\phi_i)\sin 2\alpha\right\}$$
$$+H_B\bar{H}_B\left\{(\pi-\alpha-\phi_i)\left(\sin^2\alpha+\frac{1}{2}\right)-2\sin\alpha\cos\phi_i+\frac{1}{4}(\sin 2\phi_i-3\sin 2\alpha)\right\}\Bigg] \quad (6)^*$$

where

$$\left.\begin{aligned} V_A&=\frac{\sin(\phi_j+\alpha)}{2\cos\alpha}, & V_B&=\frac{\sin(\phi_j-\alpha)}{2\cos\alpha}, & H_B&=\cos\phi_j \\ \bar{V}_A&=\frac{\sin(\phi_i+\alpha)}{2\cos\alpha}, & \bar{V}_B&=\frac{\sin(\phi_i-\alpha)}{2\cos\alpha}, & \bar{H}_B&=\cos\phi_i \end{aligned}\right\} \quad (7)$$

A similar calculation should be made for the case shown in Fig. 2(b). Taking the mean of these two deflections, we can determine a_{ij}.

The actual shape of the support is often represented by a composition of several circular arcs. It is considered to be more convenient to apply elliptic coordinates for such a shape of support. As indicated by Equations (6) and (7), a_{ij} can be statically determined. Therefore, the formula for one support is applicable to any support of the same shape, whatever the cross-section of the members and the structure may be.

Now, let us consider what kind of joint condition should be assumed to express the connection at the top of a support. If we assume that the joint is a simple hinge connection, for example, the bending moment at the point is always zero, whatever the condition of the connection may be. This differs from the facts. This is not a structure which is statically determinate from a mathematical point of view, and it suggests an undesirable situation. Thus, the authors decided to assume that a junction is a mechanism which has an axis line somewhat different from the position of the other parts, and which has a modulus of elasticity considerably smaller than that of other parts. (See Fig. 3). Such an assumption seems reasonable, judging from the destroyed conditions of the real supports.

In this case, we have only to add the deflection due to this special construction to the deflection δ_{ij} given by Equation (6). The authors omit this calculation formula here.

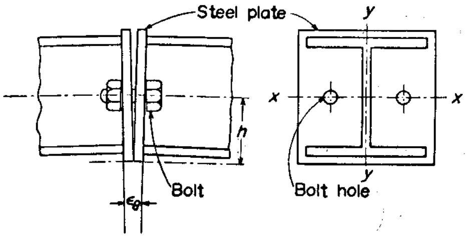

Fig. 3. Joint condition at the top of support.

* r is the radius of the arch support in Fig. 2.

4. Comparison of the Results of Calculations with Those of Experiments

In order to examine whether the above mentioned calculations agree with the facts or not, the authors first used the experimental results which were obtained in two laboratories of the Japanese National Railways[2]. These experiments were carried out as follows. As shown in the right-lower part of Fig. 4, the arch shaped support was constructed horizontally. A single active load was applied in turn on each block point indicated by the number in the same Figure, through an oil-jack. The inner

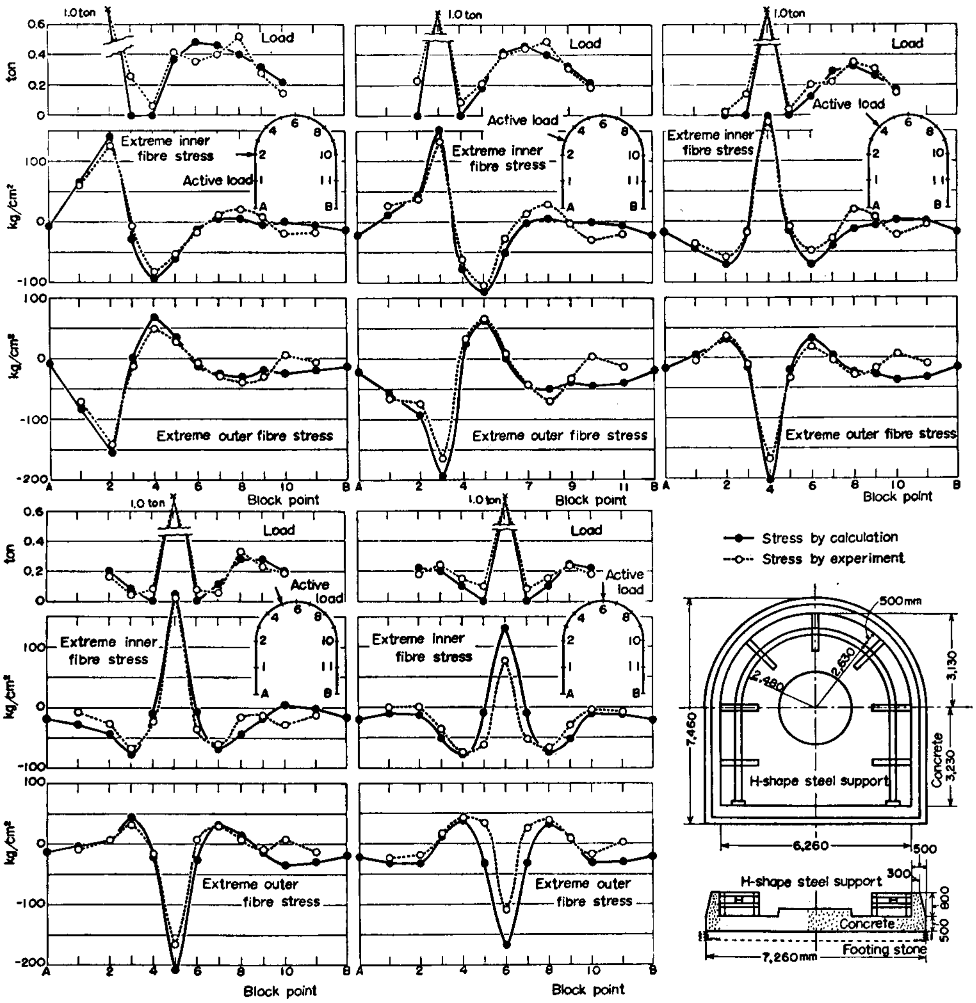

Fig. 4. Comparison of experimental and theoretical results where the active load acts on only one block point.

Table 1. Characteristic values of steel arch supports subjected to experiments.

Kind	Weight (kg/m)	Tensile Strength (kg/mm^2)	Yielding Point (kg/mm^2)	Cross-sectional Area (cm^2)	Modulus of Section (cm^3)		Modulus of Longitudinal Elasticity (t/cm^2)	Geometrical Moment of Inertia I_x (cm^4)
					W_x	W_y		
Welding 150 mm H-Shape	30.5	41～50	≧23	38.9	202	67.6	2.1×10^3	1517
200 mm H-Shape	49.9	41～50	≧23	63.5	472	160	2.1×10^3	4720

and outer extreme fibre strains were measured by electric resistance strain gauges. The experimental results were compared with the calculated values. Fig. 4 shows a comparison about the support of H-shaped 150 mm typed steel for which several characteristic values are indicated in the 2nd row of Table 1. Each top graph of Fig. 4 indicates the passive loads generated on each block point when a single active load of 1 metric ton acts respectively on the block point 2, 3, 4, 5 or 6.

In this graph, the calculated results are shown by a small black circular mark and a solid line, while the experimental results are shown by a small white circular mark and a broken line. Each middle and bottom graph indicates respectively the extreme inner and outer fibre stress. In these graphs, a positive value expresses tensile stress. Comparing the solid line with the broken line, it is noticed that both results coincide fairly well, except where the active load acts on block point 6. This calculation was carried out on the assumption that the support consisted of one member, and no account was taken that the support was connected by the joint as previous described. Hence, when the active load acts on block point 6, the stress distribution shows a considerable difference between the calculated result and the experimental result.

Subsequently, the authors examined the coincidence between the calculated and the experimental results in the case where the active loads with various magnitudes acted simultaneously on several block points. The authors took up the support which consists of the H-shaped 200 mm type steel. Its several characteristic values are indicated on the 3rd row of Table 1. Fig. 5 shows the shape of the support and the location of the block points, etc. These experiments were carried out in the Public Works Research Institute, Ministry of Construction. The authors took up two distributions of active loads, such as symmetric and asymmetric, as shown respectively in Tables 2 and 3.

Fig. 6 and Fig. 7 indicate the comparison between the calculated results and the experimental ones respectively, in the symmetrical and asymmetrical distribution of active loads. However, in the experiments, the active loads by themselves cannot have any value at the block point where active loads and passive loads act simultaneously,

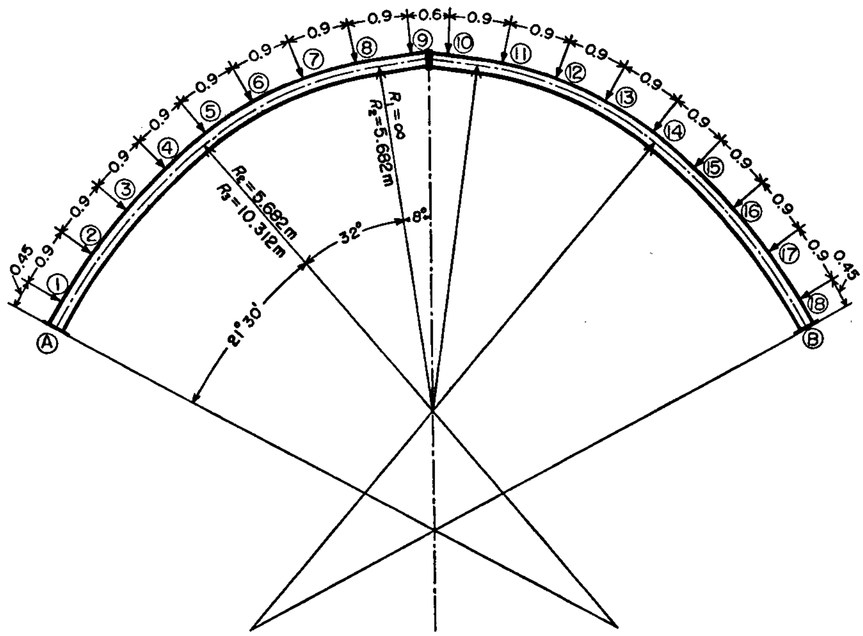

Fig. 5. H-shaped 200 mm type tunnel support.

Table 2. Symmetric distribution of active loads (ton).

Block point	1, 18	2, 17	3, 16	4, 15	5, 14	6, 13	7, 12	8, 11	9, 10
Active load	1.35	1.46	1.57	1.67	1.76	1.89	2.32	2.32	2.00

Table 3. Asymmetric distribution of active loads (ton).

Block point	1	2	3	4	5	6	7	8	9
Active load	0.81	1.05	1.24	1.37	1.49	1.62	1.75	1.89	1.66
Block point	10	11	12	13	14	15	16	17	18
Active load	1.75	2.24	2.37	2.50	2.62	2.50	2.31	1.99	1.57

but the sum of the active and passive loads can be measured. (See Fig. 1(c)). Then, according to the author's method, they calculated the passive loads which were generated by the active loads as shown in Tables 2 and 3. They found the block points, where no passive load was generated by these calculations. The active loads which acted on such block points were applied by the jacks. The loads, indicated by a cross mark in the top graph of Figs. 6 and 7, express such active loads. Accordingly, all loads, except such active loads, were measured as passive loads.

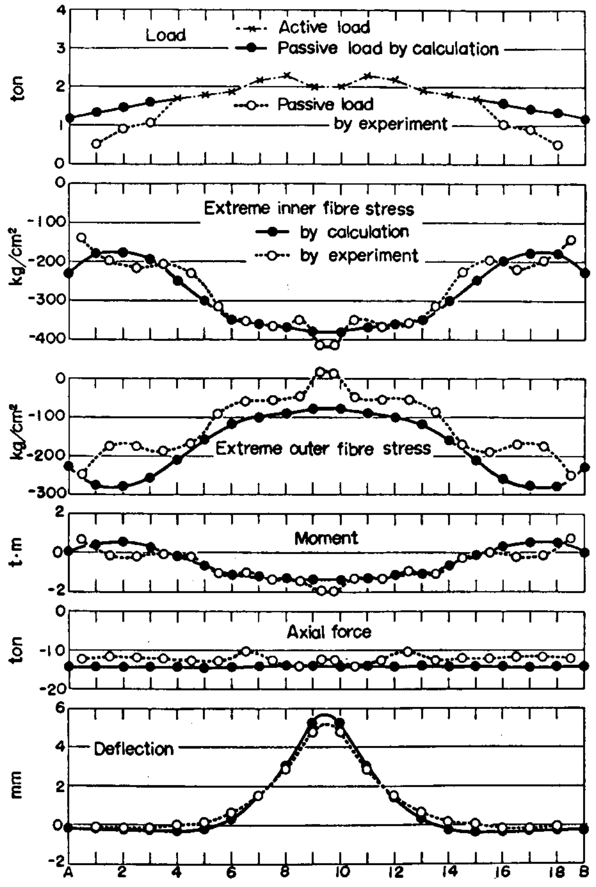

Fig. 6. Comparison of experimental and theoretical results in the case where active loads distribute symmetrically.

Though the shape of this support was a composite circular structure with four centres, they approximated it closely by an ellipse and used this representation for the calculation (see Fig. 5). The direction of movement of the support ends was regarded as horizontal, and the joint condition was represented as $\varepsilon_\theta = 24$ minutes, $E' = E/160$ and $h = 12$ cm. The coefficient of the reaction plate, K_r, in all the block points was assessed at 29 ton/cm. Moreover, they took no account of the influence of the deflection due to the shearing force.

Now, Fig. 6 and Fig. 7 represent the calculated or experimental results on the load, extreme inner fibre stress, extreme outer fibre stress, bending moment, axial force and deflection in turn from the top. It is seen that the experimental results agree with the calculated ones very satisfactorily, except for the support ends and

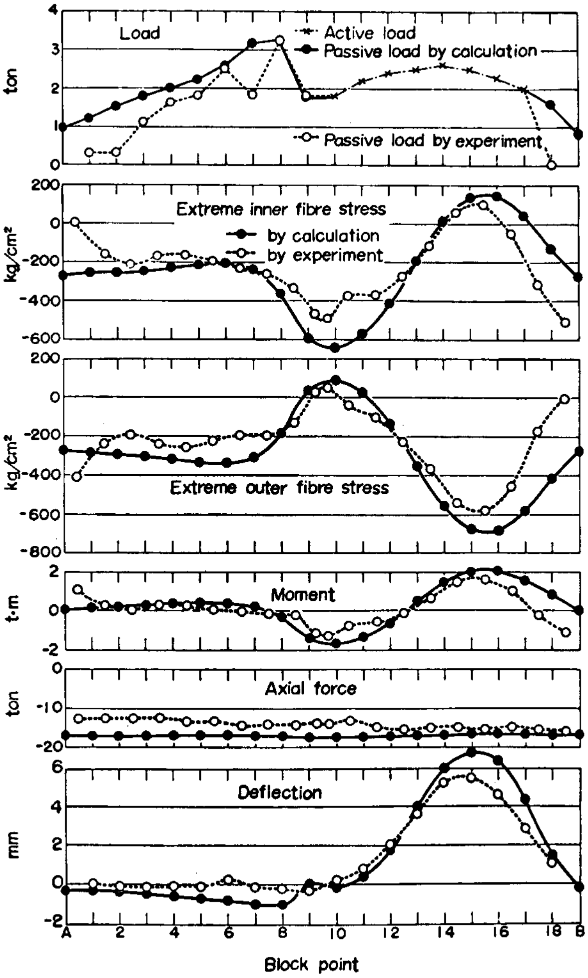

Fig. 7. Comparison of experimental and theoretical results in the case where active loads distribute asymmetrically.

their vicinity. The authors think that the discrepancy in the vicinity of the support ends is mainly due to the assumption which they adopted with regard to the direction of the movement of the supports ends. However, if we consider the fact that the measurement values at the block points near the support ends fluctuate sensitively according to the condition, this method of calculation can be recognized as a good coincidental method.

Judging from the above mentioned comparisons, it is seen that this calculation method serves the purpose of determining the passive load generated on a support, and of estimating the stress and the deformation occuring on the support.

Furthermore, comparing Fig. 6 with Fig. 7, we can indeed understand that the stresses in the case where the active loads distribute asymmetrically are far greater than for the case where the active loads distribute symmetrically. However, the passive load in Fig. 7 is generated so that the total load, namely the sum of the active and the passive loads, should become more like a symmetrical distribution. In this way, the passive load prevents a more serious condition of stress. However, the total load never becomes a perfect symmetrical distribution.[1]

5. Conclusions

The authors have investigated the mechanical values generated in steel arch supports, such as the passive load, the stress and the deformation occuring when they are subjected to the active load, by a mathematical analysis as well as by full scale laboratory tests. Consequently, a calculation method for the design of steel arch supports has been proposed.

The generation of a passive load at the block point is mainly due to the fact that the deformation caused in the support by the active load is restricted on the rock ground side. The basic equation for the determination of a passive load can be derived by considering this fact.

On analyzing the behaviour of steel arch supports, we must know the conditions of the support ends and the joint at the top. The authors proposed a unique representation for the support end condition, based on the fact that small movements of the support ends are observed, and a negligible formation of a bending moment is possible. We also proposed the joint condition as a structure which has an axis line lower than an ordinary axis line by half the thickness of the steel support, of which the modulus of the longitudinal elasticity E', is equal to 1/160 times the longitudinal elasticity of steel.

A good agreement was observed between the results from the above mentioned calculations and the experiments. This indicates the appropriateness of the authors' approach.

In cases where the active loads distribute asymmetrically, the stress state is considerably unfavorable. However, the passive loads are generated so that the total loads should become more like a symmetrical distribution. Thus, a more serious condition in stress, is prevented, though a perfect symmetrical distribution of the total load can never be obtained.[1]

References

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2) S. Sakamoto : Trans. of the Japan Society of Civil Engineers, No. 88, p. 1 (1962)