Analysis of Transient Response of Magnetic Flux in Nonlinear Magnetic Circuit

By

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Abstract

In this paper, we first introduce some new two-dimensional theoretical equations for the transient response of the magnetic flux in a fundamental nonlinear magnetic circuit which can be applied to that containing the interpole in a dc machine. Then we develop a numerical method for solving those theoretical nonlinear equations, in which a difference analogue is used, derived from both the non-uniform grid system for space and the Crank-Nicolson method for the time derivative. Next, the transient response of the flux and distribution of the magnetic field intensity and flux density in the interpole are investigated in a case where the step exciting current is supplied to the interpole winding. As a result, it is clearly seen that the magnetic saturation occurs because of a strong magnetic field intensity in the skin of the interpole. This is caused by an eddy current induced in the interpole, and the saturation has a large influence on the flux response. Hence, the flux response is too fast in the numerical calculation results by the conventional linear theory in which the saturation is not considered. Also, it is noted that the transient response is influenced very much by the conductivity of the pole as well as the lengths and cross-sectional areas of the pole and pole gap. Furthermore, we calculate the flux response in a case where a full-wave rectified magnetomotive-force is applied to the interpole winding, whereby it is clarified that the wave form of the flux response is distorted largely by the saturation.

1. Introduction

In recent years, according to the development of system control techniques, it is required that the electrical machine and apparatus are controlled accurately as elements of the sytem. In order to control the machine accurately, not only research of higher control technique, but also a control sufficiently considering the transient response of the magnetic flux in magnetic circuits in the machine are required. In particular, an analysis of the flux response is very important in a direct current(dc) machine, because deterioration of speed-torque characteristics, commutation sparks etc. may be caused when the flux response corresponding to

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the change of the exciting magnetomotive-force(mmf) of the main pole or the interpole is delayed.[1]

In a conventional dc motor, both the armature and the main pole consist of laminated cores, and the interpole and the yoke solid cores have an electrically isotropic conductivity. An eddy current is induced in the latter cores when the interpole exciting current changes. The eddy current produces an ohmic loss and also delays the flux response in the interpole. Previously Rüdenbrg, and more recently Švajcr, Sakabe, Okitsu et al. have investigated the delay of the transient response of the flux by an eddy current.[2-5] However, they studied the transient delay by means of linear theories, in which the permeability of the solid cores is assumed constant. They underestimated the delay because they neglected the concentration of the magnetic field and the magnetic saturation occuring in the skin region of the interpole core when the exciting mmf changes abruptly. Therefore, to calculate the transient response of the flux, a nonlinear theory considering the magnetic saturation has to be applied.

This paper first introduces some new two-dimensional theoretical equations for the transient response of the magnetic flux in a fundamental nonlinear magnetic circuit which is applicable to the circuit containing the interpole in a dc machine. We then develop a numerical method for solving those theoretical nonlinear equations, in which a difference analogue is used, derived from both the nonuniform grid system for space and the Crank-Nicolson method for the time derivative. Next, we examine the transient response of the flux and distributions of the magnetic field intensity and flux density in the interpole, when the step exciting current is supplied to the interpole winding.

As a result, it is clearly seen that the magnetic saturation occurs because of a strong magnetic field intensity in the skin of the pole. This is caused by an eddy current induced in the pole, and the saturation has a large influence on the flux response. Also, it is noted that the flux response is greatly affected by the conductivity of the pole and the size of the magnetic circuit. Furthermore, we calculate the flux response when a full-wave rectified mmf is applied to the interpole winding, whereby it is clarified that the wave form of the flux response is largley distorted by the saturation.

2. Theory of Transient Response of Magnetic Flux

2.1 Magnetic circuit model and assumption

We assume that the dc motor has an armature and main pole of laminated silicon steel plates and also has the interpole and yoke of solid cores. The closed

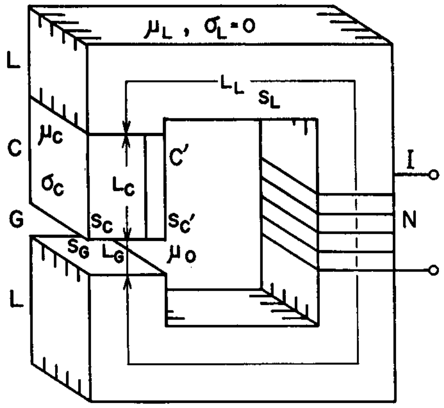

Fig. 1. Magnetic circuit model.

magnetic circuit containing the interpole is approximately expressed by the model shown in Fig. 1,[2-4] where the influences of the main pole and slots of the armature are neglected. In this figure, L, C and G represent the laminated core, the conductive core and the air gap, respectively. These correspond to the armature, the interpole and yoke, and the interpole air gap, respectively. C' is a bypass magnetic circuit of C, which corresponds to an air gap to be compensated due to the discrepancy in the cross-sectional areas of L, C and G. Also l_C, l_G and l_L are the lengths of C, G and L, respectively, S_C, $S_{C'}$, S_G and S_L the cross-sectional areas of C, C', G and L perpendicular to each longitudinal direction, μ_C and μ_L the permeabilities, σ_C and σ_L the conductivities of C and L, respectively, μ_0 the permeability of the air gap which is assumed to be equal to that of the vacuum, and I and N are the current and numbers of the interpole winding, respectively.

In order to find an accurate transient response of the flux in the magnetic circuit of Fig. 1 when the mmf NI is supplied to the circuit, the following Maxwell's equations

$$\left.\begin{array}{l} \text{rot } \boldsymbol{H} = \boldsymbol{J}, \text{ rot } \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t, \text{ div } \boldsymbol{B} = 0\,, \\ \boldsymbol{J} = \sigma \boldsymbol{E}, \boldsymbol{B} = \mu \boldsymbol{H}\,, \end{array}\right\} \qquad (1)$$

must be solved. Here $\boldsymbol{H}$, $\boldsymbol{B}$, $\boldsymbol{E}$ and $\boldsymbol{J}$ represent the magnetic field intensity, the magnetic flux density, the electric field intensity and the current density vectors, respectively, and $\mu = \mu_C$, μ_L or μ_0 and $\sigma = \sigma_C$, σ_L or 0.

It is too difficult to solve the Eqs. (1). For simplifying an analysis and yet obtaining sufficiently accurate solutions, let us assume the following:

(1) Permeabilities μ_C and μ_L are much larger than μ_0, and the leakage flux from the surfaces of C and L can be ignored.

(2) The magnetic field intensities in C, C', G and L have only the components of the longitudinal direction, which are constant in that direction. By this assumption and rot $\boldsymbol{H}=0$, the fields in C', G and L become uniform.

(3) In the skin region of C, the concentration of the magnetic field is induced by an eddy current, and therefore, μ_C is described by the following Froelich equation,[6] in which the magnetic saturation is considered,

$$\mu_C = 1/(\alpha+\beta|\boldsymbol{H}|) \tag{2}$$

where α and β are constants determined by material of C. The value of σ_C is assumed constant.

(4) Since the eddy current induced in L is very small, $\sigma_L=0$. As the magnetic field is uniform, μ_L is assumed constant.

(5) As the boundary conditions on the boundary surfaces of C, G and L, we adopt a continuity of the magnetic flux instead of that of the flux density $\boldsymbol{B}$. Though a theory considering the continuity of $\boldsymbol{B}$ has been already proposed,[2] in this case, the relation rot $\boldsymbol{H}=0$ is not satisfied in regions with $\sigma=0$, and then, the accuracy of the theory is impaired when the magnetic circuit is long.[4]

2.2 Fundamental equations

Applying Ampere's circuital law to a closed integral path which passes on the surface of C in Fig. 1, and using the previous assumptions (1) and (2), we obtain

$$NI = l_C H_{CS} + l_G H_G + l_L H_L\,, \tag{3}$$

where H_G, H_L and H_{CS} are the field intensities in G and L and on the surface of C, respectively. Since H_G and H_L are uniform, H_{CS} becomes uniform, too.

Next, let us discuss the continuity of the total flux in the assumption (5). The total flux ϕ in G and L is given by

$$\phi = \mu_0 H_G S_G = \mu_L H_L S_L\,. \tag{4}$$

Since the uniform field intensity $H_{C'}$ in C' is equal to H_{CS}, we get

$$\phi = \phi_C + \mu_0 H_{CS} S_{C'}\,, \tag{5}$$

where ϕ_C is the flux in C. From Eqs. (3) to (5), the relation among NI, H_{CS} and ϕ_C is derived as follows:

$$NI = l'_C H_{CS} + l'_G \phi_C/(\mu_0 S_G)\,, \tag{6}$$

where

$$l'_C = l_C + l'_G S_{C'}/S_G\,,$$
$$l'_G = l_G + l_L \mu_0 S_G/(\mu_L S_L)\,.$$

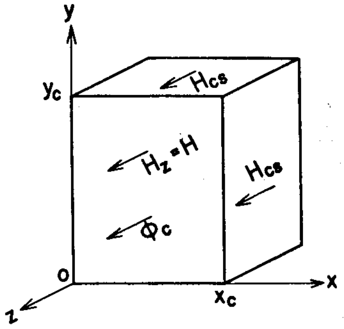

Fig. 2. Cross-section of conductive core C.

However, in order to find the values of H_{CS} and ϕ_C for some given NI, we must introduce another relation with respect to H_{CS} and ϕ_C. In the solid core C shown in Fig. 2, the magnetic field $\boldsymbol{H}$ on the cross-section perpendicular to the longitudinal direction (z-direction) has only the z-component $H_z=H$, which is independent of z according to the assumption (2). Consequently, the fundamental differential equation for H is derived from Eqs. (1) and (2) as follows:

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}=\sigma_C\frac{\partial}{\partial t}\left(\frac{H}{\alpha+\beta|H|}\right). \qquad (7)$$

Solving numerically this equation with the boundary condition that the surface field intensity is H_{CS}, we can obtain the relation between H_{CS} and H. Then, we can calculate ϕ_C in the cross-sectional area $S_C=x_C y_C$ of C by the following integral

$$\phi_C=\int_0^{x_C}\int_0^{y_C}\{H/(\alpha+\beta|H|)\}dxdy. \qquad (8)$$

From Eqs. (7) and (8), the relation between H_{CS} and ϕ_C can be evaluated. Finally, the total flux ϕ can be obtained by substituting the above values of H_{CS} and ϕ_C into Eq. (5).

Since it is usually satisfied that $l_C\gg l_G$, $S_G\gg S_{C'}$, $\mu_L\gg\mu_0$ and $\mu_C\gg\mu_0$ in Eqs. (5) and (6), the following relations are obtained

$$\phi\fallingdotseq\phi_C,\ l'_C\fallingdotseq l_C,\ l'_G\fallingdotseq l_G. \qquad (9)$$

Accordingly, it is seen that the influences of L and C' on ϕ are negligible.

Next, the eddy current $\boldsymbol{J}$ induced in C has the x- and y-components J_x and J_y given by

$$J_x=\partial H/\partial y,\ \ J_y=-\partial H/\partial x. \qquad (10)$$

Therefore, the eddy current flows along a closed path of H=constant. The total eddy current J_C in C is obtained by

$$\left.\begin{aligned} J_C &= l_C\left|\int_0^{y_C/2} J_x dy\right|_{x=x_C/2} = l_C\left|\int_0^{x_C/2} J_y dx\right|_{y=y_C/2} \\ &= l_C(H_{CS}-H_{CC}) , \end{aligned}\right\} \tag{11}$$

where H_{CC} is the magnetic field intensity at the center $(x_C/2, y_C/2)$ of the cross-section.

2.3 Numerical calculation

In this subsection, there is introduced a method for solving numerically the simultaneous equations (5) to (8) by using a finite difference analogue. Here, on the assumption that the value of the total flux ϕ at the time t is already known, we show a method to calculate that of ϕ at $t+\Delta t$. We adopt the two-dimensional difference method which uses a non-uniform grid system in an x-y plane

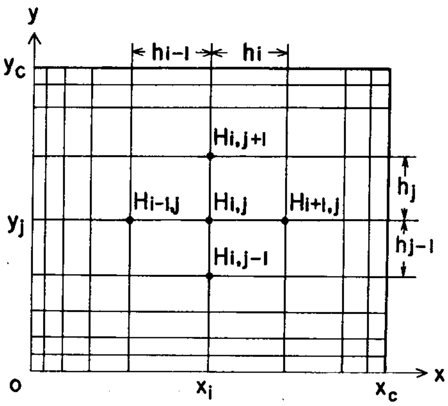

Fig. 3. Non-uniform grid system.

as shown in Fig. 3, where the number of grid points can be economized,[6] and the Crank-Nicolson method which is known as an accurate approximation for time derivative.[7] Using Eq. (7), the magnetic field intensity $H_{i,j}$ at a grid point (x_i, y_j) is given by following equations:

$$\left.\begin{aligned} &\left(\frac{H_{i,j}}{\alpha+\beta|H_{i,j}|}-\frac{\Delta t}{\sigma_C}\varepsilon g_{i,j}\right)_{t=t+\Delta t} = \left(\frac{H_{i,j}}{\alpha+\beta|H_{i,j}|}+\frac{\Delta t}{\sigma_C}(1-\varepsilon)g_{i,j}\right)_{t=t} , \\ &g_{i,j} = \frac{2(H_{i,j}-H_{i+1,j})}{h_i(h_i+h_{i-1})}+\frac{2(H_{i,j}-H_{i,j+1})}{h_j(h_j+h_{j-1})} \\ &\quad +\frac{2(H_{i,j}-H_{i-1,j})}{h_{i-1}(h_i+h_{i-1})}+\frac{2(H_{i,j}-H_{i,j-1})}{h_{j-1}(h_j+h_{j-1})} , \end{aligned}\right\} \tag{12}$$

where ε is a weight factor for time derivative and $0 \leq \varepsilon \leq 1$. Accordingly, when H_{CS} is given, the values of $H_{i,j}$ for all i and j can be obtained numerically by applying the Newton-Raphson method to Eqs. (12). When the field intensities at all grid points are obtained, the flux ϕ_C can be calculated by applying a simple trapezoid approximation to the integral in Eq. (8).[8] The calculation processes are as follows:

(1) Let us provide the n-th($n=1,2,3,\cdots$) approximate value H^n_{CS} of H_{CS}. When Eqs. (12) are solved for all i and j under the boundary condition $H_{CS}=H^n_{CS}$, the approximate distribution of the magnetic field intensity in C is obtained.

(2) The n-th approximate value of ϕ^n_C of ϕ_C for H^n_{CS} is obtained by applying the trapezoid approximation to Eq. (8).

(3) Let us assume that the following linear relation exists between H_{CS} and ϕ_C.

$$\phi_C = \frac{\phi^n_C - \phi^{n-1}_C}{H^n_{CS} - H^{n-1}_{CS}} (H_{CS} - H^n_{CS}) + \phi^n_C \,. \tag{13}$$

We obtain a new approximate value $H^{n\prime}_{CS}$ of H_{CS} by substituting the value of the above ϕ_C into Eqs. (6).

(4) The $(n+1)$-th approximate value H^{n+1}_{CS} of H_{CS} is calculated by

$$H^{n+1}_{CS} = H^n_{CS} + \omega (H^{n\prime}_{CS} - H^n_{CS}) \,, \tag{14}$$

where ω is a relaxation factor.

(5) When the digital computation in the processes (1) to (4) is iterated till $H^{n+1}_{CS} \fallingdotseq H^n_{CS}$ is satisfied, the final approximate value of ϕ_C is calculated by Eq. (13).

(6) The value of ϕ is acquired by substituting the final approximate values of H_{CS} and ϕ_C into Eq. (5).

3. Numerical Calculation Results and Discussion

3.1 Magnetic circuit model containing interpole

In this section, we apply the numerical method derived in the previous section to a closed magnetic circuit containing the interpole of a dc machine with the following specifications:[9]

$$\left.\begin{array}{l}
\text{number of poles: 4, output: 50 } kVA\text{, speed: 400 } rpm, \\
\text{terminal voltage: 220 } V\text{, armature current: 227 } A, \\
\text{mmf due to interpole exciting current: 5700 } A, \\
\text{counter mmf at interpole due to armature current: 4700 } A, \\
\text{effective mmf at interpole: 1000 } A, \\
l_C = 0.144\ m,\ \ x_C = 0.060\ m,\ \ y_C = 0.210\ m, \\
l_G = 0.006\ m,\ \ S_C = 0.0126\ m^2,\ \ S_G = 0.0126\ m^2, \\
\mu_C = (800 + 0.5\,|H|)^{-1} H/m,\ \ \sigma_C = 5 \times 10^6\ \mho/m\,.
\end{array}\right\} \tag{15}$$

Here we assume that Eqs. (9) are satisfied and the influences of L and C' on the transient response of ϕ can be ignored. Furthermore, we will estimate the effect of the yoke by changing l_C for simplicity. With respect to the interpole air gap, we treat two cases. One is where the expansion of the flux in the air gap is ignored, resulting in $S_G = S_C$. The other is where the expansion is considered, with the result that S_G and l_G are varied.

In our circuit, H_{CS} at $t=0$ and the total flux $\phi = \phi_\infty$ in the steady state where the eddy current vanishes and H is uniform are obtained by Eqs. (5), (6), (8) and (15) as follows:

$$\left.\begin{aligned} &\text{for } NI = 5700\text{ A};\ H_{CS}(t=0) = 39{,}600\ A/m, \\ &\qquad \phi_\infty = 14.3\times10^{-3}\ Wb,\ \mu_{C1} = 0.573\times10^{-3}\ H/m, \\ &\text{for } NI = 1000\ A;\ H_{CS}(t=0) = 6{,}940\ A/m, \\ &\qquad \phi_\infty = 2.57\times10^{-3}\ Wb,\ \mu_{C2} = 1.12\times10^{-3}\ H/m, \end{aligned}\right\} \tag{16}$$

where μ_{C1} and μ_{C2} are constant permeabilities used for calculating the flux response in a case where the magnetic saturation is ignored. Those values are determined so that ϕ_∞ agrees with it in the case when the saturation is considered.

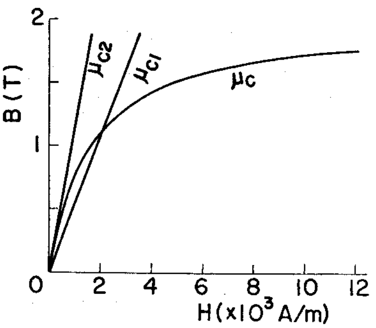

Fig. 4 Magnetization Curves of interpole core.

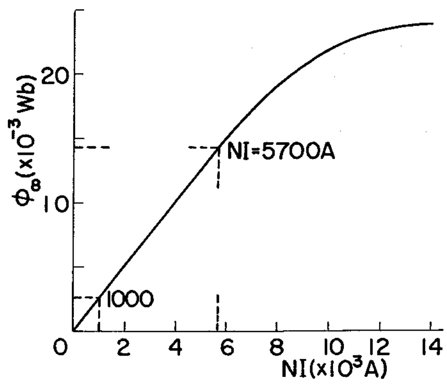

Fig. 5. Relation between NI and ϕ_∞.

Figure 4 shows the magnetization curves of the interpole core which are calculated by using μ_C in Eqs. (15) and μ_{C1} and μ_{C2} in Eqs. (16). In Fig. 5, the calculated relation between NI and ϕ_∞ for μ_C is shown.

3.2 Response of magnetic flux to step exciting mmf

Figure 6 shows the calculated results of the transient response of ϕ in a case where the step exciting mmf is supplied to the magnetic circuit, with the specifications shown in Eqs. (15). Here, the solid lines for μ_C show the results in a case where the magnetic saturation is considered, while the dotted lines for μ_{C1} and

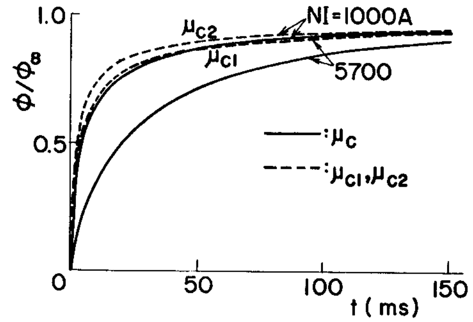

Fig. 6. Influence of magnetic saturation on step response of magnetic flux.

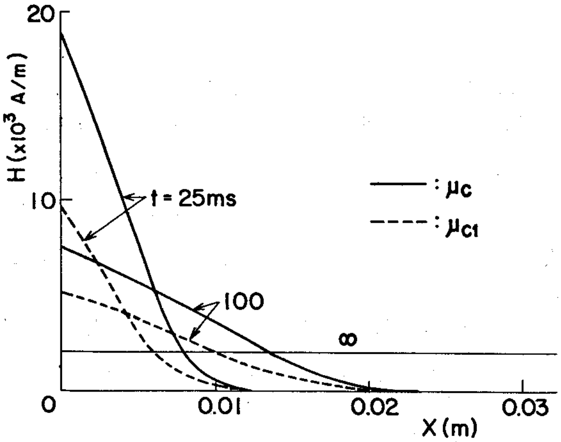

Fig. 7(a). Distribution of field intensity ($y=y_c/2$).

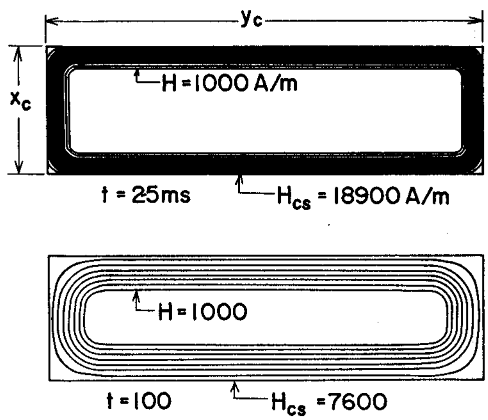

Fig. 7(b). Lines of equi-field intensity ($\Delta H = 1000\ A/m$).

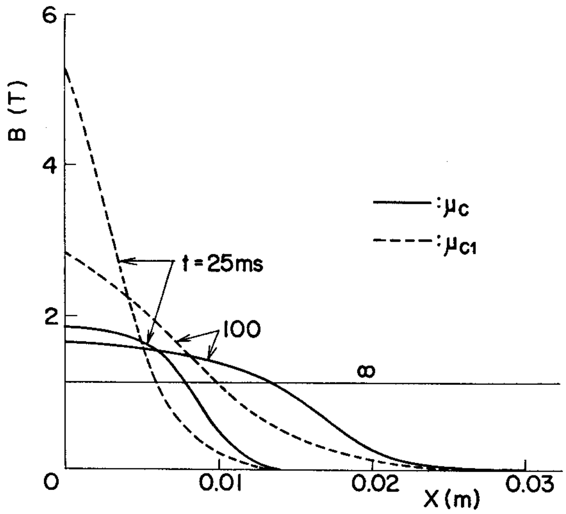

Fig. 7(c). Distribution of flux density ($y=y_c/2$).
Fig. 7. Magnetic field in interpole ($NI=5700\,A$).

μ_{C2} show the results in a case where it is not considered. From this figure, it is seen that the influence of the saturation on the flux response is very great, since the response for μ_C is delayed more noticeably than that for μ_{C1} or μ_{C2}, especially when the mmf NI is large.

Figure 7 (a) presents the distribution of the magnetic field intensity H along the x-axis for $NI=5700\,A$ and $y=y_C/2$, at $t=25$, 100 and ∞ ms. Figure 7 (b) shows the distribution of the equi-field lines at $t=25$ and 100 ms in the interpole, where the magnetic saturation is considered, and ΔH is the contour interval. In Figs. 8(a) and (b), there are shown the distributions of H and the equi-field lines, re-

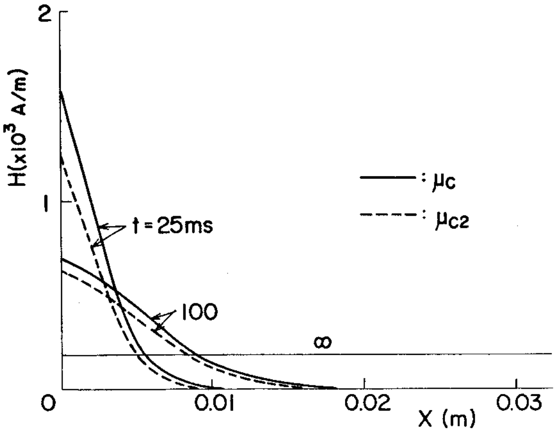

Fig. 8(a). Distribution of field intensity ($y=y_c/2$).

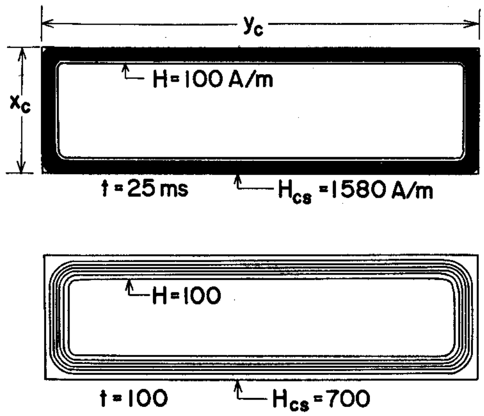

Fig. 8(b). Lines of equi-field intensity ($\Delta H = 100\ A/m$).

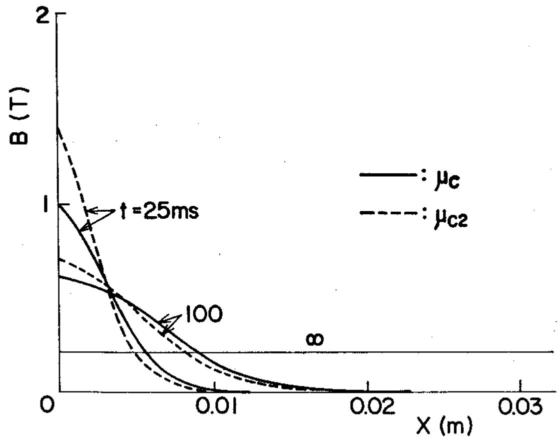

Fig. 8(c). Distribution of flux density ($y_c = y/2$).

Fig. 8. Magnetic field in interpole ($NI = 1000\ A$).

spectively, for $NI = 1000\ A$. From Figs. 7 and 8, we can see that the large field concentrates in the skin region of the interpole immediately after NI is supplied. We can also see that it takes a fairly long time for the flux to infiltrate in the interpole core. These reactions are due to the eddy current shown in Eqs. (10). Furthermore, the distributions of B are shown in Figs. 7(c) and 8(c), which correspond to the distributions of H in Figs. 7(a) and 8(a), respectively.

The solid lines for μ_C indicate that the magnetic saturation vanishes in a short time as shown in Fig. 8(c) for $NI = 1000\ A$, but the saturation is recognized for a fairly long time as shown in Fig. 7(c) for $NI = 5700$ A. On the other hand, the dotted lines for μ_{C1} in Fig. 7(c) seem to give B that is too large, because it is proportional to H in Fig. 7(a). Consequently, the linear analysis using the constant permeability overestimates the value of B, and so the flux plotted by the dotted lines in Fig. 6 responds quickly. From the above mentioned, it is clear that the saturation should be considered so as to calculate the flux response when the mmf of the interpole varies largely because of a sudden change of the motor load.

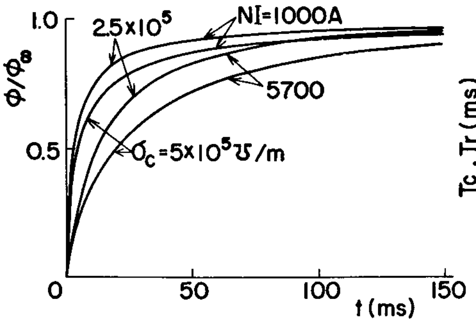

Fig. 9. Influence of σ_c on step response of magnetic flux.

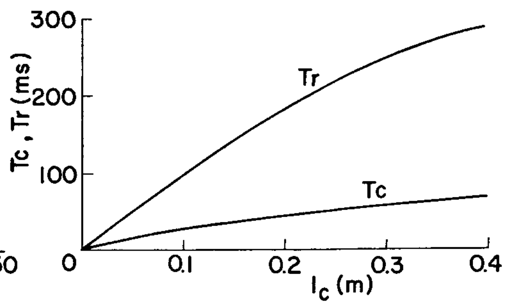

Fig. 10. Influence of l_c on T_c and T_r.

Next, we investigate the influences of σ_C, l_C, S_C, l_G, S_G, etc. of the magnetic circuit on the flux response to step mmf. When a parameter σ_C is changed and the values of the quantities, except for itself, are the same as those in Eqs. (15), the step responses of ϕ are as shown in Fig. 9, in which the delay of the flux response is greatly influenced by σ_C. The dependences of the time constant T_C and the rise time T_r on l_C and S_C are plotted in Figs. 10 and 11, respectively, where T_C and T_r are the times which it takes for ϕ/ϕ_∞ to rise from 0 to 0.63 and from 0.1 to 0.9, respectively. In the above figures, the value of NI for each l_C and S_C is chosen so that its ϕ_∞ agrees with $\phi_\infty=14.3\times10^{-3}$ Wb for $NI=5700$ A as given in Eqs. (16).

Figure 12 shows the step responses of ϕ in the case where the gap reluctance

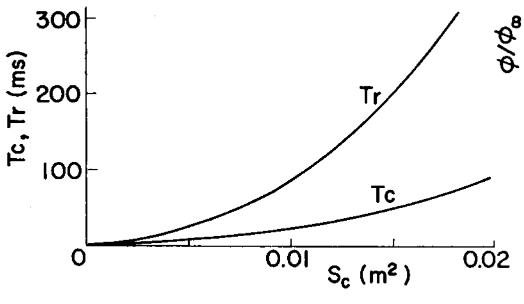

Fig. 11. Influence of S_c on T_c and T_r ($x_c+y_c=0.27$ m).

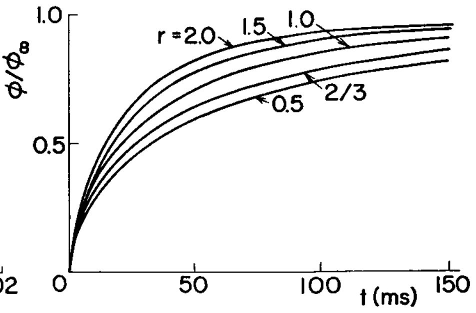

Fig. 12. Influence of interpole air gap on step response of magnetic flux.

$R_G=l_G/(\mu_0 S_G)$ is changed. In this figure, $r=R_G/R_{G0}$, R_{G0} is the reluctance for $l_G=0.006\,m$, $S_G=0.0126\,m^2$, and NI is given in the same way as in Figs. 10 and 11. In addition, let us examine the influence of the bypass circuit C' on the step response of ϕ. We calculate the step response for $r=0.5$ with the conditions $S_G=2S_C$ and $S_{C'}=S_C$, instead of Eqs. (9), and make sure that the influence of $S_{C'}$ on the response can be neglected. On the other hand, in Reference 4), Sakabe et al. attach importance to the influence of $S_{C'}$. Howerver, in our circuit it is negligible, because the permeability of the interpole core is much larger than the one of air as shown in Eqs. (15) and (16). Figure 12 is applicable to an actual dc motor, where l_G and S_G are corrected by Carter's coefficient to compensate for the expansion of the flux in the interpole air gap.[10]

3.3 Response of magnetic flux to full-wave rectified exciting mmf

Figure 13 shows the flux response in the case where the full-wave rectified mmf $NI|sin(120\pi t)|$ is applied to the interpole magnetic circuit given by Fig. 1 and Eqs. (15). This corresponds to the case where the dc motor is driven by the full-wave rectified current of a commercial frequency alternating current supply. When $NI=5700\ A$, the wave form of the flux is distorted by the magnetic satura-

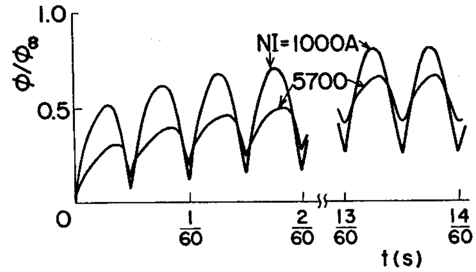

Fig. 13. Response of magnetic flux to full-wave rectified magnetomotive forces.

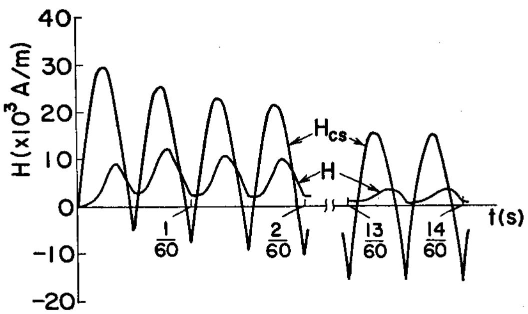

Fig. 14. Time response of H_{cs} and $H=H(x_c/24, y_c/2)$ $(NI=5700\ A)$.

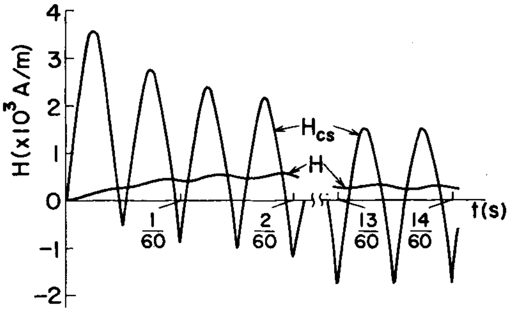

Fig. 15. Time response of H_{cs} and $H=H(x_c/24, y_c/2)$ ($NI=1000\ A$).

tion. The time behaviors of the field intensities H_{CS} on the surface, and H at the point $(x_C/24, y_C/2)$ near the surface of the interpole, are shown in Figs. 14 and 15, respectively. In this connection, since H_{CC} at the center$(x_C/2, y/_C2)$ of the corss-section of the interpole is very small, the total eddy current J_C given by Eq. (11) is proportional to H_{CS}.

4. Concluding Remarks

We have introduced new two-dimensional theoretical equations to calculate the transient response of the magnetic flux in a fundamental nonlinear magnetic circuit which simulates the circuit containing the interpole of the dc motor, we also introduced a numerical method to solve those equations, in which the non-uniform grid system for the x-y plane and the Crank-Nicolson method for the time derivative are used.

From the calculated results of the flux response and the distributions of the field intensity and flux density in the case where the step mmf to the interpole winding was applied, it has been clarified that the large field intensity concentrates in the skin region of the interpole. Consequently, the magnetic saturation occurs, and it has a strong influence on the flux response. Using the conventional linear theory in which the permeability is assumed constant, the delay of the response is underestimated, since the magnetic saturation is neglected. Also, the conductivity, length and cross-sectional area of the interpole core, the reluctance of the interpole air gap, etc. have a large influence on the flux response. Furthermore, it has been known that the response wave form of the flux is distorted remarkably by the saturation when the magnetic circuit is excited by a full-wave rectified mmf.

In the above investigation, we neglected the effects of the leakage flux from the side of the magnetic circuit, the exciting flux of the main pole, the slots of the armature etc. In the future, we intend to consider those effects.

In addition, the analytical theory derived in this paper can be applied to other magnetic circuits, such as a magnetic relay.

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