Parametrically Excited Vibration with External Constant Load and Damping

By

Hiroshi MATSUHISA*, Yutaka MIYAJI* and Susumu SATO*

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Abstract

The vibrations of gears can be expressed by differential equations of parametrically excited vibrations. In this paper, the stabilities of parametrically excited vibrations of the Meißner and Mathieu types (the former expresses the torsional vibrations of spur gears and the latter expresses those of helical gears) and their vibrational characteristics in the stable region are theoretically investigated. Through this investigation, the effects of the helix angle and the damping factor on the vibrations of gears are made clear. The vibrations of helical gears are smaller than those of spur gears. And the vibrations of helical gears with a large helix angle are smaller than those with a small helix angle. When the damping factor ζ increases, the unstable region decreases, and the peak levels of vibrations decrease remarkably, although the amplitude at the other region does not decrease. When we drive a pair of gears at a speed whose meshing frequency is two times faster than the natural frequency of the gearing system, the gears with a small damping have a very large vibration, but the gears with a large damping rotate smoothly.

1. Introduction

Gears and train rails cause large vibrations and much noise. These vibrations can be expressed by a differential equation for parametrically excited vibrations. There are many studies on the stability of parametrically excited vibrations without external loads. Since an external load is present in the case of the vibration of gears, the stability and also the behavior in the stable region become important.

In this paper, the stability of parametrically excited vibrations of a spur gear (Meißner type) and a helical gear (Mathieu type), and also the characteristics of the vibrations in the stable region are theoretically investigated.

2. Analysis

2.1 Meißner type

In this section, the torsional vibration of a pair of spur gears is analyzed.

* Department of Precision Mechanics.

The tooth stiffness of a pair of gears periodically changes with the progress of meshing. The torsional vibration of a pair of spur gears which does not have tooth errors can be formularized by a Meißner type differential equation.

$$m\ddot{x}+c\dot{x}+k(t)x = W \tag{1}$$

$$k(t) = k_0-\varepsilon k_0 \operatorname{sgn}(\cos \Omega t)$$

$$\operatorname{sgn}(a) = \begin{cases} 1 & \text{for} \quad a \geqq 0 \\ -1 & \text{for} \quad a<0 \end{cases}$$

where, x: displacement, c: damping coefficient,
k: spring constant (tooth stiffness),
W: transmitting load, Ω: meshing frequency,

We can rewrite the terms of Eq. (1) as,

$$\omega^2 = k_0/m\,,$$

$$\tau = \Omega t\,,$$

$$\eta = \omega/\Omega\,,$$

$$\zeta = c/2(mk_0)^{1/2}\,.$$

Then, the following equations are obtained.

$$\dot{x} = \frac{dx}{dt} = \frac{dx}{d\tau}\frac{d\tau}{dt} = \Omega\frac{dx}{d\tau} = \Omega x'$$

$$\ddot{x} = \frac{d\dot{x}}{d\tau} = \frac{dx}{d\tau}\frac{d\tau}{dt} = \Omega^2 x''$$

$$x''+2\zeta\eta x'+\eta^2\{1-\varepsilon \operatorname{sgn}(\cos\tau)\}x = W/m\Omega^2 \tag{2}$$

When $W/m\omega^2$ is replaced by x_0, and coordinate transformation into $y=x-x_0$ is performed, Eq. (2) may be written,

$$y''+2\zeta\eta y'+\eta^2\{1-\varepsilon \operatorname{sgn}(\cos\tau)\}y = \varepsilon\eta^2 x_0 \operatorname{sgn}(\cos\tau) \tag{3}$$

By setting the right term of Eq. (3) equal to zero, we obtain a stability criterion diagram which shows stable and unstable regions. The stability diagrams for $\zeta=0.0$, $\zeta=0.01$, $\zeta=0.05$, $\zeta=0.07$, and $\zeta=0.1$ are shown in Figs. 1–5, where η and ε are the abscissa and the ordinate, respectively.

When the right term of Eq. (3) is included, the vibration has a certain amplitude in the stable region. The vibrations in the stable region are calculated by the Runge-Kutta method, and they are shown in Figs. 6 and 7, where η and y_{max}/x_0 are the abscissa and the ordinate. Figure 6 shows the amplitudes for $\varepsilon=0.05$, 0.125,

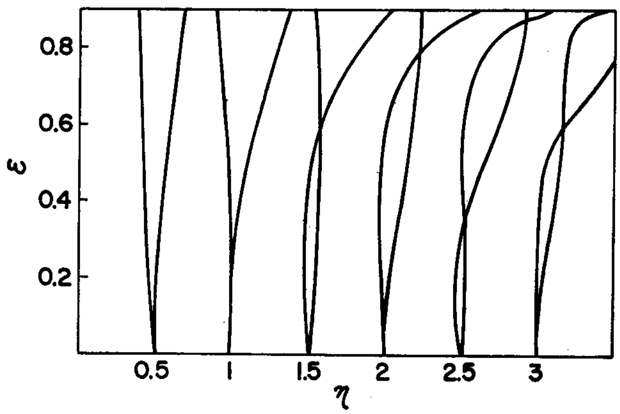

Fig. 1. Stability Diagram of Meißner Type ($\zeta=0.0$)

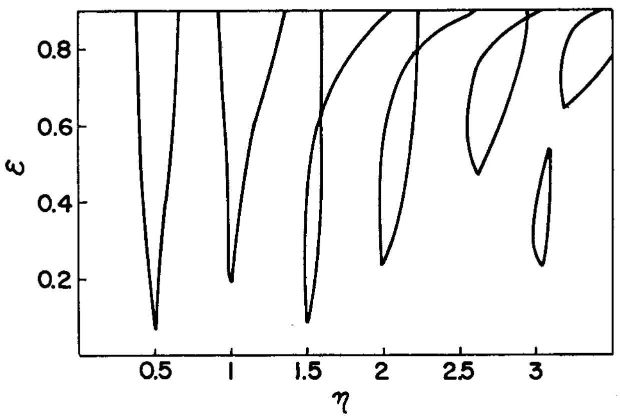

Fig. 2. Stability Diagram of Meißner Type ($\zeta=0.01$)

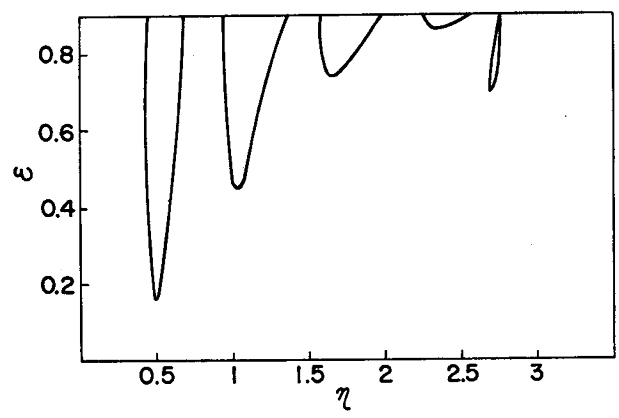

Fig. 3. Stability Diagram of Meißner Type ($\zeta=0.05$)

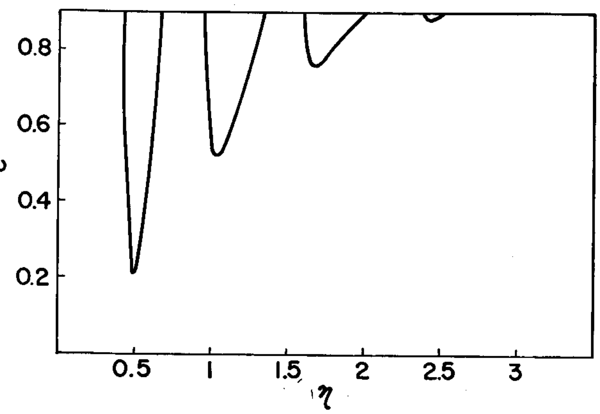

Fig. 4. Stability Diagram of Meißner Type ($\zeta=0.07$)

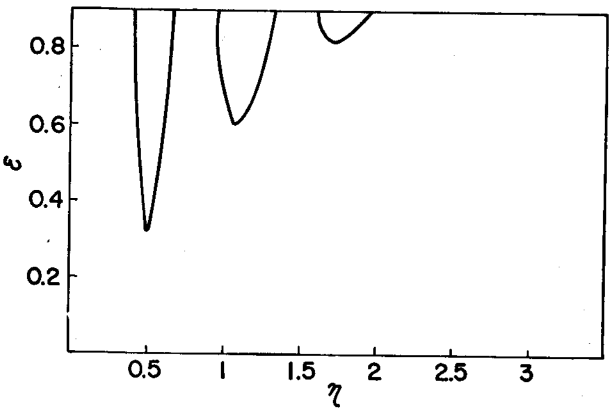

Fig. 5. Stability Diagram of Meißner Type ($\zeta=0.1$)

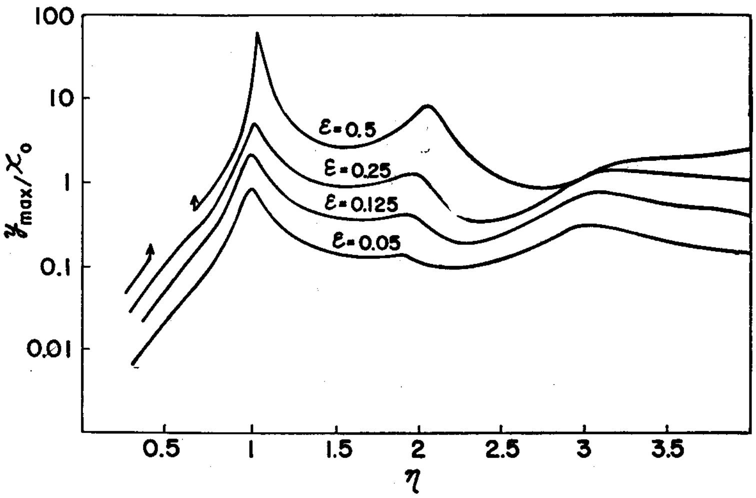

Fig. 6. Effect of ε on Amplitude of Meißner Type ($\zeta=0.07$)

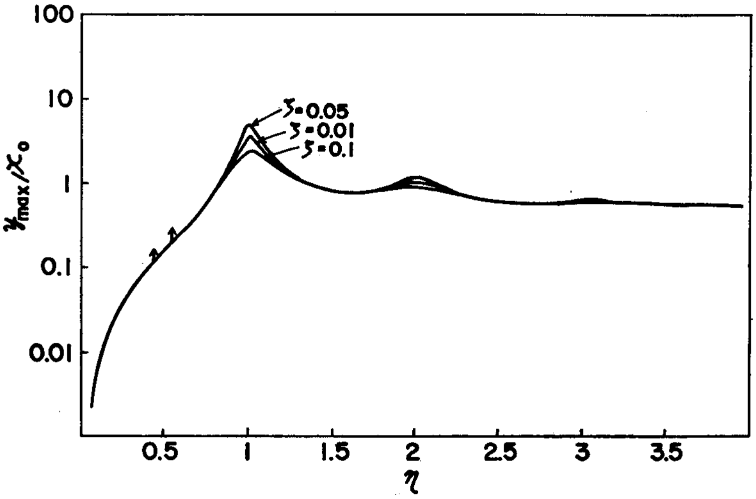

Fig. 7. Effect of Damping Factor on Amplitude of Meißner Type (ε=0.25)

0.25, and 0.5, with ζ=0.07. Figure 7 shows the amplitude for ζ=0.05, 0.07, and 0.1, with ε=0.25.

2.2 Mathieu type

In this section, we investigate the vibration of a pair of helical gears which is formularized by a Matheiu type of differenctial equation.

$$m\ddot{x}+c\dot{x}+k(t)x = W \tag{4}$$
$$k(t) = k_0-\varepsilon k_0 \cos \Omega t$$

Rewriting Eq. (4) as in the previous section, we obtain the non-dimensional

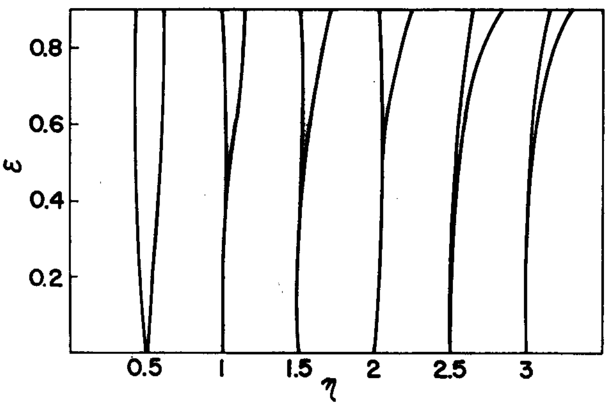

Fig. 8. Stability Diagram of Mathieu Type (ζ=0.0)

equation:

$$y''+2\zeta\eta y'+\eta^2(1-\varepsilon\cos\tau)y = \varepsilon\eta^2 x_0 \cos\tau \tag{5}$$

The stability diagrams of the homogenous equation of Eq. (5) corresponding to $\zeta=0$, 0.01, 0.05, 0.07, and 0.1 are shown in Figs. 8–12, respectively. Figures 13 and 14 show the amplitudes of the vibrations in the stable regions. The condition of Fig. 13 is $\zeta=0.07$, and the variable part of stiffness ε is equal to 0.05, 0.125, 0.25, and 0.5.

Figure 14 shows the amplitudes of the vibrations under the conditions where $\varepsilon=0.25$, and $\zeta=0.05$, 0.07, and 0.1.

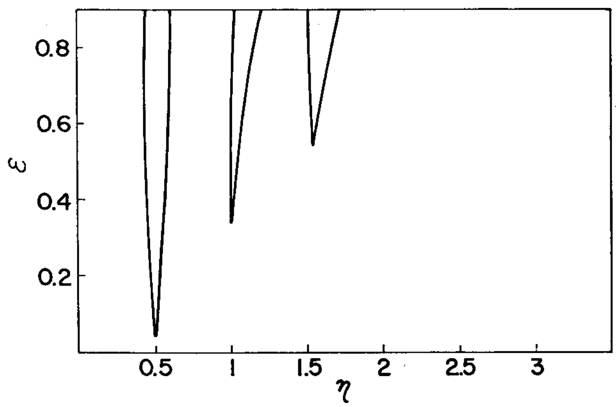

Fig. 9. Stability Diagram of Mathieu Type ($\zeta=0.01$)

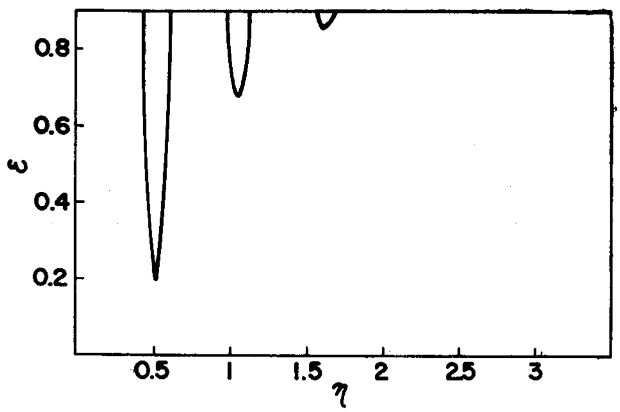

Fig. 10. Stability Diagram of Mathieu Type ($\zeta=0.05$)

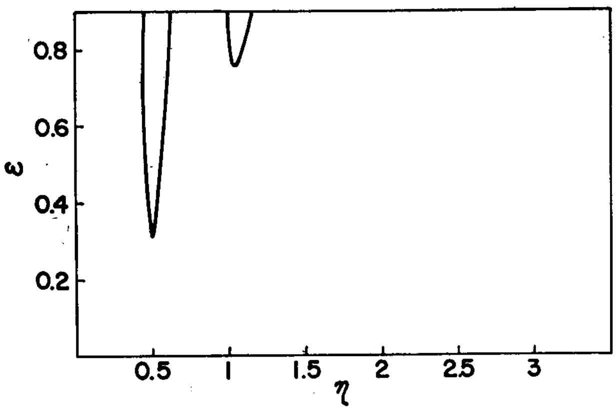

Fig. 11. Stability Diagram of Mathieu Type ($\zeta=0.07$)

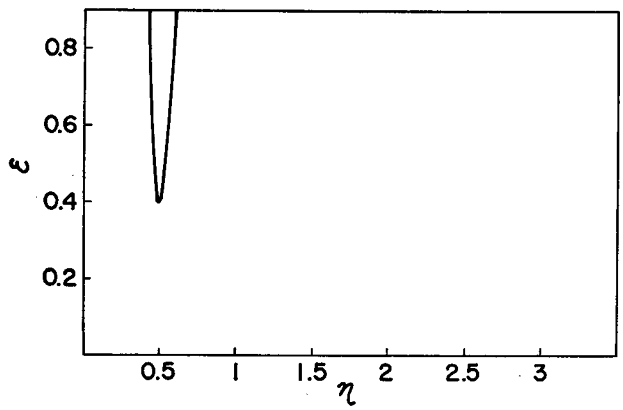

Fig. 12. Stability Diagram of Mathieu Type ($\zeta=0.1$)

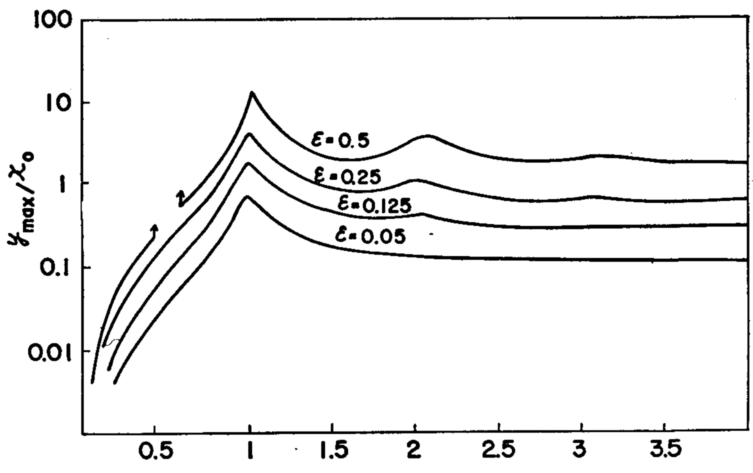

Fig. 13. Effect of ε on Amplitude of Mathieu Type ($\zeta=0.07$)

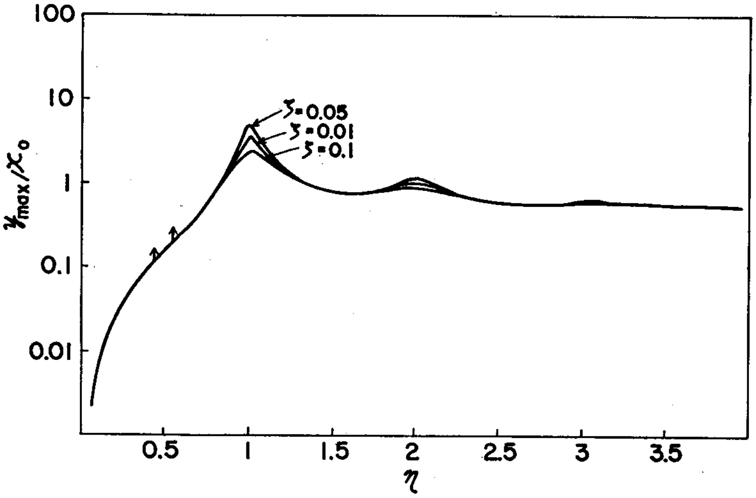

Fig. 14. Effect of Damping Factor on Amplitude of Mathieu Type ($\varepsilon=0.25$)

3. Discussion

When $\zeta=0$, the unstable regions of both the Meißner and Mathieu types have large troughs which reach $\varepsilon=0$ at $\eta=\omega/\Omega=n/2$ (n is an integer). As shown in Figs. 1–5 and Figs. 8–12, when ζ increases the unstable region decreases, and the troughs shift to the upper right side of the diagram. The decreasing of the troughs of the lower orders is smaller than that of the higher orders. Therefore, the unstable regions of the lower orders, for instance $\eta=0.5$, are still large, even when the damping is considerably large. When we compare the Meißner type with the Mathieu type, we find that the unstable region of the Meißner type, whose variable stiffness $k(t)$ has a rectangle form, is much larger than that of the Matheiu type, whose $k(t)$ is of a harmonic form.

In the stable region, the vibration has peaks in the vicinity of the troughs. We find from Fig. 6 that the peaks appear near $\eta=1$, 2, 3, that is, $\omega=n\Omega$ (n is an integer), and the vibration diverges rapidly at $\eta=0.5$. However, the vibration does not have peaks at $\eta=1.5$ and 2.5. The peak of low η is higher than that of high η. These phenomena correspond to the decreasing of the unstable region due to ζ, shown in Fig. 4. The shifts of the peaks in Fig. 6 are smaller than those of the troughs in Fig. 4. Figure 6 shows that the peaks tend to shift to the right as ε increases. Figure 7 shows that the peaks decrease when the damping factor ζ increases. By comparing Figs. 13–14 with Figs. 8–12, we can find that the vibrations of the Mathieu type have characteristics similar to the Maißner type. When the Meißner and Mathieu types have the same ε and ζ, the decreasing of the unstable region of the Mathieu type is larger than that of the Meißner type. Also, the peaks in the stable region of the mathieu type are smaller than those of the Meßiner type, as is clear from the comparisons of Fig. 4 with Fig. 11, and Fig. 7 with Fig. 14.

In the case of real gears, a pair of helical gears has a smaller ε than spur gears. Therefore, we can conclude that the tortional vibrations of the helical gears are smaller than those of the spur gears.

Figure 13 shows that the amplitudes of the vibrations decrease for all values of η when ε decreases. In general, when the helix angle increases, ε decreases. Therefore, a helical gear with a large helix angle has a smaller vibration than one with a small helix angle.

On the other hand, as shown in Figs. 7 and 14, when the damping factor ζ increases, only the peak levels decrease, while the amplitudes of the other regions do not decrease. Hence, we can say that in the gearing system, when the damping factor increases, for instance by means of adequate lubrication, only the peak levels become remarkably smaller.

When ε is equal to 0.25, as in Fig. 10 ($\zeta=0.05$), the unstable region exists near $\eta=0.5$, but in Fig. 11 ($\zeta=0.07$), the unstable region does not appear near $\eta=0.5$. This corresponds to that when $\zeta=0.05$, the vibration diverges at $\eta=0.5$. On the other hand, when $\zeta=0.07$, the amplitude of the vibration does not have a peak at $\eta=0.5$, as in Fig. 14. From this result, we can conclude that the vibrations of gears rotating with small damping have very large peaks at the rotating speed $\Omega=2\omega$, but if the damping is sufficient, the gears rotate smoothly.

4. Conclusion

Based on a theoretical investigation of the vibrations of a helical gear (Meißner type) and a spur gear (Mathieu type), the following conclusions are obtained.

(1) The vibration has peaks when $\eta=(\omega/\Omega)=n$ (n is an integer). The unstable region has troughs at $\eta=n+1/2$. When ζ increases, the locations of the troughs shift to the right (η is greater). The peaks and the troughs decrease when η increases.

(2) The peak levels of vibrations of the Mathieu type are smaller than those of the Meißner type. Therefore, we can find that the vibrations of helical gears are smaller than those of spur gears.

(3) When the amplitude of the variable part of stiffness ε decreases, the vibration in whole region and also the peak levels decrease. This explains why the vibration of a helical gear with a large helix angle is smaller than that of a

gear with a small helix angle.

(4) As the damping factor ζ increases, the unstable region decreases, and the peak levels of vibrations decrease, but the level at the other region does not decrease.

(5) The decreasing of the unstable region due to ζ is small at $\eta=0.5$. Therefore, when we drive gears at the high speed $\Omega=2\omega$, the gears with small damping have very large vibrations, but the gears with large damping rotate smoothly.