Calculation Method of the Optimum Configurations of the Extended Surfaces with Simultaneous Conductive, Convective and Radiative Heat Transfer in Heat Exchanger

By

Sadayuki TANAKA* and Takeshi KUNITOMO**

(Received October 15, 1982)

Abstract

A simultaneous conductive, convective and radiative heat transfer in the extended surface systems of longitudinally finned cylinders and circumferentially finned cylinders is treated analytically, by taking the radiative interaction into consideration. A procedure to determine the optimum configurations of extended surfaces is examined for the various kinds of combinations of six heat transfer parameters from the viewpoint of saving fin material. It is found that if the radiative interaction is not taken into consideration, wrong results are included. Instead of using the complex optimization algorithm, a graphical expression is adopted to show the numerical coefficients and indices in the formulas which give the optimum dimensionless fin height and the minimum fin volume.

1. Introduction

A simultaneous conductive, convective and radiative heat transfer in extended surface systems has been examined in earlier studies[1)~8)], taking the radiative interaction into consideration, but the design procedure was not treated in those studies. Only the convective heat transfer[4),5)], or only the radiative heat transfer[6)] was considered in some cases. Also a simultaneous convective and radiative heat transfer was treated from the viewpoint of material saving in two studies by Wilkins[7)] and Tanaka-Kunitomo[8)]. Wilkins did not consider the radiative interaction, but Tanaka-Kunitomo did consider this radiative interaction.

In the present study, the optimum configurations for material saving are investigated concerning the extended surface systems shown in Fig. 1, and a graphical method is proposed for the determination of the configuration.

* Fukui Technical College, Fukui
** Department of Engineering Science

Nomenclatures

B_i : Biot number hr_i/k
h : convective heat transfer coefficient
H, H^* : fin height, dimensionless fin height H/r_i
H^*_{opt} : dimensionless optimum fin height
k : thermal conductivity of fin material
N : total fin number of longitudinally finned cylinder
Q^* : dimensionless total heat flux
Q_0^* : dimensionless total heat flux without fin, $\varepsilon(1-T_c^{*4})+\beta(1-T_g^*)$
r_i : radius of fin base
r_0 : radius of fin tip
R_n : radiation number $\sigma T_b^3 r_i/k$
R_0 : r_0/r_i
s, s^* : fin spacing, dimensionless fin spacing s/r_i
t_i, t_i^* : a half of fin root thickness, a half of dimensionless fin root thickness t_i/r_i
t_0, t_0^* : a half of fin tip thickness, a half of dimensionless fin tip thickness t_0/r_i
T_b : absolute temperature of base surface
T_c, T_c^* : blackbody temperature of surroundings, dimensionless blackbody temperature of surroundings T_c/T_b
T_g, T_g^* : ambient gas temperature, dimensionless ambient gas temperature T_g/T_b
V^* : dimensionless fin volume, eq. (1) and eq. (2)
V^*_{min} : dimensionless minimum fin volume
β : convection-radiation parameter $B_i/R_n=h/\sigma T_b^3$
ε : total hemispherical emissivity
η : system effectiveness Q^*/Q_0^*
σ : Stefan Boltzmann constant

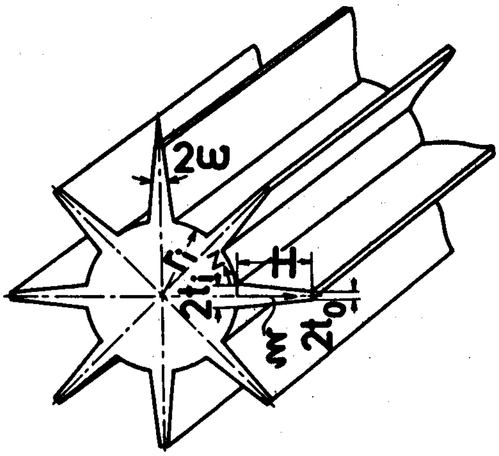
2. Heat Transfer and Configuration Parameters

When the total heat flux and other heat transfer conditions are given, the optimization process is to choose a specific combination of configuration parameters, which gives the minimum fin volume from many possible combinations which can meet the same total heat flux. The dimensionless fin volumes of trapezoidal profile are expressed as follows for the systems shown in Fig. 1.

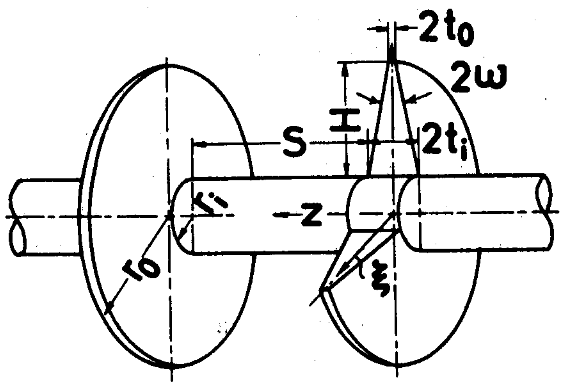
Longitudinally finned cylinder:

$$V^*=(1+t_0^*/t_i^*)t_i^*H^*N/\pi \qquad (1)$$

Circumferentially finned cylinder:



(a) longitudinally finned cylinder



(b) circumferentially finned cylinder

Fig. 1. Schematic diagram of extended heat transfer surface

$$V^*=\frac{3}{2}\{H^{*2}+3H^*+(2H^{*2}+3H^*)t_0^*/t_i^*\}/(s^*+2t_i^*) \qquad (2)$$

The configuration parameters which must be determined are the fin spacing (or the total fin number in the case of a longitudinally finned cylinder), the fin height, the fin root thickness and the fin root-tip thickness ratio. Among these parameters, the fin spacing (or the total fin number) should be determined by taking the practical production conditions into consideration and, as to the fin thickness ratio, the optimum value is surely zero as described in a previous paper[8].

Accordingly, the purpose of the present study is to express the optimum fin height and optimum fin root thickness by the functions of six heat transfer parameters, i.e. total heat flux, Biot number, emissivity, convection-radiation parameter (radiation number), ambient gas temperature and blackbody temperature of surroundings. Since the present problem includes many parameters and has nonlinear characteristics in the relation between the objective function (i.e. minimum volume) and the parameters, it is very difficult to treat these many parameters in the lamp. Therefore, the combination of the ambient gas temperature and the blackbody temperature of the surroundings is treated independently and is given in advance of the calculation. Namely, for the given combinations of these temperatures, the optimum configurations are determined by numerical experiments, and the results are expressed by the empirical equations using the remaining four heat transfer parameters. After that, the effects of the ambient temperatures upon the coefficients and the indices in these equations are investigated. In a previous paper[8], the relation between the total heat flux and the optimum configuration (that is, the optimum fin height and the minimum fin volume) was primarily considered, and the detailed results were obtained by a numerical experiment. Hence, in the present study, the fundamental relations between the total heat flux and the optimum configuration obtained in the earlier study are, firstly, expressed by simple empirical equations. Secondly, the effects of the other parameters are introduced as corrections upon the coefficients and indices in the equations.

In the following, the procedure to obtain the final relations are explained only for the circumferentially finned cylinder. A similar procedure is adopted also for the longitudinally finned cylinder.

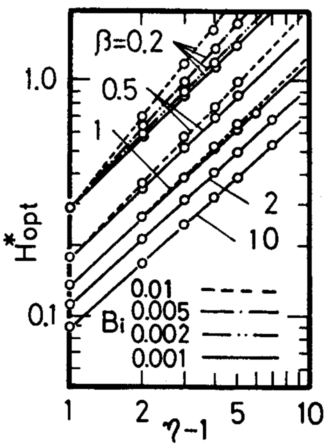
The relations between the total heat flux and the optimum configurations, i.e. H^*_{opt} and V^*_{min}/B_i, are shown in Figs. 2(a) and 2(b), respectively, by taking the system effectiveness as the abscissa instead of the total heat flux. In Fig. 2(b), the minimum fin volume divided by the Biot number is adopted as the ordinate. When we adopt the logarithmic scale for both the ordinate and abscissa, simple linear relations are obtained for the given values of β. In the region of large β, where the convective heat transfer is predominant, the magnitude of B_i scarcely affects the relation. However, in the region of small β, where the radiative heat transfer is predominant, the effect of B_i is found. Yet, the values of V^*_{min}/B_i for $\eta-1=1$ change little. The straight lines for H^*_{opt} and V^*_{min}/B_i shown in Fig. 2 are expressed by the following equations:

$$H^*_{opt}=A(\eta-1)^B \tag{3}$$

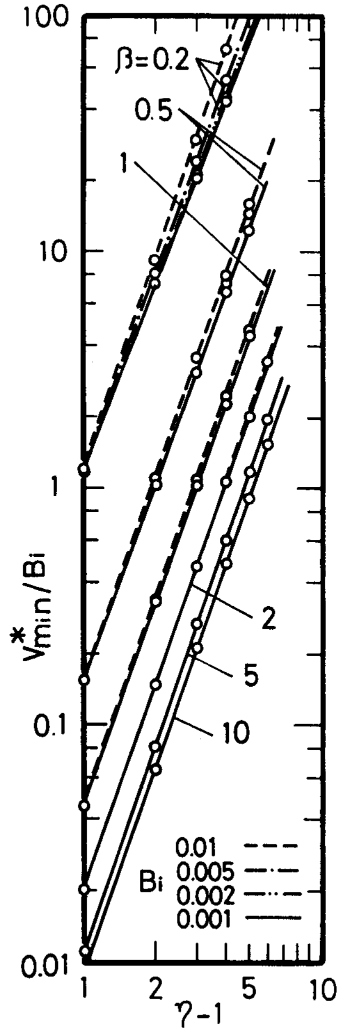
and

$$V^*_{min}/B_i=C(\eta-1)^D. \tag{4}$$

The coefficients (A and C) and the indices (B and D) in Eqs. (3) and (4) are determined



(a) Dimensionless optimum fin height



(b) Dimensionless minimum fin volume/Biot number

Fig. 2. The relation between the optimum configuration and the system effectiveness for the specific condition of $s^*=0.1$, $\varepsilon=0.5$ and $T_g^*=T_c^*=0.4$.

as the dimensionless values normalized by the values of A_0, C_0, B_0 and D_0 which are determined for the specific condition that only the convective heat transfer exists. These dimensionless values are expressed as the functions of ε, B_i and β. The values of A_0, B_0, C_0 and D_0 are independent of B_i and the ambient temperatures, and are affected only by the fin spacing as shown in Fig. 3. In the case of pure convection, the fin spacing and the fin height are mutually related under the restriction of the

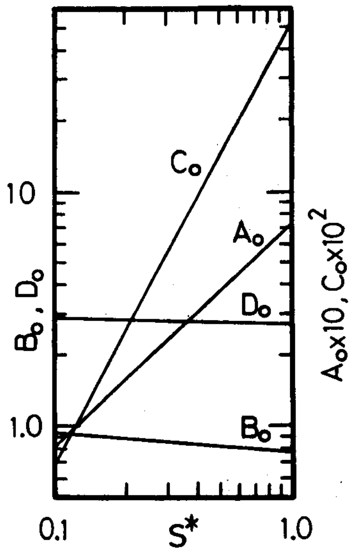


Fig. 3. The coefficients and indices in Eqs. (3) and (4) for the condition of pure convective heat transfer

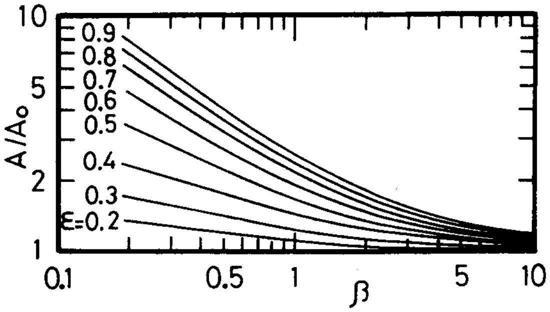


Fig. 4. The relation between A/A_0 and β for the specific condition of $s^*=0.1$, $B_i=0.001$ and $T_g^*=T_c^*=0.4$.

minimum fin volume. Therefore, the coefficients, A_0 and C_0, are affected strongly by the fin spacing. The gradients, B_0 and D_0, however, are scarcely affected by the fin spacing, since the increase of the total heat flux is roughly covered by the linear increase of the fin height. The relations between $A_0 \sim D_0$ and the dimensionless fin spacing s^* are given by the following equations:

$$A_0=0.73s^{*0.95}, \quad B_0=0.77s^{*-0.086}, \quad C_0=0.54s^{*1.89}, \quad D_0=2.7s^{*-0.03} \qquad (5)$$

The normalized values of $A/A_0 \sim D/D_0$ for $s^*=0.1$ in the case of the circumferentially finned cylinder can be expressed by the functions of emissivity ε, the Biot number B_i and the convection-radiation parameter β, as shown in the following. A similar treatment can be adopted for the other values of fin spacing s^* and for the case of the longitudinally finned cylinder. The relation between A/A_0 and β is shown in Fig. 4 for the specific case of $s^*=0.1$, $B_i=0.001$ and $T_g^*=T_c^*=0.4$. The linear relations are obtained in the region of the predominant radiation (i.e. small β) through the whole region of emissivity. As the value of β increases, the values of A/A_0 approach unity asymptotically since the convection becomes predominant. The large values of A/A_0 in the region of small β and for the large emissivities correspond to the fact that the radiative interaction occurs strongly. The curves shown in Fig. 4 are expressed by the following equations:

$$A/A_0 = f_1(\varepsilon)\beta^{f_2(\varepsilon)} + 1 - \exp[f_3(\varepsilon, \beta)] \tag{6}$$

where f_1 and f_2 are functions of ε, and f_3 is a function of ε and β. They are determined as follows;

$$f_1 = 1 + 1.7\varepsilon^{1.6}, \quad f_2 = -0.75\,\{1 - \exp(-6\varepsilon^{2.5})\}$$
$$f_3 = -0.315\,\{1 - \exp(-5\varepsilon^{2.5})\}\,\beta^{0.65} \tag{7}$$

As for the value of B/B_0, the effects of emissivity ε and the convection-radiation parameter β are examined firstly by a numerical experiment for the standard value of the Biot number (B_i=0.001). Secondly, the results obtained for the other Biot numbers are normalized by the results of the standard case. The results for the relation between B and B_i in the specific case of s^*=0.1, ε=0.5 and $T_g^* = T_c^*$=0.4 are shown in Fig. 5 for the several values of β. The values of B increase linearly with an increase of B_i, and decrease largely with an increase of β. This result indicates that, to meet the increase of the total heat flux, a further increase rate of the heat transfer surface is needed to compensate for the increase of radiative interaction when the constant heat transfer coefficient and the predominant radiative heat transfer are prescribed. Furthermore, it indicates that when fin material of lower thermal conductivity is used, a further increase of surface is also needed to meet the

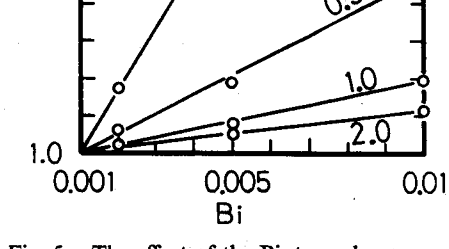


Fig. 5. The effect of the Biot number upon the index B in Eq. (1) for the specific condition of s^*=0.1, ε=0.5 and $T_g^* = T_c^*$=0.4.

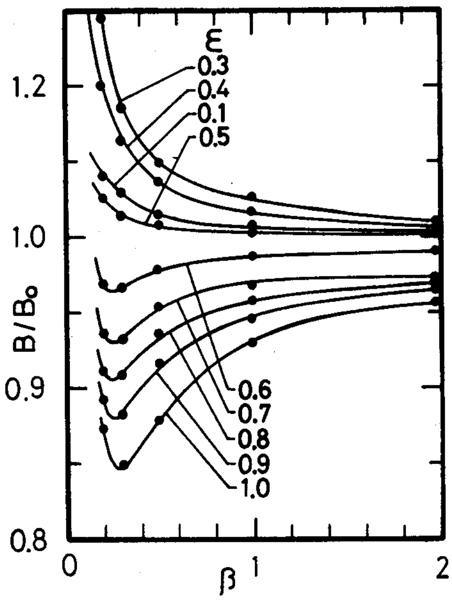


Fig. 6. The relation between B/B_0 and β for the specific condition of s^*=0.1, B_i=0.001 and $T_g^* = T_c^*$=0.4.

increase of the total heat flux.

The effect of emissivity upon the relation between B and β is shown in Fig. 6 for the specific case of $s^*=0.1$, $B_t=0.001$ and $T_g^*=T_c^*=0.4$. The effect of emissivity becomes weak when the convection-radiation parameter β becomes large, that is, when the convective heat transfer becomes predominant. In the region of small β, the increase of the fin height is not effective at the higher emissivity, due to the radiative interaction. In this case, the increase of the total heat flux should be met by the increase of the fin root thickness. The results obtained above are described by the following empirical equation:

$$B/B_0=\{1+f_4\{(B_t/0.001)-1\}\}\ \{(f_5/\beta)+\exp(f_6/\beta)\}, \tag{8}$$

where f_4 is a function of ε and β, and f_5 and f_6 are functions of ε. These functions are shown by the following:

$$f_4=0.014\varepsilon^{1.2}/\beta, \quad f_5=0.61\varepsilon(0.45-\varepsilon), \quad f_6=0.178\varepsilon^{2.2} \tag{9}$$

All of the above results are obtained for the specific condition of the ambient temperatures T_g^* and T_c^*. Lastly, the effect of the ambient temperatures must be introduced into the above equations. The final results obtained by the numerical experiment for the circumferentially finned cylinder are given by the following empirical equations (10)~(13). The coefficients and indices in the above equations (6)~(9), which are affected by the ambient temperatures, are newly expressed by the nomenclatures A_1—D_3. Since a similar procedure was adopted for obtaining C/C_0 and D/D_0, a detailed explanation about them is omitted and only the final results are given.

$$A/A_0=(1+A_1\varepsilon^{A_2})\ \beta^{-A_3\{1-\exp(-6\varepsilon^{2.5})\}}+1-\exp[-A_4\{1-\exp(-5\varepsilon^{2.5})\}\ \beta^{0.65}] \tag{10}$$

$$B/B_0=[1+(B_1\varepsilon^{1.2}/\beta)\ \{(B_t/0.001)-1\}]\ [\{0.61\varepsilon(B_2-\varepsilon)/\beta\}+\exp(B_3\varepsilon^{2.2}/\beta)] \tag{11}$$

$$C/C_0=(1+C_1\varepsilon^{C_2})\beta^{-C_3\varepsilon^{C_4}}+1/\{(-C_5\varepsilon^{0.55}/\beta^{0.5})+\exp(C_6\varepsilon^{0.45}/\beta)\} \tag{12}$$

$$D/D_0=[1+(D_1\varepsilon/\beta)\ \{(B_t/0.001)-1\}]\ [\{0.272\varepsilon^2(\varepsilon-D_2)/\beta\}+\exp(D_3\varepsilon/\beta)] \tag{13}$$

In the case of the longitudinally finned cylinder, the following equations are obtained.

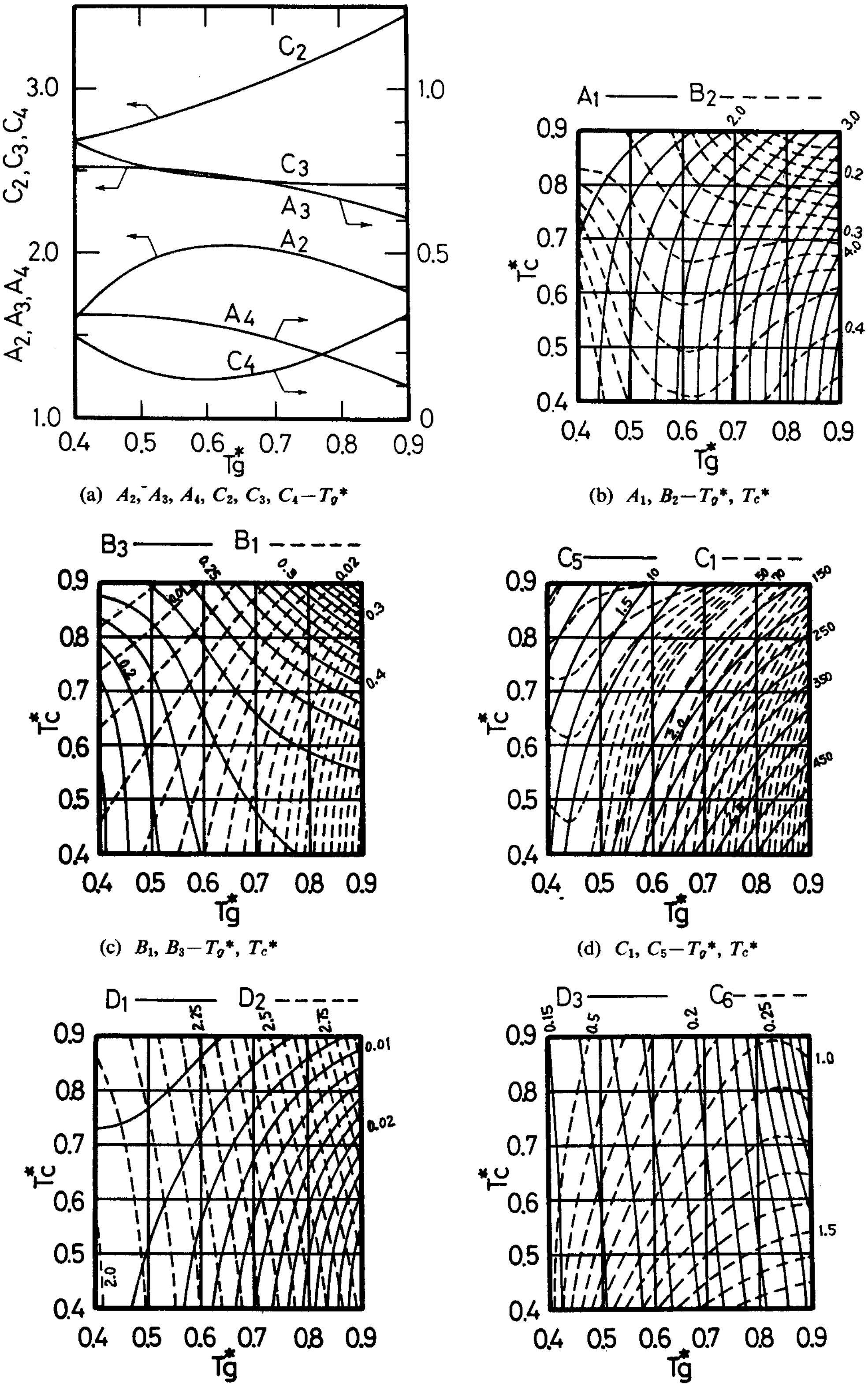
$$A/A_0=(1+A_1\varepsilon^{A_2})\beta^{-A_3\{(-\exp(-4\varepsilon^{2.5})\}}+1-\exp[-A_4\{1-\exp(-4\varepsilon^{3.5})\}\ \beta^{0.75}] \tag{14}$$

$$B/B_0=[1+(B_1\varepsilon^{1.2}/\beta)\ \{(B_t/0.001)-1\}]\ [\{0.28\varepsilon(B_2-\varepsilon)/\beta\}+\exp(B_3\varepsilon^{1.5}/\beta)] \tag{15}$$

$$C/C_0=(1+C_1\varepsilon^{C_2})\beta^{-C_3\varepsilon^{C_4}}+1/\{(-C_5\varepsilon^{0.45}/\beta^{0.5})+\exp(C_6\varepsilon^{0.4}/\beta)\} \tag{16}$$

$$D/D_0=[1+(D_1\varepsilon/\beta)\ \{(B_t/0.001)-1\}]\ [\{D_2\varepsilon^2(\varepsilon-D_3)/\beta\}+\exp(D_4\varepsilon/\beta)] \tag{17}$$

All the coefficients and indices in Eqs. (10)~(17) are shown in Fig. 7 and Fig. 8, for



(a) A_2, A_3, A_4, C_2, C_3, $C_4-T_g{}^*$

(b) A_1, $B_2-T_g{}^*$, $T_c{}^*$

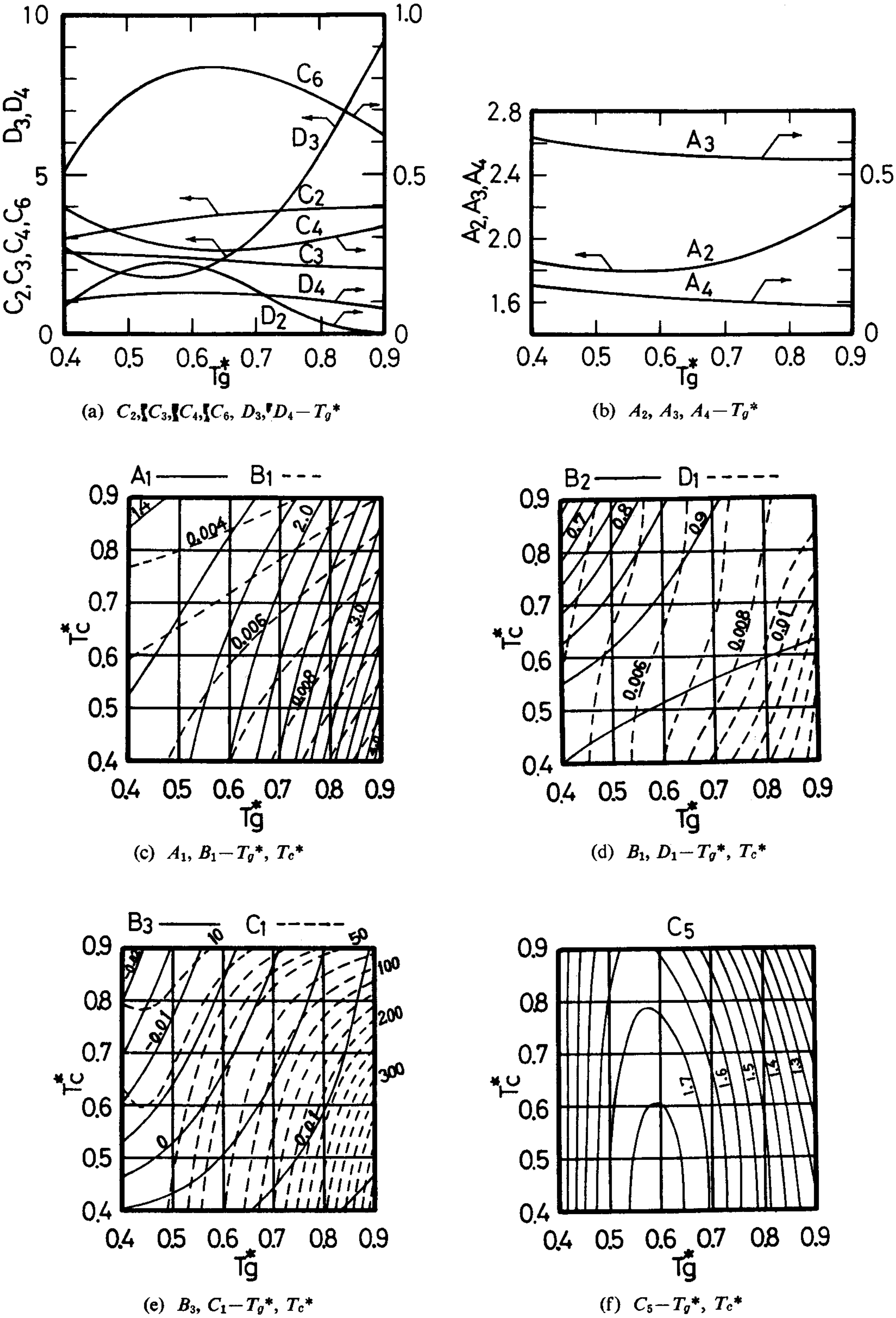
(c) B_1, $B_3-T_g{}^*$, $T_c{}^*$

(d) C_1, $C_5-T_g{}^*$, $T_c{}^*$

(e) D_1, $D_2-T_g{}^*$, $T_c{}^*$

(f) D_3, $C_6-T_g{}^*$, $T_c{}^*$

Fig. 7. The coefficients and indices $A_1 \sim D_3$ in Eqs. (10)$\sim$(13) in the case of circumferentially finned cylinder ($s^*=0.1$)



(a) C_2, C_3, C_4, C_6, D_3, $D_4-T_g{}^*$

(b) A_2, A_3, $A_4-T_g{}^*$

(c) A_1, $B_1-T_g{}^*$, $T_c{}^*$

(d) B_1, $D_1-T_g{}^*$, $T_c{}^*$

(e) B_3, $C_1-T_g{}^*$, $T_c{}^*$

(f) $C_5-T_g{}^*$, $T_c{}^*$

Fig. 8. The coefficients and indices $A_1 \sim D_4$ in Eqs. (14)~(17) in the case of longitudinally finned cylinder ($N=36$)

the circumferentially finned cylinder and for the longitudinally finned cylinder, respectively.

3. An Example of Calculation for Optimum Configuration

The calculation procedure determining the optimum configuration for the circumferentially finned cylinder is exemplified by the following, in order to show how to apply the above equations and graphs to the practical design.

The conditions given in advance for determining the fin system are as follows: $r_t=30$ mm, $T_b=500$ °C, $T_g=250$ °C, $T_c=200$ °C, $k=232.6$ W/m·K, $h=29.1$ W/m²·K, $\varepsilon=0.8$ and $Q_t=10$ KW/(1 m cylinder).

The dimensionless quantities required for optimization are calculated as follows: $B_t=0.00375$, $R_n=0.00338$, $\beta=B_t/R_n=1.1$, $Q_0^*=1.004$, $Q^*=2.62$, $\eta=2.51$, $T_g^*=0.68$ and $T_c^*=0.61$. In advance, the fin spacing is set to be $s^*=0.1$ ($s=3$ mm).

The coefficients and indices in Eqs. (10)~(13) are determined graphically by using Fig. 7 as follows: $A_1=2.82$, $A_2=2.04$, $A_3=0.71$, $A_4=0.24$, $B_1=0.0215$, $B_2=0.34$, $B_3=0.225$, $C_1=113$, $C_2=3.04$, $C_3=2.44$, $C_4=0.14$, $C_5=2.07$, $C_6=1.16$, $D_1=0.0105$, $D_2=2.29$, $D_3=0.193$.

Then the values of $A/A_0 \sim D/D_0$ can be calculated. These are $A/A_0=2.824$, $B/B_0=0.968$, $C/C_0=47.83$ and $D/D_0=0.934$. Since the values of $A_0 \sim D_0$ are obtained from Fig. 3 or Eq. (5), the values of $A \sim D$ can be determined as $A=0.0825\times 2.824=0.233$, $B=0.936\times 0.968=0.906$, $C=0.0069\times 47.83=0.33$ and $D=2.892\times 0.934=2.7$.

By introducing the above values and $\eta=2.51$ into Eqs. (1) and (2), the dimensionless optimum fin height H^*_{opt} and the dimensionless minimum fin volume V^*_{min}/B_t are given as 0.339 and 1.004, respectively. Finally, the optimum fin height is determined as $H_{opt}=0.339\times 30=10.2$ mm. From the value of the minimum fin volume, V^*_{min} ($=1.004\times 0.00375=0.00377$), the dimensionless optimum fin root thickness is given as $t_t{}^*{}_{opt}=0.005$. Hence, the optimum fin root thickness must be 0.15 mm ($=0.005\times 30$).

For the same heat transfer conditions and the ambient conditions, the optimum fin configuration was obtained by using the complex optimization algorithm developed in the earlier paper[10]. The results obtained were $H_{opt}=10.8$ mm and $t_{t\,opt}=0.141$ mm. Therefore, it is confirmed that the proposed graphical method can easily give the approximate values of the design parameters to satisfy the request of practical design. The present method is very simple and does not require any cost. However, the earlier optimization algorithm requires a complex operation plus a high cost of using the electronic computer on a large scale.

4. Conclusions

A procedure to determine the optimum configurations for the minimum volume of the extended surface with a simultaneous conductive, convective and radiative heat transfer was investigated for the various kinds of combinations of heat transfer parameters. It was made clear that if the radiative interaction was not taken into consideration, wrong results would be induced. A graphical expression was adopted to show the coefficients and indices in the formulas which give the dimensionless optimum fin height and the dimensionless minimum fin volume. When the radius of the cylinder, the three kinds of temperatures of fin base, ambient gas and surroundings, the thermal conductivity of fin material, the convective heat transfer coefficient, the emissivity and the total heat flux are given, the optimum configuration of the fin system can be easily determined by the present method.

Acknowledgement

A part of this research was supported by the scholarship offered by the Santo Kohgyosho Co. Ltd., Osaka. The calculation was carried out in the Computer Center at Kyoto University.

References

1) Donovan, R.C. and Rohler, W.M.; Trans. ASME, Ser. C, **93**, 41 (1971)
2) Tanaka, S. and Kunitomo, T.; Transactions of JSME, **43**, 2240 (1977)
3) Kunitomo, T. and Tanaka, S.; Heat and Mass Transfer Source Book, McGraw Hill, p. 276 (1977)
4) Kraus, A.D. and Kern, D.Q.; Extended Surface Heat Transfer, McGraw Hill (1972)
5) Mikk, I.; Int. J. Heat Mass Transf., **23**, 707 (1980)
6) Wilkins Jr, J.E.; J. Aeros., Sci., 145 (1960–2)
7) Wilkins Jr, J.E.; J. Franklin Inst., **297–1**, 1 (1974)
8) Tanaka, S. and Kunitomo, T.; Transactions of JSME, **48**, 2599 (1982–12)