Einstein-Maxwell Field as a Gauge-Invariant Vector-Metric Field

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Abstract

It is proved that, when the Lagrangian density of a vector-metric field is gauge-invariant, the field is reduced to Einstein-Maxwell type field, and when the coupling constant is assigned a special value, the vector field can be regarded as an electromagnetic potential.

As a preferred-frame theory of gravity based on a Lagrangian formulation, which initially assumes no prior geometry, Will and Nordtvedt[1] proposed a field which is defined by a metric field $g_{\alpha\beta}$ and a vector field h_α. The vector field may depend on motion with respect to a preferred universe rest-frame, and it may vary with the evolution of the universe.

The Lagrangian density has the form

$$L = \frac{16\pi\kappa}{c^4} L_m + R + h_\alpha h_\beta (c_1 g^{\alpha\beta} + c_2 R^{\alpha\beta}) + h_{\alpha;\beta} h_{\gamma;\delta} (c_3 g^{\alpha\gamma} g^{\beta\delta} + c_4 g^{\alpha\delta} g^{\beta\gamma} + c_5 g^{\alpha\beta} g^{\gamma\delta}) , \tag{1}$$

where κ is the gravitational constant, c is the fundamental velocity, $L_m = L_m$ ($g_{\alpha\beta}$, mater variable) is the matter Lagrangian, $R_{\alpha\beta}$ is the Ricci tensor, and $c_1, c_2, \cdots, c_5$ are coupling constants. Another vector-metric theory proposed by Hellings and Nordtvedt[2] differs from (1) by the terms containing the derivatives of the vector field, which has a special combination of the constants c_3, c_4 and c_5.

Now we assume the proposition that the Lagrangian density (1) is form-invariant under the gauge-transformation

$$\bar{h}_\alpha = h_\alpha + \phi_{,\alpha} , \tag{2}$$

where ϕ is any scalar field. Then, we will show that the restricted vector-metric field reduces to the Einstein-Maxwell type field.

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Under the transformation (2), the density (1) takes the form

$$\tilde{L} \equiv L+(2h_\alpha\phi_{,\beta}+\phi_{,\alpha}\phi_{,\beta})(c_1 g^{\alpha\beta}+c_2 R^{\alpha\beta})$$
$$+(2h_{\alpha;\beta}\phi_{;\gamma\delta}+\phi_{;\alpha\beta}\phi_{;\gamma\delta})\{(c_3+c_4)g^{\alpha\gamma}g^{\beta\delta}+c_5 g^{\alpha\beta}g^{\gamma\delta}\} . \quad (3)$$

This proposition demands that $\tilde{L}=L$ under any choice of $g_{\alpha\beta}$, h_α and ϕ, so the conditions

$$c_1 = c_2 = c_5 = 0 , \qquad c_3 + c_4 = 0 \quad (4)$$

must hold. Thus, we have the Einstein-Maxwell type Lagrangian density

$$L = \frac{16\pi\kappa}{c^4} L_m + R + \frac{1}{2} c_3 F_{\alpha\beta} F_{\gamma\delta} g^{\alpha\gamma} g^{\beta\delta} , \quad (5)$$

$$F_{\alpha\beta} \equiv h_\alpha - h_\beta . \quad (6)$$

If we specify

$$c_3 = -\frac{2\kappa}{c^4} , \quad (7)$$

the density (5) reduces to the exact density of the Einstein-Maxwell field. Then, we can say that the gauge-invariant vector-metric field must be an Einstein-Maxwell type field. Also, by giving a special value to the coupling constant, we may identify the vector field with the electromagnetic potential.

It is well known that the electromagnetic field is gauge-invariant. In this paper, we proved that the reverse relation holds, that is, the gauge-invariant vector field must be a Maxwell type field.

References

1) C.M. Will and K. Nordtvedt, Jr.; Astrophys. J. **177**, 757 (1972).
2) R.W. Hellings and K. Nordtvedt; Phys. Rev. D **7**, 3593 (1973).