Green's Tensors for Elastic Half-Spaces

——An Application of Boundary Integral Equation Method——

By

Shoichi KOBAYASHI* and Naoshi NISHIMURA*

(Received December 27, 1979)

ABSTRACT

The Fourier transform of Green's formula is utilized to obtain analytical solutions for half-space problems in the theory of elasticity. Mindlin's problem and its counterparts in transversely isotropic elastostatics and isotropic steady-state elastodynamics are solved. It is concluded that the present method is general enough to solve all these types of half-space problems.

Green's tensors thus obtained may be used effectively as kernel functions of the Boundary Integral Equation Method,

1. INTRODUCTION

Application of the computational Boundary Integral Equation Method[1] to boundary value problems seems to be one of the recent trends in solid mechanics. This method is summarized as a potential representation of the sought solution, discretization of unknown potential densities, and numerical solution of the resulting system of algebraic equations. Although the evaluation of singular integrals is required, many techniques have been proposed to overcome this difficulty, which made this method quite practical.

On the other hand, only a few analytical works[2),3)] applying potential theory to solve particular boundary value problems have been undertaken. Indeed, such a method may be generally considered impractical. However, we find it not to be the case for half-space problems, which we will discuss in what follows.

In § 2, we present the Fourier transform of Green's formula. Boundary conditions yield a system of integral equations, which reduces to algebraic equations for half-space problems. Solving these eqations, we express solutions of half-space problems in the form of Fourier integrals.

In § 3, we construct half-space Green's tensors of the 2nd kind (displacements induced by concentrated loads acting in the interior of the half-space with a traction-

* Department of Civil Engineering

free surface) for three cases, i. e., isotropic and transversely isotropic elastostatics and isotropic steady-state elastodynamics.

2. FOURIER TRANSFORM OF GREEN'S FORMULA

Let D be a domain bounded by the smooth surface ∂D. It is well-known that any solution u_i of elasticity satisfying some regularity requirements is expressed in the from[1),2)]

$$u_i(x)=\int_{\partial D}\Gamma_{ij}(x,y)\,t_j(y)\,dS_y-\int_{\partial D}\Gamma_{Iij}(x,y)\,u_j(y)\,dS_y+\int_D\Gamma_{ij}(x,y)\,b_j(y)\,dV_y\qquad x\in D$$

$$0=\int_{\partial D}\Gamma_{ij}(x,y)\,t_j(y)\,dS_y-\int_{\partial D}\Gamma_{Iij}(x,y)\,u_j(y)\,dS_y+\int_D\Gamma_{ij}(x,y)\,b_j(y)\,dV_y\qquad x\in\overline{\overline{D}},\qquad (1)^*$$

which is known as Green's formula, or Somigliana's identity. In eq. (1), t_j, b_j, Γ_{ij} and Γ_{Iij} stand for the surface traction with respect to the externally directed unit normal vector n_i, body force, fundamental solution of elasticity and double layer kernel, respectively. Γ_{Iij} is written as

$$\Gamma_{Iij}(x,y)=\Gamma_{ik}(x,y)\,\overleftarrow{\Sigma}_{yjmk}n_m(y),\qquad (2)$$

where Σ_{ijk} is the stress operator expressed by using the elasticity tensor C_{ijkl} and the del operator ∂_l as

$$\Sigma_{ijk}=C_{ijkl}\partial_l\,.\qquad (3)$$

The superposed arrow and subscript y in eq. (2) show that the differential operator Σ_{ijk} should be applied to Γ_{ij} with respect to y. By definition, we have

$$\Delta^*_{ik}\Gamma_{kj}(x,y)=-\delta_{ij}\delta(x-y),\qquad (4)$$

where Δ^*_{ik} is the operator of elasticity, or Navier's operator (see eqs. (18), (38) and (46)), δ_{ij} Kronecker delta, and δ Dirac delta.

It can be shown, using eqs. (1)-(4), that the Fourier transform of eq. (1) is written as

$$\hat{u}_i=-\Delta^{*-1}_{ij}(i\xi)\,\tilde{t}_j-\Delta^{*-1}_{ij}(i\xi)\,\Sigma_{klj}(i\xi)\,\widetilde{u_kn_l}-\Delta^{*-1}_{ij}(i\xi)\,\hat{b}_j,\qquad (5)^{**}$$

where $\wedge$ and $\sim$ are defined as

---

* Summation convention is employed for repeated indices. The Latin indices range from 1 to 3, whereas the Greek indices range from 1 to 2.

** i is used for $\sqrt{(-1)}$ instead of the usual notation i in order to avoid confusion.

$$\hat{\cdot} = \int_{R^3} \cdot (x) e^{-ix_i\xi_i} dx$$

$$\tilde{\cdot} = \int_{\partial D} \cdot (x) e^{-ix_i\xi_i} dS_x, \qquad (6)$$

$\Delta^{*-1}_{ij}(i\xi)$, $\Sigma_{klj}(i\xi)$ are matrices obtained by replacing ∂_i in their original forms by $i\xi_i$, and $\bar{u}_i$ is a function defined as

$$\bar{u}_i(x) = \begin{cases} u_i(x) & x \in D \\ 0 & x \bar{\in} \bar{D}. \end{cases} \qquad (7)$$

On the other hand, $\widetilde{u_k n_l}$ is given by $\hat{u}_i$ as

$$\begin{aligned} \widetilde{u_k n_l}(\xi) &= \int_{\partial D} e^{-i\xi_i y_i} u_k(y)\, n_l(y)\, dS_y \\ &= \frac{2}{(2\pi)^3} \int_{\partial D} e^{-i\xi_i y_i} n_l(y) \int_{R^3} e^{i\omega_i x_i} \hat{u}_k(\omega)\, d\omega dS_y \\ &= \int_{R^3} k_l(\omega - \xi)\, \hat{u}_k(\omega)\, d\omega, \end{aligned} \qquad (8)$$

where

$$k_l(\omega) = \frac{1}{4\pi^3} \int_{\partial D} e^{i\omega_i x_i} n_l(x)\, dS_x. \qquad (9)$$

In the calculation deriving eq. (8), we should be aware of the discontinuity of $\bar{u}_i$. From eqs. (5) and (8) follows a system of integral equations

$$\widetilde{u_k n_l}(\xi) = -\int_{R^3} k_l(\omega - \xi) \Big[\Delta^{*-1}_{kj}(i\omega)\, \tilde{t}_j(\omega) + \Delta^{*-1}_{kj}(i\omega)\, \Sigma_{mnj}(i\omega)\, \widetilde{u_m n_n}(\omega) + \Delta^{*-1}_{kj}(i\omega)\, \hat{b}_j(\omega) \Big] d\omega, \qquad (10)$$

which is intractable in general.

For half-space problems, eq. (10) simplifies considerably. Take the cartesian coordinate system as in Fig. 1. Then, $D = \{x \mid x_3 > 0\}$, $\partial D = \{x \mid x_3 = 0\}$ and $n = (0, 0, -1)$. Following the calculations mentioned at eq. (8), we have

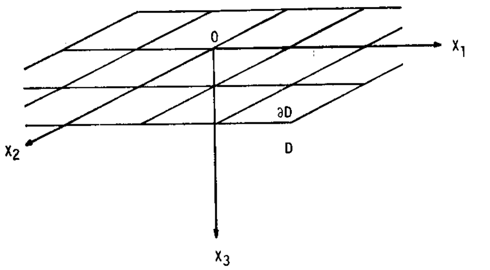

Fig. 1 ; Cartesian coordinate system used for half-space problems.

$$\tilde{u}_i(\xi_1, \xi_2) = \frac{1}{\pi} \int_p \hat{u}_i(\xi_1, \xi_2, \xi_3)\, d\xi_3 \qquad (11)$$

so that eq. (5) yields

$$\tilde{u}_i = -\frac{1}{\pi} \int_p \Delta^{*-1}_{ij}(i\xi)\, d\xi_3\, \tilde{t}_j + \frac{1}{\pi} \int_p \Delta^{*-1}_{ij}(i\xi)\, \Sigma_{k3j}(i\xi)\, d\xi_3 \tilde{u}_k - \frac{1}{\pi} \int_p \Delta^{*-1}_{ij}(i\xi)\, \hat{b}_j d\xi_3. \qquad (12)$$

In eqs. (11) and (12), the path of integration p, which extends from $-\infty$ to ∞, should be taken in the complex plane, such that the integral

$$-\frac{1}{(2\pi)^3}\int_{R^2} e^{i\xi_\alpha x_\alpha}\left(\int_p \Delta^{*-1}_{ij}(i\xi)\, d\xi_3\right) d\xi_1 d\xi_2$$

gives exactly Γ_{ij} at $x_3=0$. Also, it may be useful for some calculations to note that eq. (11) can also be written as

$$\tilde{u}_i(\xi_1,\ \xi_2)=\frac{1}{2\pi}\left(\lim_{\epsilon_1\downarrow 0}\int_p \hat{u}_i(\xi_1,\ \xi_2,\ \xi_3)\, e^{i\xi_3\epsilon_1}d\xi_3+\lim_{\epsilon_2\uparrow 0}\int_p \hat{u}_i(\xi_1,\ \xi_2,\ \xi_3)\, e^{i\xi_3\epsilon_2}d\xi_3\right), \tag{13}$$

which follows directly from the definition (eq. (7)).

If either u_i or t_i is prescribed on ∂D, the other is obtained from eq. (12) by algebraic calculations, so that all the terms appearing on the right hand side of eq. (5) are known.

After these calculations, the solution of the half-space problem is found in the from

$$u_i=\frac{1}{2\pi}F^{-1}_{1,2}\Big[-\int_p e^{i\xi_3x_3}\Delta^{*-1}_{ij}(i\xi)\, d\xi_3\tilde{t}_j+\int_p e^{i\xi_3x_3}\Delta^{*-1}_{ik}(i\xi)\,\Sigma_{j3k}(i\xi)\, d\xi_3\tilde{u}_j$$
$$-\int_p e^{i\xi_3x_3}\Delta^{*-1}_{ij}(i\xi)\,\hat{b}_j d\xi_3\Big], \tag{14}$$

where $F^{-1}_{1,2}$ stands for the Fourier inverse transform with respect to ξ_1 and ξ_2:

$$F^{-1}_{1,2}\,\cdot\ =\frac{1}{(2\pi)^2}\int_{R^2}\cdot\ (\xi_1,\ \xi_2)\, e^{ix_\alpha\xi_\alpha}d\xi_1 d\xi_2. \tag{15}$$

3. GREEN'S TENSORS

The theory presented in § 2 is applied to obtain Green's tensors of the 2nd kind for isotropic static, transversely isotropic static and isotropic harmonically oscillating elastic half-spaces.

We assume

$$t_i=0,\quad \overset{(j)}{b_i}(y)=\delta_{ij}\delta(y-y_0),\quad y_0=(0,\ 0,\ c). \tag{16}$$

Then, we have

$$\hat{b}^{(j)}_i(\xi)=\delta_{ij}e^{-ic\xi_3}. \tag{17}$$

3.1 ISOTROPIC ELASTOSTATICS

Equations of isotropic elastostatics are listed as follows:

$$\Delta^*_{ij}=\mu\delta_{ij}\partial_k\partial_k+(\lambda+\mu)\,\partial_i\partial_j, \tag{18}$$

$$\Sigma_{i3j}=\lambda\delta_{i3}\partial_j+\mu(\delta_{ij}\partial_3+\delta_{j3}\partial_i), \tag{19}$$

where λ and μ are Lamé constants. Using eqs. (18) and (19), we have

$$\Delta^{*-1}_{ij}(i\xi)=-\frac{(\lambda+2\mu)|\xi|^2\delta_{ij}-(\lambda+\mu)\xi_i\xi_j}{\mu(\lambda+2\mu)|\xi|^4}, \tag{20}$$

$$\Delta^{*-1}_{ij}\Sigma_{k3j}(i\xi)=i\left[\frac{2(\lambda+\mu)\xi_i\xi_k\xi_3-(\lambda+2\mu)|\xi|^2(\delta_{ik}\xi_3+\delta_{3i}\xi_k)-\lambda|\xi|^2\delta_{k3}\xi_i}{(\lambda+2\mu)|\xi|^4}\right], \tag{21}$$

where $|\xi|=\sqrt{(\xi_i\xi_i)}$. For the sake of generality, we suspend the assumption $t_i=0$ for the time being. As can be easily shown

$$\int_{-\infty}^{\infty}\Delta^{*-1}_{ij}(i\xi)\,d\xi_3=\begin{Vmatrix}-\frac{\pi\delta_{\alpha\beta}}{\mu r}+\frac{(\lambda+\mu)\pi}{2\mu(\lambda+2\mu)}\frac{\xi_\alpha\xi_\beta}{r^3} & 0\\ 0 & -\frac{(\lambda+3\mu)\pi}{2\mu(\lambda+2\mu)r}\end{Vmatrix}, \tag{22}$$

$$\int_{-\infty}^{\infty}\Delta^{*-1}_{ij}\Sigma_{k3j}(i\xi)\,d\xi_3=i\begin{Vmatrix}0 & \frac{\mu\pi\xi_\alpha}{(\lambda+2\mu)r}\\ -\frac{\mu\pi\xi_\beta}{(\lambda+2\mu)r} & 0\end{Vmatrix} \tag{23}$$

and

$$\int_{-\infty}^{\infty}\Delta^{*-1}_{ij}(i\xi)\,e^{-ic\xi_3}d\xi_3=\begin{Vmatrix}-\frac{\pi\delta_{\alpha\beta}}{\mu r}+\frac{(\lambda+\mu)\pi}{2\mu(\lambda+2\mu)}\left(\frac{1}{r^3}+\frac{c}{r^2}\right)\xi_\alpha\xi_\beta & -i\frac{(\lambda+\mu)\pi c\xi_\alpha}{2\mu(\lambda+2\mu)r}\\ -i\frac{(\lambda+\mu)\pi c\xi_\beta}{2\mu(\lambda+2\mu)r} & -\frac{(\lambda+3\mu)\pi}{2\mu(\lambda+2\mu)r}-\frac{(\lambda+\mu)\pi c}{2\mu(\lambda+2\mu)}\end{Vmatrix}e^{-rc}. \tag{24}$$

Eq. (12) can be written as

$$\begin{Vmatrix}\delta_{\alpha\beta} & -\frac{i\mu\xi_\alpha}{(\lambda+2\mu)r}\\ \frac{i\mu\xi_\beta}{(\lambda+2\mu)r} & 1\end{Vmatrix}\begin{Vmatrix}\tilde{u}_{\beta j}\\ \tilde{u}_{3j}\end{Vmatrix}=\begin{Vmatrix}\frac{\delta_{\alpha\beta}}{\mu r}-\frac{(\lambda+\mu)\xi_\alpha\xi_\beta}{2\mu(\lambda+2\mu)r^3} & 0\\ 0 & \frac{\lambda+3\mu}{2\mu(\lambda+2\mu)r}\end{Vmatrix}\begin{Vmatrix}\tilde{t}_{\beta j}\\ \tilde{t}_{3j}\end{Vmatrix}$$

$$+\begin{Vmatrix}\frac{\delta_{\alpha\beta}}{\mu r}-\frac{(\lambda+\mu)}{2\mu(\lambda+2\mu)}\left(\frac{1}{r^3}+\frac{c}{r^2}\right)\xi_\alpha\xi_\beta & i\frac{(\lambda+\mu)c\xi_\alpha}{2\mu(\lambda+2\mu)r}\\ i\frac{(\lambda+\mu)c\xi_\beta}{2\mu(\lambda+2\mu)r} & \frac{(\lambda+3\mu)}{2\mu(\lambda+2\mu)r}+\frac{(\lambda+\mu)c}{2\mu(\lambda+2\mu)}\end{Vmatrix}e^{-rc}, \tag{25}$$

where $r=\sqrt{(\xi_\alpha\xi_\alpha)}$. Inverting for $\tilde{u}_{ij}$, we have

$$\begin{Vmatrix}\tilde{u}_{\alpha j}\\ \tilde{u}_{3j}\end{Vmatrix}=\begin{Vmatrix}\frac{\delta_{\alpha\beta}}{\mu r}-\frac{\lambda}{2\mu(\lambda+\mu)}\frac{\xi_\alpha\xi_\beta}{r^3} & i\frac{\xi_\alpha}{2(\lambda+\mu)r^2}\\ -i\frac{\xi_\beta}{2(\lambda+\mu)r^2} & \frac{\lambda+2\mu}{2\mu(\lambda+\mu)r}\end{Vmatrix}\begin{Vmatrix}\tilde{t}_{\beta j}\\ \tilde{t}_{3j}\end{Vmatrix}$$

$$+\left\|\begin{matrix} \dfrac{\delta_{\alpha\beta}}{\mu r}-\left(\dfrac{\lambda}{2\mu(\lambda+\mu)r^3}+\dfrac{c}{2\mu}\right)\xi_\alpha\xi_\beta & i\left(\dfrac{1}{2(\lambda+\mu)r^2}+\dfrac{c}{2\mu r}\right)\xi_\alpha \\ -i\left(\dfrac{1}{2(\lambda+\mu)r^2}-\dfrac{c}{2\mu r}\right)\xi_\beta & \dfrac{\lambda+2\mu}{2\mu(\lambda+\mu)r}+\dfrac{c}{2\mu} \end{matrix}\right\| e^{-rc}\ . \tag{26}$$

Using eq. (26), we obtain $\tilde{u}_{ij}$ for the given $\tilde{t}_{ij}$. The solution of the problem may be constructed by using eq. (14) and the relations

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \Delta_{ij}^{*-1}(i\xi)\, e^{i\xi_3 x_3}d\xi_3=$$

$$\left\|\begin{matrix} -\dfrac{\delta_{\alpha\beta}}{2\mu r}+\dfrac{(\lambda+\mu)}{4\mu(\lambda+2\mu)}\left(\dfrac{|x_3|}{r^2}+\dfrac{1}{r^3}\right)\xi_\alpha\xi_\beta & i\dfrac{(\lambda+\mu)\,x_3\xi_\alpha}{4\mu(\lambda+2\mu)\,r} \\ i\dfrac{(\lambda+\mu)\,x_3\xi_\beta}{4\mu(\lambda+2\mu)\,r} & -\dfrac{\lambda+3\mu}{4\mu(\lambda+2\mu)\,r}-\dfrac{(\lambda+\mu)\,|x_3|}{4\mu(\lambda+2\mu)} \end{matrix}\right\| e^{-r|x_3|} \tag{27}$$

and

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \Delta^{*-1}_{ik}\Sigma_{j3k}(i\xi)\, e^{i\xi_3 x_3}d\xi_3=$$

$$\left\|\begin{matrix} \dfrac{\operatorname{sign}(x_3)\,\delta_{\alpha\beta}}{2}-\dfrac{(\lambda+\mu)\,x_3\xi_\alpha\xi_\beta}{2(\lambda+2\mu)\,r} & i\left(\dfrac{\mu}{2(\lambda+2\mu)\,r}-\dfrac{(\lambda+\mu)\,|x_3|}{2(\lambda+2\mu)}\right)\xi_\alpha \\ i\left(-\dfrac{\mu}{2(\lambda+2\mu)\,r}-\dfrac{(\lambda+\mu)\,|x_3|}{2(\lambda+2\mu)}\right)\xi_\beta & \dfrac{\operatorname{sign}(x_3)}{2}+\dfrac{(\lambda+\mu)\,r x_3}{2(\lambda+2\mu)} \end{matrix}\right\| e^{-r|x_3|}. \tag{28}$$

Specifically, when $t_j=0$, i. e., for a half-space free of surface traction, the last matrix on the right hand side of eq. (26) gives $\tilde{u}_{ij}$. From eqs. (14) and (28), we have

$$F_{1,2}u_{ij}=F_{1,2}\Gamma_{ij}(x_3-c)$$

$$+\left\|\begin{matrix} \dfrac{\delta_{\alpha\beta}}{2\mu r}-\left[\dfrac{(\lambda^2+2\lambda\mu-\mu^2)}{4\mu(\lambda+\mu)(\lambda+2\mu)r^3}+\dfrac{(\lambda+3\mu)(x_3+c)}{4\mu(\lambda+2\mu)r^2}-\dfrac{(\lambda+\mu)x_3c}{2\mu(\lambda+2\mu)r}\right]\xi_\alpha\xi_\beta & i\left[\dfrac{1}{2(\lambda+\mu)r^2}-\dfrac{(\lambda+3\mu)(x_3-c)}{4\mu(\lambda+2\mu)r}-\dfrac{(\lambda+\mu)x_3c}{2\mu(\lambda+2\mu)}\right]\xi_\alpha \\ i\left[-\dfrac{1}{2(\lambda+\mu)r^2}-\dfrac{(\lambda+3\mu)(x_3-c)}{4\mu(\lambda+2\mu)r}+\dfrac{(\lambda+\mu)x_3c}{2\mu(\lambda+2\mu)}\right]\xi_\beta & \dfrac{(\lambda+2\mu)^2+\mu^2}{4\mu(\lambda+\mu)(\lambda+2\mu)r}+\dfrac{(\lambda+3\mu)(x_3+c)}{4\mu(\lambda+2\mu)}+\dfrac{(\lambda+\mu)x_3cr}{2\mu(\lambda+2\mu)} \end{matrix}\right\| e^{-(x_3+c)r}, \tag{29}$$

where $\Gamma_{ij}(x)$ stands for the fundamental solution with its singularity at $(0, 0, x)$. The Fourier inverse transform of eq. (29) can be easily calculated by using known formulae of the Bessel functions. The result is

$$u_{ij}=\frac{1}{8\pi\mu}\left(\delta_{ij}\partial_k\partial_k-\frac{1}{2(1-\nu)}\partial_i\partial_j\right)R_1$$
$$+\frac{1}{16\pi\mu(1-\nu)}$$
$$\left\|\begin{array}{cc}\left[\frac{1}{R_2}+\frac{2cx_3}{R_2^3}+\frac{4(1-2\nu)(1-\nu)}{R_2+x_3+c}\right]\delta_{\alpha\beta}+\left[\frac{3-4\nu}{R_2^3}-\frac{4(1-2\nu)(1-\nu)}{R_2(R_2+x_3+c)^2}-\frac{6cx_3}{R_2^5}\right]x_\alpha x_\beta & \left[\frac{(3-4\nu)(x_3-c)}{R_2^3}+\frac{6x_3c(x_3+c)}{R_2^5}-\frac{4(1-\nu)(1-2\nu)}{R_2(R_2+x_3+c)}\right]x_\alpha \\ \left[\frac{(3-4\nu)(x_3-c)}{R_2^3}-\frac{6x_3c(x_3+c)}{R_2^5}+\frac{4(1-\nu)(1-2\nu)}{R_2(R_2+x_3+c)}\right]x_\beta & \frac{8(1-\nu)^2-(3-4\nu)}{R_2}+\frac{(3-4\nu)(x_3+c)^2-2x_3c}{R_2^3}+\frac{6x_3c(x_3+c)^2}{R_2^5}\end{array}\right\|, \tag{30}$$

where $R_1=\sqrt{(x_\alpha x_\alpha+(x_3-c)^2)}$, $R_2=\sqrt{(x_\alpha x_\alpha+(x_3+c)^2)}$ and ν is Poisson's ratio. Eq. (30) coincides with Mindlin's well-known result[4].

3.2 TRANSVERSELY ISOTROPIC ELASTOSTATICS

Assume that transversely isotropic elastic material occupies half-space $x_3>0$, with the plane of isotropy parallel to the boundary $x_3=0$. Then, we have

$$\begin{aligned}&\Delta^*_{\alpha\beta}=(c_{66}\partial_\gamma\partial_\gamma+c_{44}\partial_3\partial_3)\delta_{\alpha\beta}+(c_{11}-c_{66})\partial_\alpha\partial_\beta,\\&\Delta^*_{\alpha3}=\Delta^*_{3\alpha}=(c_{13}+c_{44})\partial_\alpha\partial_3,\qquad \Delta^*_{33}=c_{44}\partial_\gamma\partial_\gamma+c_{33}\partial_3\partial_3\end{aligned} \tag{31}$$

and

$$\Sigma_{\alpha3\beta}=c_{44}\delta_{\alpha\beta}\partial_3,\quad \Sigma_{\alpha33}=c_{44}\partial_\alpha,\quad \Sigma_{33\beta}=c_{13}\partial_\beta,\quad \Sigma_{333}=c_{33}\partial_3, \tag{32}$$

where c_{ij}'s are elastic constants. It is easy to see that

$$\Delta^{*-1}_{jk}(i\xi)=-$$
$$\left\|\begin{array}{cc}\frac{\delta_{\alpha\beta}}{c_{66}r^2+c_{44}\xi_3^2}+\left[\frac{c_{44}r^2+c_{33}\xi_3^2}{r^2c_{11}c_{44}\prod_{i=2}^{3}(r^2+\kappa_i\xi_3^2)}-\frac{1}{r^2(c_{66}r^2+c_{44}\xi_3^2)}\right]\xi_\alpha\xi_\beta & -\frac{(c_{13}+c_{44})\xi_3\xi_\alpha}{c_{11}c_{44}\prod_{i=2}^{3}(r^2+\kappa_i\xi_3^2)} \\ -\frac{(c_{13}+c_{44})\xi_3\xi_\beta}{c_{11}c_{44}\prod_{i=2}^{3}(r^2+\kappa_i\xi_3^2)} & \frac{c_{11}r^2+c_{44}\xi_3^2}{c_{11}c_{44}\prod_{i=2}^{3}(r^2+\kappa_i\xi_3^2)}\end{array}\right\| \tag{33}$$

and

$$\Delta^{*-1}_{jl}\Sigma_{k3l}(i\xi)$$

$$= -i\begin{Vmatrix} \dfrac{c_{44}\xi_3\delta_{\alpha\beta}}{c_{66}r^2+c_{44}\xi_3^2}-\left[\dfrac{(c_{13}r^2-c_{33}\xi_3^2)\xi_3}{c_{11}\prod\limits_{i=2}^{3}(r^2+\kappa_i\xi_3^2)}+\dfrac{c_{44}\xi_3}{c_{66}r^2+c_{44}\xi_3^2}\right]\dfrac{\xi_\alpha\xi_\beta}{r^2} & \dfrac{(c_{13}r^2-c_{33}\xi_3^2)\xi_\alpha}{c_{11}\prod\limits_{i=2}^{3}(r^2+\kappa_i\xi_3^2)} \\ \dfrac{(c_{11}r^2-c_{13}\xi_3^2)\xi_\beta}{c_{11}\prod\limits_{i=2}^{3}(r^2+\kappa_i\xi_3^2)} & \left[\dfrac{(c_{11}c_{33}-c_{13}(c_{13}+c_{44}))r^2+c_{33}c_{44}\xi_3^2}{c_{11}c_{44}\prod\limits_{i=2}^{3}(r^2+\kappa_i\xi_3^2)}\right]\xi_3 \end{Vmatrix}, \quad (34)$$

where κ_2 and κ_3 are the roots of the equation

$$\kappa^2-\frac{[c_{44}^2+c_{11}c_{33}-(c_{13}+c_{44})^2]}{c_{11}c_{44}}\kappa+\frac{c_{33}}{c_{11}}=0 \quad (35)$$

and $\kappa_1=c_{44}/c_{66}$. For the sake of simplicity, we assume $\kappa_2 \neq \kappa_3$ in this paper, though the degenerate case might be treated in the same way.

Because of the similarity to the isotropic case, we shall list here only the major results:

$$F_3^{-1}(\Delta^{*-1}(i\xi))_{kj}$$

$$=\frac{1}{2}\begin{Vmatrix} \dfrac{e^{-\frac{|x_3|}{\sqrt{\kappa_1}}r}}{c_{66}\sqrt{\kappa_1}\,r}\delta_{\alpha\beta}+\left[\dfrac{1}{c_{44}}\sum\limits_{i=2}^{3}K_i\dfrac{e^{-\frac{|x_3|}{\sqrt{\kappa_i}}r}}{\sqrt{\kappa_i}\,r^3}-\dfrac{e^{-\frac{|x_3|}{\sqrt{\kappa_1}}r}}{c_{66}\sqrt{\kappa_1}\,r^3}\right]\xi_\alpha\xi_\beta & -i\,\mathrm{sign}(x_3)\sum\limits_{i=2}^{3}\left(\dfrac{L_ie^{-\frac{|x_3|}{\sqrt{\kappa_i}}r}}{c_{44}\sqrt{\kappa_i}\,r^2}\right)\xi_\alpha \\ -i\,\mathrm{sign}(x_3)\sum\limits_{i=2}^{3}\left(\dfrac{L_ie^{-\frac{|x_3|}{\sqrt{\kappa_i}}r}}{c_{44}\sqrt{\kappa_i}\,r^2}\right)\xi_\beta & \sum\limits_{i=2}^{3}\dfrac{M_ie^{-\frac{|x_3|}{\sqrt{\kappa_i}}r}}{c_{44}\sqrt{\kappa_i}\,r} \end{Vmatrix}, \quad (36)$$

$$F_3^{-1}(\Delta^{*-1}\Sigma(i\xi))_{jk}$$

$$=\frac{1}{2}\begin{Vmatrix} \mathrm{sign}(x_3)\left(e^{-\frac{|x_3|}{\sqrt{\kappa_1}}r}\delta_{\alpha\beta}+\left\{\sum\limits_{i=2}^{3}\left(\dfrac{K_i}{\kappa_i}-\dfrac{L_i}{\sqrt{\kappa_i}}\right)\dfrac{e^{-\frac{|x_3|}{\sqrt{\kappa_i}}r}}{r^2}-\dfrac{e^{-\frac{|x_3|}{\sqrt{\kappa_1}}r}}{r^2}\right\}\xi_\alpha\xi_\beta\right) & i\sum\limits_{i=2}^{3}\left(\dfrac{K_i}{\sqrt{\kappa_i}}-L_i\right)\dfrac{e^{-\frac{|x_3|}{\sqrt{\kappa_i}}r}}{r}\xi_\alpha \\ -i\sum\limits_{i=2}^{3}\left(\dfrac{M_i}{\sqrt{\kappa_i}}+\dfrac{L_i}{\kappa_i}\right)\dfrac{e^{-\frac{|x_3|}{\sqrt{\kappa_i}}r}}{r}\xi_\beta & \mathrm{sign}(x_3)\sum\limits_{i=2}^{3}\left(M_i+\dfrac{L_i}{\sqrt{\kappa_i}}\right)e^{-\frac{|x_3|}{\sqrt{\kappa_i}}r} \end{Vmatrix}, \quad (37)$$

$$\left[I-\frac{1}{\pi}\int_{-\infty}^{\infty}\Delta^{*-1}\Sigma d\xi_3\right]_{jk}=\begin{Vmatrix} \delta_{\alpha\beta} & i\dfrac{A_1\xi_\alpha}{r} \\ i\dfrac{A_2\xi_\beta}{r} & 1 \end{Vmatrix}, \quad (38)$$

$$\left[I-\frac{1}{\pi}\int_{-\infty}^{\infty}\Delta^{*-1}\Sigma d\xi_3\right]_{jk}^{-1}=\left\|\begin{matrix}\delta_{\alpha\beta}-\frac{A_1A_2}{(1+A_1A_2)}\frac{\xi_\alpha\xi_\beta}{r^2} & -i\frac{A_1}{(1+A_1A_2)}\frac{\xi_\alpha}{r} \\ -i\frac{A_2}{(1+A_1A_2)}\frac{\xi_\beta}{r} & \frac{1}{1+A_1A_2}\end{matrix}\right\| \quad (39)$$

and

$$\left[-\frac{1}{\pi}\int_{-\infty}^{\infty}\Delta^{*-ic\xi_3}d\xi_3\right]_{jk}=$$

$$\left\|\begin{matrix}\frac{e^{-\frac{c}{\sqrt{\kappa_1}}r}}{c_{66}\sqrt{\kappa_1}\,r}\delta_{\alpha\beta}+\left(\sum_{i=2}^{3}\frac{K_ie^{-\frac{c}{\sqrt{\kappa_i}}r}}{c_{44}\sqrt{\kappa_i}\,r^3}-\frac{e^{-\frac{c}{\sqrt{\kappa_1}}r}}{c_{66}\sqrt{\kappa_1}\,r_3}\right)\xi_\alpha\xi_\beta & i\left(\sum_{i=2}^{3}\frac{L_ie^{-\frac{c}{\sqrt{\kappa_i}}r}}{c_{44}\sqrt{\kappa_i}\,r^2}\right)\xi_\alpha \\ i\left(\sum_{i=2}^{3}\frac{L_ie^{-\frac{c}{\sqrt{\kappa_i}}r}}{c_{44}\sqrt{\kappa_i}\,r^2}\right)\xi_\beta & \sum_{i=2}^{3}\frac{M_ie^{-\frac{c}{\sqrt{\kappa_i}}r}}{c_{44}\sqrt{\kappa_i}\,r}\end{matrix}\right\|, \quad (40)$$

where

$$K_2=\frac{\kappa_2c_{44}-c_{33}}{c_{11}(\kappa_2-\kappa_3)},\qquad L_2=\frac{(c_{13}+c_{44})\sqrt{\kappa_2}}{c_{11}(\kappa_2-\kappa_3)},\qquad M_2=\frac{c_{11}\kappa_2-c_{44}}{c_{11}(\kappa_2-\kappa_3)},$$

$$A_1=-\sum_{i=2}^{3}\left(\frac{K_i}{\sqrt{\kappa_i}}-L_i\right),\qquad A_2=\sum_{i=2}^{3}\left(\frac{M_i}{\sqrt{\kappa_i}}+\frac{L_i}{\kappa_i}\right). \quad (41)$$

L_3, K_3, M_3 are obtained by interchanging κ_2 and κ_3 in eq. $(41)_{1,2,3}$.

The fundamental solution of the transversely isotropic body may be calculated using eq. (36) to give

$$\Gamma_{jk}=\frac{1}{4\pi}$$

$$\left\|\begin{matrix}\frac{1}{c_{66}}\left(\frac{|x_3|}{\bar{R}_1\bar{R}_1^+}\delta_{\alpha\beta}+\frac{\kappa_1x_\alpha x_\beta}{\bar{R}_1(\bar{R}_1^+)^2}\right)+\sum_{i=2}^{3}\frac{K_i}{c_{44}}\left(\frac{\delta_{\alpha\beta}}{\bar{R}_i^+}-\frac{\kappa_ix_\alpha x_\beta}{\bar{R}_i(\bar{R}_i^+)^2}\right) & \mathrm{sign}(x_3)\sum_{i=2}^{3}\frac{L_i\sqrt{\kappa_i}\,x_\alpha}{c_{44}\bar{R}_i\bar{R}_i^+} \\ \mathrm{sign}(x_3)\sum_{i=2}^{3}\frac{L_i\sqrt{\kappa_i}\,x_\beta}{c_{44}\bar{R}_i\bar{R}_i^+} & \sum_{i=2}^{3}\frac{M_i}{c_{44}\bar{R}_i}\end{matrix}\right\|, \quad (42)$$

where $\bar{R}_i=\sqrt{(\kappa_ix_\alpha x_\alpha+x_3^2)}$, $\bar{R}_i^+=\bar{R}_i+|x_3|$ $(1\leqslant i\leqslant 3)$, which coincides with Kröner's result within a constant factor $-1/c_{11}c_{44}c_{66}$, which is known to be missing in his paper[6].

For the calculation of Green's tensor of the 2nd kind, we use eqs. (12), (39), (40) to have

$$\hat{u}_{jk}=$$

$$\left\| \begin{matrix} \dfrac{e^{-\frac{c}{\sqrt{\kappa_1}}r}}{c_{66}\sqrt{\kappa_1}\ r}\delta_{\alpha\beta}+\left[\dfrac{1}{(1+A_1A_2)c_{44}}\displaystyle\sum_{i=2}^{3}\left(\dfrac{K_i}{\sqrt{\kappa_i}}+\dfrac{A_1L_i}{\sqrt{\kappa_i}}\right)\dfrac{e^{-\frac{c}{\sqrt{\kappa_i}}r}}{r^3}-\dfrac{e^{-\frac{c}{\sqrt{\kappa_1}}r}}{c_{66}\sqrt{\kappa_1}\ r^3}\right]\xi_\alpha\xi_\beta & \dfrac{i\xi_\alpha}{(1+A_1A_2)c_{44}}\displaystyle\sum_{i=2}^{3}\left(\dfrac{L_i}{\sqrt{\kappa_i}}-\dfrac{A_1\kappa_i}{\sqrt{\kappa_i}}\right)\dfrac{e^{-\frac{c}{\sqrt{\kappa_i}}r}}{r^2} \\ \dfrac{i\xi_\beta}{(1+A_1A_2)c_{44}}\displaystyle\sum_{i=2}^{3}\left(\dfrac{L_i}{\sqrt{\kappa_i}}-\dfrac{A_2K_1}{\sqrt{\kappa_i}}\right)\dfrac{e^{-\frac{c}{\sqrt{\kappa_i}}r}}{r^2} & \dfrac{1}{(1+A_1A_2)c_{44}}\displaystyle\sum_{i=2}^{3}\left(\dfrac{M_i}{\sqrt{\kappa_i}}+\dfrac{A_2L_i}{\sqrt{\kappa_i}}\right)\dfrac{e^{-\frac{c}{\sqrt{\kappa_i}}r}}{r} \end{matrix} \right\|. \quad (43)$$

Using eqs. (14) and (37), we have

$$u_{kl}=\Gamma_{kl}(x_3-c)+\Gamma_{kl}(x_3+c)$$

$$+\frac{1}{2\pi c_{44}}\sum_{i,j=2}^{3}\left\| \begin{matrix} B_{ij}^1\left(\dfrac{1}{R_{ij}^+}\delta_{\alpha\beta}-\dfrac{\kappa_i x_\alpha x_\beta}{(R_{ij}^+)^2R_{ij}}\right) & -B_{ij}^2\dfrac{\sqrt{\kappa_i}\ x_\alpha}{R_{ij}^+\ R_{ij}} \\ B_{ij}^3\dfrac{\sqrt{\kappa_i}\ x_\beta}{R_{ij}^+\ R_{ij}} & B_{ij}^4\dfrac{1}{R_{ij}} \end{matrix} \right\|, \quad (44)$$

where

$$B_{ij}^1=\frac{(K_i-\sqrt{\kappa_i}\ L_i)(A_2K_j-L_j)}{\sqrt{\kappa_j}\ (1+A_1A_2)},\quad B_{ij}^2=\frac{(K_i-\sqrt{\kappa_i}\ L_i)(L_j-A_1M_j)}{\sqrt{\kappa_i}\sqrt{\kappa_j}\ (1+A_1A_2)},$$

$$B_{ij}^3=\frac{(\sqrt{\kappa_i}\ M_i+L_i)(A_2K_j-L_j)}{\sqrt{\kappa_j}\ (1+A_1A_2)},\quad B_{ij}^4=\frac{(\sqrt{\kappa_j}\ M_i+L_i)(L_j-A_1M_j)}{\sqrt{\kappa_i}\sqrt{\kappa_j}\ (1+A_1A_2)},$$

$$R_{ij}=\sqrt{\kappa_i x_\alpha x_\alpha+(x_3+\sqrt{\frac{\kappa_i}{\kappa_j}}c)^2},\quad R_{ij}^+=R_{ij}+x_3+\sqrt{\frac{\kappa_i}{\kappa_j}}c. \quad (45)$$

This problem was studied earlier by H. Okamura and I. Shimada[7)], but presented incompletely. Recently, Y. -C. Pan and T. -W. Chou[8)] solved the same problem including the degenerate case. However, as there are many misprints and undefined symbols in their paper, we gave up efforts to compare our result with theirs. Instead, we successfully checked our solution as vanishing when $x_3<0$, and to have the required reciprocal property, thus ensuring the validity of our result.

3.3 ISOTROPIC STEADY-STATE ELASTODYNAMICS

We assume $u(x)e^{-i\omega t}$ type time-harmonic solution. Then, the equation of isotropic steady-state elastodynamics takes the following form:

$$\Delta^*_{ij}=\mu\delta_{ij}\,\partial_k\,\partial_k+(\lambda+\mu)\,\partial_i\,\partial_j+\rho\omega^2\,\delta_{ij} \tag{46}$$

with $\sum$ being the same as the static case (eq. (19)). Using. (46), we have

$$\Delta^{*-1}_{ij}(i\xi)=\frac{1}{\mu}\left(\frac{\delta_{ij}}{k_T^2-|\xi|^2}-\frac{1}{k_T^2}\left(\frac{1}{k_T^2-|\xi|^2}-\frac{1}{k_L^2-|\xi|^2}\right)\xi_i\,\xi_j\right) \tag{47}$$

and

$$\Delta^{*-1}_{ik}\textstyle\sum_{j3k}(i\xi)=\frac{i}{\mu}\Big\{\frac{\lambda\xi_i\delta_{j3}+\mu\xi_3\delta_{ij}+\mu\delta_{i3}\xi_j}{k_T^2-|\xi|^2}$$

$$-\frac{1}{k_T^2}\left[\lambda\left(\frac{k_T^2}{k_T^2-|\xi|^2}-\frac{k_L^2}{k_L^2-|\xi|^2}\right)\xi_i\delta_{j3}+2\mu\left(\frac{1}{k_T^2-|\xi|^2}-\frac{1}{k_L^2-|\xi|^2}\right)\xi_i\xi_j\xi_3\right]\Big\}, \tag{48}$$

where ω, ρ, k_T and k_L are the frequency, density, and wave numbers of the transverse and longitudinal waves, respectively.

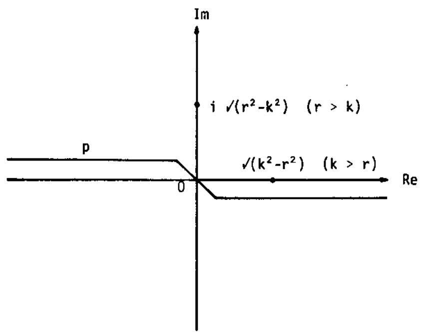

Fig. 2 ; Path of integration.

In the two previous examples, we did not need to consider what path of integration p, mentioned in eq. (11), should be taken. For steady-state elastodynamics, however, the requirement of the radiation condition for Γ_{ij} determines p. We shall take p as shown in Fig. 2. The following calculation of the fundamental solution for the Helmholtz equation illustrates this conclusion:

$$-\frac{1}{(2\pi)^3}\iint\left(\int_p\frac{e^{i\xi_3x_3}}{k^2-r^2-\xi_3^2}d\xi_3\right)d\xi_1d\xi_2=\frac{1}{4\pi}\int_0^\infty\frac{J_0(Rr)\,e^{-i\sqrt{r^2-k^2}|x_3|}\,r\,dr}{\sqrt{r^2-k^2}}$$

$$=\frac{e^{ik\sqrt{R^2+x_3^2}}}{4\pi\sqrt{R^2+x_3^2}}, \tag{49}$$

where $R=\sqrt{(x_\alpha x_\alpha)}$ and $\sqrt{(r^2-k^2)}$ is evaluated as $-i\sqrt{(k^2-r^2)}$ for $r<k$. Note that $\lim_{\epsilon\downarrow 0}\sqrt{(r^2-(k+i\epsilon)^2)}$ takes this value, if we choose for the square root a branch continuously varying near the real axis and with positive real part.

Now, we shall list the major results:

$$F_3^{-1}(\Delta^{*-1}(i\xi))_{ij}$$

$$=\frac{1}{2\mu}\begin{Vmatrix} -\frac{\delta_{\alpha\beta}}{R_T}e^{-|x_3|R_T}+\frac{\xi_\alpha\xi_\beta}{k_T^2}\left(\frac{e^{-|x_3|R_T}}{R_T}-\frac{e^{-|x_3|R_L}}{R_L}\right) & \operatorname{sign}(x_3)\frac{i\xi_\alpha}{k_T^2}(e^{-|x_3|R_T}-e^{-|x_3|R_L}) \\ \operatorname{sign}(x_3)\frac{i\xi_\beta}{k_T^2}(e^{-|x_3|R_T}-e^{-|x_3|R_L}) & -\frac{e^{-|x_3|R_T}}{R_T}-\frac{1}{k_T^2}(R_Te^{-|x_3|R_T}-R_Le^{-|x_3|R_L}) \end{Vmatrix}, \tag{50}$$

$$F_3^{-1}(\Delta^{*-1}\Sigma(i\xi))_{ij}$$

$$=\frac{1}{2}\begin{Vmatrix} \operatorname{sign}(x_3)\left[e^{-|x_3|R_T}\delta_{\alpha\beta}-\frac{2}{k_T^2}(e^{-|x_3|R_T}-e^{-|x_3|R_L})\xi_\alpha\xi_\beta\right] & -i\xi_\alpha\left[\frac{\lambda e^{-|x_3|R_L}}{(\lambda+2\mu)R_L}+\frac{2}{k_T^2}(R_Te^{-|x_3|R_T}-R_Le^{-|x_3|R_L})\right] \\ -i\xi_\beta\left[\frac{e^{-|x_3|R_T}}{R_T}+\frac{2}{k_T^2}(R_Te^{-|x_3|R_T}-R_Le^{-|x_3|R_L})\right] & \operatorname{sign}(x_3)\left[e^{-|x_3|R_L}+\frac{2r^2}{k_T^2}(e^{-|x_3|R_T}-e^{-|x_3|R_L})\right] \end{Vmatrix} \tag{51}$$

and

$$\left[I-\frac{1}{\pi}\int_p \Delta^{*-1}\Sigma(i\xi)\,d\xi_3\right]_{ij}^{-1}$$

$$=\frac{1}{G(r)}\begin{Vmatrix} \frac{(R_TR_L-r^2)}{k_T^4R_TR_L}F(r)\delta_{\alpha\beta}+\frac{\xi_\alpha\xi_\beta}{R_TR_L}\left[1+\frac{2}{k_T^2}(R_TR_L-r^2)\right]^2 & -i\frac{\xi_\alpha}{R_L}\left[1+\frac{2}{k_T^2}(R_TR_L-r^2)\right] \\ \frac{i\xi_\beta}{R_T}\left[1+\frac{2}{k_T^2}(R_TR_L-r^2)\right] & 1 \end{Vmatrix}, \tag{52}$$

where $R_T=\sqrt{(r^2-k_T^2)}, R_L=\sqrt{(r^2-k_L^2)}$ with the above mentioned sign,

$F(r)=(2r^2-k_T^2)^2-4r^2R_TR_L$ (Rayleigh's function)

and

$$G(r)=(R_TR_L-r^2)F(r)/k_T^4R_TR_L.$$

For the calcuation of Green's tensor of the 2nd kind, we use eqs. (12), (52) and

$$\left(-\frac{1}{\pi}\int_p \Delta^{*-1}\,(i\xi)\,e^{-i\xi_3 c}d\xi_3\right)_{ij}$$

$$=\frac{1}{\mu}\left\|\begin{matrix}\frac{\delta_{\alpha\beta}}{R_T}e^{-cR_T}-\frac{\xi_\alpha\xi_\beta}{k_T^2}\left(\frac{e^{-cR_T}}{R_T}-\frac{e^{-cR_L}}{R_L}\right) & \frac{i\xi_\alpha}{k_T^2}(e^{-cR_T}-e^{-cR_L}) \\ \frac{i\xi_\beta}{k_T^2}(e^{-cR_T}-e^{-cR_L}) & \frac{e^{-cR_T}}{R_T}+\frac{1}{k_T^2}(R_T e^{-cR_T}-R_L e^{-cR_L})\end{matrix}\right\| \quad (53)$$

to have

$$\tilde{u}_{ij}=\frac{1}{\mu}\left\|\begin{matrix}\frac{\delta_{\alpha\beta}}{R_T}e^{-cR_T}+\xi_\alpha\xi_\beta\left[\frac{(4R_TR_L-2r^2+k_T^2)e^{-cR_T}-2R_T^2e^{-cR_L}}{R_TF(r)}\right] & -i\xi_\alpha\left[\frac{(2r^2-k_T^2)e^{-cR_T}-2R_TR_Le^{-cR_L}}{F(r)}\right] \\ i\xi_\beta\left[\frac{(2r^2-k_T^2)e^{-cR_L}-2R_TR_Le^{-cR_T}}{F(r)}\right] & -R_L\left[\frac{2r^2e^{-cR_T}+(k_T^2-2r^2)e^{-cR_L}}{F(r)}\right]\end{matrix}\right\|. \quad (54)$$

Now, assume Im $\omega=\epsilon$ ($\epsilon>0$). Using eqs. (14), (51) and (54), we have

$$u_{ij}=\Gamma_{ij}(x_3-c)+\Gamma_{ij}(x_3+c)$$
$$+\frac{1}{2\pi\mu}\left\|\begin{matrix}-\frac{\delta_{\alpha\beta}}{R}\int_0^\infty A(r)\,r^2J_1(Rr)\,dr+\frac{x_\alpha x_\beta}{R^2}\int_0^\infty A(r)\,r^3J_2(Rr)\,dr & -\frac{x_\alpha}{R}\int_0^\infty B(r)\,r^2J_1(Rr)\,dr \\ \frac{x_\beta}{R}\int_0^\infty C(r)\,r^2J_1(Rr)\,dr & -\int_0^\infty D(r)\,r^3J_0(Rr)\,dr\end{matrix}\right\|, \quad (55)$$

where

$$A(r)=\frac{(2R_TR_Le^{-x_3R_T}-(2r^2-k_T^2)e^{-x_3R_L})(2R_TR_Le^{-cR_T}-(2r^2-k_T^2)e^{-cR_L})}{k_T^2R_LF(r)},$$

$$B(r)=\frac{(2r^2e^{-x_3R_L}-(2r^2-k_T^2)e^{-x_3R_T})(2R_TR_Le^{-cR_L}-(2r^2-k_T^2)e^{-cR_T})}{k_T^2F(r)},$$

$$C(r)=\frac{(2r^2e^{-x_3R_T}-(2r^2-k_T^2)e^{-x_3R_L})(2R_TR_Le^{-cR_T}-(2r^2-k_T^2)e^{-cR_L})}{k_T^2F(r)},$$

$$D(r)=\frac{(2R_TR_Le^{-x_3R_L}-(2r^2-k_T^2)e^{-x_3R_T})(2R_TR_Le^{-cR_L}-(2r^2-k_T^2)e^{-cR_T})}{k_T^2R_TF(r)} \quad (56)$$

and the fundamental solution Γ_{ij} is well-known to have the following from[2]

$$\Gamma_{ij}=\frac{1}{4\pi\mu}\left(\frac{e^{ik_T|x|}}{|x|}\delta_{ij}+\frac{\partial_i\partial_j}{k_T^2}\left(\frac{e^{ik_T|x|}}{|x|}-\frac{e^{ik_L|x|}}{|x|}\right)\right), \tag{57}$$

where $|x|=\sqrt{(x_i x_i)}$. The solution for real ω may be obtained by letting $\epsilon\downarrow 0$.

Eq. (55) and its extension to a layered half-space[9] are known to seismologists. Recently, O. Matsuoka and K. Yahata[10] studied this problem again.

4. CONCLUSION

Green's formula in the theory of elasticity was shown to provide an analytical method of solution for half-space problems by using Fourier transform. Half-space Green's tensors of the 2nd kind for isotropic elastostatics, transversely isotropic elastostatics and isotropic steady-state elastodynamics were constructed by the present method.

It may be said that the most remarkable feature of the present method is its constructiveness. Such penetrating insight as Mindlin had when he solved his point load problem for the first time is not required.

The present method is easily extended to problems for layered media with slight modification.

Green's tensors thus obtained may be used effectively as kernel functions of the Boundary Integral Equation Method.

REFERENCES

1) Cruse, T. A. and F. J. Rizzo (Ed.) ; "Boundary Integral Equation Method : Computational Applications in Applied Mechanics," Proc. ASME AMD vol. 11, 1975.
2) Kupradze, V. D.; "Potential Methods in the Theory of Elasticity," (Tr. H. Gotfreund), Israel program for scientific translatons, Jerusalem, 1965.
3) Jentsch, L.; ZAMM, **48**, pp 177-126, 1968.
4) Mindlin, R. D.; Physics, **7**, pp 195-202, 1936.
5) Kröner, E.; Z. Physik, **136**, pp 402-410, 1953.
6) Pan, Y. -C. and T. -W. Chou; J. Appl. Mech., **43**, pp 608-612, 1976.
7) Okamura, H. and I. Shimada; Preprint for 31st annual meeting of JSCE, I, pp 30-31, 1976.
8) Pan, Y. -C. and T. -W. Chou; Int. J. Eng. Sci., **17**, pp 545-551, 1979.
9) Harkrider, D. G. ; Bull. Seism. Soc. Am., **54**, pp 627-679, 1964.
10) Matsuoka, O. and K. Yahata; Preprint for 28th Japan Nat. Congr. Appl. Mech., pp 79-80, 1978.