Adaptive-Controlled Forecasting for Parts-Oriented Production

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Abstract

In Parts-Oriented Production Systems, in which multi-products are produced from several multi-parts, the forecast demand for the parts and products is different from each other. The demand for products depends on the consumers' preference, but the demand for parts depends on the demand for all kinds of various products which contain certain kinds of parts. This paper is concerned with the adaptive-controlled forecasting technique for this kind of production system to fit the actual demand for each part. Namely, it is to decrease the rate of modification of production planning for parts and the total stock, and to stabilize the supply of parts for assembling into products.

1. Introduction

As the consumers' preference for products becomes diversified, it is necessary for the production manager to produce many kinds of products. Under these circumstances, it is more preferable for the manager to have common parts for many products as inventory in a parts-storage area, and to assemble these parts into finished products as the consumers' needs arise, rather than to have finished products as inventory. This kind of production system is called "Parts-Oriented Production System", abbreviated POPS. This is an important type of modern production system. The advantage of Parts-Oriented Production Systems over Products-Oriented Production Systems is that the manufacturing lead times are much shorter. With this method, an increase of parts inventory reduces a lack of parts that may occur when orders of products are received. However, it is accompanied by an increase in inventory costs. On the other hand, a decrease of parts inventory reduces the inventory cost, but highly increases the

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possibility of a lack of parts. Therefore, it is necessary for the production manager to forecast the amount of parts needed, and then to decide an appropriate level of parts inventory.

In order to forecast the demand for parts, there are two forecasting methods available for the managar. One is to forecast the demand for parts directly by using a time series of the actual demand for parts, which had been observed until that time. The other is to forecast the demand for products by using a time series of the actual demand for products, and then to calculate the amount of parts necessary to assemble that amount of products. For simplicity, the former forecasting method is denoted as ($\tilde{\mathrm{F}}$), and the latter is denoted as ($\tilde{\tilde{\mathrm{F}}}$).

Since each kind of product has its own peculiar demand pattern, ($\tilde{\tilde{\mathrm{F}}}$) reflects the demand pattern of each kind of product directly in the demand for parts. At the same time, ($\tilde{\tilde{\mathrm{F}}}$) also reflects the random component of the demand for each kind of product in the demand for parts. On the other hand, if the demand for parts does not fluctuate heavily period by period, or if the demand for one kind of product depends heavily on that for another kind of product, ($\tilde{\mathrm{F}}$) will seem to be better than ($\tilde{\tilde{\mathrm{F}}}$). Therefore, ($\tilde{\tilde{\mathrm{F}}}$) is sometimes superior and also sometimes inferior to ($\tilde{\mathrm{F}}$). From this point of view, a method is proposed in this paper to more precisely forecast the demand for parts in the future by considering both the time series of demand for parts and those for products.

2. Parts-Oriented Production Systems

In the factory concerned in this paper, M kinds of products, Q_j ($j=1, 2, \cdots, M$), composed of N kinds of parts, P_i ($i=1, 2, \cdots, N$), are produced. Let $e_{i,j}$ denote the amount of P_i which is needed to produce a unit of Q_j. $e_{i,j}$ is a non-negative integer, and $e_{i,j}=0$ means that product Q_j does not contain part P_i. Let $X_{j,t}$ and $Y_{i,t}$ be the amounts of demands for product Q_j and for part P_i at period t, respectively. It is assumed that S_i periods are necessary to fabricate or purchase part P_i, and also that L_j periods are necessary to assemble product Q_j. As mentioned briefly in section 1, in POPS, parts are fabricated or purchased in advance and held as inventory. As soon as the order for products is received, the products are assembled from the parts stocked, and shipped. Some kinds of parts are common for several kinds of products. The framework of this system is depicted in Figure 1. The characteristics and effectiveness of this kind of production system were investigated by Hitomi et al.[1] and Kuriyama et al.[2] POPS causes a decrease of lead time, a reduction of product inventory, and an increase of service ratio, where service ratio means the percentage of the amount shipped over the amount of demand.

To begin with, a special case where $S_1=S_2=\ldots=S_N=S$ and $L_1=L_2=\ldots=L_M=L$,

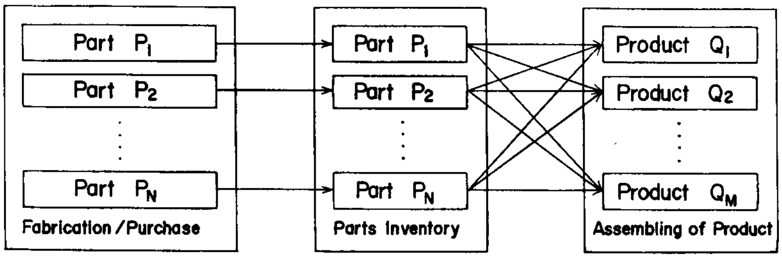

Fig. 1. Framework of Parts-Oriented Production System

is considered. Since the demand for Q_j at period t is $X_{j,t}$, $e_{i,j}X_{j,t}$ pieces of P_i are required at period $t-L$ to satisfy the demand at period t. Therefore, the following relation must be satisfied between the amount of demand for parts and that for products:

$$\mathrm{Y}_{t-L}=\mathrm{E}\mathrm{X}_t \tag{1}$$

where E, X_t, and Y_{t-L} are defined as follows:

$$\mathrm{E}=\begin{bmatrix} e_{1,1} & e_{1,2} & \cdots & e_{1,M} \\ e_{2,1} & e_{2,2} & \cdots & e_{2,M} \\ \vdots & \vdots & & \vdots \\ e_{N,1} & e_{N,2} & \cdots & e_{N,M} \end{bmatrix},\quad \mathrm{X}_t=\begin{bmatrix} X_{1,t} \\ X_{2,t} \\ \vdots \\ X_{M,t} \end{bmatrix},\quad \mathrm{Y}_{t-L}=\begin{bmatrix} Y_{1,t-L} \\ Y_{2,t-L} \\ \vdots \\ Y_{N,t-L} \end{bmatrix}. \tag{2}$$

At period T, the manager wants to forecast the amount of parts required to produce the products demanded at period $T+S+L$. Because of the necessity of lead time for assembling Q_j, the actual demand for Q_j at period $T+L$ must be confirmed at period T. Hence, the information available for the manager at period T is two time series: Y_T, Y_{T-1}, Y_{T-2}, $\cdots$, and X_{T+L}, X_{T+L-1}, X_{T+L-2}, $\cdots$. These time series satisfy the relation (1).

Let $\tilde{X}_{T+S+L}(T)$ be the column vector whose ith component $\tilde{X}_{j,T+S+L}(T)$ is the amount of demand for product Q_j at period $T+S+L$, forecast at period T by using $X_{j,T+L}$, $X_{j,T+L-1}$, $X_{j,T+L-2}$, $\cdots$. Let $\tilde{\breve{Y}}_{T+S}(T)$ be the column vector whose ith component is the amount of part P_i needed at period $T+S$ in order to assemble $\tilde{X}_{j,T+S+L}(T)$ pieces of product Q_j required at period $T+S+L$. Futhermore, let $\tilde{Y}_{T+S}(T)$ be the column vector whose ith component $\tilde{Y}_{i,T+S}(T)$ is the amount of demand for part P_i at period $T+S$, forecast at period T by using $Y_{i,T}$, $Y_{i,T-1}$, $Y_{i,T-2}$, $\cdots$.

Since the fluctuation of demand for each kind of product has its own characteristics, the fluctuation of demand for each kind of part is complicated. The reason is that common parts are included in various kinds of products. Therefore, $\tilde{Y}_{i,T+S}(T)$ is generated differently from $\tilde{\breve{Y}}_{i,T+S}(T)$. By taking both forecast values, $\tilde{Y}_{i,T+S}(T)$ and $\tilde{\breve{Y}}_{i,T+S}(T)$, into consideration, the future demand for parts can be forecast more precisely than by considering one of these forecast values. From this point of view, a method of forecasting the amount of parts needed in the future is proposed in this paper, and the effectiveness of this method is discussed by computer simulation.

3. Forecasting Methods for Parts Demand

Two forecasting methods, $(\tilde{\mathrm{F}})$ and $(\tilde{\tilde{\mathrm{F}}})$, for parts demand introduced in section 1 are defined precisely as follows: $(\tilde{\mathrm{F}})$ is a forecasting method for parts demand in which parts demand at period $T+S$ is forecast at period T, only by using the time series of the actual parts demand which had been observed until period T. $(\tilde{\tilde{\mathrm{F}}})$ is a forecasting method for parts demand in which the parts demand of period $T+S$ is forecast at period T, only by exploding the forecast amount of the products demand for period $T+S+L$. In this paper, another effective forecasting method $(\hat{\mathrm{F}})$ is proposed, in which the trend of parts demand extended over a long period of time is considered. The trend of products demand is reflected as precisely and also as quickly as possible.

Since the fluctuation of demand for each kind of product has its own characteristics, it is preferable for the manager to use a forecasting technique appropriate to each kind of product when he forecasts that product demand in the future. By the same reason, it is preferable for him to use an appropriate forecasting technique for each kind of part when he forecasts the value of $\tilde{Y}_{i,T+S}(T)$, $(i=1, 2, \cdots, N)$.

Once the values of $\tilde{X}_{j,T+S+L}(T)$, $(j=1, 2, \cdots, M)$ are forecast by using $X_{j,T+L}$, $X_{j,T+L-1}$, $X_{j,T+L-2}$, $\cdots$, the value of $\tilde{\tilde{Y}}_{i,T+S}(T)$ is calculated by taking all the values of $\tilde{X}_{1,T+S+L}(T)$, $\tilde{X}_{2,T+S+L}(T)$, $\cdots$, $\tilde{X}_{M,T+S+L}(T)$ into consideration. On the other hand, $\tilde{Y}_{i,T+S}(T)$ is calculated only by using $Y_{i,T}$, $Y_{i,T-1}$, $Y_{i,T-2}$, $\cdots$. Therefore, the value of $\tilde{Y}_{i,T+S}(T)$ is different from that of $\tilde{\tilde{Y}}_{i,T+S}(T)$. In order to get a more precise forecast value for parts demand, the following forecasting method $(\hat{\mathrm{F}})$ is proposed.

$$(\hat{\mathrm{F}}) : \hat{Y}_{T+S}(T) = \Gamma_T \tilde{Y}_{T+S}(t) + (\mathrm{I} - \Gamma_T) \tilde{\tilde{Y}}_{T+S}(T),$$

where $\hat{Y}_{T+S}(T)$ is a column N-vector whose ith component $\hat{Y}_{i,T+S}(T)$ is the demand of P_i for period $T+S$, forecast by method $(\tilde{\mathrm{F}})$ at period T. Γ_T is a diagonal matrix of order N whose ith diagonal component is $\gamma_{i,T}$ and I is an identity matrix of order N. The framework of this forecasting method is depicted in Figure 2. The forecasting method $(\hat{\mathrm{F}})$ combines two kinds of forecasts for parts demand. The combination of forecasts is proposed by Bates and Granger[3], and discussed further by Dickinson[4),5)]. In this paper, the applicability of the combination of forecasts to parts-oriented production systems is proposed and discussed.

In order to specify $\gamma_{i,T}$ in $(\hat{\mathrm{F}})$, the following two methods are employed.

(M-I) : If $Y_{i,T} = \tilde{Y}_{i,T}(T-S) = \tilde{\tilde{Y}}_{i,T}(T-S)$, then $\gamma_{i,T} = \gamma_{i,T-1}$. Otherwise, let $\beta_{i,T} = |Y_{i,T} -$

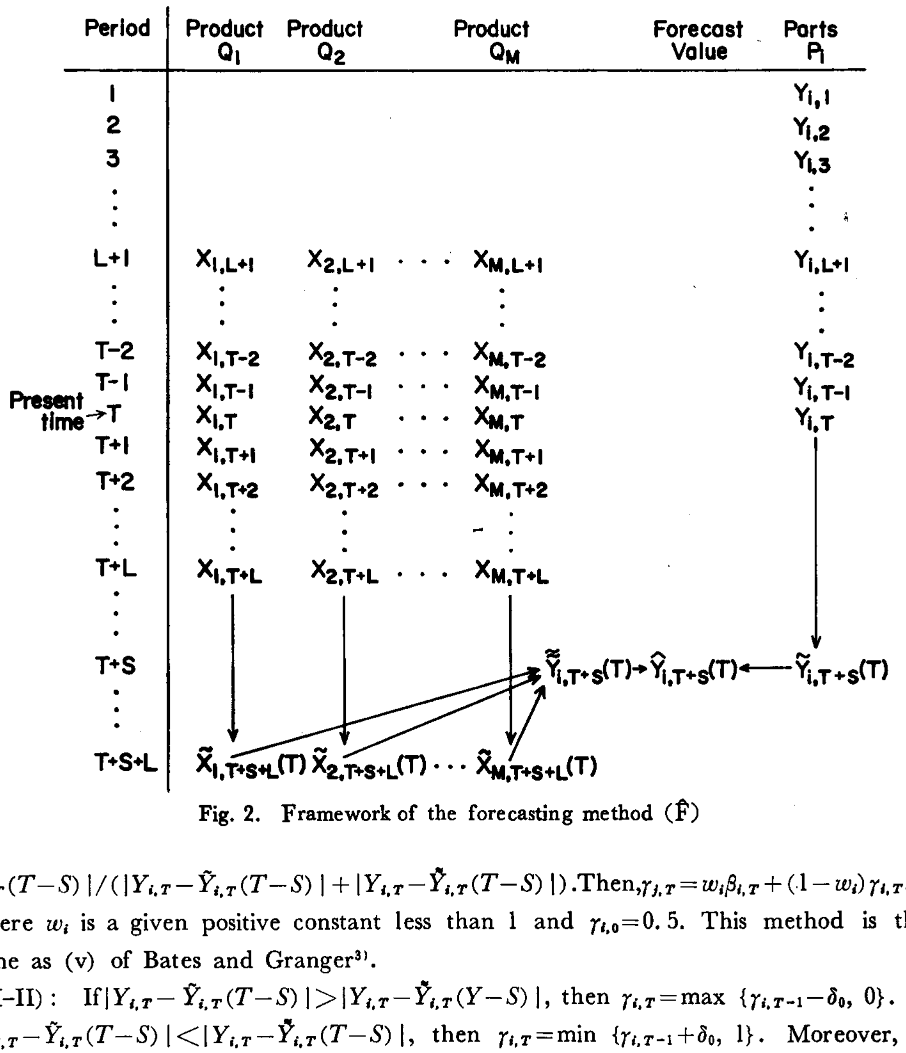

Fig. 2. Framework of the forecasting method ($\hat{F}$)

$\tilde{\tilde{Y}}_{i,T}(T-S)|/(|Y_{i,T}-\tilde{Y}_{i,T}(T-S)|+|Y_{i,T}-\tilde{\tilde{Y}}_{i,T}(T-S)|)$.Then,$\gamma_{j,T}=w_i\beta_{i,T}+(1-w_i)\gamma_{i,T-1}$, where w_i is a given positive constant less than 1 and $\gamma_{i,0}=0.5$. This method is the same as (v) of Bates and Granger[3].

(M-II): If $|Y_{i,T}-\tilde{Y}_{i,T}(T-S)|>|Y_{i,T}-\tilde{\tilde{Y}}_{i,T}(Y-S)|$, then $\gamma_{i,T}=\max\{\gamma_{i,T-1}-\delta_0, 0\}$. If $|Y_{i,T}-\tilde{Y}_{i,T}(T-S)|<|Y_{i,T}-\tilde{\tilde{Y}}_{i,T}(T-S)|$, then $\gamma_{i,T}=\min\{\gamma_{i,T-1}+\delta_0, 1\}$. Moreover, if $|Y_{i,T}-\tilde{Y}_{i,T}(T-S)|=|Y_{i,T}-\tilde{\tilde{Y}}_{i,T}(T-S)|$, then $\gamma_{i,T}=\gamma_{i,T-1}$, where $\gamma_{i,0}=0.5$ and δ_0 is a small positive constant less than 1.

In these two methods, $\gamma_{i,T}$ is determined by $\gamma_{i,T-1}$, $\tilde{Y}_{i,T}(T-S)$, $\tilde{\tilde{Y}}_{i,T}(T-S)$, and $Y_{i,T}$ for $T>1$. $\gamma_{i,0}$ is any positive value less than 1, and is predetermined at the outset by the manager.

Both (M-I) and (M-II) are proposed from the standpoint that, if the difference between the present value of parts demand observed and the forecast value by using ($\tilde{F}$) is smaller than that by using ($\tilde{\tilde{F}}$), then a new value of γ_i is adaptively made larger for the next period, that is, more weight is assigned to ($\tilde{F}$).

In the following sections, the forecasting methods ($\hat{F}$) by using (M-I) and

(M-II) are denoted by ($\hat{F}$-I) and ($\hat{F}$-II), respectively.

4. Simulation

In this section, the characteristics and the efficiency of the forecasting methods proposed in the previous section are investigated by computer simulation. For this purpose, one kind of part P_1 is considered as a representative for five kinds of products, Q_j ($j=1, 2, 3, 4, 5$). Then, the amount of this part needed in a future period is forecast. For simplicity, let E$=(1, 1, 1, 1, 1)$, $S=3$, and $L=2$. Furthermore, let $w_i=0.1$, and $\delta_0=0.1$.

As an evaluation criterion for comparing four forecasting methods, ($\tilde{F}$), ($\tilde{\tilde{F}}$), ($\hat{F}$-I), and ($\hat{F}$-II), the mean squared error of the forecasting value from the value of the actual demand is adopted. Let MSE_1, MSE_2, MSE_3, and MSE_4 denote the mean squared errors accompanied by ($\tilde{F}$), ($\tilde{\tilde{F}}$), ($\hat{F}$-I), and ($\hat{F}$-II), respectively.

Since the demand for each kind of product has its own peculiar pattern of fluctuation, for example, trend, periodicity, etc., it is preferable for the manager to choose and use an appropriate forecasting method in order to forecast the demand for each kind of product. (With respect to various forecasting methods, see, for example, Johnson and Montgomery[6].) Products demand is forecast by using Trigg and Leachs' adaptive forecasting procedure with a tracking signal[7]. The same procedure is used to forecast parts demand $\tilde{Y}_{i,T+S}(T)$.

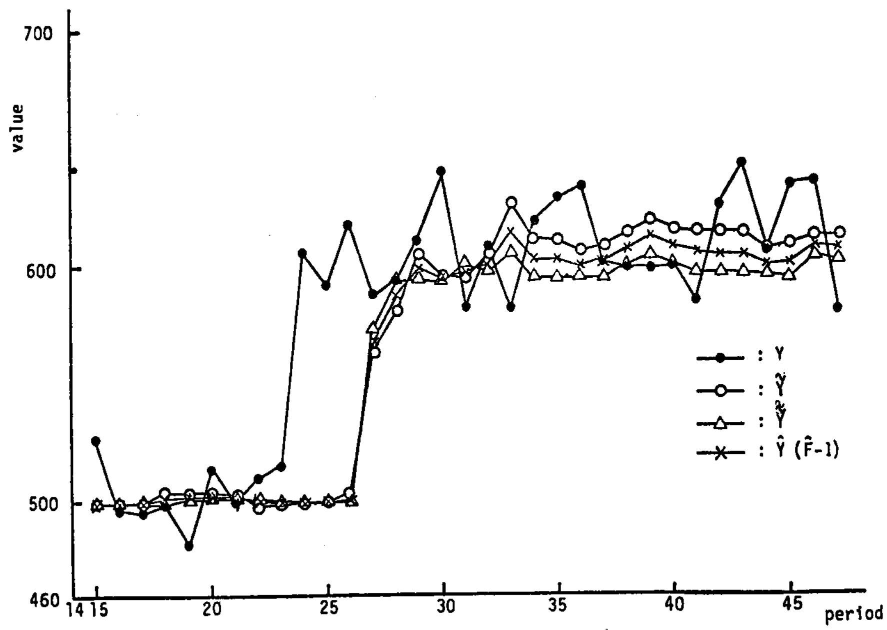

Fig. 3. Comparison of actual demand and forecast value for model A. (S=3, L=2)

In order to investigate the efficiency of ($\hat{F}$), the following two demand models, model A and model B, are introduced:

(Model A): The time horizon is fixed at 47. From period 3 to period 25, the product demand for Q_1 is generated from $N(\mu_1, \sigma_1^2)$, and from period 26 to period 49, the product demand for Q_1 is generated from $N(\mu_1', \sigma_1^2)$, where $N(\mu, \sigma^2)$ means the normal distribution with a mean μ and a variance σ^2. The demand for the other four kinds of products $Q_j(j=2, 3, 4, 5)$ is generated from $N(\mu_j, \sigma_j^2)$ from period 3 to period 49.

(Model B): The time horizon is also fixed at 49. The product demand for Q_1 is generated similarly as in the case of model A. However, the product demand for Q_j $(j=2, 3, 4, 5)$ is generated from $N(\mu_j, \sigma_2^2)$ from period 3 to period 49, except for the $(26+j-2)$th period when the demand for Q_j at that period is generated from $N(\mu_j',\sigma_j^2)$.

As a numerical example for model A, the case where $\mu_1=100$, $\mu_1'=200$, $\mu_j=100$ $(j=2, 3, 4, 5)$ and $\sigma_j^2=25$ $(j=1, 2, 3, 4, 5)$ is considered and examined by computer simulation. The data generated from the computer simulation and the forecast amount of demand are depicted in Figure 3. The mean squared errors are: $MSE_1=846.34$, $MSE_2=817.09$, $MSE_3=798.60$, and $MSE_4=811.77$. The computer simulation for model A was repeated forty times. The values of $MSE_i(i=1, 2, 3, 4)$ and MSE_i/MSE_j

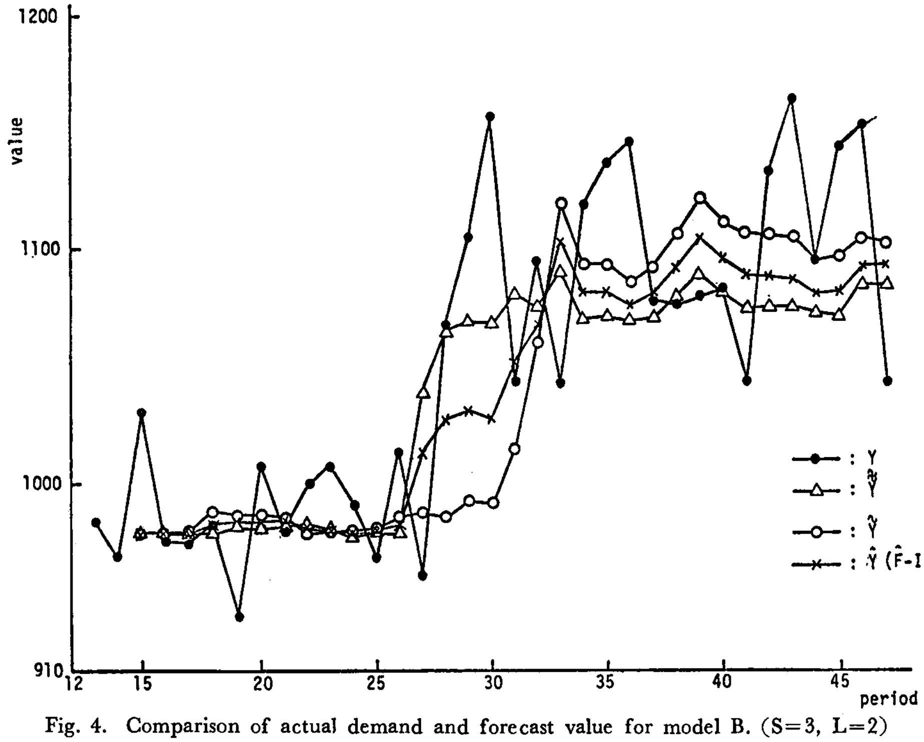

Fig. 4. Comparison of actual demand and forecast value for model B. (S=3, L=2)

Table 1. Results of computer simulation for model A.

No.	MSE_1	MSE_2	MSE_3	MSE_4	MSE_3/MSE_1	MSE_3/MSE_2	MSE_4/MSE_1	MSE_4/MSE_2
1	846.34	817.09	798.60	811.77	0.9436	0.9774	0.9592	0.9935
2	964.14	576.34	686.09	735.01	0.7116	0.1904	0.7623	1.2753
3	683.11	742.24	650.17	641.39	0.9518	0.8760	0.9389	0.8641
4	793.24	555.15	619.87	699.55	0.7814	1.1166	0.8819	1.2601
5	1039.41	1081.48	998.40	1034.93	0.9605	0.9232	0.9957	0.9570
6	790.47	532.76	615.20	672.90	0.7783	1.1547	0.8513	1.2631
7	1060.95	810.34	915.14	972.76	0.8626	1.1263	0.9169	1.2004
8	1082.06	948.62	992.49	1040.20	0.9172	1.0462	0.9613	1.0965
9	604.69	586.10	567.32	593.99	0.9382	0.9680	0.9823	1.0135
10	870.58	942.98	882.63	874.23	1.0138	0.9360	1.0042	0.9271
11	1063.83	806.05	874.76	951.27	0.8223	1.0852	0.8942	1.1802
12	1305.06	1095.71	1169.72	1252.76	0.8936	1.0675	0.9599	1.1433
13	1009.53	944.83	946.48	981.27	0.9375	1.0017	0.9720	1.0386
14	541.82	491.41	480.43	505.22	0.8867	0.9776	0.9324	1.0281
15	1157.25	688.25	864.69	978.65	0.7472	1.2564	0.8457	1.4219
16	856.34	817.09	798.60	811.77	0.9436	0.9774	0.9592	0.9935
17	820.87	591.08	665.74	742.99	0.8110	1.1263	0.9051	1.2570
18	1035.14	978.38	995.05	1015.43	0.9613	1.0170	0.9810	1.0379
19	768.08	627.53	655.23	672.68	0.8531	1.0441	0.8758	1.0720
20	804.60	637.01	685.15	752.21	0.8515	1.0756	0.9349	1.1808
21	865.07	659.61	714.99	760.72	0.8265	1.0840	0.8794	1.1533
22	934.25	862.37	887.13	910.26	0.9496	1.0287	0.9743	1.0555
23	967.25	852.14	871.16	911.76	0.9007	1.0223	0.9426	1.0700
24	888.14	931.89	879.21	900.96	0.9900	0.9435	1.0144	0.9668
25	1050.50	904.38	948.83	1008.84	0.9032	1.0491	0.9603	1.1155
26	762.99	593.38	643.21	676.54	0.8430	1.0840	0.8867	1.1401
27	779.16	686.22	710.07	752.07	0.9113	1.0348	0.9652	1.0960
28	980.29	674.52	808.08	971.46	0.8243	1.1980	0.9910	1.4402
29	1204.40	731.03	849.50	906.28	0.7053	1.1621	0.7525	1.2397
30	930.71	606.48	750.22	822.34	0.8061	1.2370	0.8836	1.3559
31	784.13	564.97	607.40	694.88	0.7746	1.0751	0.8862	1.2300
32	903.96	842.75	849.19	860.51	0.9383	1.0065	0.9519	1.0211
33	931.62	527.28	653.33	825.86	0.7013	1.2391	0.8865	1.5663
34	809.81	671.91	690.40	658.86	0.8526	1.0275	0.8136	0.9806
35	735.52	598.37	633.08	635.69	0.8607	1.0580	0.8643	1.0624
36	702.47	819.20	752.87	748.41	1.0718	0.9109	1.0654	0.9136
37	719.91	522.97	575.81	666.33	0.7998	1.1010	0.9256	1.2714
38	588.20	551.01	509.15	571.17	0.8656	0.9240	0.9710	1.0366
39	1096.84	985.46	953.81	1083.59	0.8696	0.9679	0.9879	1.0996
40	878.26	643.06	719.47	713.35	0.8192	1.1188	0.8122	1.1093
				Total	34.7830	42.2271	36.9288	45.1304
				Mean	0.8696	1,0557	0,9232	1.1283

Table 2. Results of computer simulation for model B.

No.	MSE_1	MSE_2	MSE_3	MSE_4	MSE_3/MSE_1	MSE_3/MSE_2	MSE_4/MSE_1	MSE_4/MSE_2
1	2270.04	2319.97	1997.75	2140.58	0.8801	0.8611	0.9430	0.9227
2	3323.98	2041.71	2132.75	1957.92	0.6416	1.0446	0.5890	0.9590
3	3008.59	2575.20	2272.97	2263.44	0.7555	0.8826	0.7523	0.8789
4	2681.31	2202.13	2058.24	2059.94	0.7676	0.9347	0.7683	0.9354
5	2395.15	2032.09	1924.51	2170.79	0.8035	0.9471	0.9063	1.0683
6	3112.04	2682.62	2461.76	3023.10	0.7910	0.9177	0.9714	1.1269
7	2344.36	2530.42	2013.99	2130.98	0.8591	0.7959	0.9090	0.8421
8	2331.42	2415.95	2103.34	2102.92	0.9022	0.8706	0.9020	0.8704
9	1893.51	1842.68	1732.91	1750.79	0.9152	0.9404	0.9246	0.9501
10	2545.12	2219.29	2183.83	2305.66	0.8580	0.9840	0.9059	1.0389
11	3123.85	2258.94	2314.61	2608.90	0.7409	1.0246	0.8352	1.1549
12	2275.02	2063.91	1885.47	2009.25	0.8288	0.9135	0.8832	0.9735
13	2363.54	2138.70	1976.55	1965.10	0.8363	0.9242	0.8314	0.9188
14	2847.44	2275.60	2155.72	2225.90	0.7571	0.9473	0.7817	0.9782
15	2927.29	2459.85	2226.70	2447.28	0.7607	0.9052	0.8360	0.9949
16	2270.04	2319.97	1997.75	2140.58	0.8801	0.8611	0.9430	0.9227
17	2087.39	2117.76	1679.13	2915.89	0.8044	0.7929	0.9657	0.9519
18	2258.40	1920.06	1866.18	2060.77	0.8263	0.9719	0.9125	1.0733
19	2664.26	2986.50	2328.88	2564.97	0.8741	0.7798	0.9627	0.8589
20	2113.83	1767.41	1655.38	1732.23	0.7831	0.9366	0.8195	0.9801
21	2220.59	2372.29	1765.05	1925.23	0.7949	0.7440	0.8670	0.8116
22	2014.18	2046.27	1849.15	1912.33	0.9181	0.9037	0.9494	0.9345
23	2479.85	2253.43	2014.16	2387.02	0.8122	0.8938	0.9626	1.0593
24	1650.91	1511.47	1372.97	1439.87	0.8316	0.9084	0.8733	0.9526
25	2162.27	2209.72	1887.61	2038.22	0.8730	0.8542	0.9426	0.9224
26	1993.80	1964.35	1628.53	1807.63	0.8168	0.8290	0.9066	0.9202
27	2485.15	2503.43	2299.23	2288.12	0.9252	0.9184	0.9207	0.9140
28	2569.49	2594.69	2189.56	2463.35	0.8521	0.8436	0.9587	0.9494
29	3133.90	1978.18	1927.23	1875.32	0.6150	0.9742	0.5984	0.9480
30	1960.85	1612.02	1426.94	1619.07	0.7277	0.8852	0.8257	1.0044
31	1758.92	2458.72	1436.52	1664.78	0.8167	0.5843	0.9465	0.6771
32	2062.12	1844.13	1696.11	1799.09	0.8225	0.9197	0.8724	0.9756
33	2162.10	2107.49	1589.10	2076.31	0.7350	0.7540	0.9603	0.9852
34	1936.73	2226.35	1580.68	1636.10	0.8162	0.7100	0.8448	0.7349
35	2373.90	2576.10	2121.14	2299.31	0.8935	0.8234	0.9686	0.8926
36	1473.11	2124.56	1629.33	1622.91	1.1060	0.7669	1.1017	0.7639
37	2163.27	1543.67	1368.56	1385.47	0.6326	0.8866	0.6405	0.8975
38	2715.90	2996.14	2165.25	2246.21	0.7973	0.7227	0.8271	0.7497
39	2738.17	4149.38	2461.74	2727.99	0.8990	0.5933	1.0127	0.6683
40	2303.63	2454.81	1959.35	1980.60	0.8506	0.7982	0.8598	0.8068
				Total	32.8015	34.5498	35.1809	36.9678
				Mean	0.8200	0.8637	0.8795	0.9242

($i=3, 4$; $j=1, 2$) are tabulated in Table 1. It is easily found from this table that both ($\hat{F}$–I) and ($\hat{F}$–II) are superior to ($\tilde{F}$), but in some cases superior and in other cases inferior to ($\tilde{\tilde{F}}$).

As a numerical example for model B, the case where $\mu_1=100$, $\mu_2=200$, $\mu_3=220$, $\mu_4=250$, $\mu_5=210$, $\mu'_1=200$, $\mu'_2=100$, $\mu'_3=120$, $\mu'_4=150$, $\mu'_5=110$, and $\sigma_j^2=100$ ($j=1$, 2, 3, 4, 5) is considered and examined by computer simulation. The data generated from the computer simulation and the forecast amount of demand are depicted in Figure 4. The mean squared errors are: $MSE_1=2270.04$, $MSE_2=2319.97$, $MSE_3=1997.75$, and $MSE_4=2140.58$. The computer simulation for model B was also repeated forty times. The values of $MSE_i(i=1, 2, 3, 4)$ and $MSE_i/MSE_j(i=3, 4;\ j=1, 2)$ are tabulated in Table 2. Both ($\hat{F}$–I) and ($\hat{F}$–II) are usually superior to ($\tilde{F}$) and ($\tilde{\tilde{F}}$). About 10 to 15 percent of the average of MSE is reduced.

It is concluded from these two kinds of experiments that the forecasting method ($\hat{F}$–I) is effective in forecasting the parts demand for parts-oriented production.

5. Effects of Lead Time

In section 3, it was assumed that the lead time to fabricate products from parts is the same for all products, and that to produce parts is fixed for all parts. Since this assumption is not always satisfied in actual cases, in this section it is removed, while keeping the forecasting method described in the previous section.

The matrix E_τ is redefined as a generalized form of E introduced in section 2 as follows:

$$E_\tau=\begin{bmatrix} e_{1,1,\tau} & e_{1,2,\tau} & \cdots e_{1,M,\tau} \\ e_{2,1,\tau} & e_{2,2,\tau} & \cdots e_{2,M,\tau} \\ \vdots & \vdots & \vdots \\ e_{N,1,\tau} & e_{N,2,\tau} & \cdots e_{N,M,\tau} \end{bmatrix} \quad \text{for } \tau=1, 2, \cdots, L_{max},$$

where $L_{max}=\max_{1\leq j\leq M} L_j$ and $e_{i,j,\tau}$ pieces of parts are needed to assemble a piece of Q_j, when τ periods are required to assemble product Q_j. Therefore, if τ_0 periods are needed to assemble product Q_j, then $e_{i,j,\tau}=0$ for any $\tau\neq\tau_0$ ($i=1, 2, \cdots, N$; $j=1, 2, \cdots, M$). By using the above matrix, the following relation is satisfied between the parts demand and the products demand:

$$Y_T=\sum_{\tau=1}^{L_{max}} E_\tau X_{T+\tau}$$

This is a generalized form of Equation (1). In this case, since the forecast of the cumulative amount of parts demand at period T is affected by the forecast of each kind of product, the forecasting procedure becomes complex.

It is not necessary to consider the case where the assumption $S_1=S_2=\cdots=S_n=S$

proposed in section 2 is not satisfied, because the purpose of this paper is to forecast the future demand for each kind of part.

6. Conclusion

In this paper, forecasting methods which combine two kinds of forecasts for parts demand are proposed for the Parts-Oriented Production System. The effectiveness of the proposed methods was compared with that of conventional forecasting methods. It was found that the proposed forecasting methods are not inferior to, but according to circumstances, are superior to the conventional forecasting methods. The selection of the forecasting method depends heavily on what kind of Parts-Oriented Production System is concerned with in reality, and also on what kinds of parts are common for a variety of products.

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