Mathematical Model of Ship Collision Probability

By

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Abstruct

A mathematical model for estimating the probability of collision of ships passing through a uniform channel is proposed. The model takes into account the give-way motions of the encountered ships and the wake position characteristics under the given channel conditions such as width, length and centerline like buoys.

The probability of collision is defined as the probability of an event where a ship fails to give way. Using this probability of collision, the "Collision Risk of Channel" is defined. The proposed model is examined by the collision statistics of some channels and straits in Japan. According to these statistics, the present model gives a good estimation of the collision risk of a channel.

Since the proposed model takes account of traffic characteristics such as traffic volume, ship size distribution, and sailing velocity distribution, as well as channel conditions such as width, length and centerline mark like buoys, the effects of their change or control on the probability of collision can be easily predicted. Therefore, the proposed model is quite useful for the engineering planning and design of any channel.

1. Introduction

In the planning and design of channels, the safety consideration is an important aspect as well as the economical one. Especially in recent Japan, this becomes more and more important since ships like tankers carrying hazardous materials tend to increase in number and size. Therefore, once a traffic accident happens, there will be tremendous economical losses and environmental damages.

In the light of these circumstances, researches associated with marine traffic accidents are strongly required. However, no research can be seen to analyze them systematically in order to intend to apply to planning and design of channels.

Marine traffic accidents include collision, grounding, capsizing, fire damage, engine trouble, foundering, etc. Among these, collisions and grounding account

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for more than 60 % of the accidents, and will be anticipated to give serious damages. These two types of accidents are strongly related to the design of channels and to traffic volume.

Taking these into account, this paper treats with the estimation model of collisions as the first step.

Previous studies on ship collisions can be categorized into two groups. One is a statistical analysis, and other is concerned with mathematical estimation models. In the statistical analysis, studies by Fujii (1966)[1], Shiobara (1967)[2], Fujii (1971)[3], Fujii and et al. (1971)[4], Kandori (1971, 1972)[5,6], Yamanouch and Fujii (1978)[7] should be notified. Through their researches, it is made clear that the probability of collisions depends on the ship size and traffic volume. Although these researches give valuable information about the probability of ship collisions they can not be applied directly to the planning and design of channels because they do not treat the effects of the design variables of channels such as width, depth, curvature etc. on the collision probability.

On the other hand, Fujii (1968)[8] and Hara (1971, 1973)[9,10] present models of ship collisions. Fujii[8] presents a mathematical model of ship collisions based on the idea of collisions of molecules with random motion. Since this model does not explicitly include the design variables of channels, it is difficult to use this model for the prediction of collision probability in case of changing the variables. In addition, Fujii's model does not consider the give-way motion of ships and assumes that a ship will collide over two times. These assumptions will result in an over-estimation of collision probability. Hara applies the Queueing Theory to the estimation of ship collisions. His model introduces the give-way motion of ships as the service time. However, since the characteristics of traffic flow such as the distribution of ship sizes, sailing velocity and wake position, traffic volume etc. are represented in Hara's model only by the term of "Collision Avoidance Intensity", it is difficult to estimate the effect of any change of these factors. In his model, the collision probability is assumed as the probability that ships wait longer than a certain "limit value of waiting time τ". Though collision probability is sensitive to the "limit of waiting time τ", the value of τ is decided only to fit the records and has little theoretical support.

Because of these assumptions introduced into these models, they give little contribution to the planning and design of water channels. Based on these previous researches, this paper presents a probability model of ship collisions including the operational variables such as width, length, depth and so forth.

In Chap. 2, the factors influencing a ship collision are listed and classified, and in Chap. 3, the estimation model is constructed. Then, in Chap. 4, the effectiveness of the model is examined.

2. Factors Influencing Ship Collision

In Table 1 are shown the factors which influence ship collisions. These

Table 1. Factors Influencing Collisions

Operational		Non-Operational		
Channel Characteristics	Traffic characterstics	Navigators Characteristics	Ship Characteristics	Natural Conditions
1. Fairway Width 2. Fairway Length 3. Depth 4. Curvature 5. Fairway Crossing 6. Navigation Mark 7. Obstacles 8. Channel Side Shape	1. Ship Size Distribution 2. Sailing Velocity Distribution 3. Total Traffic Volume 4. Traffic Volume Ratio in Different Directions 5. Crossing Traffic Volume 6. Wake Position Distribution 7. Headway Distribution	1. Quality 2. Illegal Sailing 3. Bad Watching 4. With or Without Pilot	1. Ship Size 2. Speed Performance 3. Steering Performance 4. Stopping Performance 5. Radar Equipment	1. Tidal Stream 2. Wave 3. Sight Distance 4. Wind Direction 5. Wind Force 6. Weather 7. Time

factors are divided into two groups: operational factors and non-operational factors. Operational factors are concerned with the channel design conditions and the traffic characteristics. Non-operational factors are associated with the characteristics of navigators, the physical conditions of ships and the natural environmental conditions. All the operational factors are introduced into the present model. The natural conditions are partly taken into consideration as the sailing velocity distribution. The influence of the factors associated with navigators are treated as a random variable. That is, they are represented as the probability distribution of the distance at which two ships start to give way in order to avoid a collision.

3. Probability Model of Collision

3-1 Modeling Process

As shown in Fig. 1, the basic consideration starts with the modeling of the give-way motion of two specific ships of particular ship sizes k and k', respectively, and with the particular sailing velocities V_k, and $V_{k'}$, respectively. The collision of these two ships is the event resultant from the failure of give-way. This is given by the function of k, k', V_k and $V_{k'}$ and the distance, $l_{kk'}$, between two ships, where

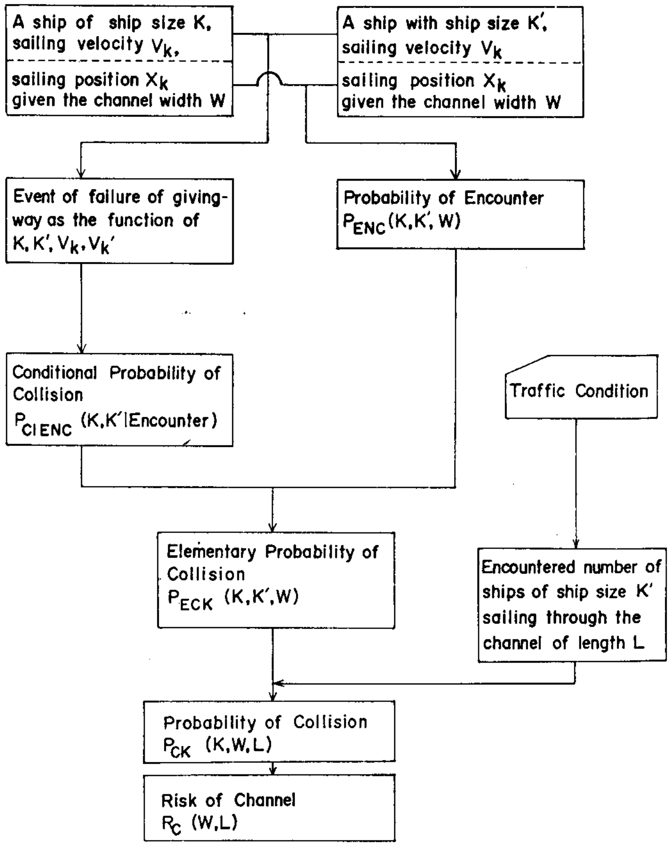

Fig. 1. Modeling Process.

they start give-way. Hereafter, the distance $l_{kk'}$ is called as the give-way starting distance (GWS-distance). Since the GWS-distance $l_{kk'}$ can be considered as a random variable, the event of a failure of give-way is considered a random event. The probability of this event is defined as the conditional collision probability, denoted by $P_{C|ENC}$, under the condition of being encountered. This conditional collision probability, $P_{C|ENC}$, is the function of the ship sizes, k and k'. On the other hand, the event that two ships are encountered is the function of their ship sizes and the relative sailing position in the direction perpendicular to the fairway. Since this relative sailing position is also considered as a random variable peculiar to the channel width W, the event of the encounter can be regarded as a random event. The probability of this event is denoted by P_{ENC}.

By multiplying $P_{C|ENC}$ by P_{ENC}, the elementary collision probability, P_{eCK} (k, k', W), can be given. This elementary collision probability of two ships is peculiar to their ship sizes, the channel width W and other natural conditions.

The probability of collision when a ship of ship size k is sailing through a channel of width W and length L is calculated by the elementary collision probability, P_{ECK}, and the given traffic volume, Q. This is denoted by $P_C(k, W, L, Q)$. Finally, the collision risk $R_C(W, L, Q)$ of the given channel and the traffic volume is calculated.

3-2 Probability of two ships' collision

As discussed in 3–1, modeling give-way motions is the basis of a mathematical treatment of ship collisions. For the modeling of give-way motions, three cases

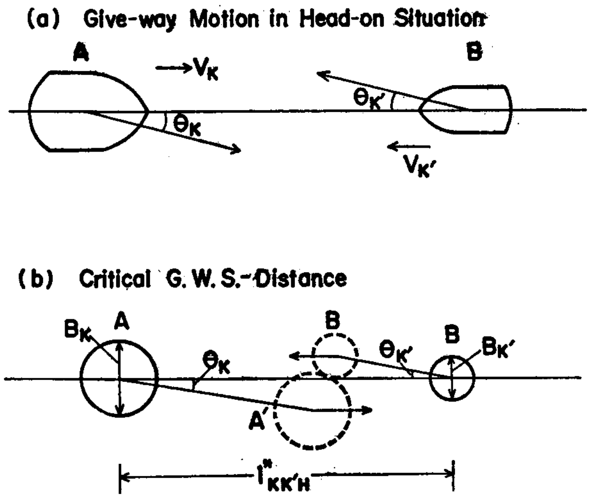

Fig. 2. Head-on Situation.

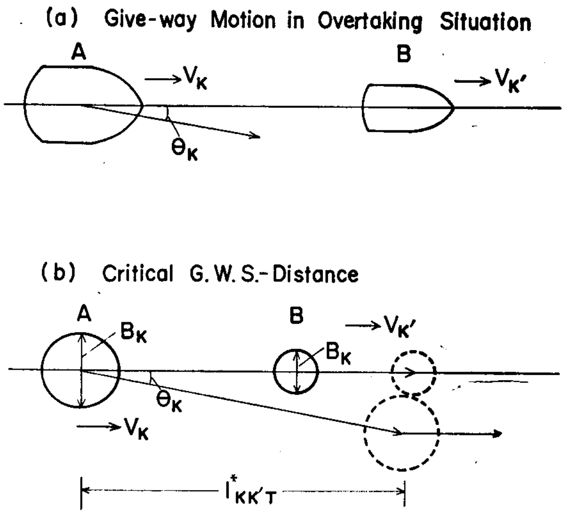

Fig. 3. Overtaking Situation.

should be taken into account, that is, the head-on situation (See Fig. 2 (a)), the overtaking situation (Fig. 3 (a)) and the overtaken situation (Fig. 4 (a)).
overtaking situation (Fig. 3 (a)) and the overtaken situation (Fig. 4 (a)). In a head-on situation, both of the encountering ships must make a give-way motion, while only the overtaking ship must make a give-way motion in an overtaking or overtaken situation. Therefore, collision events are modeled for these three cases.

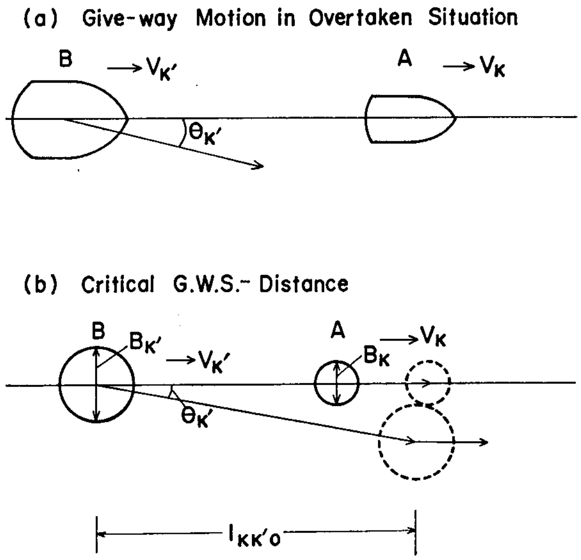

Fig. 4. Overtaken Situation.

(*a*) *Head-on situation*

Suppose that two ships, A and B, are encountered in a head-on situation. The ship sizes and the sailing velocities of these two ships are k, k' and V_k, $V_{k'}$, respectively. These two ships will collide with each other if they do not make a give-way motion. In general, the give-way motion includes altering the course by steering, speed-down, anchoring and so forth. However, the present model considers only the steering motion because speed-down, anchoring and other motions are quite rare comparing with the steering motion. Let $l_{kk'}$, be the distance between the ships A and B when both of the ships start give-way by the steering angle θ_k and $\theta_{k'}$, respectively. (See Fig. 2(b)). The distance, $d_{ABH}(t)$, between the centers of the ships A and B when time t is passed after starting the give-way motions is given by

$$d_{ABH}(t) = \{[V_k^2+V_{k'}^2+2V_kV_{k'}\cos(\theta_k-\theta_{k'})\}t^2-2l_{kk}(V_k\sin\theta_k+V_{k'}\sin\theta_{k'})t+l_{kk'}]^{1/2} \qquad \cdots\cdots\cdots\cdots(3.1)$$

This gives the minimum distance d^*_{ABH} as

$$d^*_{ABH} = l_{kk'} \cdot \frac{(V_k \sin \theta_k + V_{k'} \sin \theta_{k'})}{[V_k^2 + V_{k'}^2 + 2V_k V_{k'} \cos (\theta_k - \theta_{k'})]^{1/2}} \qquad \cdots\cdots(3.2)$$

For simplicity, the ship size is often represented by a circle with a diameter B which is equivalent to the width of the ship. Then collision of two ships of sizes B_k and $B_{k'}$ is defined as the event that the distance between the centers of the ships is less than equal to the "Collision Diameter $D_{kk'}$" which is defined by

$$D_{kk'} = (B_k + B_{k'})/2 \qquad \cdots\cdots\cdots\cdots(3.3)$$

By the definition of collision mentioned above, the condition of failure of give-way, that is, the condition of two ships' collision, is given by

$$d^*_{ABH} \leq D_{kk'} \qquad \cdots\cdots\cdots\cdots(3.4)$$

Applying Eq. (3.2) to Eq. (3.4), the critical GWS-distance, l^*_{kkH}, between two ships of sizes k and k' is given by the following equation:

$$l^*_{kkH} = D_{kk'} \cdot \frac{[V_k^2 + V_{k'}^2 + 2V_k V_{k'} \cos (\theta_k - \theta_{k'})]^{1/2}}{(V_k \sin \theta_k + V_{k'} \sin \theta_{k'})} \qquad \cdots\cdots(3.5)$$

According to Fujii (1966)[1], most of the ships use the constant steering angle θ (=15°). For such a case, Eq. (3.5) yields

$$l^*_{kk'H} = D_{kk'} \cdot \frac{(V_k + V_{k'})}{\sin \theta} \qquad \cdots\cdots\cdots\cdots(3.6)$$

According to the observational data by the Ministry of Transportation of Japan (M.T.J.) (1973)[11], the GWS-distance, $l_{kk'}$, in a head-on situation can be approximated by the form;

$$l_{kk'H} = \alpha_H + \beta_H (V_k + V_{k'}) + \varepsilon_H \qquad \cdots\cdots\cdots\cdots(3.7)$$

where α_H, β_H are the regression constants, and ε_H is the normal random variable with mean zero and standard deviation $\sigma_{\varepsilon H}$. The standard deviation $\sigma_{\varepsilon H}$ could be a function of the condition of navigators, climate, instruments of ships and so forth.

Since the GWS-distance can be regarded as the normal random variable, the conditional probability of two ships' collision $P_{CH|ENC}$ is defined by the probability that $l_{kk'H}$ is less than equal to the critical give-way distance, $l^*_{kk'H}$. That is,

$$P_{CH|ENC}(k, k', V_k, V_{k'} | \text{Encounter}) = \text{Prob.} [l_{kk'H} \leq l^*_{kk'H} | \text{Encounter}]$$

$$= \int_0^{l^*_{kk'H}} \frac{1}{\sqrt{2\pi}\sigma_{\varepsilon H}} \exp\left[-\frac{1}{2}\left(\frac{l_{kk'H} - \overline{l_{kk'H}}}{\sigma_{\varepsilon H}}\right)^2\right] dl_{kk'H} \qquad \cdots\cdots\cdots\cdots(3.8)$$

where

$$\overline{l_{kk'H}} = \alpha_H + \beta_H (V_k + V_{k'}) \qquad \cdots\cdots\cdots\cdots(3.9)$$

(*b*) *Overtaking or Overtaken Situations*

Referring to Fig. 3(b) for an overtaking situation, the distance $d_{ABT}(t)$ between the ships A and B at time t after ship A starts giving way is expressed as

$$d_{ABT}(t) = [(V_k^2+V_{k'}^2-2V_kV_{k'}\cos\theta_k)t^2-2l_{kk'}(V_{k'}-V_k\cos\theta_k)t+l_{kk'}^2]^{1/2}$$
$$\text{for} \quad V_k\cos\theta_k > V_{k'} \qquad \cdots\cdots\cdots\cdots(3.10)$$

This gives the minimum distance d_{ABT}^* as

$$d_{ABT}^* = V_k\sin\theta_k\cdot\frac{l_{kk'}}{(V_k^2+V_{k'}^2-2V_kV_{k'}\cos\theta_k)^{1/2}} \qquad \cdots\cdots\cdots\cdots(3.11)$$

Applying Eq.(3.11) to the condition of two ships' collision;

$$d_{ABT}^* \leq D_{kk'}$$

the critical GWS-distance, $l_{kk'T}^*$ is given by

$$l_{kk'T}^* = D_{kk'}\cdot\frac{(V_k^2+V_{k'}^2-2V_kV_{k'}\cos\theta_k)^{1/2}}{V_k\sin\theta_k} \quad \text{for} \quad V_k\cos\theta_k > V_{k'} \cdots\cdots\cdots\cdots(3.12)$$

For overtaken situations, the following condition must be held: (See Fig. 4(b)),

$$V_k \leq V_{k'}\cos\theta_{k'} \qquad \cdots\cdots\cdots\cdots(3.13)$$

Referring to the reduction process of Eq. (3.12), the critical GWS-distance, $l_{kk'O}^*$, is given by the equation;

$$l_{kk'O}^* = D_{kk'}\cdot\frac{(V_k^2+V_{k'}^2-2V_kV_{k'}\cos\theta_{k'})^{1/2}}{V_{k'}\sin\theta_{k'}} \quad \text{for} \quad V_k < V_{k'}\cos\theta_{k'} \cdots\cdots\cdots(3.14)$$

According to the observational data by MTJ (1973)[11], the GWS-distances $l_{kk'T}$ and $l_{kk'O}$ are given by the forms:

$$l_{kk'T} = \alpha_T+\beta_T(GT)_k+\gamma_T(GT)_{k'}+\varepsilon_T \qquad \cdots\cdots\cdots\cdots(3.15)$$

and

$$l_{kk'O} = \alpha_T+\beta_T(GT)_k+\gamma_T(GT)_{k'}+\varepsilon_T \qquad \cdots\cdots\cdots\cdots(3.16)$$

where α_T, β_T and γ_T are the regression coefficients, $(GT)_k$ and $(GT)_{k'}$ are the gross-tonnage of the two ships A and B, and ε_T is the normal random variable with mean zero and standard deviation σ_{ε_T}.

Using Eqs. (3.12) and (3.15), the conditional probability, $P_{CT|ENC}$, of two ships' collision for an overtaking situation can be calculated as

$$P_{CT|ENC}(k, k', V_k, V_{k'}|\text{Encounter}) = \text{Prob.}\,[l_{kk'T}\leq l_{kk'T}^*\,|\,\text{Encounter}]$$
$$= \int_0^{l_{kk'T}^*}\frac{1}{\sqrt{2\pi}\,\sigma_T}\exp\left[-\frac{1}{2}\left(\frac{l_{kk'T}-\overline{l_{kk'T}}}{\sigma_T}\right)^2\right]dl_{kk'T}$$
$$\text{for} \quad V_k\cos\theta_k > V_{k'} \qquad \cdots\cdots\cdots\cdots(3.17)$$

where

$$\overline{l_{kk'T}} = \alpha_T + \beta_T (GT)_k + \gamma_T (GT)_{k'} \qquad \cdots\cdots\cdots\cdots(3.18)$$

Similarly, the conditional probability of two ships' collision for an overtaken is calculated by Eqs. (3.14) and (3.16) as

$$P_{CO|ENC}(k, k', V_k, V_{k'} | \text{Encounter}) = \text{Prob.} [l_{kk'O} \leq l^*_{kk'O} | \text{Encounter}]$$
$$= \int_0^{l^*_{kk'O}} \frac{1}{\sqrt{2\pi}\sigma_T} \exp\left[-\frac{1}{2}\left(\frac{l_{kk'O}-\overline{l_{kk'O}}}{\sigma_T}\right)^2\right] dl_{kk'O}$$
$$\text{for} \quad V_k < V_{k'} \cos \theta_{k'} \qquad \cdots\cdots\cdots\cdots(3.19)$$

where

$$\overline{l_{kk'O}} = \alpha_T + \beta_T (GT)_{k'} + \gamma_T (GT)_k \qquad \cdots\cdots\cdots\cdots(3.20)$$

As can be seen in Eqs. (3.8), (3.17) and (3.19), the conditional probability of two ships' collision is the function of the sailing velocities V_k and $V_{k'}$. According to Fujii (1971)[3], the sailing velocity V_k of a ship of size k can be approximated by normal distribution, $N(\mu_{V_k}, \sigma^2_{V_k})$, as shown in Fig. 5. Taking this into account, the conditional probabilities, $P_{CH|ENC}(k, k' | \text{Encounter})$, $P_{CT|ENC}(k, k' | \text{Encounter})$ and $P_{CO|ENC}(k, k' | \text{Encounter})$ are re-expressed as follows;

$$P_{CH|ENC}(k, k' | \text{Encounter})$$
$$= \int_0^\infty \int_0^\infty \text{Prob.} [l_{kk'H} \leq l^*_{kk'H} | \text{Encounter}] f_k(V_k) f_{k'}(V_{k'}) dV_k dV_{k'} \qquad \cdots\cdots\cdots\cdots(3.21)$$

$$P_{CT|ENC}(k, k | \text{Encounter})$$
$$= \int_0^\infty \int_0^{V_k \cos\theta} \text{Prob.} [l_{kk'T} \leq l^*_{kk'T} | \text{Encounter}] f_{k'}(V_{k'}) f_k(V_k) dV_{k'} dV_k \qquad \cdots\cdots\cdots\cdots(3.22)$$

and

$$P_{CO|ENC}(k, k' | \text{Encounter})$$
$$= \int_0^\infty \int_{V_k/\cos\theta_k}^\infty \text{Prob.} [l_{kk'O} \leq l^*_{kk'O} | \text{Encounter}] f_{k'}(V_{k'}) f_k(V_k) dV_{k'} dV_k \qquad \cdots\cdots\cdots\cdots(3.23)$$

where $f_k(V_k)$ and $f_{k'}(V_{k'})$ are the probability density functions of the sailing velocity.

3-3 Probability of Encounter

For defining the event of encounter, assume that a ship is expressed by a circle with the diameter B_k, which is equivalent to its width. Then, when two ships of sizes k and k' are supposed, they are expressed by two circles of diameters B_k and $B_{k'}$, respectively. "Encounter" is defined as the event that the circle with the diameter B_k will intersect the area made by drawing the other circle of diameter

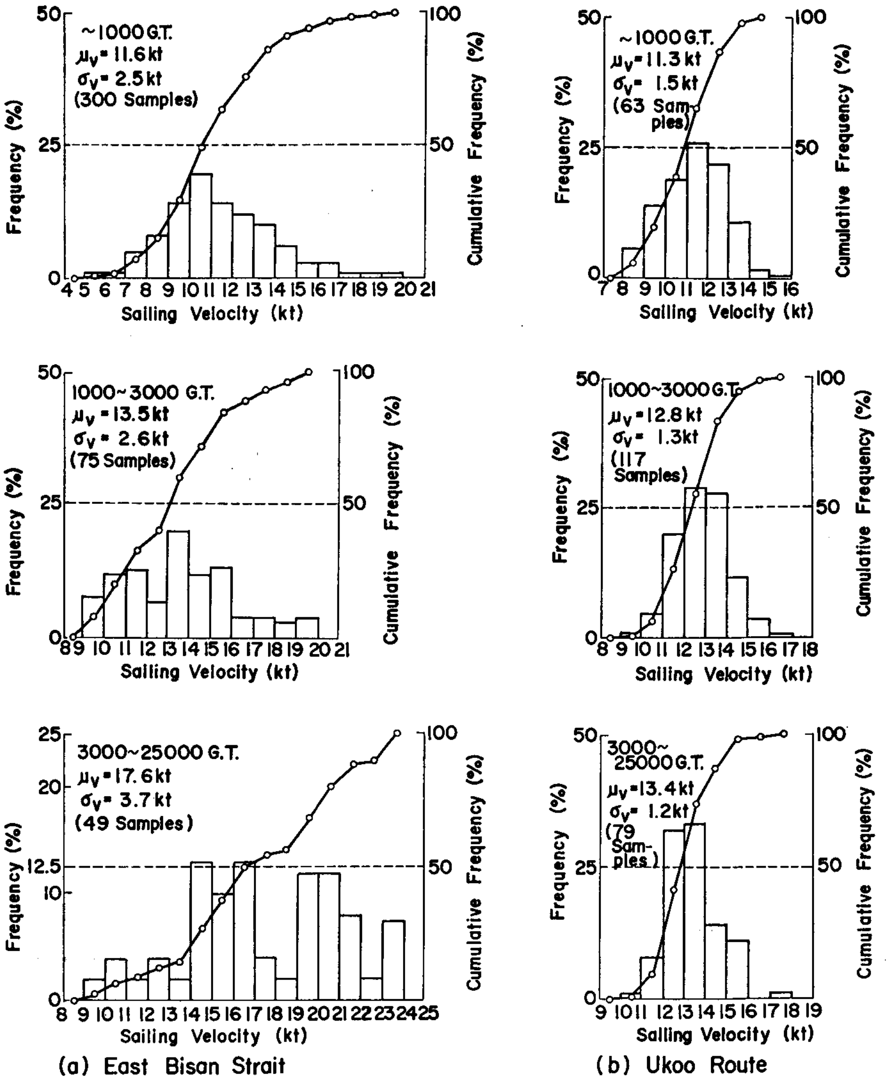

Fig. 5. Distribution of the Sailing Velocity of Each Ship Size.

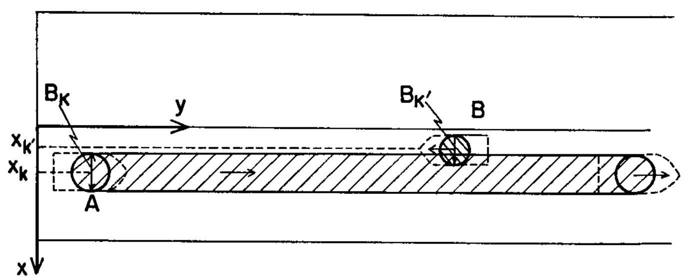

Fig. 6. The Event of Encounter.

B_k along its sailing course. (See Fig. 6). This is described by the positions x_k and $x_{k'}$ of the two ships in the coordinate, shown in Fig. 6, and the "collision diameter, $D_{kk'}$" as

$$|x_k - x_{k'}| \leq \frac{1}{2}(B_k + B_{k'}) = D_{kk'} \qquad \cdots\cdots\cdots\cdots(3.24)$$

or using the relative distance, $L_{kk'}$, defined by

$$L_{kk'} = x_k - x_{k'}$$

Eq. (3.24) is rewritten as

$$-D_{kk'} \leq L_{kk'} \leq D_{kk'} \qquad \cdots\cdots\cdots\cdots(3.25)$$

Eq. (3.25) means that the considered two ships of sizes k and k' will encounter when their relative distance, $L_{kk'}$, happens to enter the range. Usually, ships will take their wake positions at their option when they enter the channel. Therefore, as this result, the relative distance, $L_{kk'}$, will be a random variable. According to Inoue (1977)[12], the probability distribution of wake position in a two-way traffic channel of width W can be approximated by the normal distribution as shown in Fig. 7. He shows that the average wake position, $\bar{x}$, for the traffic in one direction is given by

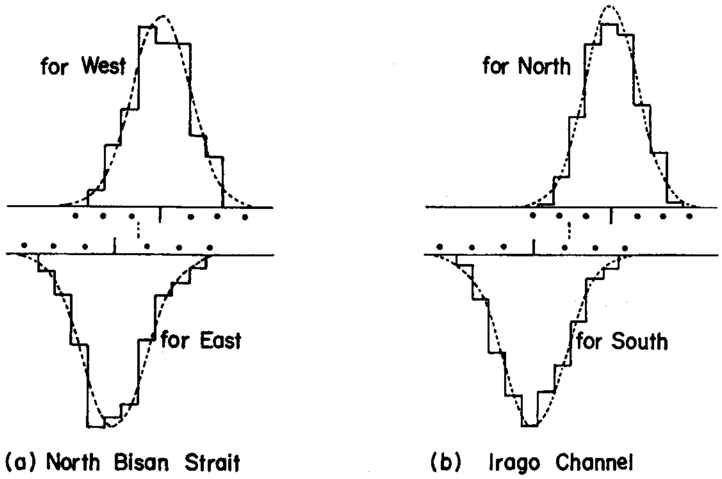

Fig. 7. Wake Distribution in the Channel.

$$\bar{x} = aW \qquad \cdots\cdots\cdots\cdots(3.26)$$

where a is the constant peculiar to the channel conditions, that is,

$a=0.2$ for two-way traffic in the channel with a centerline mark,

$a=0.1$ for two-way traffic in the channel without a centerline mark.

He also concludes that this average wake position is independent of the sailing velocity and ship size.

The standard deviation, σ_x, of a wake position of the traffic in a certain direction is shown as the function of ship size and the channel width as follows;

$$\sigma_x = -7.170+0.105W+2.1680Q_L \qquad \cdots\cdots\cdots\cdots(3.27)$$

where W is the channel width measured by meters, and Q_L is the traffic volume modified by the ship length when the standard ship length is employed as 35 m.

Consulting Eqs. (3.26) and (3.27), the probability density function of the relative distance, $L_{kk'}$, is given by

$$f_H(L_{kk'}) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\sigma_{x_k}^2+\sigma_{x_{k'}}^2}}\exp\left[-\frac{1}{2}\left(\frac{L_{kk'}-2aW}{\sqrt{\sigma_{x_k}^2+\sigma_{x_{k'}}^2}}\right)^2\right] \qquad \cdots\cdots\cdots\cdots(3.28)$$

for the case that a ship of size k and a ship of size k' sail in opposite directions,

and

$$f_{SM}(L_{kk'}) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\sigma_{x_k}^2+\sigma_{x_{k'}}^2}}\exp\left[-\frac{1}{2}\left(\frac{L_{kk'}}{\sqrt{\sigma_{x_k}^2+\sigma_{x_{k'}}^2}}\right)^2\right] \qquad \cdots\cdots\cdots\cdots(3.29)$$

for the case that ships of sizes k and k' sail in the same direction

in which

$$\sigma_{x_k} = -7.170+0.105W+2.168Q_{Lk} \qquad \cdots\cdots\cdots\cdots(3.30)$$

and

$$\sigma_{x_{k'}} = -7.170+0.105W+2.1680Q_{Lk'} \qquad \cdots\cdots\cdots\cdots(3.31)$$

In Eqs. (3.30) and (3.31), Q_{Lk} and $Q_{Lk'}$ represent the traffic volume for each direction of ships of sizes k and k', respectively, modified by their ship length when the standard ship length is 35 m.

Substituting Eq. (3.28) into (3.25), the probability, P_{ENC}^H, of encounter of the two ships of sizes k and k' sailing in opposite directions is calculated by

$$\begin{aligned} P_{ENC}^H(k, k') &= \text{Prob.}\,[-D_{kk'} \leq L_{kk'} \leq D_{kk'}] \\ &= \int_{-D_{kk'}}^{D_{kk'}} f_H(L_{kk'})dL_{kk'} \end{aligned} \qquad \cdots\cdots\cdots\cdots(3.32)$$

For the cases of overtaking and overtaken, Eqs. (3.29) and (3.25) give the probabilities of encounter, P^T_{ENC} for the former case, and P^O_{ENC} for the later case, by the same from:

$$P^T_{ENC}(k, k') = P^O_{ENC}(k, k') = \text{Prob. } [-D_{kk'} \leq L_{kk'} \leq D_{kk'}] = \int_{-D_{kk'}}^{D_{kk'}} f_{SM}(L_{kk'}) dL_{kk'} \qquad \cdots\cdots\cdots\cdots(3.33)$$

It should be notified that Eq.(3.32) means the potential possibility of encountering one ship of size k' when a certain ship of size k enters the channel of width W. It does not mean the relative frequency of the number of ships of size k' encountered while sailing through the channel. This will be discussed again in later sections.

3-4 Elementary Collision Probability

In the previous sections, the conditional probability of two ships' collision under the event of encounter and the potential probability of encoutner of the two ships are formulated. These two kinds of probability give the elementary collision probability, that is, the potential collision probability when two ships of sizes k and k' enter the channel. This can be given by the multiplication of the above two probabilities. From Eqs. (3.21) through (3.23) and Eqs. (3.32) and (3.33), the elementary collision probabilities for three cases are calculated as follows;

(*a*) *Head-on situation*

$$P_{eCH}(k, k') = P_{CH|ENC}(k, k' \,|\, \text{Encounter}) \cdot P^H_{ENC}(k, k') = \int_0^\infty \int_0^\infty \text{Prob. } [l_{kk'H} \leq l^*_{kk'H} \,|\, \text{Encounter}] f_{k'}(V_{k'}) f_k(V_k) dV_{k'} dV_k \times \int_{-D_{kk'}}^{D_{kk'}} f_H(L_{kk'}) dL_{kk'} \qquad \cdots\cdots\cdots\cdots(3.34)$$

(*b*) *Overtaking situation*

$$P_{eCT}(k, k') = P_{CT|ENC}(k, k' \,|\, \text{Encounter}) \cdot P^T_{CEN}(k, k') = \int_0^\infty \int_0^{V_k \cos\theta_k} \text{Prob. } [l_{kk'T} \leq l^*_{kk'T} \,|\, \text{Encounter}] f_{k'}(V_{k'}) f_k(V_k) dV_{k'} dV_k \times \int_{-D_{kk'}}^{D_{kk'}} f_{SM}(L_{kk'}) dL_{kk'} \qquad \cdots\cdots\cdots\cdots(3.35)$$

(*c*) *Overtaken situation*

$$P_{eCO}(k, k') = P_{CO|ENC}(k, k' \,|\, \text{Encounter}) \cdot P^O_{ENC}(k, k') = \int_0^\infty \int_{V_k/\cos\theta_k}^\infty \text{Prob. } [l_{kk'O} \leq l^*_{kk'O} \,|\, \text{Encounter}] f_{k'}(V_{k'}) f_k(V_k) dV_{k'} dV_k \times \int_{-D_{kk'}}^{D_{kk'}} f_{SM}(L_{kk'}) dL_{kk'} \qquad \cdots\cdots\cdots\cdots(3.36)$$

It should be again noticed that the Eqs. (3.34) through (3.36) mean the potential collision probability when only two ships of sizes k and k' enter the channel. Thus, they give the elementary collision probability peculiar to the channel and to the considered two ships.

In the next section is analyzed the total probability of collision for a one-way-trip of a certain ship of size k sailing through a channel of width W and length L under the steady traffic flow of a certain volume, and the total collision risk peculiar to the channel under given traffic volume.

3-4 Number of Encountered Ships

For analyzing the collision probability for a one-way-trip of a certain ship of size k sailing through a channel, the number of encountered ships must be specified under given traffic characteristics. For this reason, suppose a sea traffic which satisfies following asumptions:

(1) Ship size of the total traffic through the channel follows the log-normal distribution.

This is supported by the data of Fujii (1971)[3] as illustrated in Fig. 8 which

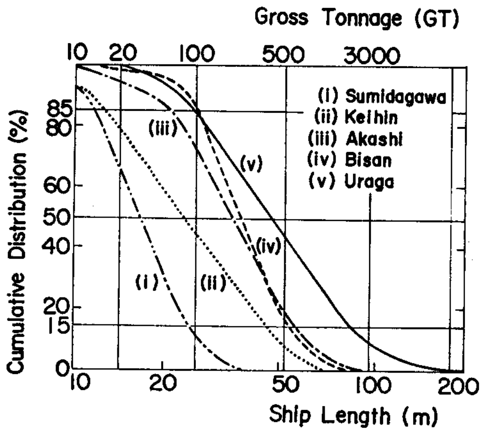

Fig. 8. Cumulative Distribution of Ship Size of the Traffic in the Narrow Channel.

shows the cumulative distribution of ship sizes of the traffic in some narrow channels in Japan.

Since the ship size has a unique relationship with the ship length, the ship size k can be represented by the ship length L_k. Denoting the logarithm of ship length L by ω, that is,

$$\omega = \log_{10} L$$

the probability density function of ω is given by

$$\phi(\omega) = \frac{1}{\sqrt{2\pi}\sigma_\omega} \exp\left[-\frac{1}{2}\left(\frac{\omega-\mu_\omega}{\sigma_\omega}\right)^2\right] \qquad \cdots\cdots\cdots\cdots(3.37)$$

Based on this equation, the ship size of the traffic in one direction of the channel is assumed to follow the log-normal distribution.

(2) Traffic flow through the channel is assumed to be steady.

The ratio, λ_1, of the traffic volume in a certain direction '1' to the other '2' is constant.

(3) All the ships sail with constant velocity.

From the assumptions (1) and (2), the traffic volume, Q_{1k}, of the ship size k in the direction '1' and $Q_{2k'}$ of the ship size k' in the direction '2' are given by Eq. (3.38) when the total traffic volume is Q.

$$Q_{1k} = \lambda_1 Q\phi(\omega_k)d\omega_k \qquad \cdots\cdots\cdots\cdots(3.38a)$$

$$Q_{2k} = (1-\lambda_1)Q\phi(\omega_{k'})d\omega_{k'} \qquad \cdots\cdots\cdots\cdots(3.38b)$$

Under these traffic characteristics, the number of encountered ships for the three cases discussed in the previous sections are calculated as follows:

(a) *number of ships encountered in a head-on situation*

When a certain ship of the size k, say ship A, with the velocity V_k enters a channel of length L, it takes the time

$$t_k = L/V_k$$

for sailing through the channel. On the other hand, any ship of size k' and velocity $V_{k'}$ at the position of the distance $L+\Delta L_H$ from ship A at the instance when ship A arrives at an entrance of the channel takes the time

$$t_{k'} = \Delta L_H/V_{k'}$$

for arrival at the other entrance of the channel.

Thus, the condition that the ship of size k' is encountered by ship A during ship A's sailing through the channel is given by

$$t_k = \frac{L}{V_k} \geq \frac{\Delta L_H}{V_{k'}}$$

or

$$\Delta L_H \leq L \cdot \frac{V_{k'}}{V_k} \qquad \cdots\cdots\cdots\cdots(3.39)$$

Therefore, any ships within the distance, $L+\Delta L_H$, will be in a head-on situation with ship A while ship A sails through the channel. (See Fig. 9.) Thus, the number, $n_{Hkk'}$, of the ships of size k' being in a head-on situation with ship A during ship

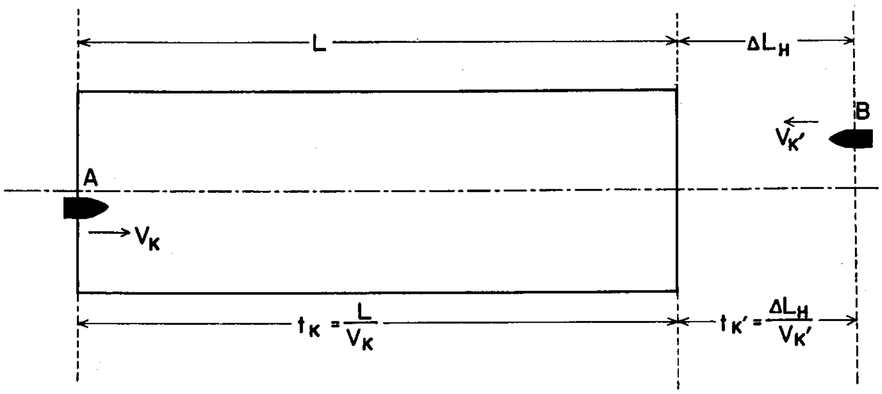

Fig. 9. Maximum Distance between Ship A and B for Encountering Condition (Head-on Situation).

A's sailing through the channel can be given by

$$n_{Hkk'} = \frac{Q_{2k'}}{V_{k'}}(L+\Delta L_H) \qquad \cdots\cdots\cdots\cdots(3.40)$$

Applying Eqs. (3.38b) and (3.39) to Eq. (3.40), the above equation can be expressed as

$$n_{Hkk'} = (1-\lambda_1)Q\phi(\omega_{k'})d\omega_{k'}L\left(\frac{1}{V_{k'}}+\frac{1}{V_k}\right) \qquad \cdots\cdots\cdots\cdots(3.41)$$

This gives the total number of the ships of size k' which will be in a head-on situation with ship A. The expected total number, $\bar{n}_{Hkk'}$, of the ships of size k', thus, can be given by

$$\bar{n}_{Hkk'} = \int_0^\infty \int_0^\infty n_{Hkk'} f_{k'}(V_{k'}) f_k(V_k) dV_{k'} dV_k \qquad \cdots\cdots\cdots\cdots(3.42)$$

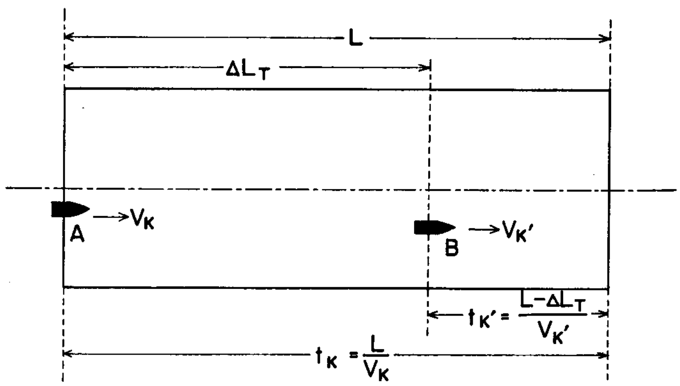

Fig. 10. Maximum Distance between Ship A and B for Encountering Condition (Overtaking Situation).

(*b*) *number of ships encountered in an overtaking situation*

Referring to Fig. 10, the critical distance in an overtaking situation, corresponding to Eq. (3.39), is given by

$$\Delta L_T \leq L\left(1-\frac{V_{k'}}{V_k}\right) \quad \text{for } V_k > V_{k'} \qquad \cdots\cdots\cdots\cdots(3.43)$$

Therefore, the number, $n_{Tkk'}$, of the ships of size k' being overtaken by ship A during A's sailing through the channel is given as

$$n_{Tkk'} = \frac{Q_{1k'}}{V_{k'}}\Delta L_T \qquad \cdots\cdots\cdots\cdots(3.44)$$

Applying Eqs. (3.38 a) and (3.43) to Eq. (3.45), $n_{Tkk'}$ is rewritten as

$$n_{Tkk'} = \lambda_1 Q\phi(\omega_{k'})d\omega_{k'}L\left(\frac{1}{V_{k'}}-\frac{1}{V_k}\right) \quad \text{for } V_k > V_{k'} \qquad \cdots\cdots\cdots\cdots(3.45)$$

Then, the total expected number, $\bar{n}_{Tkk'}$ of the ships of size k' being in an overtaking situation with ship A can be calculated as follows:

$$\bar{n}_{Tkk'} = \int_0^\infty \int_0^{V_k} n_{Tkk'} f_{k'}(V_{k'}) f_k(V_k) dV_{k'} dV_{k'} \qquad \cdots\cdots\cdots\cdots(3.46)$$

(*c*) *number of ships encountered in an overtaken situation*

Fig. 11 shows the mathematical condition that ship A of size k with a velocity

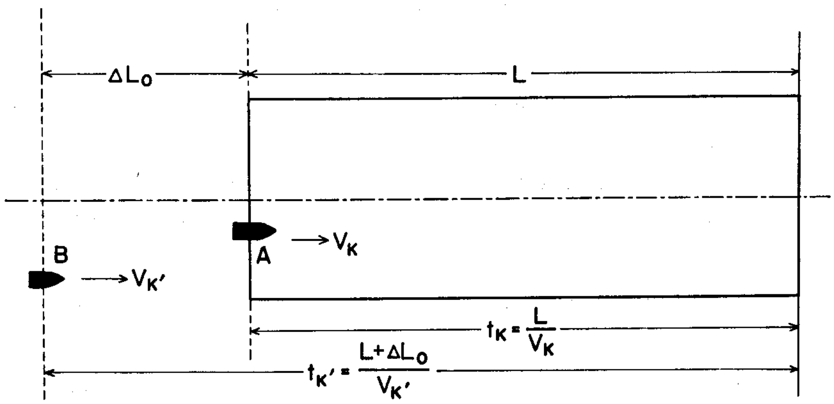

Fig. 11. Maximum Distance between Ship A and B for Encountering Condition (Overtaken Situation).

V_k is overtaken by a certain ship of size k' with a velocity $V_{k'}$ during ship A's sailing through the channel. That is,

$$\Delta L_O \leq L\left(\frac{V_{k'}}{V_k}-1\right) \quad \text{for } V_{k'} \geq V_k \qquad \cdots\cdots\cdots\cdots(3.47)$$

Using Eqs. (3.38) and (3.47), the total number, $n_{Okk'}$, of the ships of size k' overtaking ship A during ship A's sailing through the channel can be given by

$$n_{Okk'} = \lambda_1 Q \phi(\omega_{k'}) d\omega_{k'} L \left(\frac{1}{V_k} - \frac{1}{V_{k'}}\right) \quad \text{for } V_{k'} > V_k \qquad \cdots\cdots\cdots\cdots(3.48)$$

The total expected number, $\bar{n}_{Okk'}$ of the ships of size k' being in an overtaken situation can be calculated by the equation:

$$\bar{n}_{Okk'} = \int_0^\infty \int_0^\infty n_{Okk'} f_{k'}(V_{k'}) f_k(V_k) dV_{k'} dV_k \qquad \cdots\cdots\cdots\cdots(3.49)$$

3-5 Collision Risk of Channel

The collision risk of a channel under a certain traffic condition is defined as the probability that all the ships sailing through the channel will collide during a one-way-trip. For the calculation of this collision risk of the channel, the non-collision probability of a one-way-trip of a certain ship of size k is required.

Denoting $P_{STkk'}$ as the probability that a certain ship of size k will sail through the channel without collision with any of the encountered ships of size k' in a head-on situation, of which number is expected as $\bar{n}_{Hkk'}$, it is given as

$$P_{SHkk'} = [1 - P_{eCH}(k, k')]^{\bar{n}_{Hkk'}} \qquad \cdots\cdots\cdots\cdots(3.50)$$

Thus the probability, P_{SHk}, that a ship of size k can sail through the channel without collision with any of the encountered ships of any size in a head-on situation is given by the multiplication of the probability $P_{SHkk'}$ for all k'. That is,

$$P_{SHk} = \prod_{k'} P_{SHkk'}$$
$$= \prod_{k'} [1 - P_{eCH}(k, k')^{\bar{n}_{Hkk'}}] \qquad \cdots\cdots\cdots\cdots(3.51)$$

Similarly, the probability, P_{STk}, of sailing through the channel safely in an overtaking situation and the probability P_{SOk}, for an overtaken situation are calculated, respectively, as

$$P_{STk} = \prod_{k'} [1 - P_{eCT}(k, k')]^{\bar{n}_{Tkk'}} \qquad \cdots\cdots\cdots\cdots(3.52)$$

and

$$P_{SOk} = \prod_{k'} [1 - P_{eCO}(k, k')]^{\bar{n}_{Okk'}} \qquad \cdots\cdots\cdots\cdots(3.53)$$

Employing Eqs. (3.51) through (3.53), the collision probability, P_{Ck}, of a one-way-trip of a ship of size k can be represented as

$$P_{Ck} = 1 - P_{SHk} \cdot P_{STk} \cdot P_{SOk}$$
$$= 1 - \prod_{k'} \{[1 - P_{eCH}(k, k')]^{\bar{n}_{Hkk'}} \cdot [1 - P_{dCT}(k, k')]^{\bar{n}_{Tkk'}} \cdot [1 - P_{eCO}(k, k')]^{\bar{n}_{Okk'}}\}$$
$$\cdots\cdots\cdots\cdots(3.54)$$

Eq. (3.54) gives the collision probability when a particular ship of size k sails through a channel of length L under given traffic conditions. Thus, the collision probability, P_C, when the size of the ship sailing through the channel is not specified can be considered as follows:

$$P_C = \int_k P_{Ck}\phi(\omega_k)d\omega_k \qquad \cdots\cdots\cdots\cdots(3.55)$$

This is reduced on the basis of the assumption (1) discussed in the section 3–4.

Eq. (3.55) gives the collision probability for one-way-trips of all the ships sailing to a particular direction in the channel. Using the suffix '1' for a particular direction and the suffix '2' for another direction, the collision probability for each case is expressed as

$$P_{C1} = \int_k P_{Ck1}\phi(\omega_k)d\omega_k \qquad \cdots\cdots\cdots\cdots(3.56)$$

and

$$P_{C2} = \int_k P_{Ck2}\phi(\omega_k)d\omega_k \qquad \cdots\cdots\cdots\cdots(3.57)$$

Based on the assumption (2) given in the section 3–4, the "Collision Risk of Channel", R_C, can be represented by

$$R_C = \lambda_1 P_{C1} + (1-\lambda_1)P_{C2} \qquad \cdots\cdots\cdots\cdots(3.58)$$

4. Examination of Collision Probability Model

4-1 Accuracy of Model

Accuracy of the proposed model for estimating collision probability is examined with the collision statistics of three channels in Japan. Those are from Uraga Channel, Akashi Channel and Bisan Straits. The conditions of these channels and traffics are shown in Table 2. The channel conditions are referred to Hara (1973)[9], and the traffic condition concerned with the total volume Q is from the data given by Fujii (1971)[4]. The ratio, λ_1, of the traffic volume to a certain direction to another is also shown in Table 2. These values are based

Table 2. Channel and Traffic Conditions

	L (km)	W (m)	Q (ships/hr)	λ_1
AKASHI	18.5	4000	61	0.53
BISAN (west+east)	67.4	2000	68	0.51
URAGA	27.8	2000	27	0.53

after Hara (1973)[10]

Table 3. Parameter of Sailing Velocity Distribution

Rank	~100	100~ 500	500~ 3000	3000~
V	358.1	358.1	416.7	543.3
σ_v	77.2	77.2	80.3	114.2

(m/min.)

on the data[11] observed by M.T.J. in 1973. The values of the statistical parameters of the ship size distribution are assumed to be same as those shown in Fig. 8. The parameters of the sailing velocity distribution of each ship size rank are shown in Table 3. These values are read from Fig. 5.
The regression coefficients in Eqs. (3.7) and (3.15) are based on the data[11] by M.T.J. and shown in Table 4.

Table 4. Regression Coefficients in Eq. (3.7) and (3.15)

	α	β	γ	σ_t
Head-on*	−931.4	4.59	—	437
Overtaking**	326	−1.39	3.44	102
Overtaken**	326	−1.39	3.44	102

* V_k and $V_{k'}$ are measured in meter/min.
** $(GT)_k$, $(GT)_{k'}$, are measured in Gross tonnage

after M.T.J.[11]

Table 5. Estimated and Observed Values of Collision Risk of Channel

	Estimated	Observed by L.M.V*
AKASHI	0.30×10^{-4}	0.69×10^{-4}
BISAN	0.40×10^{-4}	1.58×10^{-4}
URAGA	1.36×10^{-4}	1.93×10^{-4}

* L.M.V is the traffic volume modified by ship length

Using these numerical data in the proposed model, the estimated values of the collision risk of each channel are shown in Table 5. In this table, the observed value of the collision risk is calculated by

$$R_C = \frac{Q_{CL}}{Q_L} \qquad \cdots\cdots\cdots\cdots(4.1)$$

where Q_{CL} is the collision ship number modified by the ship size, and Q_L is the total traffic volume modified by the ship size.

Table 5 shows that the estimated values tend to be less than the observed values. The main reasons can be considered as follows:

(1) The present model does not consider ships crossing the channel and fishing boats.

(2) Collisions with ships during give-way are neglected.

(3) Weather effects are not taken into account.

(4) Effects of unsteady traffic flow are not taken into account.

However, it can be said that the present model gives a satisfactory estimation under the current situation of proposed models. Hence, this is the first model to be applicable to engineering planning and design of channels.

4-2 Estimation Process by the Proposed Model

For practical uses of the proposed model, the process for estimation shown in Fig. 12 is required. For the estimation of the collision risk of a channel, the basic

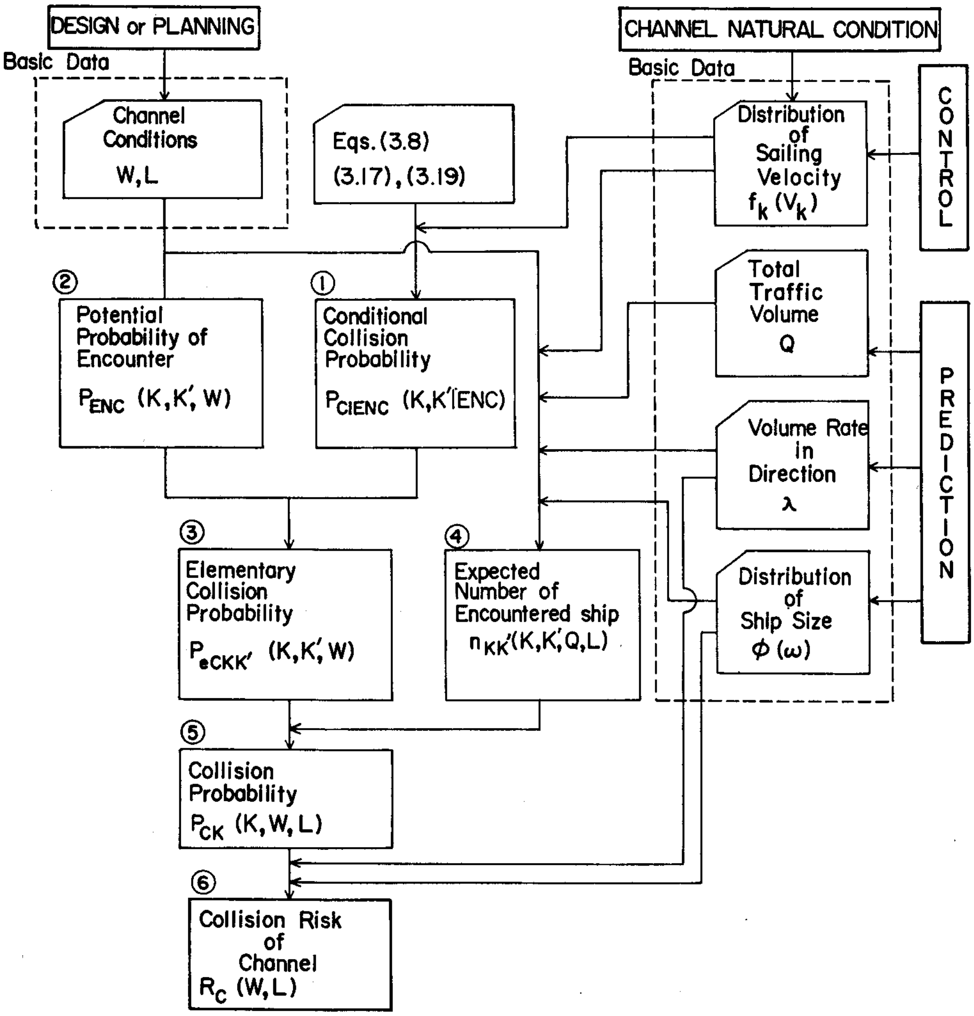

Fig. 12. Estimation Process.

data associated with traffic conditions and the channel design condition must be prepared. At the first step, the conditional collision probability should be computed by using the traffic data and Eqs. (3.8), (3.17) and (3.19). At the second step, the potential probability of encounter is computed on the basis of the channel conditions. At the third step, the elementary collision probability is computed by using the results of the first and second steps. At the fourth step, the expected number of encountered ships should be computed from the traffic and the channel data. At the fifth step, the collision probability is computed by using the results of the third and fourth steps. Finally, at the sixth step, the collision risk of the channel can be computed by applying the traffic data to the result of the fifth step.

In these processes, it should be noticed that some of the traffic data will be given through prediction, and the probability distribution of sailing velocity can be partly controlled by the speed limitation. It should be also noticed that the channel conditions are, in general, given through design or planning or improvement of the existing conditions.

5. Concluding Remarks and Discussions

A model is presented in order to estimate the collision risk of a channel. The proposed model is distinguished from other models previously presented in the following points:

(1) The present model is made on the basis of the give-way motion of ships.

(2) The present model formulates the collision as the events resultant from failure of give-way by two ships concerned, while previous models do not make this clear in a meaningfull way.

(3) The present model can estimate not only the collision risk of a channel, but also the collision probability of any number of trips of one ship of any size.

(4) Since the present model is based on the traffic conditions such as total volume, ship size distribution, sailing velocity distribution, and also on the channel conditions such as width, length and centerline mark, the effects on the collision probability by controlling or changing these conditions are easily obtained.

Comparing the estimated values of collision risk for some channels in Japan with their collision statistics, the present model tends to give an under-estimation. However, the estimated values give a satisfactory approximation from the enginnering point of view. According to the estimated values and collision statistics, the collision risk of main channels and straits in Japan has a value about $10^{-4} \sim 10^{-5}$.

As discussed in the previous chapter, the present model does not consider ships crossing the channel nor weather conditions. These should be taken into account to improve the present model.

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