Vibration Damping of Beams and Plates by Mastic Deadner

By

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Abstract

Mastic deadner is widely used for the vibration reduction of thin plates and beams because of its easy treatment and low cost. In this paper, the damping capacity of the mastic deadner is investigated theoretically and experimentally. The vibrations of a beam and a plate covered with the mastic deadner under an external exciting force were analyzed by making use of an eigenfunction expansion method. Experiments with the beams and the plates were carried out, and the relation between the vibration amplitude and the thickness of the mastic deadner was investigated. From the theoretical and the experimental results, the following conclusions were obtained. The vibration amplitudes at the natural frequencies are remarkably decreased by the mastic deadner. The loss factor of the plate covered with the mastic deadner is proportional to the square of the ratio of the thickness of the mastic deadner to that of the basic beam and plate. The vibration amplitude at the natural frequency is inversely proportional to the square of the thickness ratio.

1. Introduction

Machines which radiate sound are in many cases covered with thin metal plates in order to reduce the noise. However, the plates become new noise sources due to their own resonance and radiation properties. In order to reduce the noise generated by the plate, mastic deadner is widely used because of its easy treatment and low cost. The vibration damping capacity of the mastic deadner has been investigated by many researchers[1-8]. Most of them discussed it from the viewpoint of the decay rate. However, it is desired to know how much the vibration is reduced by the mastic deadner, and the relation between the amplitude of the vibration and the thickness of the mastic deadner. In this study, the vibrations of beams and plates covered with the mastic deadner under an external force were analyzed theoretically by making use of an eigenfunction expansion method. Those vibrations were also investigated experimentally. The beams and plates were excited by an electromagnetic exciter with a frequency sweep, and their vibration accelerations were measured. On the basis of the theoretical and the experimental investigation, the

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vibration damping capacity of the mastic deadner was discussed in relation with the avibrtion amplitude.

2. Theoretical Analysis

2.1 Bending rigidities and loss factors of a layered beam and plate

In order to analyze the vibrations of a thin beam and a plate covered with a damping layer, their bending rigidities and damping loss factors must be determined. When the thickness of the plate is small enough, compared with the wave length of the flexural vibration, and the loss factor of the basic plate is negligible, the bending rigidity and the loss factor of the layered plate shown in Fig. 1 are given by[3]

$$D=\kappa D_1, \tag{1}$$

$$D_1=\frac{E_1h_1^3}{12(1-\nu^2)},$$

$$\kappa=\frac{1+2a(2\xi+3\xi^2+2\xi^3)+a^2\xi^4}{1+a\xi}$$

$$\eta=\frac{a}{1+a}\,\frac{3+6\xi+4\xi^2+2a\xi^3+a^2\xi^4}{1+2a(2\xi+3\xi^2+2\xi^3)+a^2\xi^4}\,\eta_2 \tag{2}$$

where $a=E_2/E_1$, $b=\rho_2/\rho_1$, $\xi=h_2/h_1$,
E: Young's modulus, ρ: density, h: thickness and η; loss factor The suffixes 1 and 2 denote the plate and the damping layer, respectively.

In the case of the beam, the bending rigidity is

$$B=\kappa B_1, \tag{3}$$

where $B_1=\dfrac{E_1b_1h_1^3}{12}$ and b_1: width of the beam.

The complex bending rigidities of the layered plate and beam are given by

$$D^*=D(1+i\eta), \tag{4}$$

$$B^*=B(1+i\eta). \tag{5}$$

2.2 Vibration of a layered beam

The lateral vibration of a beam with the internal damping under an external exciting force will be analyzed in this section. The differential equation of the

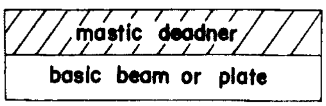

Fig. 1. Layered plate.

deflection curve becomes

$$B^*\frac{\partial^4 w}{\partial x^4}+A\rho\frac{\partial^2 w}{\partial t^2}=P, \tag{6}$$

where w: deflection, A: cross sectional area, and P: external force. When the external force is harmonic and acts on a point (x_0), P is given by Dirac's delta function as

$$P=P_0e^{i\omega t}\delta(x-x_0). \tag{7}$$

The deflection w is assumed to be given in the form of a series of the eigenfunctions w_m's corresponding to the normal modes of vibration as

$$w=\sum_m C_m w_m e^{i\omega t}. \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (6), we obtain

$$B^*\frac{d^4(\sum C_m w_m)}{dx^4}-A\rho\omega^2\sum C_m w_m=p_0\delta(x-x_0). \tag{9}$$

The eigenfunction satisfies the following homogeneous differential equation

$$B\frac{dw_m{}^4}{dx^4}-A\rho\omega_m{}^2w_m=0, \tag{10}$$

where ω_m is the natural frequency.
Substitution of Eq. (10) into Eq. (9) gives

$$\sum C_m\omega_m{}^{*2}A\rho w_m-A\rho\omega^2\sum C_m w_m=P_0\delta(x-x_0), \tag{11}$$

where $\omega_m{}^{*2}=\omega_m{}^2(1+i\eta)$.
The eigenfunctions satisfy the following orthogonal relation.

$$\int_0^l w_m w_n dx=0 \qquad \text{for } m\neq n. \tag{12}$$

where l: length of the beam. Application of Eqs. (11) and (12) gives

$$A\rho C_m(\omega_m{}^{*2}-\omega_m{}^2)\int w_m{}^2dx=P_0w_m(x_0),$$

$$C_m=\frac{P_0w_m(x_0)}{A\rho(\omega_m{}^{*2}-\omega^2)\int w_m{}^2dx}e^{i\omega t}. \tag{13}$$

Then, the deflection is obtained by substituting Eq. (13) into Eq. (8) as

$$w(x)=\sum_m\frac{P_0w_m(x)w_m(x_0)}{A\rho(\omega_m{}^{*2}-\omega^2)\int w_m{}^2dx}e^{i\omega t}. \tag{14}$$

In this study, a cantilever has been considered as an example because of its easy treatment in the experiment. It is well known that the eigenfunctions of the cantilever are given by

$$w_m=\cosh(k_m x)-\cos(k_m x)-\alpha_m\{\sinh(k_m x)-\sin(k_m x)\}, \tag{15}$$

$$\alpha_m=\frac{\cosh(lk_m)+\cos(lk_m)}{\sinh(lk_m)+\sin(lk_m)}.$$

where k_m is determined from the boundary condition as

$$\cos(lk_m)\cosh(lk_m)=-1.$$

The natural frequency is given by

$$\omega_m=k_m^2\sqrt{B/A\rho}. \tag{16}$$

The integral part of Eq. (14) can be easily calculated[4]. When the right end ($x=l$) of the beam is free,

$$\int_0^l w_m^2 dx=\frac{l}{4}(w_m^2)_{x=l}. \tag{17}$$

2.3 Vibration of a layered plate

The equation of the motion of the thin plate with the internal damping under an external force becomes

$$D^*\nabla^4 w+\rho h\frac{\partial^2 w}{\partial t^2}=P \tag{18}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ and w: deflection.

When the external force P acts on a point (x_0, y_0), P is given by Dirac's delta function as

$$P=P_0\delta(x-x_0)\delta(y-y_0)e^{i\omega t}. \tag{19}$$

The deflection w is given in the form of a series of the eigenfunctions as

$$w=\sum_m\sum_n C_{mn}w_{mn}e^{i\omega t}. \tag{20}$$

The eigenfunction w_{mn} satisfies

$$D\nabla^4 w_{mn}-\omega_{mn}^2\rho h w_{mn}=0, \tag{21}$$

where ω_{mn} is the natural frequency.

The eigenfunctions also satisfy the orthogonal relation as

$$\int\int w_{mn}w_{ij}dxdy=0 \qquad \text{for } m,n \neq i,j. \tag{22}$$

Substituting Eqs. (19) to (22) into Eq. (18), the deflection w is obtained as

$$w(x, y)=\sum_m\sum_n \frac{P_0 w_{mn}(x, y)\, w_{mn}(x_0, y_0)}{\rho h(\omega_{mn}^{*2}-\omega^2)\int\int w_{mn}^2 dxdy} e^{i\omega t}, \tag{23}$$

where $\omega_{mn}^{*2}=\omega_{mn}^2(1+i\eta)$.

In the case of a rectangular plate $(a_1\times b_1)$ with simply supported edges, the eigenfunction and the natural frequency are given by

$$w_{mn}=\sin\frac{m\pi x}{a_1}\sin\frac{n\pi y}{b_1}, \tag{24}$$

$$\omega_{mn}=\pi^2\sqrt{\frac{D}{\rho h}}\left(\frac{m^2}{a_1^2}+\frac{n^2}{b_1^2}\right). \tag{25}$$

3. Results of Experiments and Numerical calculations

We carried out the experiments with cantilevers and rectangular plates. The experimental system is shown in Fig. 2. In this study, the mastic deadner "Sadanok FK-100" was used. It was composed of asphalt, synthetic resin, rubber and asbest. The cantilevers have a length $l=308$ mm, a width $b_1=19$ mm and thicknesses $h_1=3$ mm and 4.5 mm. The rectangular plates have dimensions $a_1=405$ mm, $b_1=308$ mm and $h_1=0.8$ mm, 1.2 mm, and 1.6 mm.

In the experiment of the plate, the boundary was clamped, though it was simply supported in the theoretical analysis. The purpose of this study is to investigate the damping capacity of the mastic deadner, and the boundary condition does not have much influence on the damping characteristics. Then, the plate was clamped for easy treatment.

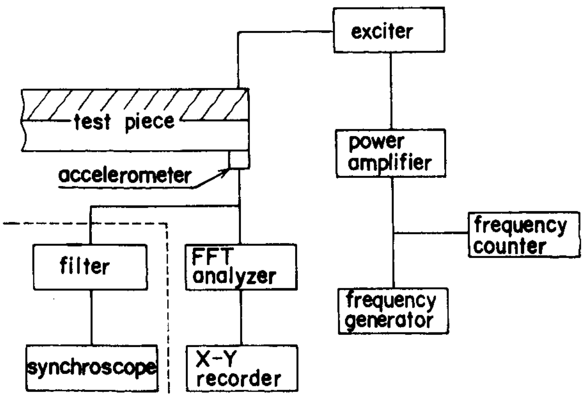

Fig. 2. Measuring system.

Impulse shocks were given to the beams and the plates, and the vibration acceleration was measured by an accelerometer. The loss factor at each natural frequency was obtained by making use of a frequency filter. Figure 3 shows the relation between the loss factor η and the thickness ratio β. From this figure, the relation can be denoted by

$$\eta=3.4\times10^{-3}\beta^2. \tag{26}$$

This figure is composed from the data of the lowest modes. The loss factors of the higher modes were slightly larger than those of the lowest modes. However, since this difference was negligible, the loss factor in the numerical calculation was determined from Eq. (26).

Young's modulus of the mastic deadner was determined by the comparison of the natural frequencies obtained from the experimental and the theoretical calculation as

$$E_2=20\ \text{kgf/mm}^2. \tag{27}$$

Figure 4 shows the vibration acceleration of the cantilever ($h_1=3$ mm), where a harmonic force acts on the free end of the beam. The accelerometer is also located at the free end of the beam. Figure 5 shows the calculated results.

The comparison of these two figures shows that the experimental results have smaller peaks than the calculated ones. This is because of the sound radiation loss and the damping at the clamped edges. However, the tendencies of the vibration reduction due to the mastic deadner are in good agreement. The vibrations at natural frequencies decrease remarkably with an increase in the thickness of the mastic deadner. However, the vibrations at the non-natural frequencies do not

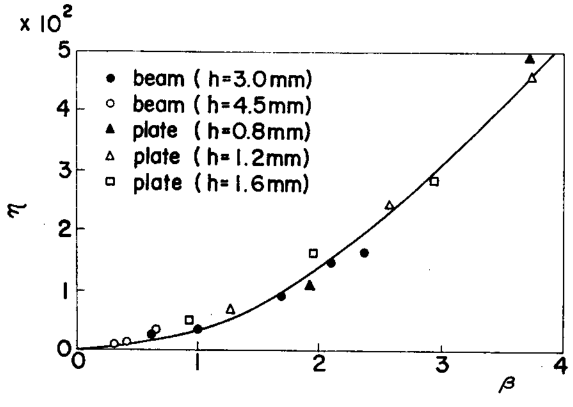

Fig. 3. Relation between the loss factor and the thickness ratio.

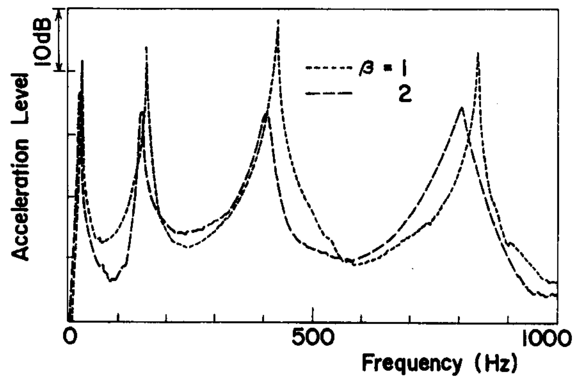

Fig. 4. Experimental results of the vibration acceleration of the layered cantilever (h_1=3 mm).

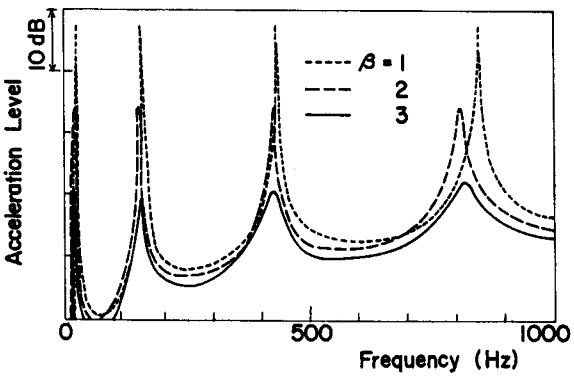

Fig. 5. Theoretical results of the vibration acceleration of the layered cantilever (h_1=3 mm).

becrease. Since the vibration amplitudes at the natural frequencies are larger than those at the non-natural frequencies by more than 30 dB, as shown in Figs. 4 and 5, we can conclude that the mastic deadner decreases the vibration amplitude very effectively. The experiments with other cantilevers had results similar to those mentioned above.

The experiments with the plates were carried out. A typical result with a plate (h_1=1.6 mm) is shown in Fig. 6. The calculated results are shown in Fig. 7. The experimental results which show the decrease in vibration amplitude as a result of the mastic deadner is smaller than those of the theoretical results. This is attributed to the sound radiation loss[4].

Finally, we will discuss the approximate relation between the amplitude of the vibration and the thicknesses of the beam and the plate. In the case of the beam,

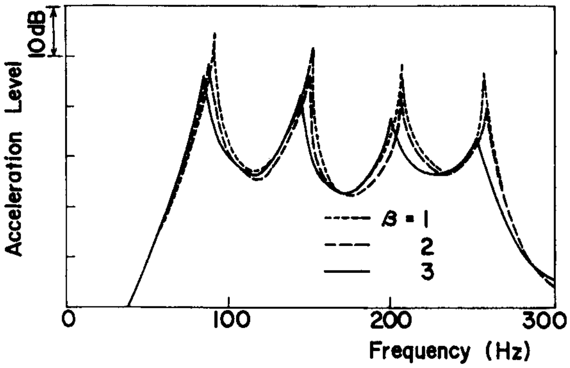

Fig. 6. Experimental results of the vibration acceleration of the layered plate ($h_1=1.6$ mm).

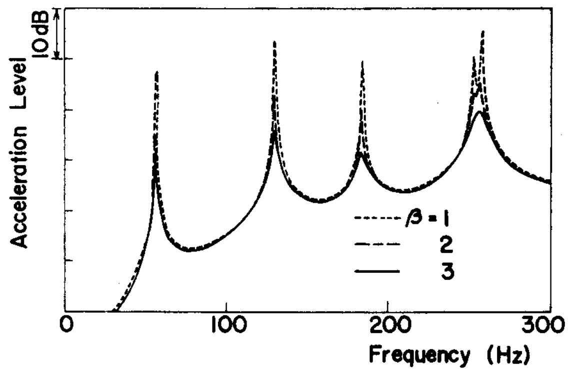

Fig. 7. Theoretical results of the vibration acceleration of the layered plate ($h_1=1.6$ mm).

when $\eta \ll 1$ and the time term of the external force is $\exp(i\omega_m t)$, Eq. (14) becomes

$$w(x) \fallingdotseq \frac{P_0 w_m(x) w_m(x_0)}{A\rho(\omega_m{}^{*2}-\omega_m{}^2)\int w_m{}^2 dx} e^{i\omega_m t}. \tag{28}$$

The complex natural frequency is given by

$$\omega_m{}^{*2}=\omega_m{}^2(1+i\eta). \tag{29}$$

Substitution of Eq. (29) into Eq. (28) gives

$$w(x) \fallingdotseq \frac{P_0 w_m(x) w_m(x_0)}{A\rho\omega_m{}^2 i\eta \int w_m{}^2 dx} e^{i\omega_m t}. \tag{31}$$

Then, the following approximate relation between the deflection and the loss factor is given as

$$w(x) \propto 1/\eta \,. \tag{31}$$

Thus, the amplitude of the vibration is inversely proportional to the loss factor. The loss factor is proportional to the square of the thickness ratio. Therefore, we can say that the amplitude of the vibration is inversely proportional to the square of the thickness of the mastic deadner as

$$w(x) \propto 1/b_2^2 \,. \tag{32}$$

For example, the amplitude of a beam with $\beta=1$ is 1/4 of that with $\beta=2$. This decreasing level is 12 dB. This agrees well with the experimental and the theoretically calculated results shown in Figs. 3 and 4.

In the case of the plate, we can have the same theoretical discussion that the vibration amplitude is inversely proportional to the square of the thickness of the mastic deadner. However, the practical vibration is not reduced so much, because the sound radiation loss is considerably large compared with the internal loss.

4. Conclusions

The damping capacity of the mastic deadner depends on many factors, such as temperature, composing materials, condition of drying and others. However, the relation between the vibration amplitude and the thickness of the mastic deadner is the most important for practical usage. With respect to this, theoretical and experimental investigations were carried out, and the following general tendencies were obtained.

1) The vibrations of the beam and the plate are dominated by the vibrations of the normal modes. The vibrations at the natural frequencies are remarkably decreased by the mastic deadner, though the vibrations at the non-natural frequencies are not.
2) The loss factors of the beam and plate covered with the mastic deadner are proportional to the square of the thickness ratio of the mastic deadner and the basic beam and plate.
3) In the case of the beam, the vibrational amplitudes at the natural frequencies are inversely proportional to the square of the thickness of the mastic deadner. That is to say, the vibration amplitude is reduced to one-fourth by doubling the thickness of the mastic deadner. However, the vibration of the plate is not reduced so much, because of the sound radiation loss.

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