Strength and Ductility of Reinforced High Strength Concrete Beams

By

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Abstract

Complete stress-strain curves for different concrete strengths up to 920 kg/cm² were measured, and the coefficients of stress block were calculated. Meanwhile, a total number of 114 reinforced concrete beams, made with normal or high strength concrete, were tested in shear and flexure.

It is shown that though high strength concrete is less ductile than normal strength concrete, reinforced concrete beams made with high strength concrete can show more ductility than might be expected from the ductility of concrete itself. Also, the influences of different factors such as shear span to depth ratio, reinforcement ratio and cross-section on the strength of singly reinforced high strength concrete beams are discussed.

1. Introduction

Interest in high strength concrete (HSC) has been increasing over the past several years, and a few projects have been constructed by using HSC[1,2,3]. By increasing its compressive strength, concrete could not only be more economical, but also be a more effective construction material, especially when reduced weight is necessary, for example, as in long span concrete bridges or high rise buildings. These advantadges of HSC might more widely increase its use in the future.

Generally, to achieve HSC, lower water-cement ratio with a large amount of cement and a good quality of aggregates are to be used. Besides these, to get enough workability, high-range water-reducing agents[4] are used.

Studies on the mechanical and deformation properties of HSC indicate that it shows somewhat different behaviors compared with normal strength concrete (NSC)[5]. The most important characteristics of the physical properties of HSC are as follows: it is more elastic under compression, showing less plastic deformation until failure, and the ratio of the tensile to the compressive strength ($\sigma t/\sigma c$) becomes relatively small. These might reduce the ductility of the structures made

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with HSC. Also, since the tensile strength of concrete plays an important role on the shear strength of the concrete members, investigation of the strength and ductility of concrete members made with HSC is quite important.

There have been very few studies on HSC members, especially from the viewpoint of ductility. Hence, two series of experimental works were carried out to obtain additional data on the strength and behavior of HSC members.

2. Experimental Works

2-1 Materials

Ordinary Portland cement, sand from a local mountain having a fineness modulus of 2.8, and crushed stone with a maximum size of 15 mm and a fineness modulus of 6.1 were used for making NSC. To make HSC, high early-strength Portland cement, sand from a local river having a fineness modulus of 3.0, crushed stone with a maximum size of 15 mm and with a fineness modulus of 6.6, and a high-range water-reducing agent of sulfonated polyalkylaryl type compound (NL-1400)[6] were used. For reinforcement of beams, deformed bar of 10 mm dia. with a yield strength of 3970 kg/cm^2 was used for the NSC beams. Deformed bars, of 16 mm dia. and 19 mm dia. with yield strengths of 3430 kg/cm^2 and 3330 kg/cm^2, respectively, were used for the HSC beams.

2-2 Specimens

Cylindrical specimens of $\phi 10\times 20$ cm were used for obtaining compressive stress-strain curves, and singly reinforced beams having a cross-section of 10×20 cm (effective depth=17 cm) and a length of 160 cm were cast for test series I. In this series all beams were fully reinforced against shear. For the purpose of comparison, concrete with three strength levels, 280, 700 and 920 kg/cm^2, were used for both cylinders and beams. In order to keep the tensile reinforcement index, q, $\left(q=p\dfrac{\sigma_{sy}}{\sigma_{su}}\right)$, constant, reinforcement ratios, p, of the foresaid beams were chosen as 0.84, 2.34 and 3.37%, ($q=0.114\sim 0.122$) respectively. In test series II, the major test was conducted on singly reinforced beams without any web reinforcement. However, for determining the value of the flexural strength experimentally, two beams fully reinforced against shear in the web, were used for each combination of different concrete strengths. The materials used were the same as in test series I, but the concrete strength was 340 or 380 kg/cm^2 in the NSC beams and 820 kg/cm^2 in the HSC beams. The reinforcement ratios varied from 0.84 to 3.37% for beams having a 10×20 cm cross-section (effective depth=17 cm) (group 1), and from 1.14 to 3.18% for beams having a 10×15 cm cross-section (effectiv e depth=12.5 cm) (group 2), respectively. The shear span to depth ratio (a/d) was chosen as 1.5, 2.0, 2.5 and 3.0 for beams in group 1, and 1.5, 2.0, 2.5, 30, 5.0 and 6.0 for beams in group 2.

2-3 Testing apparatus

Cylindrical specimens at the age of 28 days were tested by a stiff testing machine[7] under a uniform compressive deformation rate of 0.25 mm/min. The beams were simply supported and loaded with two symmetrical concentrated loads, also at the age of 28 days. The beams in test series I were broken in flexure; and those in test series II were done primarily in flexural shear. Mid span deflection, section curvature, tensile and compressive strains of concrete and tensile strain of steel were measured during the tests.

3. Results and Discussion

The results of the tests of series I are shown in Table 1. The average stress-strain curves obtained from the cylinder tests are given in Fig. 1. Fig. 2 is the non-dimensionized curve of Fig. 1, in which σ/σ_{cu} is taken as the ordinate, and $\varepsilon/\varepsilon_{cu}$ as the obscissa. From these figures it can be noted that: (1) As the concrete strength increases the strain at the maximum stress also increases, and (2) In higher sterngth concrete the descending rate of stress beyond the maximum stress becomes remarkable. These observations confirm the results of the studies by Muguruma[8] and Matsumoto[9]. As was mentioned before, HSC was a vrey rich mix with an extremely low water-cement ratio. Moreover, it contained a high-range water-reducing agent, as shown in Table 2 for the typical proportioning of $\sigma_{cu}=900$ kg/cm^2. Thus compared to NSC, the mortar matrix was stronger in HSC, and as a result the strength difference between the matrix and coarse aggregate might be small. Naturally, in order to achieve HSC it is important to improve the strength of the binder[10], but it is considered that the proximity of

Table 1. Test Results of Series I

Beams No.	σ_c kg/cm^2	σ_t kg/cm^2	σ_b kg/cm^2	p (%)	q	k_2/k_1k_3	M_y (t·m)	M_{f1}* (t·m)	M_{f1}** (t·m)	M_{f1}*/ M_{f1}**
No. 1	280	25.8	40	0.84	0.115	0.52	1.01	1.31	0.90	1.46
No. 2	"	"	"	"	"	"	1.01	1.28	"	1.42
No. 3	700	49.7	62	2.34	0.114	0.55	2.47	2.59	2.17	1.19
No. 4	"	"	"	"	"	"	2.45	2.63	"	1.21
No. 5	920	50.3	75	3.37	0.122	0.58	3.20	3.52	3.02	1.16
No. 6	"	"	"	"	"	"	3.25	3.50	"	1.16

* Measured, ** Calculated (19)

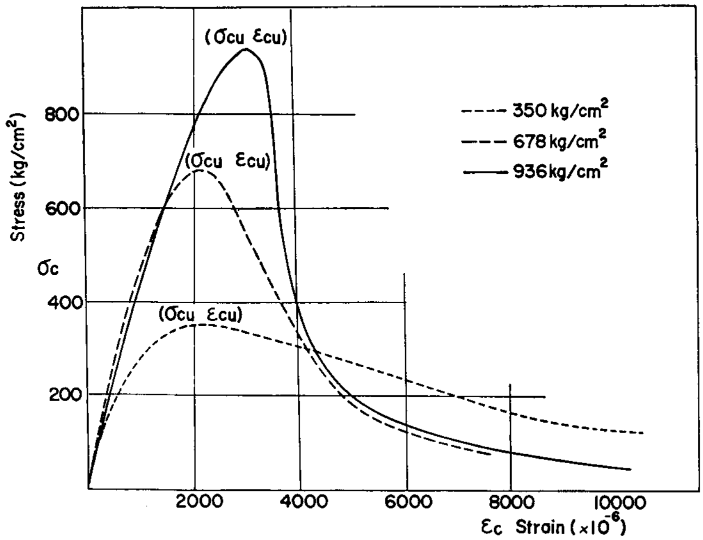

Fig. 1. Stress-strain Relationships of Concretes having Various Strengths.

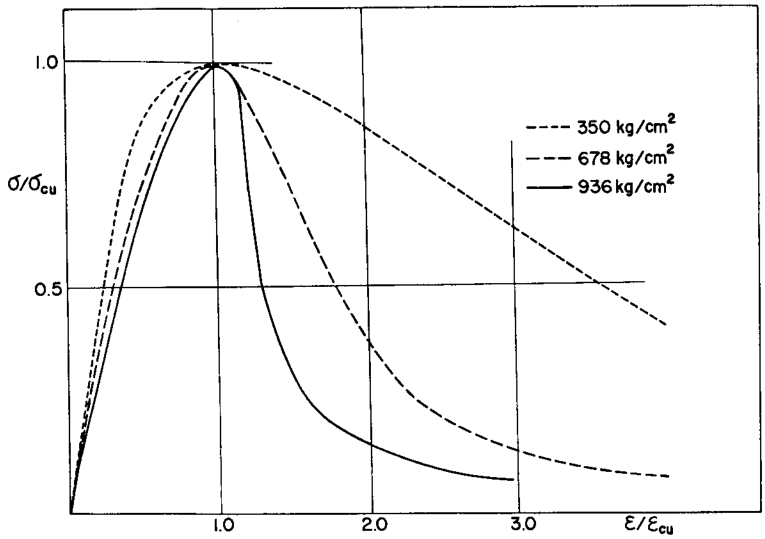

Fig. 2. Normalized Stress-Strain Curves Calculated from Fig. 1.

strengths between the binder or mortar matrix and coarse aggregate is responsible for the foresaid characteristics of HSC. Tanikawa[11] reports that as the concrete strength increases, the strain

Table 2. Typical Mix. Prop. of $\sigma_{cu}=900$ kg/cm²

W/C (%)	*s/a* (%)	(kg/m³) *W*	*C*	*S*	*G*	(*l*/m³) *NL*
28	28	168	600	476	1115	21.5

at the maximum stress also increases, but the ductility as well as the relative strain enregy, m ($m=A/A_0$, where A_0 and A are the strain energies at the strain of maximum stress and at 10×10^{-3} respectively.) decrease. In the studies by Hiramatsu and Okada[7] the similar characteristics of the stress-strain curves of HSC and light weight aggregate concrete (LWC) are reported. These also account for the proximity of strength between the mortar matrix and coarse aggregate in those two types of concrete.

Next, the ultimate flexural moment M_{fl} of a singly reinforced concrete beam is given by:

$$M_{fl}=qbd^2\sigma_{cu}\left(1-\frac{k_2}{k_1k_3}q\right) \tag{1}$$

In this study, k_1k_3 as well as k_2 were determined from the stress-strain curves of the corresponding concrete specimens, and their ratio, $\frac{k_2}{k_1k_3}$, is shown in Table 1. From this table, it is clear that as the concrete strnegth increases, the ratio of k_2 to k_1k_3 also increases, and therefore is unfavorable to the ultimate moment, as

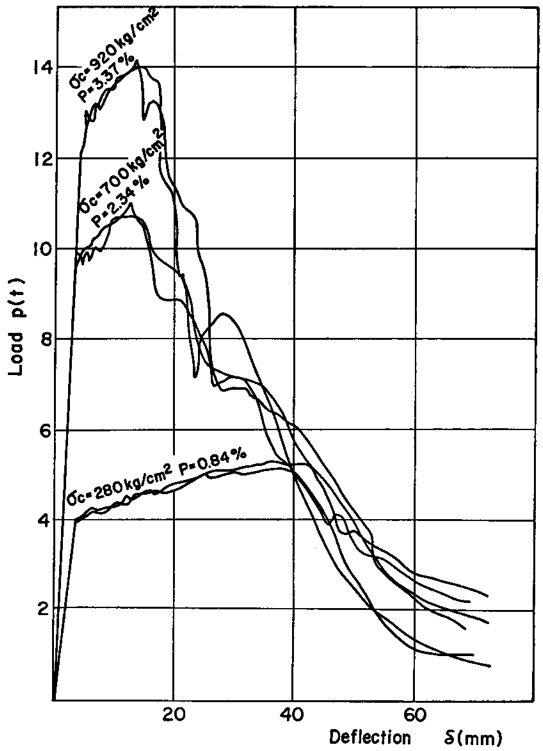

Fig. 3. Load-Deflection Curves of Beams.

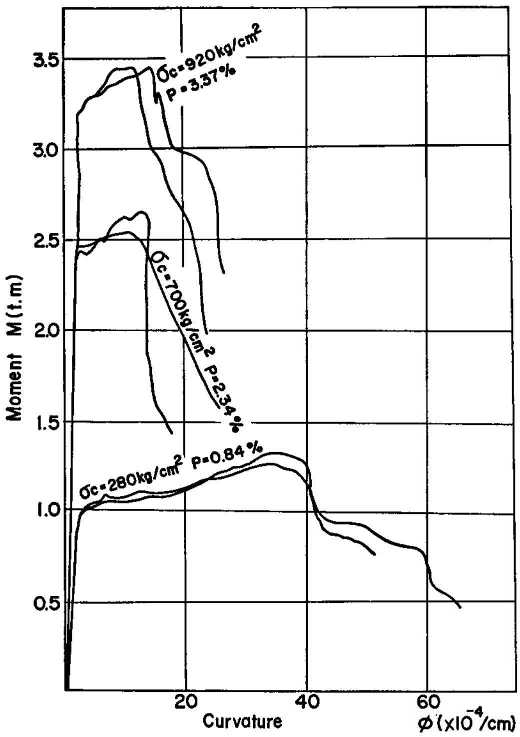

Fig. 4. Moment-Curvature Curves of Beams.

shown in Eq. 1. Muguruma[8] has achieved the same result. In his study, Muguruma indicates that as the strength of concrete becomes higher, its contribution to the ultimate flexural moment lessens.

On the other hand, the typical lcad-deflection as well as moment-curvature relations obtained from the tests of beams are shown in Figs. 3 and 4, respectively. Generally, up to the yilde of reinforcing steel, no differences are observed between the behaviors of the NSC and HSC-under-reinforced beams. With an increase of the reinforcement ratio, as shown in Table 1, the yielding load or yielding moment increases. For the same reinforcement ratio, the ductility of the beam also increases as the concrete strength increases[12]. As mentioned before, the reinforcement index, q, which corresponds to the depth of the neutral axis when the steel yields and has a direct relation upon the ductility of beams, was chosen as almost constant in this study. Therefore, regardless of the concrete strength, the beam's deflection as well as the curvature increase linearly up to the yield of reinforcement. To clarify the behavior of beams made with HSC beyond the yield of reinforcement, the variation in the relative values of the compressive strain of concrete $(\varepsilon/\varepsilon_{cu})$, and of the deflection or curvature (δ/δ_y) or (ϕ/ϕ_y), each in the

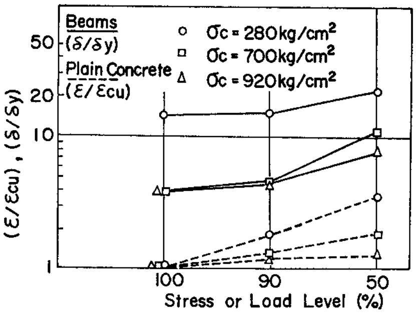

Fig. 5. Relative Values of Compressive Strain of Concretes ($\varepsilon/\varepsilon_{cu}$) and of Deflection of Beams (δ/δ_y) versus Stress or Load Level (%) in Falling Branch Calculated from Figs. 2, 3 and 7.

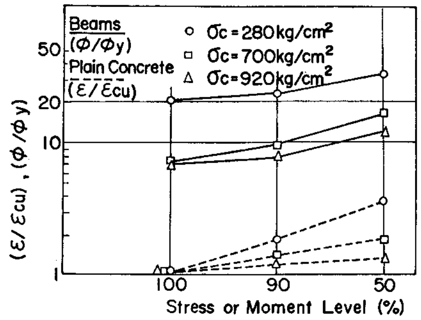

Fig. 6. Relative Values of Compressive Strain of Concrete ($\varepsilon/\varepsilon_{cu}$) and of Curvature of Beams (ϕ/ϕ_y) versus Stress or Load Level (%) in Falling Branch Calculated from Figs. 2, 4 and 7.

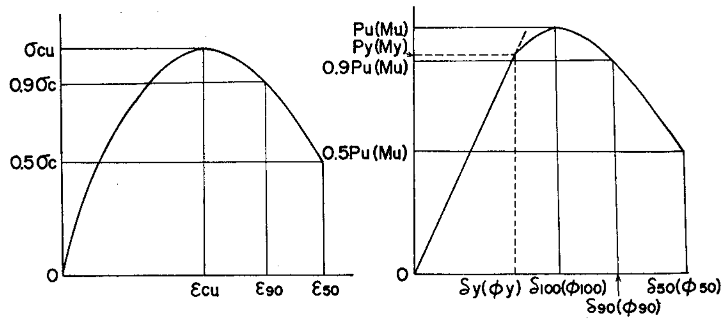

Fig. 7. Skematical Diagrams of Stress-strain Curve of Concrete and of Load-Deflection or-Curvature of Beam.

falling branch of Figs. 2, 3 and 4, are compared and shown in Figs. 5 and 6 by referring to Fig. 7.

From these figures it can be noted that: since the reinforcement ratio, p, is larger in HSC beams, the relative deflection (δ_{100}/δ_y) or relative curvature (ϕ_{100}/ϕ_y) at the maximum load is smaller compared with those of NSC beams. These facts confirm the results of the study by Leslie[13]. Leslie recommends that for assuring the proper ductility of HSC beams, the reinforcement ratio, p, should be limited to $0.35p_b$, where p_b is the balanced reinforcement ratio calculated by the triangular stress block, with the extreme fiber stress at σ_{cu} and the zero stress at the neutral axis. The ductility index, $\frac{\delta_{100}}{\delta_y}$, of HSC beams, as given by Leslie's experiments, is reexpressed in terms of the reinforcement index, q, $\left(q=p\frac{\sigma_{sy}}{\sigma_{cu}}\right)$, in Fig. 8, including the results of this study. If the increments of the relative deflection or curvature beyond the yielding load or yielding moment respectively,

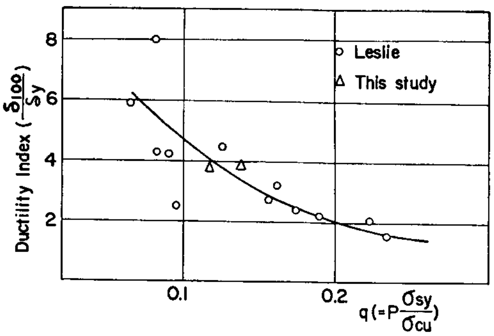

Fig. 8. Ductility Index versus Reinforcement Index Relationship.

Table 3. Concrete Strength (Series II)

Strength \ Type		(kg/cm²) σ_c	σ_t	σ_b
HSC	Group 1	820	50.0	80
	Group 2	820	49.4	80
NSC	Group 1	380	35.0	45
	Group 2	340	34.0	41

are taken as an index for the ductility of reinforced concrete beams, it is clear from Figs. 5 and 6 that for the same reinforcement index, the ductility of HSC beams is lower than that of NSC. However, when compared with the ductility of plain concrete, $(\varepsilon/\varepsilon_{cu})$, satisfactory ductility is assured for HSC beams. When considering the reserved ductility byeond the yield of reinforcement, singly reinforced HSC beams having a reinforcement index, q, of 0.15~0.2, are supposed to show enough ductility.

Next, the concrete strengths in test series II are shown in Table 3. Typical load-deflection curves of tested beams without web reinforcement, as measured by an X-Y recorder, can be divided into three types, which are schematically shown in Fig. 9.

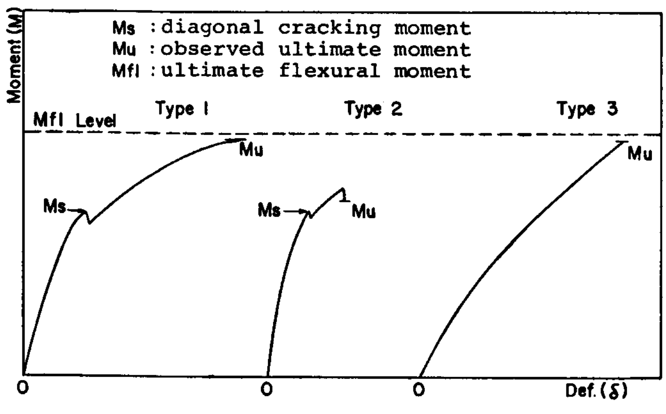

Fig. 9. Typical Load-Deflection Curves of Beams without Web Reinforcement.

Type 1. For beams with a shear span to depth ratio (a/d) of less than 2, the inclined crack was generally generated following the occurrence of one of the flexural cracks. This crack extended almost vertically from the extreme tensile fiber of the beam to just above the reinforcement level, and then progressed toward the nearest applied loading point. After the formation of the diagonal crack, a sudden fall in the applied load was observed, but failure of the beam was avoided by the arch action of the web concrete. As the load increased again, the inclined crack propagated downward, meeting the reinforcement, and in some csaes extended along the reinforcement toward the support. After the formation of this secondary crack, failure was caused because of either a diagonal compression crushing or a splitting of the concrete. In most of the beams, flexural capacity was reached.

Type 2. For beams with $2 \leqq a/d \leqq 3$, and a higher percentage of reinforcement than in type 1, the inclined crack was formed directly in the shear span. Then, with a small increase in the load, it extended both toward the applied loading point and along the level of reinforcement. In most cases the failure was very sudden, and flexural capacity was not reached.

Type 3. For beams with $a/d \geqq 3$, no distinctive inclined cracks were formed. Just prior to failure, the flexural cracking spread to the shear regions, but failure occurred in the section of maximum moment. Flexural capacity was attained in most beams.

All the beams which were reinforced in the web against shear, failed in flexure.

Concerning the characteristics of load-deflection curves, no appreciable difference could be found between the behaviors of HSC and HSC beams in this test series.

As to the strength of beams, generally, from the viewpoint of the design of beams without web reinforcement, the inclined cracking load is considered as the ultimate capacity[14]. However, as mentioned above, in beams with a relatively small a/d, a further increase in the load carrying capacity can be expected even after the formation of the inclined cracks. Therefore, in this study, the effects of differnet factors are discussed on both the formation of the inclined crack and on the ultimate strength of beams.

a/d Effect

The nominal shear stress at the inclined cracking load (τ_s), as well as at the observed ultimate load (τ_u), for beams of group 1 (Series II) are shown in Figs. 10 and 11. Shown in Fig. 12 is the relation between a/d and Mu/Mfl where Mu is the observed ultimate moment of beams without web reinforcement, and Mfl is the ultimate flexural moment obtained from the beams fully reinforced against shear in the web. Fig. 10 shows that as a/d increases from 1.5 to 2.5, τ_s decreases almost linearly in HSC as well as in NSC beams regardless of the reinforcement ratio.

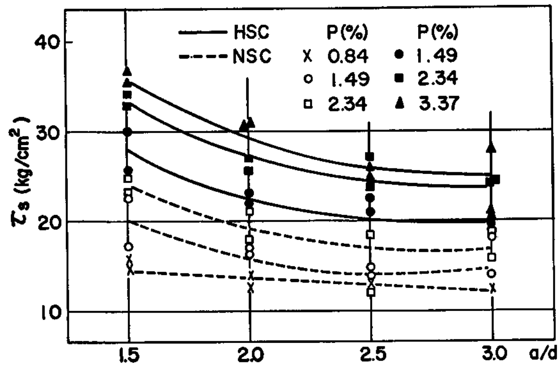

Fig. 10. Nominal Sracking Shear Stress (τ_s) versus a/d.

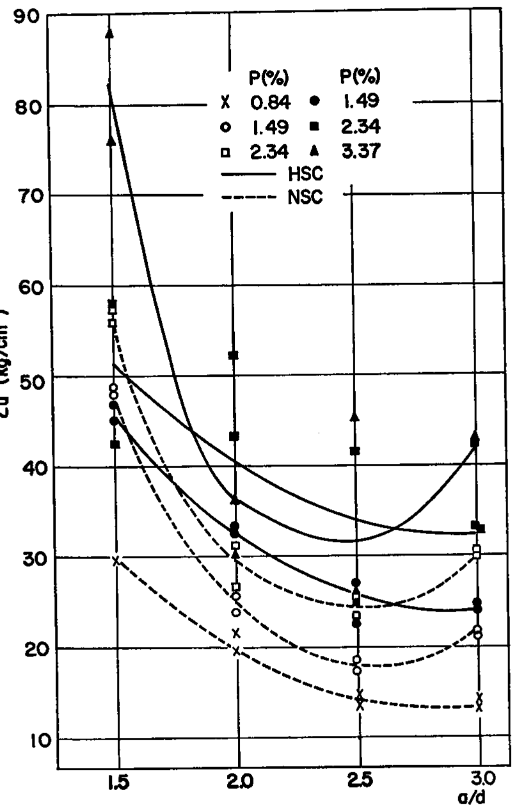

Fig.. 11. Nominal Ultimate Shear Stress (τ_u) versus a/d.

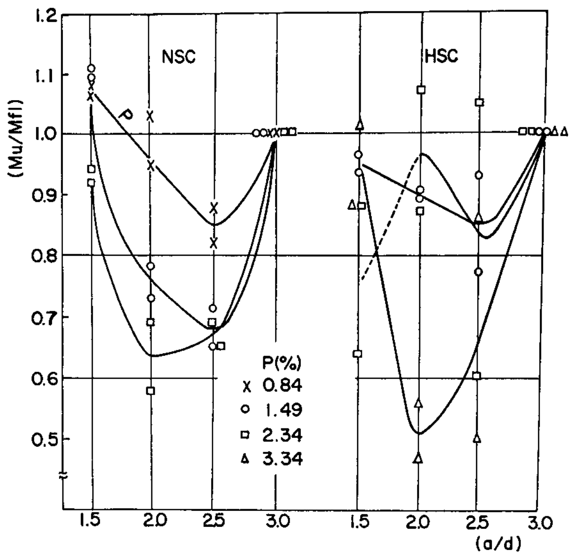

Fig. 12. M_u/M_{fl} versus a/d.

Beyond $a/d=2.5$, no appreciable variations in τ_s value are noted.

For the same reinforcement ratio, τ_s of HSC beams is about 1.4 times as high as that of NSC beams. This value corresponds almost to the ratio of the tensile strengths of these two types of concrete. On the other hand, it can be seen from Figs. 11 and 12 that the τ_u- and Mu/Mfl-a/d relationships show a relatively large scattering, especially in HSC beams. τ_u as well as Mu/Mfl values sharply decrease when a/d increases from 1.5 to 2.0.

For $a/d=3.0$, as mentioned before, the beams were fully reinforced in the web in order to obtain the ultimate flexural moment expermentally. Therefore, some increase in τ_u and Mu/Mfl values are noted when a/d increases from 2.5 to 3.0.

When taking into account the effects of shear reinforcement in beams with $a/d=3.0$, the above results agree with those obtained by Kani[15] and Leonhardt[16]. The same results were obtained from the beams in group 2.

Effect of Reinforcement Ratio

The effects of the tensile reinforcement ratio, p, on τ_s and τ_u are shown in Figs. 13 and 14, respectively. These figures show that as the reinforcement ratio increases, τ_s and τ_u increase almost linearly in HSC beams as well as in NSC beams. Hofbeck and Mattock[17] obtained similar results.

Size Effect

The size of a beam's section seems to have some influence on its τ_s-value. For

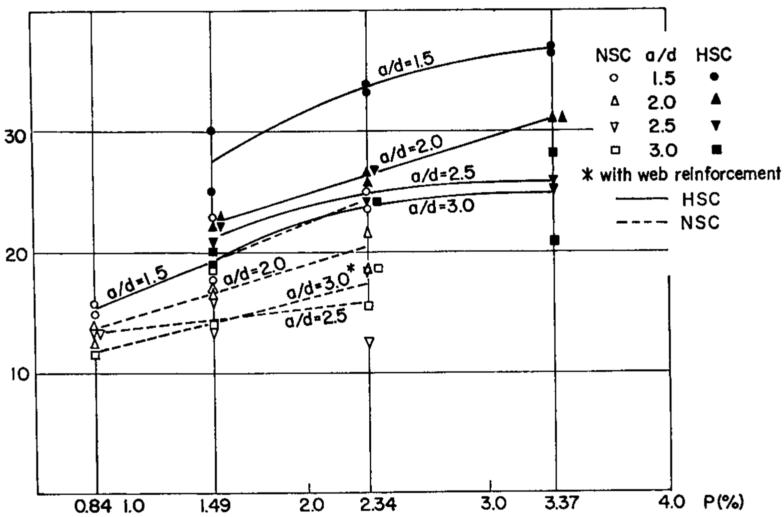

Fig. 13. τ_s versus p.

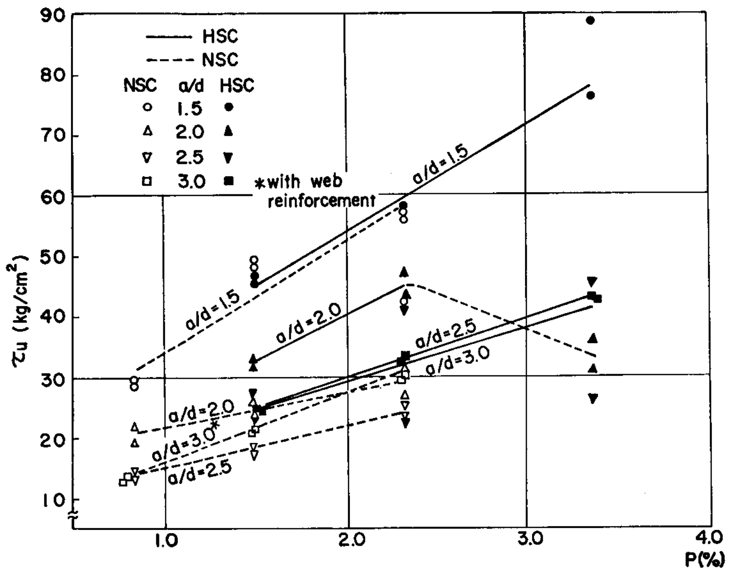

Fig. 14. τ_u versus p.

example, Fig. 15 shows the relationship between the effective depth, d, and the shear stress at the inclined cracking, τ_s, obtained in beams having the same width and almost same reinforcement ratio. This figure indicates that τ_s is larger in both HSC and NSC beams having a larger depth. This tendency is also observed on τ_u, although not so clearly. On the other hand Mu/Mfl generally decreases as the effective depth increases, as shown in Fig. 16.

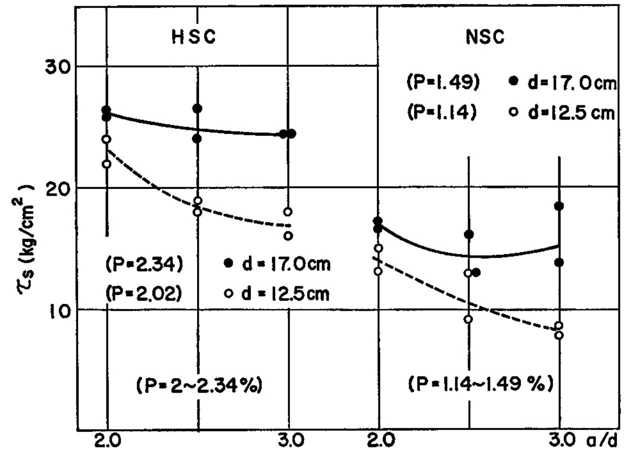

Fig. 15. τ_s versus Effective Depth d.

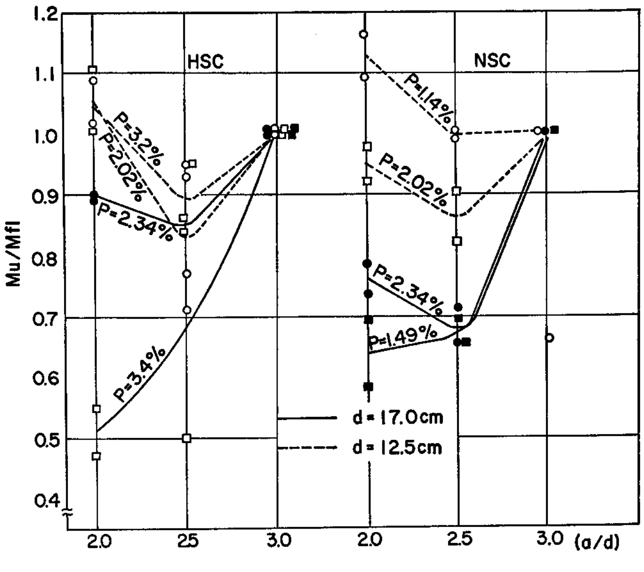

Fig. 16. M_u/M_{fl} versus Effective Depth d.

Though the tensile reinforcement ratios are not necessarily equal in this study, *Mu*/*Mfl* sharply falls as the depth of the beams increases. Kani[18] reports that *Mu*/*Mfl* is proportional to $\sqrt[4]{\frac{1}{d}}$—that is—a 10% decrease in *Mu*/*Mfl* of the larger beams in this study.

Considering the difference in the reinforcement ratio, Kanis' suggestion could give a good approximation to the results of this study.

8. Conclusions

In this study, the strength and behavior of reinforced concrete beams made with HSC were investigated through comparison with NSC beams. The following results were obtained.

(1) As the concrete strength increases the strain at the maximum stress also increases, and the descending rate of stress beyond the maximum stress becomes remarkable.

(2) HSC is not as ductile as NSC, but when it is used in reinforced concrete beams, HSC beams can show more ductility than might be expected from the plain concrete itself.

(3) As the shear span to depth ratio increases up to about 2.5, the shear stresses at the formation of the inclined crack and at the ultimate strength, as well as Mu/Mfl values decrease.

(4) For the shear span to depth ratio of less than about 2, both in HSC beams as well as in NSC beams, a remarkable increase in the load carrying capacity could be expected even after the formation of the inclined crack.

(5) As the tensile reinforcement ratio increases, the shear stresses at the formation of the inclined crack as well as at the ultimate strength increase.

(6) As the depth of the beam increases, the shear stress at the formation of the inclined crack as well as at the ultimate strength also increases, but Mu/Mfl decreases.

(7) When all other factors are kept constant, the shear stress at the formation of the inclined crack for HSC beams is about 1.4 times as high as that of NSC beams, but this ratio drops to 1.1 to 1.2 for the ultimate shear stress.

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