Transient Stability Analysis of Multi-machine Power System with Automatic Voltage Regulators via Lyapunov's Direct Method

By

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Abstract

In this paper, Lyapunov's direct method is applied to a multi-machine power system where generators are installed with atuomatic voltage regulators. The automatic voltage regulator and the thyrister exciter are represented by a third order transfer function. The stability of the power system is checked according to a generalized Popov criterion. This criterion guarantees that the system is stable if the gains of the voltage regulators are lower than the limit values. A Lur'e type Lyapunov function is constructed by the systematic method established by J.L. Willems. The obtained Lyapunov function is used in a transient stability analysis of a 10-machine power system. The direct method yields results which are very close to those obtained by simulations. It is concluded that Lyapunov's direct method is applicable with sufficient accuracy to transient stability analyses of power systems, where automatic voltage regulators are installed in generators on the condition that the gains of the automatic voltage regulators must be enlarged to practically used values in the future.

1. Introduction

Lyapunov's direct method is one of the promising methods of analyzing the transient stability of power systems. This method enables us to determine the stability or instability of power systems without simulating the entire transient. It has already reached a practical level for a simple system model, where generators are represented by constant voltages behind transient reactances [1]-[8]. One aim of the research on this method is to improve the mathematical models of power systems. There are several works along this direction. In particular, the dynamics of automatic voltage regulators and exciters is one very important item which should be incorporated in power system models, because such dynamics are closely related to the transient stability of power systems, and they are positively useful for improving the transient stability.

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In order to apply Lyapunov's direct method to power systems, it is necessary to construct Lyapunov functions for these systems. There are several works on Lyapunov's direct method for systems where voltage regulators are installed in generators. M.W. Siddiqee constructed a Lyapunov function for a one-machine connected to an infinite bus system with a forced voltage regulator through trial and error in 1968 [9]. Also, M.A. Pai and V. Rai derived the same function with a method based on a generalized Popov criterion in 1974 [10]. However, this type of voltage regulator is not one of feedbacking terminal voltages of generators. V.K. Verma et al. constructed a Lyapunov function with a variable gradient method for a system using an automatic voltage regulator in 1975 [11], and T. Taniguchi and H. Miyagi constructed a Lyapunov function with a method based on a Lagrangean function in 1977 [12]. These two papers are original works which incorporated automatic voltage regulators in system models in Lyapunov's direct method. However, their systems are a one-machine connected to infinite bus systems, and it is very difficult to apply their construction methods to multi-machine power systems. Besides, their modeling of automatic voltage regulators and exciters are very simple, that is, only automatic voltage regulators are represented by zero or first order transfer functions. In order to construct a Lyapunov function for a system in which generators are installed with automatic voltage regulators and thyristor exciters represented by third order transfer functions, we use a generalized Popov criterion derived from our previous paper [13]. Thyristor exciters are prevailingly applied to improve the transient stability of power systems. Their time responses are so fast that the excitation voltages of generators are rapidly increased with any occurrence of a fault, and the electric power outputs of generators are kept at a high level. Accordingly, investigations of their effects on transient stability are important.

In this paper, a Lyapunov function is constructed for a system in which automatic voltage regulators and thyristor exciters are installed in generators. Firstly, the system equations are derived, and the stability of the system is checked with a generalized Popov criterion. After this manipulation, a Lur'e type Lyapunov function is constructed with the systematic method established by J.L. Willems and other researchers [2], [14], [15]. Lastly, a 10-machine power system is analyzed as a numerical example, and features of the obtained Lyapunov function are illustrated.

2. System equation

In this section, system equations are described for a multi-machine power system in which automatic voltage regulators and thyristor exciters are installed in all generators. Motions of generators are expressed by

$$m_i \frac{d^2\delta_i'}{dt^2} + d_i \frac{d\delta_i'}{dt} = P_{mi} - \sum_{j=1}^{n} Y_{ij} E_i E_j \sin(\delta_{ij} + \theta_{ij}) \quad (1)$$

$$\text{for } i = 1, 2, \cdots, n$$

Variations of voltages E_{qi}' are expressed by

$$\frac{dE_{qi}'}{dt} = (1/T_{doi}')[E_{fdi} - E_{qi}' - (x_{di} - x_{di}')i_{di}] \quad (2)$$

$$\text{for } i = 1, 2, \cdots, n$$

Variations of variables in automatic voltage regulators and exciters are expressed by

$$\begin{aligned} \frac{dE_{ai}}{dt} &= -(1/T_{ai})E_{ai} - (K_{ai}/T_{ai})E_{di} + (K_{ai}/T_{ai})(V_{refi} - V_{ti}) \\ \frac{dE_{ei}}{dt} &= (K_{ei}/T_{ei})E_{ai} - (1/T_{ei})E_{ei} \\ \frac{dE_{di}}{dt} &= (K_{di}K_{ei}/T_{ei})E_{ai} - (K_{di}/T_{ei})E_{ei} - (1/T_{di})E_{di} \end{aligned} \quad (3)$$

$$\text{for } i = 1, 2, \cdots, n$$

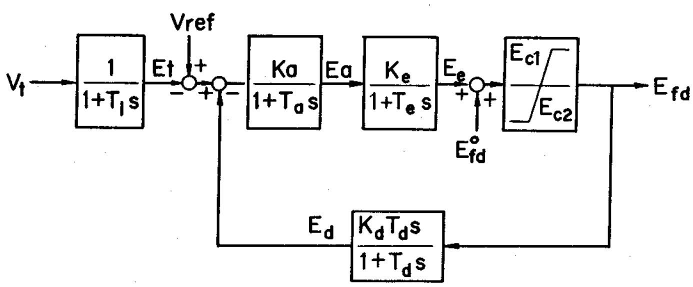


Fig. 1. Excitation system model

where a block diagram of the thyristor excitation system is shown in Fig. 1, but the time constant T_1 is assumed to be zero. The terminal voltages $\dot{V}_{ti}$ can be expressed by

$$\begin{aligned} \dot{V}_{ti} &= \dot{E}_i - jx_{di}' \dot{I}_i \\ &= \dot{E}_i - jx_{di}' \sum_{j=1}^{n} \dot{Y}_{ij} \dot{E}_j \qquad \text{for } i = 1, 2, \cdots, n \end{aligned} \quad (4)$$

where, for generator i,

P_{mi} : mechanical power input

m_i : angular momentum constant

d_i : damping power coefficient

$Y_{ij}\angle\phi_{ij}$: post-fault transfer admittance between the ith and jth generator nodes (obtained after reduction of a network retaining only generator nodes)

θ_{ij} : complement of ϕ_{ij}, i.e., $\theta_{ij}=\pi/2-\phi_{ij}$

$E_i\angle\delta_i$: internal voltage

δ_{ij} : $\delta_i-\delta_j$

$E'_{qi}\angle\delta'_i$: voltage related to the internal voltage as in Fig. 2. δ'_i indicates a rotor angle relative to a reference frame rotating at a synchronous speed.

E_{fdi} : excitation voltage

i_{di} : d-axis current

x_{di}, x'_{di} : d-axis synchronous, transient reactances, respectively.

T'_{doi} : d-axis transient open-circuit time constant

In order to construct a Lyapunov function, three basic assumptions are necessary:

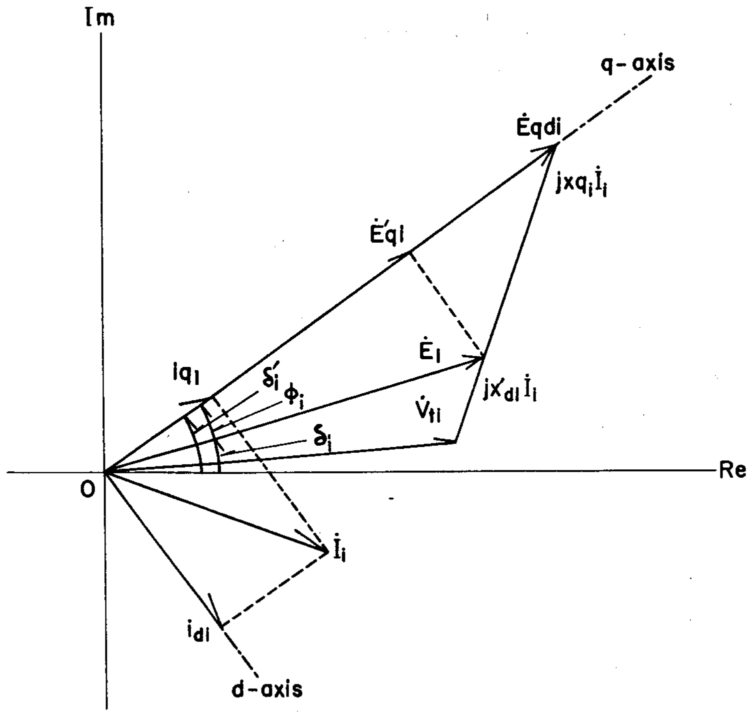


Fig. 2. Relations between generator variables

(i) Each internal voltage lags behind the q-axis of each generator by a constant angle ϕ_i all the time [17].

(ii) The transfer conductances in the reduced admittance matrix are negligible.

(iii) The magnitude of the terminal voltage can be approximately expressed by

$$V_{ti} \simeq E_i + x'_{di} \sum_{j=1}^{n} Y_{ij} E_j \cos(\delta_{ij} + \theta_{ij}) \tag{5}$$

where (5) is derived from Appendix A.

Under these assumptions, (1), (2) and (3) change to the following equations:

$$\begin{aligned}
m_i \frac{d^2\delta_i}{dt^2} + d_i \frac{d\delta_i}{dt} &= \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij}(E_i^o E_j^o \sin \delta_{ij}^o - E_i E_j \sin \delta_{ij}) \\
\frac{dE_i}{dt} &= (1/T'_{doi} \cos \phi_i)(E_{fdi} - E_{fdi}^o) - (1/T'_{doi})[1 - (x_{di} - x'_{di})B_{ii}](E_i - E_i^o) \\
&\quad - (1/T'_{doi})(x_{di} - x'_{di}) \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} E_j (\cos \delta_{ij}^o - \cos \delta_{ij}) \\
\frac{dE_{ai}}{dt} &= -(1/T_{ai})(E_{ai} - E_{ai}^o) - (K_{ai}/T_{ai})(E_{di} - E_{di}^o) \\
&\quad - (K_{ai}/T_{ai})(1 + x'_{di} B_{ii})(E_i - E_i^o) \\
&\quad + (K_{ai}/T_{ai}) x'_{di} \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} (\cos \delta_{ij}^o - \cos \delta_{ij}) \\
\frac{dE_{ei}}{dt} &= (K_{ei}/T_{ei})(E_{ai} - E_{ai}^o) - (1/T_{ei})(E_{ei} - E_{ei}^o) \\
\frac{dE_{di}}{dt} &= (K_{di} K_{ei}/T_{ei})(E_{ai} - E_{ai}^o) - (K_{di}/T_{ei})(E_{ei} - E_{ei}^o) - (1/T_{di})(E_{di} - E_{di}^o) \\
&\qquad \text{for } i = 1, 2, \cdots, n
\end{aligned} \tag{6}$$

where the superscript "o" denotes the stable equilibrium point in the post-fault state, and accordingly, (6) applies to the post-fault state.

Eq. (6) can be rewritten in a form of state equation, that is,

$$\begin{aligned}
\dot{x} &= Ax - BF(\sigma) \\
\sigma &= C'x
\end{aligned} \tag{7}$$

where

$$A = \begin{pmatrix}
0 & K'_{n(n-1)} & 0 & 0 & 0 & 0 \\
0 & -M^{-1}D_{nn} & 0 & 0 & 0 & 0 \\
0 & 0 & -H_1 & 0 & H_2 & 0 \\
0 & 0 & -H_3 & -H_4 & 0 & -H_5 \\
0 & 0 & 0 & H_6 & -H_7 & 0 \\
0 & 0 & 0 & H_8 & -H_9 & -H_{10}
\end{pmatrix} \tag{8}$$

$$B=\begin{pmatrix} 0 & 0 \\ -M^{-1}T & 0 \\ 0 & \alpha \\ 0 & -\beta \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad C=\begin{pmatrix} G & 0 \\ 0 & 0 \\ 0 & I \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

in which

$$\begin{aligned} K_{n(n-1)} &= \begin{bmatrix} 1_{1(n-1)} \\ -I_{(n-1)(n-1)} \end{bmatrix} \\ G_{(n-1)m} &= \begin{bmatrix} I_{(n-1)(n-1)} & -T_{(n-1)(m-n+1)} \end{bmatrix} \\ T_{nm} &= \begin{bmatrix} 1_{1(n-1)} & 0_{1(m-n+1)} \\ -I_{(n-1)(n-1)} & T_{(n-1)(m-n+1)} \end{bmatrix} \end{aligned} \tag{9}$$

and

$$T_{nm} = K_{n(n-1)} G_{(n-1)m} \tag{10}$$

The row vector $1_{1(n-1)}$ and $0_{1(m-n+1)}$ has all its elements equal to unity and zero, respectively. The number m is defined by $m=n(n-1)/2$. H_i $(i=1, 2, \cdots, 10)$ is an $n\times n$ matrix of a diagonal form, and its elements are defined as follows:

$$\begin{aligned} h_{1i} &= (1/T'_{doi})[1-(x_{di}-x'_{di})B_{ii}] \\ h_{2i} &= 1/(T'_{doi}\cos\phi_i) \\ h_{3i} &= (K_{ai}/T_{ai})(1+x'_{di}B_{ii}) \\ h_{4i} &= 1/T_{ai} \\ h_{5i} &= K_{ai}/T_{ai} \\ h_{6i} &= K_{ei}/T_{ei} \\ h_{7i} &= 1/T_{ei} \\ h_{8i} &= K_{di}K_{ei}/T_{ei} \\ h_{9i} &= K_{di}/T_{ei} \\ h_{10i} &= 1/T_{di} \\ \alpha_i &= (x_{di}-x'_{di})/T'_{doi} \\ \beta_i &= K_{ai}x'_{di}/T_{ai} \qquad \text{for } i = 1, 2, \cdots, n \end{aligned} \tag{11}$$

The state vector x is a $(6n-1)$ dimensional vector consisting of six vectors defined by

$$x = [\delta'_r, \omega', \Delta E', \Delta E'_a, \Delta E'_e, \Delta E'_d]' \tag{12}$$

where elements of δ_r, ω, ΔE, ΔE_a, ΔE_e and ΔE_d are defined by

$$
\begin{aligned}
&\delta_{ri} = \delta_{1(i+1)} - \delta^o_{1(i+1)} && \text{for } i = 1, 2, \cdots, n-1 \\
&\omega_i = \dot{\delta}_i && \text{for } i = 1, 2, \cdots, n \\
&\Delta E_i = E_i - E^o_i && \text{for } i = 1, 2, \cdots, n \\
&\Delta E_{ai} = E_{ai} - E^o_{ai} && \text{for } i = 1, 2, \cdots, n \\
&\Delta E_{ei} = E_{ei} - E^o_{ei} && \text{for } i = 1, 2, \cdots, n \\
&\Delta E_{di} = E_{di} - E^o_{di} && \text{for } i = 1, 2, \cdots, n
\end{aligned}
\tag{13}
$$

The non-linearity $F(\sigma)$ consists of two vectors, i.e.,

$$F(\sigma) = [f_1(\sigma)', f_2(\sigma)']' \tag{14}$$

where $f_1(\sigma)$ and $f_2(\sigma)$ are m and n dimensional vectors defined by

$$
\begin{aligned}
f_{1k}(\sigma) = B_{ij}[E_iE_j \sin(\sigma_k + \delta^o_{ij}) - E^o_iE^o_j \sin \delta^o_{ij}] \\
\text{for } i=1, 2, \cdots, n-1, \; j = i+1, \cdots, n, \; k = 1, 2, \cdots, m
\end{aligned}
\tag{15}
$$

$$f_{2i}(\sigma) = \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij}E_j(\cos \delta^o_{ij} - \cos \delta_{ij}) \qquad \text{for } i = 1, 2, \cdots, n \tag{16}$$

In (15), k is related to i and j by

$$k = (i-1)n - i(i+1)/2 + j \tag{17}$$

The output σ is an $(m+n)$ dimensional vector defined by

$$
\begin{aligned}
&\sigma_k = \delta_{ij} - \delta^o_{ij} && \text{for } k = 1, 2, \cdots, m \\
&\sigma_k = E_i - E^o_i && \text{for } k = m+1, \cdots, m+n
\end{aligned}
\tag{18}
$$

where k is related to i and j by (17) for $k=1, 2, \cdots, m$. Eq. (7) describes the multi-machine power system as a multivariable dynamical system with linear elements in the forward path, and multiple, memory-less and coupled non-linear elements in the feedback path, that is, a system in a form as shown in Fig. C. (Appendix B)

3. Stability Check of System

In this section, it is investigated with a generalized Popov criterion whether the system described by (7) is stable or not stable (Appendix B). The transfer matrix $W(s)$ for the linear part of the system is written as follows:

$$
\begin{aligned}
W(s) &= C'(sI-A)^{-1}B \\
&= \begin{bmatrix} T'[s(sI+M^{-1}D)]^{-1}M^{-1}T & 0 \\ 0 & \Delta^{-1}(s^3\varepsilon_1 + s^2\varepsilon_2 + s\varepsilon_3 + \varepsilon_4) \end{bmatrix} \\
&= \begin{bmatrix} W_1(s) & 0 \\ 0 & W_2(s) \end{bmatrix}
\end{aligned}
\tag{19}
$$

where $\Delta(s)$ is an $n\times n$ matrix defined by

$$\Delta(s) = s^4I+s^3\gamma_1+s^2\gamma_2+s\gamma_3+\gamma_4 \tag{20}$$

$$\begin{aligned}
\gamma_1 &= H_1+H_4+H_7+H_{10}\\
\gamma_2 &= H_1(H_4+H_7+H_{10})+H_4(H_7+H_{10})+H_7H_{10}+H_5H_8\\
\gamma_3 &= H_1(H_4H_7+H_7H_{10}+H_{10}H_4+H_5H_8)+H_4H_7H_{10}+H_2H_3H_6\\
\gamma_4 &= (H_1H_4H_7+H_2H_3H_6)H_{10}
\end{aligned} \tag{21}$$

and ε_1, ε_2, ε_3 and ε_4 are $n\times n$ matrices defined by

$$\begin{aligned}
\varepsilon_1 &= \alpha\\
\varepsilon_2 &= \alpha(H_4+H_7+H_{10})\\
\varepsilon_3 &= \alpha(H_4H_7+H_7H_{10}+H_{10}H_4+H_5H_8)-\beta H_2H_6\\
\varepsilon_4 &= (\alpha H_4H_7-\beta H_2H_6)H_{10}
\end{aligned} \tag{22}$$

For the system to be stable, there must exist matrices N and Q such that

$$Z(s) = (N+Qs)W(s) \tag{23}$$

is positive real (Appendix B).

In this problem, N is chosen as follows:

$$N = \begin{bmatrix} (1/q)I_{mm} & 0 \\ 0 & 0_{nn} \end{bmatrix} \tag{24}$$

The inequality in (B2) with this N is equivalent to the following inequalities:

$$f_{1k}(\sigma)\sigma_k \geq 0 \qquad \text{for all } \sigma_k \in R \text{ and } k=1, 2, \cdots, m \tag{25}$$

However, the above inequalities are satisfied not for all $\sigma_k \in R$, but for ranges of σ_k, i.e.,

$$\sigma_{\min} \leq \sigma_k \leq 0 (\text{or } \sigma_s), \quad \text{and} \quad \sigma_s (\text{or } 0) \leq \sigma_k \leq \sigma_{\max} \tag{26}$$

where

$$\sigma_{\min} = -\pi-(\delta^o_{ij}+\delta^s_{ij}), \quad \sigma_s = \delta^s_{ij}-\delta^o_{ij}, \quad \sigma_{\max} = \pi-(\delta^o_{ij}+\delta^s_{ij})$$

and

$$\delta^s_{ij} = \sin^{-1}(E^o_iE^o_j \sin \delta^o_{ij}/E_iE_j)$$

As observed from (B10), $F(\sigma)'N\sigma$ has an influence on $\dot{V}(x)$, so it is desirable to make its influence zero by letting $q\rightarrow\infty$. However, this selection of q causes a pole-zero cancellation between $(N+Qs)$ and $W(s)$, because $W_1(s)$ has a pole at $s=0$. In order to avoid the pole-zero cancellation, we give q a finite value in constructing a Lyapunov function, and once it is obtained, we let $q \to \infty$.

The function $V_1(\sigma)$ in (B4) is chosen as follows:

$$\begin{aligned} V_1(\sigma) &= \sum_{k=1}^{m} \int_0^{\sigma} f_{1k}(\sigma) d\sigma_k \\ &= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} B_{ij}[E_i E_j (\cos \delta_{ij}^0 - \cos \delta_{ij}) - (\delta_{ij} - \delta_{ij}^0) E_i^0 E_j^0 \sin \delta_{ij}^0] \end{aligned} \tag{27}$$

From (27), it is clear that $V_1(\sigma)$ is positive not for all σ, but for a range of σ about $\sigma=0$. Accordingly, the global stability of the system can not be guaranteed with this $V_1(\sigma)$. However, it is possible to estimate the domain of attraction by using the Lyapunov function obtained with this $V_1(\sigma)$. One more thing to be noted is that $V_1(\sigma)$ can have negative values about $\sigma=0$ if $E_i = E_i^0$ is not satisfied for all E_i, where $i=1, 2, \cdots, n$. This fact may have significant influence on the stability of the system, but its influence is assumed to be negligible in this paper. The partial derivatives of $V_1(\sigma)$ are given as follows:

$$\begin{aligned} \frac{\partial V_1}{\partial \sigma_k} &= B_{ij}[E_i E_j \sin(\sigma_k + \delta_{ij}^0) - E_i^0 E_j^0 \sin \delta_{ij}^0] && \text{for } k = 1, 2, \cdots, m \\ \frac{\partial V_1}{\partial \sigma_k} &= \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} E_j (\cos \delta_{ij}^0 - \cos \delta_{ij}) && \text{for } k = m+1, \cdots, m+n \end{aligned} \tag{28}$$

that is,

$$\nabla V_1(\sigma) = I_{(m+n)(m+n)} F(\sigma) \tag{29}$$

Accordingly, Q is given by

$$Q = I_{(m+n)(m+n)} \tag{30}$$

Substituting (19), (24) and (30) into (23) gives the expression of $Z(s)$ as follows:

$$\begin{aligned} Z(s) &= \begin{bmatrix} (1/q+s) T'[s(sI+M^{-1}D)]^{-1} M^{-1} T & 0 \\ 0 & s\Delta^{-1}(s^3\varepsilon_1 + s^2\varepsilon_2 + s\varepsilon_3 + \varepsilon_4) \end{bmatrix} \\ &= \begin{bmatrix} Z_1(s) & 0 \\ 0 & Z_2(s) \end{bmatrix} \end{aligned} \tag{31}$$

The conditions for $Z(s)$ to be positive real are

1) $Z(s)$ has elements which are analytic for $Re\ s>0$,
2) $Z^*(s) = Z(s^*)$ for $Re\ s>0$,
3) $Z'(s^*) + Z(s)$ is positive semi-definite for $Re\ s>0$.

Since $Z(s)$ is a direct sum of $Z_1(s)$ and $Z_2(s)$, they are investigated independently of each other. The first two conditions clearly hold for both $Z_1(s)$ and $Z_2(s)$.

For condition 3) to be satisfied, it is sufficient in this case to show that $Z_i(j\omega)+Z_i'(-j\omega)$ is positive semi-definite for each scalor ω, where $i=1, 2$. After some manipulation, they are realized as follows:

$$
\begin{aligned}
Z_1(j\omega)+Z_1'(-j\omega) &= 2T'diag\left(\frac{d_i-m_i/q}{m_i^2\omega^2+d_i^2}\right)T \\
Z_2(j\omega)+Z_2'(-j\omega) &= 2\omega^2 diag\left(\frac{\xi_{1i}\omega^6+\xi_{2i}\omega^4+\xi_{3i}\omega^2+\xi_{4i}}{\Delta_i(j\omega)\Delta_i(-j\omega)}\right)
\end{aligned}
\tag{32}
$$

where

$$
\begin{aligned}
\xi_1 &= \varepsilon_1 \\
\xi_2 &= -\varepsilon_1\gamma_2+\varepsilon_2\gamma_1-\varepsilon_3 \\
\xi_3 &= \varepsilon_1\gamma_4-\varepsilon_2\gamma_3+\varepsilon_3\gamma_2-\varepsilon_4\gamma_1 \\
\xi_4 &= -\varepsilon_3\gamma_4+\varepsilon_4\gamma_3
\end{aligned}
\tag{33}
$$

Both right hands in (32) are positive semi-definite if the following conditions are satisfied:

$$q>m_i/d_i \qquad \text{for } i = 1, 2, \cdots, n \tag{34}$$

and

$$\xi_{1i}\geq 0, \quad \xi_{2i}\geq 0, \quad \xi_{3i}\geq 0, \quad \xi_{4i}\geq 0 \qquad \text{for } i = 1, 2, \cdots, n \tag{35}$$

Under these conditions, $Z_1(s)$, $Z_2(s)$, and accordingly, $Z(s)$ are positive real. According to the theorem in Appendix B, the system proves to be stable.

4. Solution of Matrix Equations

Since the system described by (7) is stable, there exists a Lyapunov function

$$V(x) = x'Px+2V_1(\sigma) \tag{36}$$

where P is a $(6n-1)\times(6n-1)$ positive definite symmetric matrix satisfying the following matrix equations:

$$
\begin{aligned}
PA+A'P &= -LL' \\
PB &= CN'+A'CQ'-LW_o \\
W_o'W_o &= QC'B+B'CQ'
\end{aligned}
\tag{37}
$$

where L and W_o are $(6n-1)\times(m+n)$ and $(m+n)\times(m+n)$ matrices. Since $Z(s)$ is a direct sum of $Z_1(s)$ and $Z_2(s)$, P can be expressed as follows:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \tag{38}$$

where P_1 and P_2 are $(2n-1)\times(2n-1)$ and $4n\times 4n$ matrices corresponding to $Z_1(s)$ and $Z_2(s)$, respectively.

The transfer matrix $W_1(s)$ is rewritten as follows:

$$W_1(s) = C_1'(sI-A_1)^{-1}B_1 \tag{39}$$

where

$$A_1 = \begin{bmatrix} 0 & K' \\ 0 & -M^{-1}D \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ M^{-1}T \end{bmatrix} \quad C_1 = \begin{bmatrix} G \\ 0 \end{bmatrix} \tag{40}$$

Since the relation $C_1'B_1=0$ holds, (37) reduces to

$$\begin{aligned} P_1A_1+A_1'P_1 &= -L_1L_1' \\ P_1B_1 &= C_1N_1'+A_1'C_1Q_1' \end{aligned} \tag{41}$$

P_1 and L_1 are partitioned as follows:

$$P_1 = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad L_1 = \begin{bmatrix} L_{11} \\ L_{12} \end{bmatrix} \tag{42}$$

where P_{11}, P_{12}, P_{21}, P_{22}, L_{11} and L_{12} are $(n-1)\times(n-1)$, $(n-1)\times n$, $n\times(n-1)$, $n\times n$, $(n-1)\times m$ and $n\times m$ matrices, respectively. Substituting (40) and (42) into (41) gives

$$0 = -L_{11}L_{11}' \tag{43}$$

$$P_{11}K'-P_{12}M^{-1}D = 0 \tag{44}$$

$$P_{21}K'+KP_{12}-P_{22}M^{-1}D-DM^{-1}P_{22} = -L_{12}L_{12}' \tag{45}$$

$$P_{12}M^{-1}T = (1/q)G \tag{46}$$

$$P_{22}M^{-1}T = T \tag{47}$$

Eqs. (43)-(47) are solved after some manipulation [14].

$$\begin{aligned} KP_{11}K' &= (1/q)D+\rho DUD \\ KP_{12} &= (1/q)M+\rho DUM \\ P_{22} &= M+\mu MUM \end{aligned} \tag{48}$$

where ρ is a non-negative scalar, and U is an $n\times n$ matrix with all elements equal to 1. Substituting (48) into (45) gives

$$2(D-M/q)+(\mu-\rho)(DUM+MUD)\geq 0 \tag{49}$$

This inequality is satisfied if the following condition is satisfied:

$$(\mu^*)^2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{(d_im_j-d_jm_i)^2}{4(d_i-m_i/q)(d_j-m_j/q)}-\mu^*\sum_{i=1}^{n}\frac{d_im_i}{d_i-m_i/q}-1\leq 0 \tag{50}$$

that is, μ^* must lie between the two roots of the quadratic equation, where $\mu^*=\mu-\rho$. If the damping torques of generators are uniform, then μ^* reduces to μ_o^*, where

$$\mu_o^* = -1/[\sum_{i=1}^{n} \{m_i d_i/(d_i-m_i/q)\}] \tag{51}$$

The transfer matrix $W_2(s)$ is rewritten as follows:

$$W_2(s) = C_2'(sI-A_2)^{-1}B_2 \tag{52}$$

where

$$A_2=\begin{pmatrix} -H_1 & 0 & H_2 & 0 \\ -H_3 & -H_4 & 0 & -H_5 \\ 0 & H_6 & -H_7 & 0 \\ 0 & H_8 & -H_9 & -H_{10} \end{pmatrix} \quad B_2=\begin{pmatrix} \alpha \\ -\beta \\ 0 \\ 0 \end{pmatrix} \quad C_2=\begin{pmatrix} I \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{53}$$

Since $Z_2(s)$ is positive real, $Z_2(s)+Z_2'(-s)$ is factorized as follows:

$$Z_2(s)+Z_2'(-s) = Y_2'(-s)Y_2(s) \tag{54}$$

where

$$Y_2(s) = \mathrm{diag}\left(\frac{\sqrt{2\xi_{1i}}s(s+\zeta_{1i})(s+\zeta_{2i})(s+\zeta_{3i})}{\Delta_i(s)}\right) \tag{55}$$

and ζ_{1i}, ζ_{2i} and ζ_{3i} are determined by the following equations:

$$\xi_{1i}\omega^6+\xi_{2i}\omega^4+\xi_{3i}\omega^2+\xi_{4i} = \xi_{1i}(\omega^2+\zeta_{1i}^2)(\omega^2+\zeta_{2i}^2)(\omega^2+\zeta_{3i}^2) \tag{56}$$

$Y_2(s)$ can be represented as follows [18]:

$$Y_2(s) = L_2(sI-A_2)^{-1}B_2 \tag{57}$$

Solving this equation with A_2 and B_2 in (53) gives a $4n\times n$ matrix L_2,

$$L_2 = (L_{21}', L_{22}', L_{23}', L_{24}')' \tag{58}$$

where L_{21}, L_{22}, L_{23} and L_{24} are $n\times n$ diagonal matrices:

$$\begin{aligned} L_{21} &= \mathrm{diag}(l_{1i}) & L_{22} &= \mathrm{diag}(l_{2i}) \\ L_{23} &= \mathrm{diag}(l_{3i}) & L_{24} &= \mathrm{diag}(l_{4i}) \end{aligned} \tag{59}$$

Elements l_{1i}, l_{2i}, l_{3i} and l_{4i} are determined by solving the following equation:

$$\Lambda_i l_i = \tau_i \qquad \text{for } i = 1, 2, \cdots, n \tag{60}$$

where Λ_i, l_i and τ_i are 4×4 matrix and 4 dimensional vectors, respectively, defined by

$$\begin{aligned}
\lambda_{i11} &= \alpha_i \\
\lambda_{i12} &= -\beta \\
\lambda_{i13} &= 0 \\
\lambda_{i14} &= 0 \\
\lambda_{i21} &= \alpha_i(h_{4i}+h_{7i}+h_{10i}) \\
\lambda_{i22} &= -\alpha_i h_{3i}-\beta_i(h_{1i}+h_{7i}+h_{10i}) \\
\lambda_{i23} &= -\beta_i h_{6i} \\
\lambda_{i24} &= -\beta_i h_{8i} \\
\lambda_{i31} &= \alpha_i(h_{4i}h_{7i}+h_{7i}h_{10i}+h_{10i}h_{4i}+h_{5i}h_{8i})-\beta_i h_{2i}h_{6i} \\
\lambda_{i32} &= -\alpha_i h_{3i}(h_{7i}+h_{10i})-\beta_i(h_{1i}h_{7i}+h_{7i}h_{10i}+h_{10i}h_{1i}) \\
\lambda_{i33} &= -\alpha_i h_{3i}h_{6i}-\beta_i h_{6i}(h_{1i}+h_{10i}) \\
\lambda_{i34} &= -(\alpha_i h_{3i}+\beta_i h_{1i})h_{8i} \\
\lambda_{i41} &= (\alpha_i h_{4i}h_{7i}-\beta_i h_{2i}h_{6i})h_{10i} \\
\lambda_{i42} &= -(\alpha_i h_{3i}+\beta_i h_{1i})h_{7i}h_{10i} \\
\lambda_{i43} &= -(\alpha_i h_{3i}+\beta_i h_{1i})h_{6i}h_{10i} \\
\lambda_{i44} &= 0 \qquad \text{for } i = 1, 2, \cdots, n
\end{aligned} \tag{61}$$

$$\begin{aligned}
\tau_{1i} &= \sqrt{2\xi_{1i}}(\zeta_{1i}+\zeta_{2i}+\zeta_{3i}-\gamma_{1i}) \\
\tau_{2i} &= \sqrt{2\xi_{1i}}(\zeta_{1i}\zeta_{2i}+\zeta_{2i}\zeta_{3i}+\zeta_{3i}\zeta_{1i}-\gamma_{2i}) \\
\tau_{3i} &= \sqrt{2\xi_{1i}}(\zeta_{1i}\zeta_{2i}\zeta_{3i}-\gamma_{3i}) \\
\tau_{4i} &= \sqrt{2\xi_{1i}}(-\gamma_{4i}) \qquad \text{for } i = 1, 2, \cdots, n
\end{aligned} \tag{62}$$

and

$$l_i = (l_{1i}, l_{2i}, l_{3i}, l_{4i}) \qquad \text{for } i = 1, 2, \cdots, n \tag{63}$$

Since L_2 is related with P_2 as follows:

$$P_2A_2+A_2'P_2 = -L_2L_2' \tag{64}$$

P_2 can be obtained by solving (64) with L_2 in (58) as follows:

$$P_2 = \begin{pmatrix} P_{33} & P_{34} & P_{35} & P_{36} \\ P_{43} & P_{44} & P_{45} & P_{46} \\ P_{53} & P_{54} & P_{55} & P_{56} \\ P_{63} & P_{64} & P_{65} & P_{66} \end{pmatrix} \tag{65}$$

where $P_{ij}(i, j=3, 4, 5, 6)$ is an $n \times n$ diagonal matrix defined by

$$P_{ij} = \mathrm{diag}(p_{ijk}) \qquad \text{for } i, j = 3, 4, 5, 6, \ k = 1, 2, \cdots, n \tag{66}$$

Elements p_{ijk} are determined by solving the following equation:

$$\Gamma_i p_i = \nu_i \qquad \text{for } i = 1, 2, \cdots, n \tag{67}$$

where Γ_i, p_i and ν_i are 10×10 matrix and 10 dimensional vectors, respectively, defined by

$$\Gamma_i = \begin{pmatrix} 2h_1 & 2h_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & h_1+h_4 & -h_6 & -h_8 & h_3 & 0 & 0 & 0 & 0 & 0 \\ -h_2 & 0 & h_1+h_7 & h_9 & 0 & h_3 & 0 & 0 & 0 & 0 \\ 0 & h_5 & 0 & h_1+h_{10} & 0 & 0 & h_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2h_4 & -2h_6 & -2h_8 & 0 & 0 & 0 \\ 0 & -h_2 & 0 & 0 & 0 & h_4+h_7 & h_9 & -h_6 & -h_8 & 0 \\ 0 & 0 & 0 & 0 & h_5 & 0 & h_4+h_{10} & 0 & -h_6 & -h_8 \\ 0 & 0 & -2h_2 & 0 & 0 & 0 & 0 & 2h_7 & 2h_9 & 0 \\ 0 & 0 & 0 & -h_2 & 0 & h_5 & 0 & 0 & h_7+h_{10} & h_9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2h_5 & 0 & 0 & 2h_{10} \end{pmatrix}_i$$

$$\text{for } i = 1, 2, \cdots, n \tag{68}$$

$$p_i = (p_{33i}, p_{34i}, p_{35i}, p_{36i}, p_{44i}, p_{45i}, p_{46i}, p_{55i}, p_{56i}, p_{66i})'$$
$$\text{for } i = 1, 2, \cdots, n \tag{69}$$

and

$$\nu_i = (l_{1i}l_{1i}, l_{1i}l_{2i}, l_{1i}l_{3i}, l_{1i}l_{4i}, l_{2i}l_{2i}, l_{2i}l_{3i}, l_{2i}l_{4i}, l_{3i}l_{3i}, l_{3i}l_{4i}, l_{4i}l_{4i})'$$
$$\text{for } i = 1, 2, \cdots, n \tag{70}$$

Thus P_1 and P_2, and accordingly, P are obtained.

5. Lyapunov Function

Since P is obtained, we can get an expression of the Lyapunov function as follows:

$$V(x) = [\delta_r', \omega', \Delta E', \Delta E_a', \Delta E_e', \Delta E_d'] \begin{pmatrix} P_{11} & P_{12} & 0 & 0 & 0 & 0 \\ P_{21} & P_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{33} & P_{34} & P_{35} & P_{36} \\ 0 & 0 & P_{43} & P_{44} & P_{45} & P_{46} \\ 0 & 0 & P_{53} & P_{54} & P_{55} & P_{56} \\ 0 & 0 & P_{63} & P_{64} & P_{65} & P_{66} \end{pmatrix} \begin{pmatrix} \delta_r \\ \omega \\ \Delta E \\ \Delta E_a \\ \Delta E_e \\ \Delta E_d \end{pmatrix}$$
$$+2V_1(\sigma)$$
$$= \delta_r' P_{11}\delta_r + 2\delta_r' P_{12}\omega + \omega' P_{22}\omega + 2V_1(\sigma) + \Delta E' P_{33}\Delta E + \Delta E_a' P_{44}\Delta E_a$$
$$+ \Delta E_e' P_{55}\Delta E_e + \Delta E_d' P_{66}\Delta E_d + 2(\Delta E' P_{34}\Delta E_a + \Delta E' P_{35}\Delta E_e$$
$$+ \Delta E' P_{36}\Delta E_d + \Delta E_a' P_{45}\Delta E_e + \Delta E_a' P_{46}\Delta E_d + \Delta E_e' P_{56}\Delta E_d) \tag{71}$$

Now that the Lyapunov function is obtained, we let $q \to \infty$ because q is introduced only in order not to cause a pole-zero cancellation between $(N+Qs)$ and $W(s)$, as mentioned before. Substituting (27) into (71), and expanding and rearranging terms in (71), we obtain the following expression:

$$
\begin{aligned}
V(x) = {} & (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i-\omega_j)^2 \\
& +(\mu^*-\mu_o)(\sum_{i=1}^{n} m_i\omega_i)^2+\rho\{\sum_{i=1}^{n}[d_i(\delta_i-\delta_i^o)+m_i\omega_i]\}^2 \\
& +\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}[E_iE_j(\cos\delta_{ij}^o-\cos\delta_{ij})-(\delta_{ij}-\delta_{ij}^o)E_i^oE_j^o\sin\delta_{ij}^o] \\
& +\sum_{i=1}^{n}[p_{33i}(E_i-E_i^o)^2+p_{44i}(E_{ai}-E_{ai}^o)^2+p_{55i}(E_{ei}-E_{ei}^o)^2 \\
& +p_{66i}(E_{di}-E_{di}^o)^2+2p_{34i}(E_i-E_i^o)(E_{ai}-E_{ai}^o) \\
& +2p_{35i}(E_i-E_i^o)(E_{ei}-E_{ei}^o)+2p_{36i}(E_i-E_i^o)(E_{di}-E_{di}^o) \\
& +2p_{45i}(E_{ai}-E_{ai}^o)(E_{ei}-E_{ei}^o)+2p_{46i}(E_{ai}-E_{ai}^o)(E_{di}-E_{di}^o) \\
& +2p_{56i}(E_{ei}-E_{ei}^o)(E_{di}-E_{di}^o)]
\end{aligned}
\tag{72}
$$

where μ_o is equal to $(-1/\sum m_i)$. The first and the second terms in (72) represent kinetic energy. If the damping torques are uniform, μ^* equals μ_o, and the kinetic energy depends only on relative angular velocities. ρ in the third term is an arbitrary non-negative scalar, but it is chosen as zero because the term narrows and complicates any estimations of the transient stability regions. The fourth term is potential energy which is stored in the system owing to some deviations of the rotor angles from those at the stable equilibrium point. The potential energy plays an important role in defining the transient stability region of the system [6], [13]. The fifth term is a new term which is related to field flux linkages and excitation system variables.

If the damping torques are uniform or zero, (72) reduces to

$$
\begin{aligned}
V(x) = {} & (1/2\sum_{i=1}^{n} m_i)\sum_{i=1}^{n}\sum_{j=1}^{n} m_i m_j(\omega_i-\omega_j)^2 \\
& +\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}[E_iE_j(\cos\delta_{ij}^o-\cos\delta_{ij})-(\delta_{ij}-\delta_{ij}^o)E_i^oE_j^o\sin\delta_{ij}^o] \\
& +\sum_{i=1}^{n}[p_{33i}(E_i-E_i^o)^2+p_{44i}(E_{ai}-E_{ai}^o)^2 \\
& +p_{55i}(E_{ei}-E_{ei}^o)^2+p_{66i}(E_{di}-E_{di}^o)^2 \\
& +2p_{34i}(E_i-E_i^o)(E_{ai}-E_{ai}^o)+2p_{35i}(E_i-E_i^o)(E_{ei}-E_{ei}^o) \\
& +2p_{36i}(E_i-E_i^o)(E_{di}-E_{di}^o)+2p_{45i}(E_{ai}-E_{ai}^o)(E_{ei}-E_{ei}^o) \\
& +2p_{46i}(E_{ai}-E_{ai}^o)(E_{di}-E_{di}^o)+2p_{56i}(E_{ei}-E_{ei}^o)(E_{di}-E_{di}^o)] \\
= {} & V_k(\omega)+V_p(\delta, E)+V_f(E, E_a, E_e, E_d)
\end{aligned}
\tag{73}
$$

where ρ is chosen as zero. V_k and V_p are kinetic energy and potential energy, respectively. The time derivatives of V_k, V_p and V_f are written as follows:

$$\frac{dV_k}{dt} = -(1/\sum_{i=1}^{n} d_i) \sum_{i=1}^{n}\sum_{j=1}^{n} d_i d_j (\omega_i - \omega_j)^2$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin \delta_{ij}^o - E_i E_j \sin \delta_{ij})(\omega_i - \omega_j) \tag{74}$$

$$\frac{dV_p}{dt} = -\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}(E_i^o E_j^o \sin \delta_{ij}^o - E_i E_j \sin \delta_{ij})(\omega_i - \omega_j)$$
$$+2\sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij} E_j (\cos \delta_{ij}^o - \cos \delta_{ij}) \tag{75}$$

$$\frac{dV_f}{dt} = -2\sum_{i=1}^{n} (dE_i/dt) \sum_{j=1}^{n} B_{ij} E_j (\cos \delta_{ij}^o - \cos \delta_{ij})$$
$$-\sum_{i=1}^{n} [l_{1i}(E_i - E_i^o) + l_{2i}(E_{ai} - E_{ai}^o) + l_{3i}(E_{ei} - E_{ei}^o)$$
$$+ l_{4i}(E_{di} - E_{di}^o) - \sqrt{2\xi_{1i}}\, f_{2i}(\sigma)]^2 \tag{76}$$

The first term of (74) is due to the damping torques of generators, and it is non-positive. A part of the kinetic energy is dissipated by the damping torques. The second term of (74) and the first term of (75) are of the same magnitude, and are opposite signs of each other, which imply that there is an exchange of energy between the kinetic and the potential energy. Hence, these terms do not contribute to the damping rate of V. Similarly, the second term of (75) and the first term of (76) are of the same magnitude, and are opposite signs of each other. There is an exchange of energy between V_p and V_f, too. The second term of (76) is due to field flux linkages and excitation system variables, and it is non-positive. As a whole, V dampens according to

$$\frac{dV}{dt} = -(1/\sum_{i=1}^{n} d_i) \sum_{i=1}^{n}\sum_{j=1}^{n} d_i d_j (\omega_i - \omega_j)^2$$
$$-\sum_{i=1}^{n} [l_{1i}(E_i - E_i^o) + l_{2i}(E_{ai} - E_{ai}^o) + l_{3i}(E_{ei} - E_{ei}^o)$$
$$+ l_{4i}(E_{di} - E_{di}^o) - \sqrt{2\xi_{1i}}\, f_{2i}(\sigma)]^2 \tag{77}$$

while V_k and V_p, V_p and V_f are interacting respectively.

6. Numerical Example

The transient stability of a 10-machine 39-bus system is investigated. A line diagram and generator parameters are provided in Fig. 3 and Table 1. As is observed from Table 1, all generators are installed with automatic voltage regulators and thyristor exciters, and their parameters are the same for all genera-

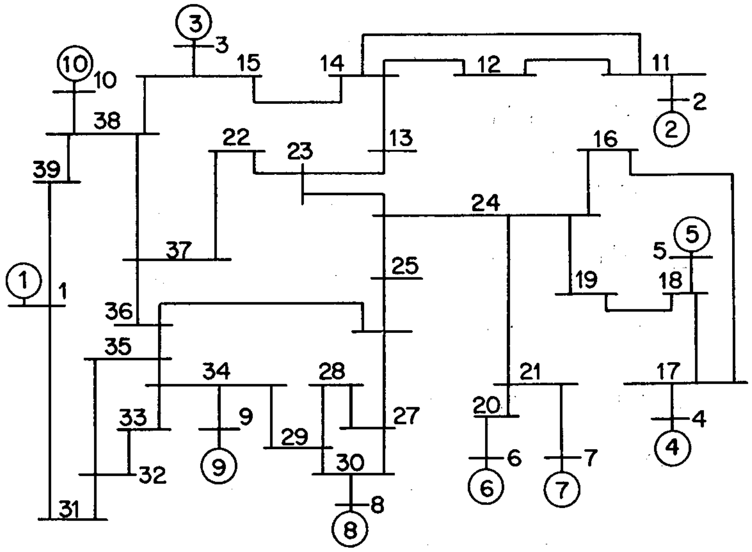


Fig. 3. Configuration of a 10-machine system

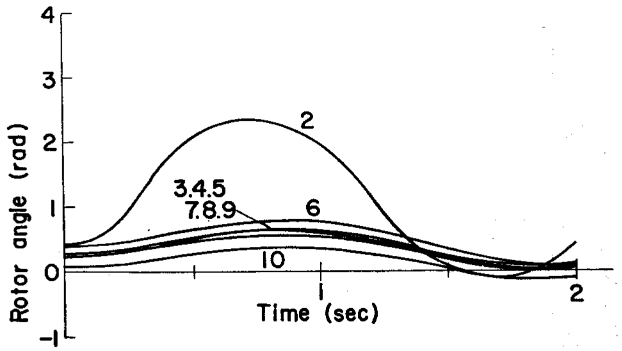
Table 1. Generator parameters

Unit	H	x_d	x'_d	x_q	T'_{d0}	Exciter & AVR
1	500.0	0.0200	0.0060	0.0190	7.00	$K_a=1.000$
2	34.5	0.2106	0.0570	0.2050	4.79	$K_e=1.000$
3	24.3	0.2900	0.0570	0.2800	6.70	$K_d=0.004$
4	26.4	0.2950	0.0490	0.2920	5.66	$T_a=0.020$
5	34.8	0.2540	0.0500	0.2410	7.30	$T_e=0.040$
6	26.0	0.6700	0.1320	0.6200	5.40	$T_d=0.500$
7	28.6	0.2620	0.0436	0.2580	5.69	$T_1=0.0$
8	35.8	0.2495	0.0531	0.2370	5.70	$E_{c1}=7.0$
9	30.3	0.2950	0.0697	0.2820	6.56	$E_{c2}=0.0$
10	42.0	0.1000	0.0310	0.0690	10.20	

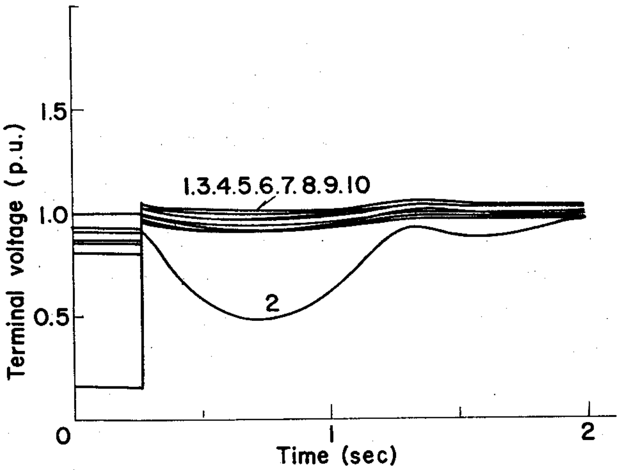
tors. The system is assumed to be disturbed by a 3-phase short-circuit, which occurs at a terminal x of a transmission line x—y. The fault is cleared by opening the line at both terminals after a certain lapse of time.

6.1 Time variations of generator variables

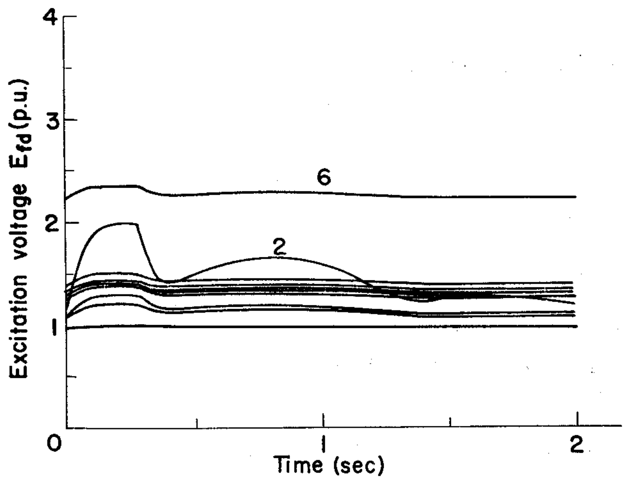
Fig. 4 shows time variations of generator variables for a case where a fault 11–12 is cleared at $t=0.26$ sec. Fig. 4(a) represents swing curves of generators, which indicate that the system is stable for this fault. In this case, the No. 2 generator suffers large disturbances because the fault bus is near this generator. Fig. 4(b) shows the time variations of terminal voltages. The terminal voltage of



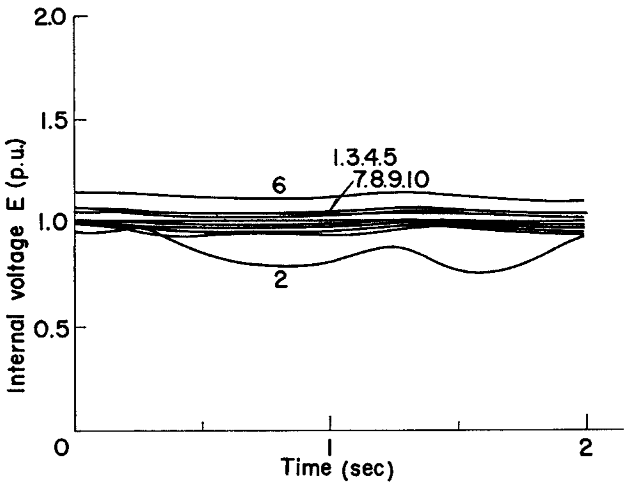
(a) rotor angle



(b) terminal voltage



(c) excitation voltage



(d) internal voltage

Fig. 4. Time variations of generator variables

the No. 2 generator falls to values smaller than 0.20 p.u. during the fault period. After the clearance of the fault, the terminal voltage recovers its magnitude to values of about 1.0 p.u., but a large voltage dip appears again because of an excursion of the rotor angle of the generator. Fig. 4(c) shows the variations of excitation voltages. The excitation voltages vary rapidly according to the variations of the terminal voltages. However, their values do not reach their ceiling voltages because of the low gains of automatic voltage regulators. Fig. 4 (d) shows the variations of internal voltages. They decreases during the first swing of generators. However, their extent is relatively small, compared with cases, where automatic voltage regulators are not installed.

6.2 Stable equilibrium point

In order to apply Lyapunov's direct method, a stable equilibrium point must be calculated for a post-fault condition. If the damping torques of generators are zero, then the stable equilibrium point can be obtained by solving the following equations:

$$\begin{aligned}
&(P_{m1}-P_{e1})/m_1-(P_{mi}-P_{ei})/m_i = 0\\
&E_{fdi}-E'_{qi}-(x_{di}-x'_{di})i_{di} = 0\\
&E_{ai}+K_{ai}E_{di}-K_{ai}(V_{refi}-V_{ti}) = 0\\
&K_{ei}E_{ai}-E_{ei} = 0\\
&(K_{di}K_{ei}/T_{ei})E_{ai}-(K_{di}/T_{ei})E_{ei}-(1/T_{di})E_{di} = 0 \qquad \text{for } i = 1, 2, \cdots, n \qquad (78)
\end{aligned}$$

The above equations are solved by the Newton-Raphson method iteratively in

Table 2. Equilibrium point (fault 11–12)

Unit	Prefault condition		Postfault condition (voltage regulator)		Postfault condition (variable flux)	
	δ (rad)	E (p.u.)	δ (rad)	E (p.u)	δ (rad)	E (p.u.)
1	0.0	1.014	0.0	1.014	0.0	1.010
2	0.387	0.983	0.514	0.968	0.571	0.925
3	0.258	0.991	0.263	0.989	0.282	0.979
4	0.276	1.064	0.280	1.063	0.302	1.054
5	0.273	1.079	0.278	1.076	0.299	1.069
6	0.396	1.149	0.401	1.146	0.425	1.140
7	0.249	0.997	0.254	0.996	0.276	0.987
8	0.277	1.004	0.280	1.003	0.297	0.996
9	0.280	1.016	0.283	1.016	0.299	1.009
10	0.093	1.057	0.104	1.056	0.110	1.049

this paper, and its details are provided in Appendix C.

Table 2 shows the equilibrium points for a fault 11–12. There are three cases, i.e., a case of pre-fault condition, a case of post-fault condition where automatic voltage regulators are installed in generators and a case of post-fault condition where automatic voltage regulators are not installed. From this table, we can extract the following features:

(a) Under post-fault conditions, the rotor angles are larger than those under pre-fault conditions. Its extent is large for generators around the fault, i.e., the No. 2 generator in this case. Its rotor angle changes from 0.387 rad to 0.514 rad, or from 0.387 rad to 0.571 rad.

(b) Under post-fault conditions, the magnitudes of internal voltages are smaller than those under pre-fault condition. Its extent is large for generators around the fault, i.e., the No. 2 generator in this case. Its internal voltage changes from 0.983 p.u. to 0.968 p.u., or from 0.983 p.u. to 0.925 p.u.

(c) If automatic voltage regulators are installed, then an increase in rotor angles and a decrease in magnitudes of internal voltages are suppressed when the system state changes from a pre-fault condition to a post-fault condition.

The feature (c) is characteristic to systems where automatic voltage regulators are installed. It is clear that this feature leads to an improvement of the transient stability of systems.

6.3 Stability condition

What values of the automatic voltage regulator gain may be allowed? This section is addressed to this problem. According to a generalized Popov criterion,

(34) and (35) must be satisfied for the system to be stable. The inequalities in (35) are equivalent to the following inequalities:

$$\xi_{1i} = \alpha_i \geq 0 \tag{79-a}$$

$$\xi_{2i} = \eta_{1i} K_{ai} + \eta_{2i} \geq 0 \tag{79-b}$$

$$\xi_{3i} = \eta_{3i} K_{ai}^2 + \eta_{4i} K_{ai} + \eta_{5i} \geq 0 \tag{79-c}$$

$$\xi_{4i} = \eta_{6i} K_{ai}^2 + \eta_{7i} K_{ai} + \eta_{8i} \geq 0 \tag{79-d}$$

where

$$\begin{aligned}
\eta_{1i} &= \beta_i' h_{2i} h_{6i} - 2\alpha_i h_{5i}' h_{8i} \\
\eta_{2i} &= \alpha_i (h_{4i}^2 + h_{7i}^2 + h_{10i}^2) \\
\eta_{3i} &= (\alpha_i h_{5i}' h_{8i} - \beta_i' h_{2i} h_{6i}) h_{5i}' h_{8i} \\
\eta_{4i} &= \alpha_i [2h_{5i}' h_{8i} (h_{4i} h_{7i} + h_{7i} h_{10i} + h_{10i} h_{4i}) - h_{2i} h_{3i}' h_{6i} (h_{4i} + h_{7i})] \\
&\quad + \beta_i' (h_{10i}^2 - h_{1i} h_{4i} - h_{4i} h_{7i} - h_{7i} h_{1i}) h_{2i} h_{6i} \\
\eta_{5i} &= \alpha_i (h_{4i}^2 h_{7i}^2 + h_{7i}^2 h_{10i}^2 + h_{10i}^2 h_{4i}^2) \\
\eta_{6i} &= -(\alpha_i h_{3i}' + \beta_i' h_{1i}) h_{2i} h_{5i}' h_{6i} h_{8i} h_{10i} \\
\eta_{7i} &= -[\alpha_i (h_{4i} + h_{7i}) h_{3i}' + \beta_i' (h_{1i} h_{4i} + h_{4i} h_{7i} + h_{7i} h_{1i})] h_{2i} h_{6i} h_{10i}^2 \\
\eta_{8i} &= \alpha_i (h_{4i} h_{7i} h_{10i})^2 \qquad \text{for } i = 1, 2, \cdots, n
\end{aligned} \tag{80-a}$$

and

$$\begin{aligned}
h_{3i}' &= h_{3i}/K_{ai} = (1/T_{doi}')(1 - x_{di}' B_{ii}) \\
h_{5i}' &= h_{5i}/K_{ai} = 1/T_{ai} \\
\beta_i' &= \beta_i/K_{ai} = x_{di}'/T_{ai} \qquad \text{for } i = 1, 2, \cdots, n
\end{aligned} \tag{80-b}$$

The inequalities (79-a) and (79-b) hold for the usual generator parameters. From (79-c), K_{ai} has to stay in the following interval:

$$\xi_{3i\,\min} \leq K_{ai} \leq \xi_{3i\,\max} \tag{81}$$

where

$$\xi_{3i\,\min} \simeq -\frac{(x_{di} T_{di}^2 - x_{di}' T_{ai} T_{doi}') T_{ei} + (x_{di} T_{ai} + x_{di}' T_{doi}') T_{di}^2}{x_{di}' K_{di} K_{ei} T_{doi}' T_{di}^2} \tag{82-a}$$

$$\xi_{3i\,\max} \simeq \frac{(x_{di} - x_{di}')(T_{ai}^2 + T_{ei}^2 + T_{di}^2) T_{doi}' \cos\phi_i}{K_{ei}[(x_{di} T_{di}^2 - x_{di}' T_{ai} T_{doi}') T_{ei} + (x_{di} T_{ai} + x_{di}' T_{doi}') T_{di}^2]} \tag{82-b}$$

From (79-d), K_{ai} has to stay in the following interval, too:

$$\xi_{4i\,\min} \leq K_{ai} \leq \xi_{4i\,\max} \tag{83}$$

where

$$\xi_{4i\,\min} \simeq -\frac{x_{di} T_{ai} + x_{di}' T_{doi}' + x_{di} T_{ei}}{x_{di} K_{di} K_{ei} T_{di}} \tag{84-a}$$

$$\xi_{4i\max} \simeq \frac{(x_{di}-x'_{di})\,T'_{doi}\cos\phi_i}{K_{ei}(x_{di}T_{ai}+x'_{di}T'_{doi}+x_{di}T_{ei})} \tag{84-b}$$

It is clear from (82-a) and (84-a) that $\xi_{3i\min}$ and $\xi_{4i\min}$ are both negative, and accordingly, that the left hand inequalities in (81) and (83) are satisfied if K_{ai} has a positive value. On the other hand, $\xi_{3i\max}$ and $\xi_{4i\max}$ are both positive. They are proportional to $\cos\phi_i$, and are inversely proportional to K_{ei}. Fig. 5 shows

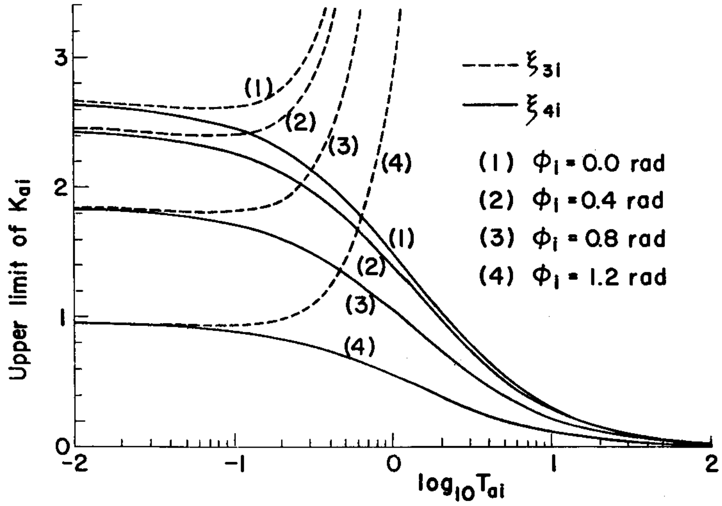


Fig. 5. Variations of $\xi_{3i\max}$ and $\xi_{4i\max}$

variations of $\xi_{3i\max}$ and $\xi_{4i\max}$ with T_{ai}. It is observed from the figure that $\xi_{3i\max}$ is greater than $\xi_{4i\max}$, so the maximum value of K_{ai} is determined by $\xi_{4i\max}$. $\xi_{4i\max}$ becomes larger as T_{ai} approaches zero. In order to simplify the expression of $\xi_{4i\max}$, we let T_{ai}, $T_{ei}\to 0$, then (84-b) reduces to

$$\xi_{4i\max} = K_{ai\max}\cos\phi_i \tag{85}$$

where $K_{ai\max}$ is defined by

$$K_{ai\max} = \frac{x_{di}-x'_{di}}{x'_{di}} \tag{86}$$

and K_{ei} is assumed to be 1.0. $K_{ai\max}$ is the maximum value of K_{ai} which is allowed for the system to be stable. It is determined only by x_{di} and x'_{di}. Table 3 shows the values of $K_{ai\max}$ for all generators in the system. They are in a range of 2.0~6.0. These values are

Table 3. Value of $K_{ai\max}$

Unit	$K_{ai\max}$
1	2.333
2	2.695
3	4.088
4	5.020
5	4.080
6	4.076
7	5.009
8	3.699
9	3.232
10	2.226

somewhat smaller than the practically used values.

Why is K_{ai} suppressed to such low values by the stability criterion? It will be useful to answer this question. First of all, let T_{ai}, T_{ei} and T_{di} be zero, then the variations of the terminal voltages are directly transmitted to the excitation voltages, and the internal voltages vary according to the following equations:

$$\frac{dE_i}{dt} = -\alpha_i^*(E_i - E_i^0) - \beta_i^* \sum_{\substack{j=1 \\ j \neq i}}^{n} B_{ij} E_j (\cos \delta_{ij}^0 - \cos \delta_{ij}) \qquad \text{for } i = 1, 2, \cdots, n \tag{87}$$

where

$$\begin{aligned} \alpha_i^* &= (1/T'_{doi})[1-(x_{di}-x'_{di})B_{ii}+(1+x'_{di}B_{ii})K_{ai}K_{ei}/\cos\phi_i] \\ \beta_i^* &= (1/T'_{doi})(x_{di}-x'_{di}-x'_{di}K_{ai}K_{ei}/\cos\phi_i) \end{aligned} \tag{88}$$

Eq. (87) is formally equivalent to that of a system where automatic voltage regulators are not installed. (See eq. (10) in [13].) From the result of [13], β_i^* must be non-negative for the system to be stable. If K_{ai} is 0, then β_i^* equals β_i in [13], and it is positive. According to an increase in the magnitude of K_{ai}, β_i^* takes smaller values, and at last, it comes to take negative values. The value β_i^* is zero when K_{ai} is K_{ai}^*, where

$$K_{ai}^* = \frac{x_{di}-x'_{di}}{x'_{di}} \cos\phi_i \tag{89}$$

This equation is equivalent to (85), that is, (85) shows the value where β_i^* becomes zero. In other words, the system is stable if K_{ai} is zero, but its stability margin decreases with an increase in the magnitude of K_{ai}, and the margin becomes zero when K_{ai} reaches K_{ai}^*. In any event, the above limitation of K_{ai} is not desirable from the viewpoint of applying Lyapunov's direct method to practical power system transient stability analyses, so it should be removed. This problem is still open.

6.4 Calculation of P_2 matrix

Matrix P_2, a part of the P matrix, is obtained by solving (67). Table 4 shows P_2 matrices for all generators in the system for a fault 11–12. It can be observed from the table that the element P_{33i} takes greater values than the other elements by multipliers $10^2 \sim 10^4$. Since the deviations of E_i, E_{ai} and E_{ei} are the same degree, and since the deviation of E_{di} is much smaller than the other variables, V_f, a component of V, is determined mainly by the deviation of E_i. The above facts are due to the same reason as given in the previous section. When T_{ai} and T_{ei} are small, variations of internal voltages can be approximately expressed by

Table 4. P_2-matrix

Unit	P_2 Matrix			
1	268.90	0.1679	0.4214	−0.4994
	0.1679	0.0224	0.0448	−0.0788
	0.4214	0.0448	0.1238	−0.1258
	−0.4994	−0.0788	−0.1258	2.0020
2	32.020	0.0319	0.0798	−0.0989
	0.0319	0.0043	0.0087	−0.0155
	0.0798	0.0087	0.0238	−0.0248
	−0.0989	−0.0155	−0.0248	0.3928
3	23.520	0.0099	0.0253	−0.0240
	0.0099	0.0019	0.0037	−0.0060
	0.0253	0.0037	0.0106	−0.0093
	−0.0240	−0.0060	−0.0093	0.1509
4	21.020	0.0086	0.0221	−0.0229
	0.0086	0.0018	0.0037	−0.0057
	0.0221	0.0037	0.0106	−0.0086
	−0.0229	−0.0057	−0.0086	0.1417
5	25.340	0.0082	0.0211	−0.0211
	0.0082	0.0016	0.0032	−0.0051
	0.0211	0.0032	0.0092	−0.0078
	−0.0211	−0.0051	−0.0078	0.1270
6	11.170	0.0048	0.0124	−0.0131
	0.0048	0.0009	0.0019	−0.0030
	0.0124	0.0019	0.0053	−0.0046
	−0.0131	−0.0030	−0.0046	0.0742
7	25.300	0.0104	0.0269	−0.0247
	0.0104	0.0022	0.0044	−0.0068
	0.0269	0.0044	0.0127	−0.0105
	−0.0247	−0.0068	−0.0105	0.1716
8	26.600	0.0126	0.0322	−0.0324
	0.0126	0.0023	0.0046	−0.0074
	0.0322	0.0046	0.0130	−0.0116
	−0.0324	−0.0074	−0.0116	0.1868
9	23.520	0.0104	0.0264	−0.0285
	0.0104	0.0017	0.0035	−0.0057
	0.0264	0.0035	0.0098	−0.0090
	−0.0285	−0.0057	−0.0090	0.1448
10	67.860	0.0250	0.0625	−0.0773
	0.0250	0.0032	0.0063	−0.0113
	0.0625	0.0063	0.0174	−0.0180
	−0.0773	−0.0113	−0.0180	0.2871

(87), and V_f is approximated as follows:

$$V_f \simeq \sum_{i=1}^{n} (\alpha_i^* / \beta_i^*)(E_i - E_i^0)^2 \tag{90}$$

where α_i^* and β_i^* are defined by (88). Table 5 shows the values of P_{33i} and (α_i^*/β_i^*). It is observed from the table that there is a good agreement between them for all generators.

Table 5. P_{33i} and (α_i^*/β_i^*)

Unit	P_{33i}	(α_i^*/β_i^*)
1	268.80	267.41
2	32.02	31.74
3	23.52	23.46
4	21.02	20.98
5	25.34	25.29
6	11.17	11.35
7	25.30	25.24
8	26.61	26.51
9	23.52	23.45
10	67.86	67.63

6.5 Time variations of Lyapunov function

Fig. 6 shows the time variations of V and its components for a sustained fault 11–12. Three cases are shown: one where the field flux linkages of genarators are constant, one where the variations of the field flux linkages are taken into account, but automatic voltage regulators are not installed in generators, and one where the variations of the field flux linkages are taken into account, and automatic voltage regulators are installed in generators. The kinetic energy V_k

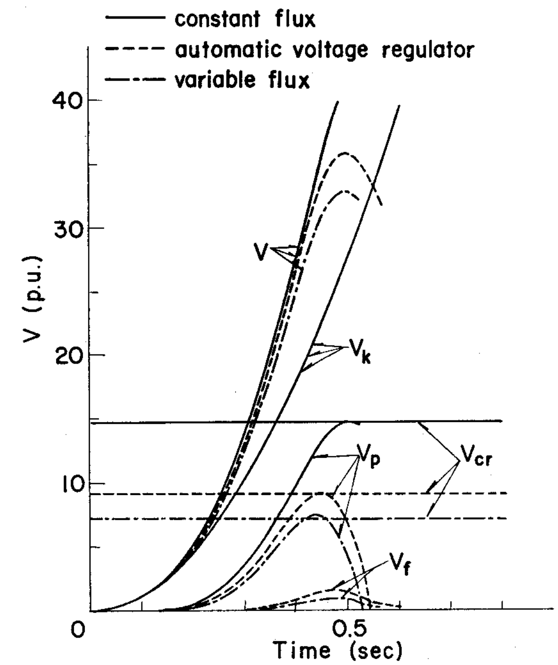


Fig. 6. Time variations of V and its components: sustained fault

increases monotonously, and it takes almost the same values each time in all these cases. These facts imply that all generators receive similar accelerations in all cases. On the other hand, the potential energy takes different values each time for these cases. The values of V_p become smaller in the case where the field flux linkages are constant, in the case where automatic voltage regulators are installed and in the case where only the variations of the field flux linkages are taken into account. Since the new term takes small values in all cases, the differences of V among these cases are mainly due to those of V_p. In the figure, the critical values of V are also shown. The method of determining them is described in [6] and [13] in detail. The critical value becomes smaller in the same order of cases as for V_p. The time when V reaches the critical value is adopted as an estimation value of the critical fault clearing time. In these cases, the estimation values are 0.30, 0.26 and 0.25 sec. These values are very close to the results by simulations.

Fig. 7 shows the time variations of V and its components for a fault 11–12 which is cleared at $t=0.26$ sec. After the fault clearance, the kinetic energy

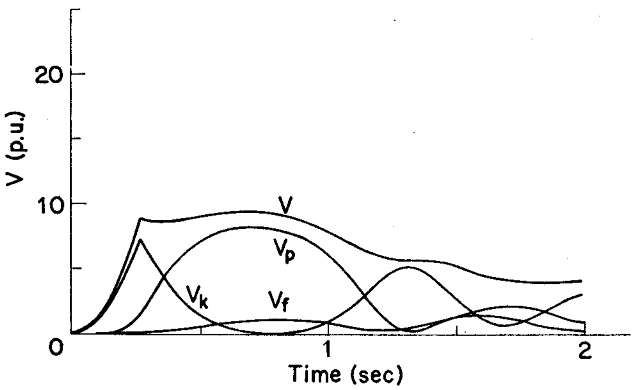


Fig. 7. Time variations of V and its components: critical fault

decreases and the potential energy increases, and they reach their minimum and maximum values at almost the same times. Afterwards, they oscillate with their amplitudes decreasing with time. The new term V_f remains at a low level. Its variation corresponds to that of the internal voltage of the No. 2 generator. The Lyapunov function is nearly constant during the first swing of generators, and it begins to decrease with the backswing of the No. 2 generator. Afterwards, it decreases monotonously. In the figure, transfer conductances are counted in by appropriately modifying the Lyapunov function, the details of which are provided in [7], [13].

6.6 Estimation results

In order to evaluate Lyapunov's direct method, a reasonably comprehensive series of stability analyses are performed by simulations, and the results are compared with those obtained by Lyapunov's direct method. Critical fault clearing times are used as a measure of the transient stability.

Table 6 shows critical fault clearing times estimated by Lyapunov's direct method along with those obtained by simulations. A range of faults which occurs at various buses in the system are tested. As is observed from the table, there is a good agreement between the results obtained by the direct method and the simulations for all faults. Differences between them are within 0.02 sec. for all faults. It is concluded from these results that the direct method yields results of practical significance.

Table 7 shows critical fault clearing times for three cases, i.e., one where the field flux linkages of generators are constant, one where the variations of the field

Table 6. Estimation results of critical clearing time

Fault	V_{cr}	T_{es}	T_{cr}
11–12	9.12	0.26	0.26
15–14	31.37	0.37	0.38
17–18	32.57	0.44	0.43
18–17	39.23	0.47	0.47
24–16	42.66	0.39	0.39
30–27	32.60	0.46	0.45
34–29	35.93	0.45	0.45
38–15	38.56	0.51	0.49

T_{es}: results by direct method
T_{cr}: results by simulations
V_{cr}: critical value of V

Table 7. Critical clearing times for three cases

Fault	Variable flux	Voltage regulator	Constant flux
11–12	0.25	0.26	0.30
15–14	0.38	0.38	0.41
17–18	0.42	0.43	0.47
18–17	0.46	0.47	0.50
24–16	0.37	0.39	0.45
30–27	0.44	0.45	0.47
34–29	0.44	0.45	0.46
38–15	0.44	0.49	0.65

flux linkages are taken into account, but automatic voltage regulators are not installed in generators, and one where the variations of the field flux linkages are taken into account, and automatic voltage regulators installed in generators. Comparing the critical clearing times, we can extract the following features:

(a) Critical clearing times take the smallest values for all faults when only the variations of field flux linkages are taken into account.

(b) Critical clearing times take the largest values for all faults when the field flux linkages of generators are constant.

(c) Critical clearing times take values between those of the foregoing two cases.

From these results, two conclusions are drawn. One conclusion is that the transient stability of the system is improved by using automatic voltage regulators. The other is that the improvement of the transient stability is not so large that the stability surpasses that of the system where the field flux linkages are constant. This is due to the fact that high automatic voltage regulator gains are not allowed according to the Popov criterion. Table 8 shows the critical values of the Lyapunov functions for the three cases. From this table, we can also extract the above mentioned two conclusions.

Table 8. Critical values of V for three cases

Fault	Variable flux	Voltage regulator	Constant flux
11–12	7.23	9.12	14.75
15–14	28.85	31.37	47.11
17–18	28.77	32.57	50.17
18–17	36.17	39.23	58.61
24–16	39.71	42.66	66.41
30–27	29.49	32.60	45.89
34–29	33.00	35.93	49.17
38–15	33.14	38.56	53.06

7. Conclusions

In this paper, a Lur'e type Lyapunov function is constructed in a systematic way for a system where automatic voltage regulators are installed in generators. The main features of the Lyapunov function are as follows:

(a) It consists of three terms: potential energy V_p, kientic energy V_k and a new term V_f which is related to internal voltages, automatic voltage regulator and exciter variables.

(b) The potential energy includes internal voltages as its variable, so it varies not only with rotor angles but also with internal voltages.

(c) The new term is in a quadratic form of internal voltages and other variables. Its coefficient matrix is calculated separately for each generator. This feature is desirable from a viewpoint of calculation for systems which consist of many generators.

(d) When excitation systems have fast time responses, V_f can be represented by that for a system where only field flux linkages variations are taken into account with changes of coefficients.

In order to investigate the applicability of Lyapunov's direct method to power system stability analyses, a 10-machine power system was studied. From the results, we can extract the following features:

(e) The critical fault clearing times estimated by Lyapunov's direct method are very close to those obtained by simulations.

(f) The transient stability of the system is improved by using automatic voltage regulators.

(g) However, the improvement in the transient stability is small because the automatic voltage regulator gains must be low according to the generalized Popov stability criterion.

The main contribution of this paper is a systematic construction of a Lyapunov function for a multi-machine power system where automatic voltage regulators and thyristor exciters represented by third order transfer functions are installed in generators. However, the gains of the automatic voltage regulators must be low compared with the practically used values according to the Popov criterion. Enlargement of the gains are indispensable for Lyapunov's direct method to be applied to practical systems. This problem is difficult, but it must be resolved.

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A. *Approximation of Terminal Voltage*

In this section, an approximate expression of a terminal voltage is derived. The terminal voltage of the ith generator is given as follows:

$$\begin{aligned}\dot{V}_{ti} &= \dot{E}_i - jx'_{di}\dot{I}_i \\ &= \dot{E}_i - jx'_{di}\sum_{j=1}^{n}\dot{Y}_{ij}\dot{E}_j \qquad \text{for } i = 1, 2, \cdots, n\end{aligned} \tag{A1}$$

From (A1), we can extract the feature that $\dot{V}_{ti}$ is in a circle with its center at $\dot{E}_i$ and with a radius r_i as is shown in Fig. A, where

$$r_i = x'_{di}\sum_{j=1}^{n} Y_{ij}E_j \qquad \text{for } i = 1, 2, \cdots, n \tag{A2}$$

If the radius r_i is small, then the magnitude of the terminal voltage $\dot{V}_{ti}$ can be approximately expressed by projecting $\dot{V}_{ti}$ onto $\dot{E}_i$ as follows:

$$V_{ti} \simeq E_i + x'_{di}\sum_{j=1}^{n} Y_{ij}E_j\cos(\delta_{ij}+\theta_{ij}) \tag{A3}$$

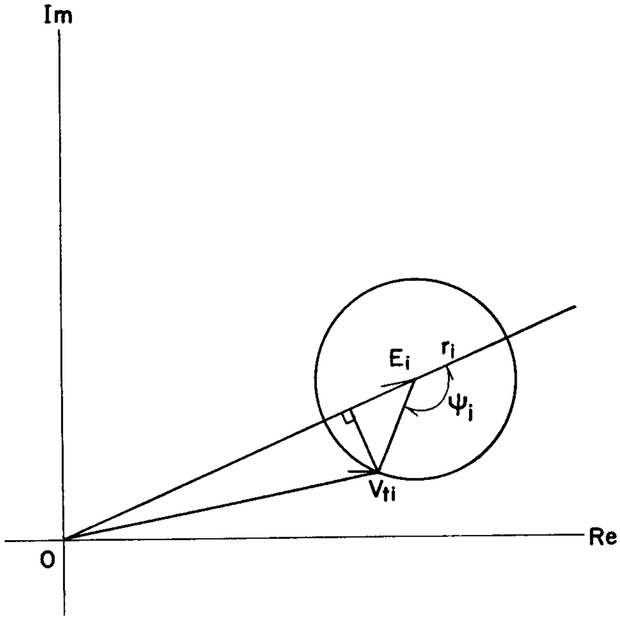

Fig. A. Approximation of terminal voltages

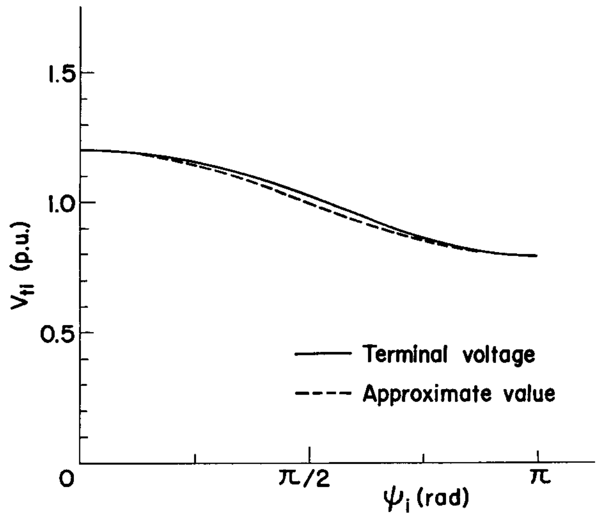

Fig. B. Variation of terminal voltage with ψ_i

Fig. B shows variations of V_{ti} and its approximation by (A3) with ψ_i, where E_i is assumed to be 1.0 p.u. and r_i is 0.2 p.u. The maximum approximation error is $(r_i^2/2)$. In the case where r_i equals 0.2 p.u., the maximum error is 0.02, and its ratio to r_i is 0.10. These values are adequately small. However, the maximum error is 0.125 when r_i equals 0.5 p.u., and its ratio to r_i is 0.25. In this case, the accuracy of the approximation is somewhat low. From the above considerations, it is concluded that V_{ti} is approximated by using (A3) with adequate accuracy when r_i is small. However, its approximation accuracy becomes lower with an increase of r_i.

B. Stability Criterion

The non-linear system considered here is that whose form is shown in Fig. C. The Lyapunov stability is considered, so the inputs are not indicated. The system is assumed to satisfy the following conditions:

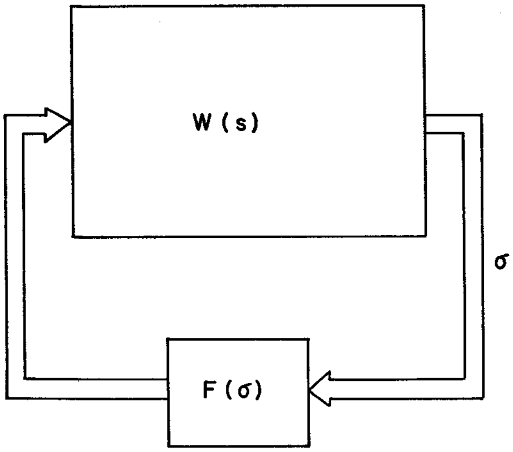

Fig. C. Non-linear system model

1) The matrix $W(s)$ is an $m \times m$ matrix of stable rational transfer functions, assumed to be such that

$$W(\infty) = 0 \tag{B1}$$

2) The non-linearity $F(\sigma)$ satisfies the following conditions:

(i) $F(\sigma)$ is continuous and maps R^m into R^m.

(ii) For a constant real matrix N,

$$F(\sigma)' N\sigma \geq 0 \qquad \text{for all } \sigma \in R^m \tag{B2}$$

and

$$F(0) = 0 \tag{B3}$$

(iii) There is a function $V_1 \in C_1^1$ mapping R^m into R such that

$$V_1(\sigma) \geq 0 \qquad \text{for all } \sigma \in R^m \tag{B4}$$

and

$$V_1(0) = 0 \tag{B5}$$

and for a constant real matrix Q

$$\nabla V_1(\sigma) = Q'F(\sigma) \qquad \text{for all } \sigma \in R^m \tag{B6}$$

The stability criterion for this system is given as follows:
[Theorem]

If there exist real matrices N and Q such that

$$Z(s) = (N+Qs)W(s) \tag{B7}$$

is positive real, then the system shown in Fig. C is stable, where $(N+Qs)$ does not cause pole-zero cancellations with $W(s)$.

If a system proves to be stable according to the theorem, then there exists a Lyapunov function as follows:

$$V(x) = x'Px+2V_1(\sigma) \tag{B8}$$

where x is the state vector of the system, and P is a positive definite and symmetric matrix which satisfies the following equations:

$$\begin{aligned} PA+A'P &= -LL' \\ PB &= CN'+A'CQ'-LW_0 \\ W_0'W_0 &= QC'B+B'CQ' \end{aligned} \tag{B9}$$

where (A, B, C) is regarded as the minimal realization of $W(s)$. The time derivative of V is given as follows:

$$\dot{V}(x) = -[x'L-F(\sigma)'W_0'][L'x-W_0F(\sigma)]-2F(\sigma)'N\sigma \tag{B10}$$

As is observed from (B10), $\dot{V}(x)$ is non-positive.

C. Calculation of Equilibrium Point

In applying Lyapunov's direct method to transient stability analyses, it is necessary to calculate the stable equilibrium point of the system in a post-fault operating state. The equilibrium point is obtained by solving the following equations:

$$g_{1i} = (P_{m1}-P_{e1})/m_1-(P_{m(i+1)}-P_{e(i+1)})/m_{(i+1)} = 0 \qquad \text{for } i = 1, 2, \cdots, n-1 \tag{C1-a}$$

$$g_{2i} = E_{fdi}-E_{qi}'-(x_{di}-x_{di}')i_{di} = 0 \tag{C1-b}$$

$$g_{3i} = E_{ai}+K_{ai}E_{di}-K_{ai}(V_{refi}-V_{ti}) = 0 \tag{C1-c}$$

$$g_{4i} = K_{ei}E_{ai} - E_{ei} = 0 \tag{C1-d}$$

$$g_{5i} = (K_{di}K_{ei}/T_{ei})E_{ai} - (K_{di}/T_{ei})E_{ei} - (1/T_{di})E_{di} = 0 \quad \text{for } i = 1, 2, \cdots, n \tag{C1-e}$$

where the damping torques of the generators are neglected. From (C1-c), (C1-d) and (C1-e), we get the following equations:

$$E_{ai} = K_{ai}(V_{refi} - V_{ti}) \tag{C1-c)$'$}$$

$$E_{ei} = K_{ai}K_{ei}(V_{refi} - V_{ti}) \tag{C1-d)$'$}$$

$$E_{di} = 0 \tag{C1-e)$'$}$$

and accordingly,

$$g_{2i} = [E^{o}_{fdi} + K_{ai}K_{ei}(V_{refi} - V_{ti})] - E'_{qi} - (x_{di} - x'_{di})i_{di} = 0 \tag{C1-b)$'$}$$

Consequently, the stable equilibrium point can be obtained if (C1-a) and (C1-b)$'$ are solved. In the following part, we derive the necessary relations to solve these equations.

Voltages and currents of generators are related by

$$\dot{I}_i = \sum_{j=1}^{n} \dot{Y}'_{ij}\dot{E}_{qdj} \qquad \text{for } i = 1, 2, \cdots, n \tag{C2}$$

where $\dot{Y}'$ is the admittance matrix which relates $\dot{I}$ with $\dot{E}_{qd}$ behind the q-axis reactance x_q. Since $\dot{E}_{qdi}$ is parallel with the q-axis of the ith machine, $\dot{I}'_i$ in the ith machine frame is given by

$$\begin{aligned} \dot{I}'_i &= \dot{I}_i e^{j(\pi/2 - \delta_i)} \\ &= \sum_{j=1}^{n} Y'_{ij}E_{qdj}\angle \pi - (\delta_{ij} + \theta_{ij}) \end{aligned} \tag{C3}$$

Accordingly, the d- and q-axis components of the ith machine are given as follows:

$$i_{di} = -\sum_{j=1}^{n} Y'_{ij}E_{qdi}\cos(\delta_{ij} + \theta_{ij}) \tag{C4-a}$$

$$i_{qi} = \sum_{j=1}^{n} Y'_{ij}E_{qdj}\cos(\delta_{ij} + \theta_{ij}) \tag{C4-b}$$

Since E_{qdi} is defined as

$$E_{qdi} = E'_{qi} + (x_{qi} - x'_{di})i_{di} \tag{C5}$$

we obtain the following equations by substituting (C5) into (C4):

$$Ai_d = f_d \tag{C6-a}$$

$$i_q = Bi_d + f_q \tag{C6-b}$$

where A, B are $n \times n$ matrices, and f_d, f_q are n dimensional vectors defined by

$$A_{ij} = Y'_{ij}(x_{qj}-x'_{dj})\cos(\delta_{ij}+\theta_{ij}) \qquad (j \neq i) \tag{C7-a}$$

$$A_{ii} = Y'_{ii}(x_{qi}-x'_{di})\cos\theta_{ii}+1 \tag{C7-b}$$

$$B_{ij} = Y'_{ij}(x_{qj}-x'_{dj})\sin(\delta_{ij}+\theta_{ij}) \tag{C7-c}$$

$$f_{di} = -\sum_{j=1}^{n} Y'_{ij}E'_{qj}\cos(\delta_{ij}+\theta_{ij}) \tag{C7-d}$$

$$f_{qi} = \sum_{j=1}^{n} Y'_{ij}E'_{qj}\sin(\delta_{ij}+\theta_{ij}) \tag{C7-e}$$

Currents i_d and i_q are obtained by solving (C6). The terminal voltage of the ith generator is given by

$$\dot{V}_{ti} = \dot{E}_{qdi}-jx_{qi}\dot{I}_i \tag{C8}$$

and the d- and the q-axis components of the voltage are given by

$$v_{di} = x_{qi}i_{qi} \tag{C8-a}$$

$$v_{qi} = E'_{qi}-x'_{di}i_{di} \tag{C8-b}$$

Hence the magnitude of the terminal voltage is given by

$$V_{ti} = (v_{di}^2+v_{qi}^2)^{1/2} \tag{C9}$$

The active power of the ith generator is given by

$$\begin{aligned} P_{ei} &= E_{qdi}i_{qi} \\ &= [E'_{qi}+(x_{qi}-x'_{di})i_{di}]i_{qi} \end{aligned} \tag{C10}$$

Now all necessary equations are obtained, so we linearize them in the following part.

Eq.(C6) is linearized as follows:

$$(\Delta A)i_d+A(\Delta i_d) = \Delta f_d \tag{C11-a}$$

$$\Delta i_q = (\Delta B)i_d+B(\Delta i_d)+\Delta f_q \tag{C11-b}$$

where

$$(\Delta A)i_d = S_1\Delta\delta_r \tag{C12-a}$$

$$(\Delta B)i_d = S_2\Delta\delta_r \tag{C12-b}$$

$$\Delta f_d = S_3\Delta\delta_r - T_1\Delta E'_q \tag{C12-c}$$

$$\Delta f_q = S_4\Delta\delta_r - T_2\Delta E'_q \tag{C12-d}$$

and S_1, S_2, S_3 and S_4 are $n\times(n-1)$ matrices, and T_1, T_2 are $n\times n$ matrices defined as follows:

$$S_{1i(j-1)} = -\sum_{\substack{k=1\\k\neq i}}^{n} Y'_{ik}(x_{qk}-x'_{dk})\sin(\delta_{ik}+\theta_{ik})i_{dk} \qquad (j=i) \tag{C13-a}$$

$$S_{1i(j-1)} = Y'_{ij}(x_{qj}-x'_{dj})\sin(\delta_{ij}+\theta_{ij})i_{dj} \qquad (j\neq i) \tag{C13-b}$$

$$S_{2i(j-1)} = \sum_{\substack{k=1\\k\neq i}}^{n} Y'_{ik}(x_{qk}-x'_{dk})\cos(\delta_{ik}+\theta_{ik})i_{dk} \qquad (j=i) \tag{C13-c}$$

$$S_{2i(j-1)} = -Y'_{ij}(x_{qj}-x'_{dj})\cos(\delta_{ij}+\theta_{ij})i_{dj} \qquad (j\neq i) \tag{C13-d}$$

$$S_{3i(j-1)} = \sum_{\substack{k=1\\k\neq i}}^{n} Y'_{ik}E'_{qk}\sin(\delta_{ik}+\theta_{ik}) \qquad (j=i) \tag{C13-e}$$

$$S_{3i(j-1)} = -Y'_{ij}E'_{qj}\sin(\delta_{ij}+\theta_{ij}) \qquad (j\neq i) \tag{C13-f}$$

$$S_{4i(j-1)} = \sum_{\substack{k=1\\k\neq i}}^{n} Y'_{ik}E'_{qk}\cos(\delta_{ik}+\theta_{ik}) \qquad (j=i) \tag{C13-g}$$

$$S_{4i(j-1)} = -Y'_{ij}E'_{qj}\cos(\delta_{ij}+\theta_{ij}) \qquad (j\neq i) \tag{C13-h}$$

for $i = 1, 2, \cdots, n,\quad j = 2, 3, \cdots, n$

and

$$T_{1ij} = Y'_{ij}\cos(\delta_{ij}+\theta_{ij}) \tag{C13-i}$$

$$T_{2ij} = Y'_{ij}\sin(\delta_{ij}+\theta_{ij}) \tag{C13-j}$$

for $i = 1, 2, \cdots, n,\quad j = 2, 3, \cdots, n$

By substituting (C12) into (C11), we get

$$\Delta i_d = U_1\Delta\delta_r - U_2\Delta E'_q \tag{C14-a}$$

$$\Delta i_q = U_3\Delta\delta_r - U_4\Delta E'_q \tag{C14-b}$$

where

$$U_1 = A^{-1}(S_3-S_1) \tag{C15-a}$$

$$U_2 = A^{-1}T_1 \tag{C15-b}$$

$$U_3 = S_2+S_4+BU_1 \tag{C15-c}$$

$$U_4 = T_2-BU_2 \tag{C15-d}$$

Eq. (C9) is linearized as follows:

$$\Delta v_{di} = x_{qi}\Delta i_{qi} \tag{C16-a}$$

$$\Delta v_{qi} = \Delta E'_{qi} - x'_{di}\Delta i_{di} \tag{C16-b}$$

$$\Delta V_{ti} = (v_{di}\Delta v_{di}+v_{qi}\Delta v_{qi})V_{ti} \tag{C16-c}$$

By substituting (C14) and (C16-a,b) into (C16-c), we get

$$\Delta V_t = U_5\Delta\delta_r + U_6\Delta E'_q \tag{C17}$$

where

$$U_5 = \mathrm{diag}\,(x_{qi}v_{di}/V_{ti})\,U_3 - \mathrm{diag}\,(x'_{di}v_{qi}/V_{ti})\,U_1 \tag{C18-a}$$

$$U_6 = \mathrm{diag}\,(x_{qi}v_{di}/V_{ti})\,U_4 + \mathrm{diag}\,(x'_{di}v_{qi}/V_{ti})\,U_2 + \mathrm{diag}(v_{qi}/V_{ti}) \tag{C18-b}$$

Eq.(C10) is linearized as follows:

$$\Delta P_{ei} = [\Delta E'_{qi} + (x_{qi}-x'_{di})\Delta i_{di}]i_{qi} + [E'_{qi} + (x_{qi}-x'_{di})i_{di}]\Delta i_{qi} \quad \text{for } i = 1, 2, \cdots, n \tag{C19}$$

From this equation, we get

$$\Delta P_e = U_7 \Delta\delta_r + U_8 \Delta E'_q \tag{C20}$$

where

$$U_7 = \mathrm{diag}\,[(x_{qi}-x'_{di})i_{qi}]\,U_1 + \mathrm{diag}\,[E'_{qi} + (x_{qi}-x'_{di})i_{di}]\,U_3 \tag{C21-a}$$

$$U_8 = \mathrm{diag}\,(i_{qi}) - \mathrm{diag}\,[(x_{qi}-x'_{di})i_{qi}]\,U_2 + \mathrm{diag}\,[E'_{qi} + (x_{qi}-x'_{di})i_{di}]\,U_4 \tag{C21-b}$$

Eq. (C1) is linearized as follows:

$$\Delta g_1 = (\partial g_1/\partial\delta_r)\Delta\delta_r + (\partial g_1/\partial E'_q)\Delta E'_q \tag{C22-a}$$

$$\Delta g_2 = (\partial g_2/\partial\delta_r)\Delta\delta_r + (\partial g_2/\partial E'_q)\Delta E'_q \tag{C22-b}$$

where

$$\partial g_1/\partial\delta_r = -K'M^{-1}U_7 \tag{C23-a}$$

$$\partial g_1/\partial E'_q = -K'M^{-1}U_8 \tag{C23-b}$$

$$\partial g_2/\partial\delta_r = -\mathrm{diag}\,(x_{di}-x'_{di})\,U_1 - \mathrm{diag}\,(K_{ai}K_{ei})\,U_5 \tag{C23-c}$$

$$\partial g_2/\partial E'_q = -I + \mathrm{diag}\,(x_{di}-x'_{di})\,U_2 - \mathrm{diag}\,(K_{ai}K_{ei})\,U_6 \tag{C23-d}$$

Now that (C22) is obtained, it is easy to solve (C1) iteratively by the well-known Newton-Raphson method.

The value of δ_r and E'_q at the stable equilibrium point can be obtained by iterating the following equation:

$$\begin{bmatrix} \delta_r \\ E'_q \end{bmatrix}_{(i+1)} = \begin{bmatrix} \delta_r \\ E'_q \end{bmatrix}_{(i)} - \begin{bmatrix} \partial g_1/\partial\delta_r & \partial g_1/\partial E'_q \\ \partial g_2/\partial\delta_r & \partial g_2/\partial E'_q \end{bmatrix}^{-1}_{(i)} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}_{(i)} \tag{C24}$$

where the subscript "(i)" denotes the iteration number. The initial values of δ_r and E'_q are chosen as follows:

$$\begin{aligned} \delta_r &= 0_{(n-1)1} \\ E'_q &= 1_{n1} \end{aligned} \tag{C25}$$

where $0_{(n-1)1}$ and 1_{n1} are $(n-1)$ and n dimensional vectors with all the elements equal to zero and unity, respectively. A sufficiently accurate solution of the stable equilibrium point can be obtained in 4 or 5 iterations.