Digital Simulation of Hysteretic Loop by Preisach Diagram

By

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Abstract

This paper proposes a method for a digital simulation of hysteretic loops based on the use of the Preisach diagram. The hysteretic characteristic is expressed by an integral equation the kernel of which is discretely given. As examples, using the method proposed, we obtain the major loop, minor loop and so forth.

1. Introduction

The hysteretic characteristics of the magnetic materials can be studied by the Preisach diagram or the electrical circuit containing bimetal switches[1),2)]. In these methods, the hysteretic characteristics are expressed by an integral equation. The distribution function in the integral equation is given by a continuous function with two independent variables. However, in a practical problem, we obtain the approximate values of the distribution function by an experiment discretely. Therefore, it is significant for the hysteretic characteristics to be obtained by a discrete distribution function. In this paper, we develop a method for simulating the hysteretic characteristics by means of the Preisach diagram, and give an algorithm. By this algorithm, we can obtain the hysteretic characteristics, for examples, a major loop, a minor loop and so forth. We express the method by using the terminology of the magnetic materials.

2. Elementary dipole

We assume that the magnetic material consists of a large number (not infinite) of the magnetic dipoles. Each dipole is characterized by a rectangular magnetizing loop in the HJ plane, where H and J are the strength of the magnetic field and the intensity of the magnetization, respectively. As shown in Fig. 1, the dipole has the

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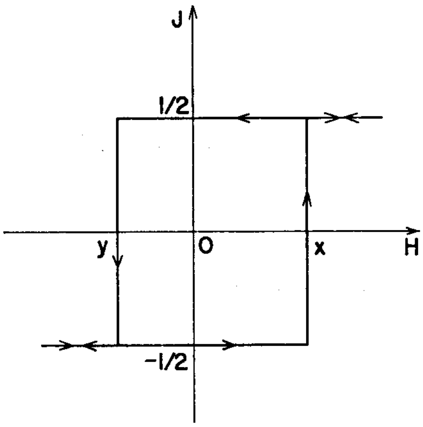


Fig. 1 Magnetizing characteristics of elementary dipole.

magnetic coercivities x and y corresponding to the ascending and descending sides, respectively. The remanent intensities of the magnetization are $\pm 1/2$. We call this magnetic dipole the 'elementary dipole' or simply the 'dipole'. When the strength of the magnetic field H is less than y, the dipole is called negatively magnetized. When the strength of the magnetic field H is greater than x, the dipole is called positively magnetized. Here, because we assume that the magnetic material is composed of a finite number of dipoles, the coercivities x and y are assumed to be distributed from $-H_s$ to $+H_s$ of the magnetic field, where H_s is the saturating strength of the magnetic field.

3. Distribution function

We define a distribution function $K(x, y)$ where the variables x and y are independent each other. Because we assume that the magnetic material is made up of a large number of dipoles, the number of dipoles in the intervals $[x, x+dx]$ and $[y, y+dy]$ is given by $K(x, y)\,dxdy$. This function $K(x, y)$ is referred to as the distribution function. Here, we also assume that the distribution function is independent of the external magnetic field applied to the magnetic material. This function represents the distribution of the density of the dipoles for various values of the magnetic field.

In a practical problem, we can evaluate the values of the distribution function approximately by an experiment[3]. Therefore, we can represent the values of the distribution function by the constant values on the small two-dimensional region $dxdy$.

Because we assume that there are no dipoles which tread reversely the loop shown in Fig. 1, we have $K(x, y)=0$ for the region $x<y$. For the magnetic saturation, we

assume $K(x, y)=0$ in the region $|x|>H_s$, $|y|>H_s$. Under these assumptions, the distribution function $K(x, y)$ is defined in the following three regions:

(a) $0\leq x\leq H_s$, $y\leq x$

(b) $0\leq x\leq H_s$, $-H_s\leq y\leq 0$,

(c) $-H_s\leq x\leq 0$, $y\leq x$.

4. Magnetization process

When the strength of the magnetic field is zero at the begining and when the magnetic material is magnetically neutral, there are as many dipoles in the state 1/2 (positively magnetized dipoles) as those in the state $-1/2$ (negatively magnetized dipoles). Therefore, we can determine the result of applying a given time sequence of the strength of the magnetic field to the magnetic material by the following rules:

(1) If an increasing magnetic field H is applied, all the dipoles in the state $-1/2$ switch to the state 1/2 when H becomes greater than x.

(2) If a decreasing magnetic field H is applied, all the dipoles in the state 1/2 switch to the state $-1/2$ when H becomes less than y.

Applying these rules, we first consider the intensity of magnetization J when we increase the strength of the magnetic field H. In this magnetizing process, all the dipoles are in the state 1/2 on the left side of the line $x=H$, and over it in the state $-1/2$. When the strength of the magnetic field H increases, this line shifts parallel to itself to the right. Therefore, we integrate the distribution function $K(x, y)$ over the region swept by the line. This value of integration means the incremental intensity of magnetization dJ corresponding to dH in the external field H. Therefore, the incremental intensity of magnetization dJ is given by an integration of the distribution function over the trapezoidal region, swept successively by the parallel lines to the y-axis. Therefore, the incremental intensity of the magnetization is given by

$$dJ=\int_{-H_s}^{H}\int_{H}^{H+dH}K(x, y)\,dxdy \tag{1}$$

and the intensity of magnetization J is given by

$$J=J_{min}+\int_{-H_s}^{H}\int_{-H_s}^{H}K(x, y)\,dxdy \tag{2}$$

where J_{min} is the minimal intensity of magnetization defined by

$$J_{min}\triangleq -\frac{1}{2}\int_{-H_s}^{H_s}\int_{-H_s}^{H_s}K(x, y)\,dxdy$$

In a similar way, when the strength of the magnetic field H decreases, all the dipoles are in the state $-1/2$ over the line $y=H$, and below it in the state 1/2. This line shifts parallel to itself to the lower side. The incremental intensity of the magnetization corresponding to the decrease $-dH$ in the external field H is given by

$$dJ=-\int_{H-dH}^{H}\int_{H}^{H_s}K(x,\ y)\,dxdy \tag{3}$$

and the intensity of the magnetization is given by

$$J=J_{max}-\int_{H}^{H_s}\int_{H}^{H_s}K(x,\ y)\,dxdy \tag{4}$$

where J_{max} is the maximal intensity of magnetization defined by

$$J_{max}\triangleq\frac{1}{2}\int_{-H_s}^{H_s}\int_{-H_s}^{H_s}K(x,\ y)\,dxdy.$$

5. Computation of intensity of magnetization J

We describe a method for computing the intensity of the magnetization J when the distribution function $K(x, y)$ is discretely given. Let us consider the finite region on the xy plane where $K(x, y)$ has already defined. Then, we partition the interval $I_x\triangleq[-H_s, H_s]$ and $I_y\triangleq[-H_s, H_s]$ on the x-axis and y-axis into M subintervals, respectively. We express these subintervals on the x-axis and y-axis respectively by

$$\left.\begin{aligned} &h_x(i)\triangleq[x_i,\ x_{i+1}] \\ &h_y(j)\triangleq[y_j,\ y_{j+1}] \quad i,\ j=0,\ 1,\ \cdots\cdots,\ M\text{-}1. \end{aligned}\right\} \tag{5}$$

where i is the integral position of node x_i counting along the x-axis and j the position along the y-axis. Therefore, we denote the two-dimensional subregion corresponding to these intervals as

$$h(i,\ j)\triangleq h_x(i)\otimes h_y(j),\quad i,\ j=0,\ 1,\cdots\cdots,\ M\text{-}1. \tag{6}$$

Here, we call the integers i and j the interval number on the x-axis and y-axis, respectively. On the subregion $h(i, j)$, we denote the distribution function as $K(i, j)$ which takes a constant value. Using these notations, we have the maximal and minimal intensity of magnetization J_{max} and J_{min}, respectively. These are given by

$$J_{max}=\frac{1}{2}\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}K(i,\ j)\,h(i,\ j) \tag{7}$$

$$J_{min}=-\frac{1}{2}\sum_{i=0}^{M-1}\sum_{j=0}^{M-1}K(i,\ j)\,h(i,\ j). \tag{8}$$

Let the strength of the external magnetic field be $H(t)$ dependent on time t. Then, we can obtain a discrete time sequence $\{H(t_p)\}$, in which $t_p\triangleq p\Delta t$ $(p=0,\ 1,\ \cdots\cdots)$. First, we deal with the case of the increasing strength of the magnetic field. For any increasing subsequence $\{H(t_p)\}$, we can obtain the intensity of manetization $\{J(t_p)\}$ in the following way. For each sampling point t_p, we search the subinterval

$h_x(i)$ to which the value of $H(t_p)$ belongs. Therefore, we can obtain the relation between p and i. We denote this relation as

$$i=i(p). \tag{9}$$

Therefore, the incremental intensity of magnetization $\triangle J$ is given by

$$\triangle J(p)=\sum_{j=0}^{i(p)} K(i(p), j)\, h(i(p), j). \tag{10}$$

In the same manner, we can obtain the incremental intensity of magnetization $\triangle J$ for any decreasing subsequence $\{H(t_p)\}$. For each value of $H(t_p)$, we search the sub-interval $h_y(j)$ to which the value of $H(t_p)$ belongs, and obtain the relation

$$j=j(p). \tag{11}$$

Therefore, the incremental intensity of the magnetization is given by

$$\triangle J(p)=-\sum_{i=j(p)+1}^{M-1} K(i, j(p)+1)\, h(i, j(p)+1). \tag{12}$$

Thus, for the increasing strength of magnetic field $H(t_p)$, the sequence of the intensity of magnetization $J(p)$ is given by

$$J(p)=J_{min}+\sum_{i=0}^{i(p)}\sum_{j=0}^{i} K(i, j)\, h(i, j). \tag{13}$$

and for the decreasing strength of magnetic field $H(t_p)$, $J(p)$ is given by

$$J(p)=J_{max}-\sum_{j=j(p)+1}^{M-1}\sum_{i=j}^{M-1} K(i, j)\, h(i, j). \tag{14}$$

6. Introduction of array D

In order to facilitate the programing technique and to infer the process of magnetization, we introduce a two-dimensional array D. The array D has the same dimension as the discrete distribution function $K(i, j)$, and its elements are zero or unity. The position of zero or unity varies with the time sequence $\{t_p\}$. When the sequence of the strength of the magnetic field is increasing, we search the interval number $i(p)$ corresponding to the value of $H(t_p)$, and put the column less than or equal to $i(p)$ of D to unity. When the sequence of the strength of the magnetic field is decreasing, we search the interval number $j(p)$ corresponding to the value of $H(t_p)$, and put the row more than $j(p)$ of D to zero. From the position of zero and unity for each time point t_p, we can infer the magnetizing process.

Hereafter, for simplicity, we denote $K(i, j)h(i, j)$ as $K(i, j)$. Using the array D, we can rewrite Eqs (13) and (14) as

$$J(p)=J_{min}+\sum_{i=0}^{i(p)}\sum_{j=0}^{i} K(i, j)\, D(i, j). \tag{15}$$

and

$$J(p) = J_{max} - \sum_{j=j(p)+1}^{M-1} \sum_{i=j}^{M-1} K(i, j) D(i, j). \tag{16}$$

By using the array D, we need not classify the various cases of the distribution of positively or negatively magnetized dipoles for each time point t_p. Consequently, we can easily compute the intensity of the magnetization.

7. Algorithm

From the above consideration, we have the following algorithm to draw the hysteretic loops for the given discrete distribution function.

S 0 : Give the distribution function $K(i, j)$ for $i, j=0, 1, \cdots\cdots, M-1$. Set the array $D(i, j) \leftarrow 0$ for $i, j=0, 1, \cdots\cdots, M-1$.

S 1 : Partition the interval $[-H_s, H_s]$ of the strength of the magnetic field into M subintervals, and assign numbers i and j to the subintervals $[x_i, x_{i+1}]$ and $[y_j, y_{j+1}]$ $(i, j=0, 1, \cdots\cdots, M-1)$.

S 2 : Compute the maximal and minimal intensity of magnetization J_{max} and J_{min} by Eqs. (7) and (8), respectively.

S 3 : Give the time sequence $H(t_p)$ for $p=0, 1, \cdots\cdots, N-1$.

S 4 : Set $p \leftarrow 0$.

S 5: Examine whether $H(t_p)$ is increasing or not. If $H(t_p)$ is increasing, go to S6. Otherwise, go to S8.

S 6 : Check the interval number $i(p)$ to which $H(t_p)$ belongs.

S 7 : Set $D(i, j) \leftarrow 1$ for $i, j=0, 1, \cdots\cdots, i(p)$ and go to S11.

S 8 : Check the interval number $j(p)$ to which $H(t_p)$ belongs.

S 9 : Set $D(i, j) \leftarrow 0$ for $i, j=j(p)+1, \cdots\cdots, M-1$.

S10 : Set $J(p) \leftarrow J_{max} - \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} K(i, j) D(i, j)$ and go to S12.

S11 : Set $J(p) \leftarrow J_{min} + \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} K(i, j) D(i, j)$.

S12 : If $p > N-1$, then set $p \leftarrow p+1$ and return to S5. Otherwise, go to the next step.

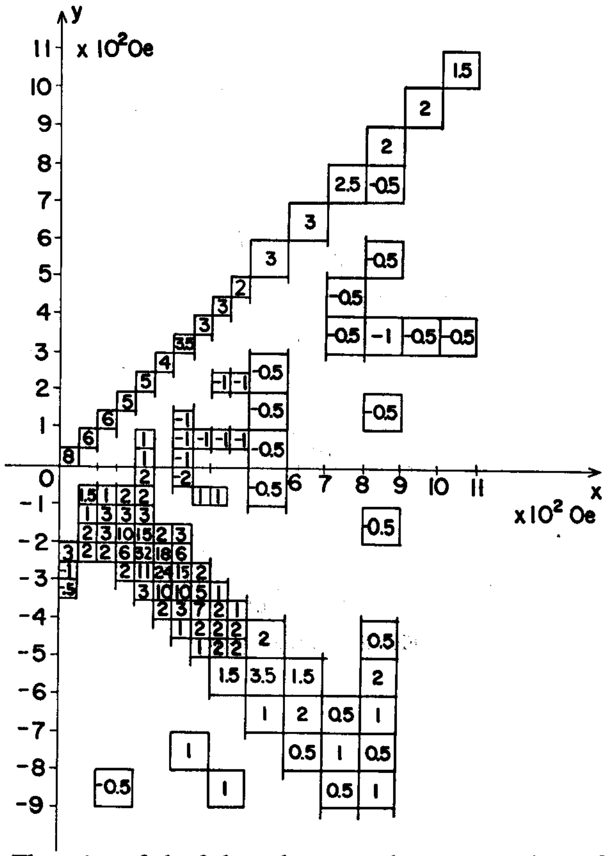
S13 : Draw the relation between $H(t_p)$ and $J(t_p)$ for $p=0, 1, \cdots\cdots, N-1$ on the HJ plane by some interpolation method.

S14 : Stop.

8. Some examples

Using the above algorithm, we draw the hysteretic loops on the HJ plane. We use the distribution function given in Table 1. Tanaka and Ohotsubo obtained this

Table 1. Distribution function



Note: The values of the 3rd quadrant are the same as those of the 1st and are given symmetrically with respect to the line $y=-x$.

table for the magnetic tape (Scotch-111-12) by the experiment[3]. We deal with the following three cases:

(a) $H(t)=1200\,cos(2\pi t)$, $N=5001$, step size $\Delta t=5.0\times10^{-5}$.

(b) $H(t)=700\,cos(100\,\pi t)+500\,cos(300\,\pi t)$, $N=401$, $\Delta t=5.0\times10^{-5}$.

(c) $H(t)=1310\,exp(-15.5\,t)\,cos(100\,\pi t)$, $N=1001$, $\Delta t=2.5\times10^{-4}$.

These correspond to the major loop, minor loop, and the perfect demagnetizing process, respectively. The results are shown in Fig. 2-a to 2-c.

9. Conclusion

We propose a method for the digital simulation of the hysteretic loops and give the algorithm, based on the use of Preisach diagram. When we compute the intensity of the magnetization for each time point, we find it convenient to introduce the array

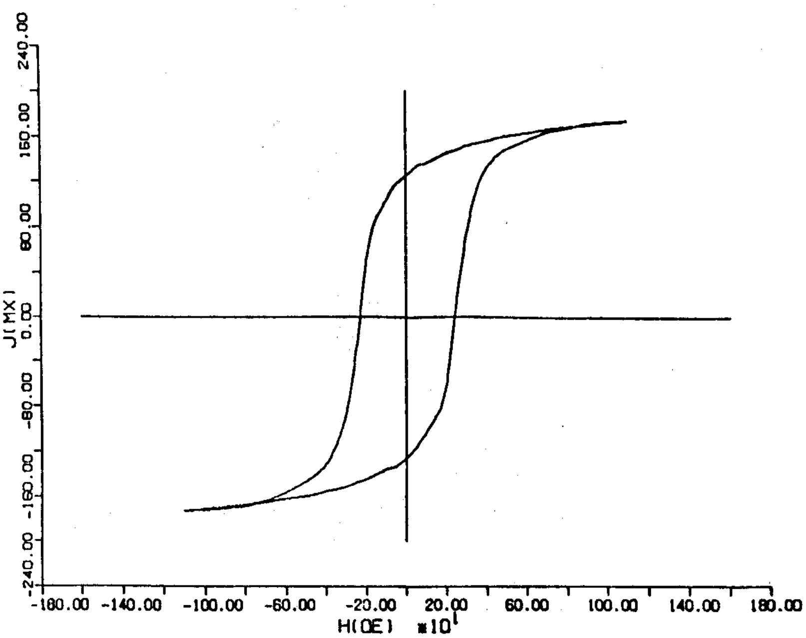


Fig. 2-a Major loop.

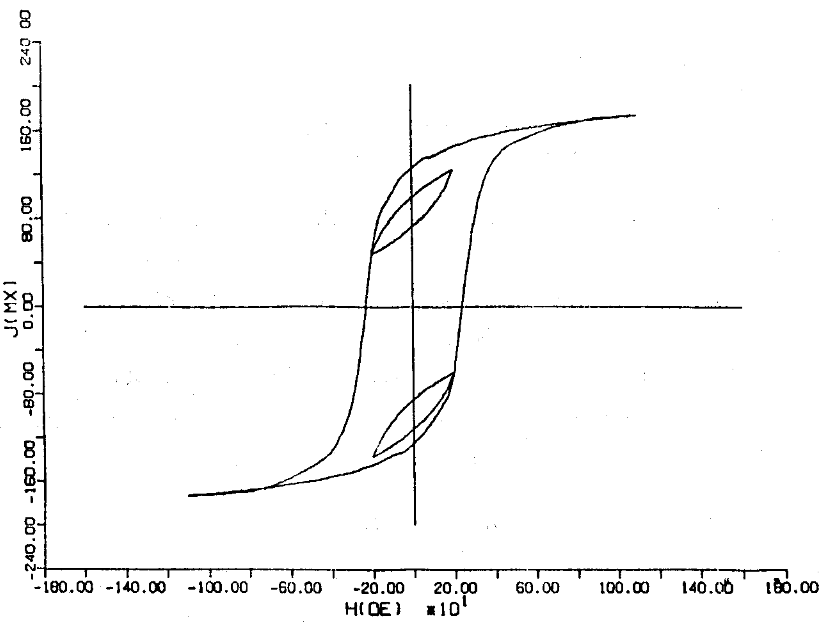


Fig. 2-b Minor loop.

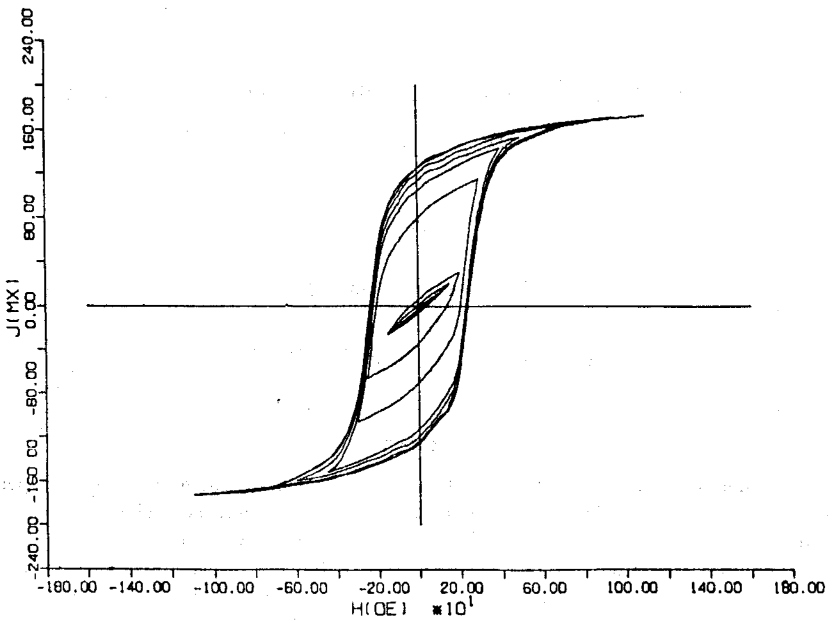


Fig. 2-c Perfect demagnetizing process.

D. By memorizing the array *D*, we can infer the state of the magnetizing process. The introduction of the array *D* facilitates the technique of the programing.

References

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