Modified Budyko Model of Global Climate and its Response to Cloudiness Variation

By

Shin YABUSHITA

(Received June 24, 1983)

Abstract

Budyko's global climate model where the meridional heat transfer of the form $\beta(T-T_p)$ is shown to be equivalent to North's model, where the heat transfer is by diffusion. The coeffecient of diffusion is specified so as to be consistent with Budyko's empirical data. Here, T and T_p stand for the local and planetary temperatures, respectively.

The global climatic model so obtained is used to calculate the response of T_p for a for a variation of the solar constant, Q. It has been found that $\Delta T_p = 0.56°C$ for a one peacent increase in Q, which is far less than Budyko's and Schneider's results. Again, the albedo change, owing to a cloudiness variation obtained by Henderson-Sellers, is used to calculate the planetary temperature in terms of the cloudiness, n for a fixed Q. Under the present climatic condition, $\Delta T_p = -0.8°C$ for a one percent increase of cloudiness.

1. Introduction

Recently, much attention has been paid to global climatic models which permit analytical or semi-analytical treatment. Among them, the best known is the one due to Budyko[1], which gives a relationship between the variation in the solar constant, Q, and the latitude for the edge of the ice cap. The Budyko model is based on the empirical relation that the meridional transfer of heat is proportional to the difference between the local temperature and the average temperature of the Earth (planetary temperature, T_p). On the other hand, North[2] approximated the meridional heat transfer by a diffusion. Both Budyko and North calculated the temperature distribution by balancing the heat input due to solar radiation with the (infra-red) out-going radiation and meridional heat transfer, and obtained the latitude of the ice-cap as a function of the solar constant. The relation between the solar constant and the latitude of the ice-cap boundary is known as the ice feedback mechanism. The diuffsion approximation was later adopted by Suarez and Held[3] to calculate the climatic response due to the orbital variation of the Earth.

It has been found by the authors cited above that the overall character of the ice cap edge latitude-solar constant relations calculated by the Budyko model and by the diffusion approximations-are similar. These authors assumed that the cloudiness factor, n, remains constant even if the solar constant is allowed to vary.

The object of the present paper is twofold. We first show that Budyko's empirical formula for the meridional heat transfer is equivalent to a diffusion approximation, and the coefficient of diffusion can be obtained from Budyko's empirical constant.

Second, we will investigate the effect of variation in cloudiness on the global climate using the heat diffusion model. When the result is incorporated with a relation between the solar constant and cloudiness, it is possible to obtain a relation between the solar constant and the planetary temperature. An implication of the result wil be briefly discussed in Section 5.

2. Empirical relationship

Budyko's global climatic model[1] is such that the meridional heat transport per unit length per unit time is given by $\beta(T-T_p)$, where T and T_p are the local and average temperature, respectively, and β is an empirical constant. He obtained the relation by estimating the transport of heat by oceanic and atmospheric circulations. For the present earth, $T_p=14°C$.

On the other hand, the diffusion term of heat the transfer, $\nabla^2 T$, can be calculated from the temperature distribution at present. To do so, we adopt the result given by Budyko (Fig. 3. 11, Ref. 4), which gives the surface temperature averaged over longitude as a function of the latitude. It is easy to read temperatures from the figures, and the results are shown in Table 1.

Table 1
Earth surface temperature averaged over longitude.

θ (latitude)	T (temperature)	$\delta^2 T\cdot(\Delta\theta)^2$
0°	29°C	
15	25	−3.10
30	17.5	−1.68
45	7.5	2.25
60	−3.5	5.92
75	−12	10.31
90	−15.5	

Now, if θ' is the co-latitude, the meridional part of $\nabla^2 T$, apart from the radius factor, is given by $\partial^2 T/\partial\theta'^2+\cot\theta'\cdot\partial T/\partial\theta'$. Therefore, we have

$$\nabla^2 T=\frac{\partial^2 T}{\partial\theta^2}-\tan\theta\frac{\partial T}{\partial\theta}.$$

If $\Delta\theta$ denotes the grid interval, this may be approximated by the central difference, namely

$$\delta^2 T=\left\{\frac{T(\theta+\Delta\theta)+T(\theta-\Delta\theta)-2T(\theta)}{(\Delta\theta)^2}-\tan\theta\cdot\frac{T(\theta+\Delta\theta)-T(\theta-\Delta\theta)}{2\Delta\theta}\right\}$$

From the temperatures given in Table 1, it is straight forward to calculate $\delta^2 T$, which is also given in Table 1. In Fig. 1, the empirical value of $\delta^2 T$ is plotted against T.

It may be seen that $\delta^2 T$ is almost a linear function of T, and that $\delta^2 T$ vanishes near $T=14°C$, which is extremely close to the present mean temperature of the Earth. Therefore one may write

$$\delta^2 T = a + bT \tag{2.1}$$

and estimate the unknown constants by the least square method. The normal equations so obtained are :

$$\begin{aligned} &5a + 34.5b = 13.7 \\ &34.5a + 1143.75b = -234.465 \end{aligned} \tag{2.2}$$

so that the constnts a and b are given by

$$a = 5.246, \quad b = -0.363$$

Thus, equation (2.1) can be written as

$$\delta^2 T = -0.363(T - 14.4)(\Delta\theta)^{-2}$$

where T is measured in degrees Celsius. In our case, $\Delta\theta = 0.2618$, so that

$$\delta^2 T = -5.29(T - 14.4).$$

On the other hand, the empirical heat transfer obtained by Budyko is $3.69(T - T_p)$

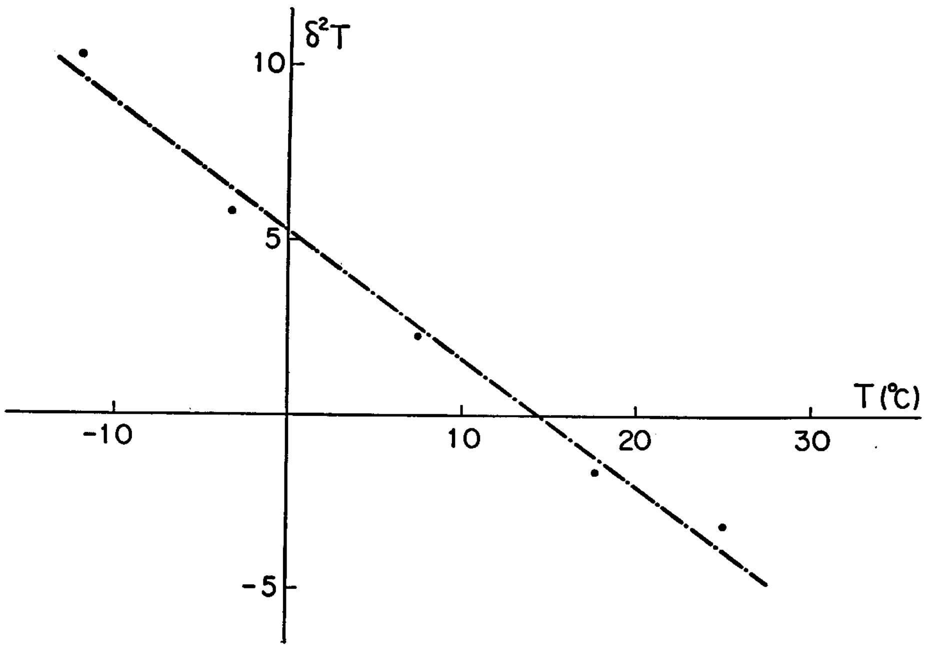

Fig. 1. Relation between the zonally averaged temperature and the second difference of the temperature. The temperature distribution is taken from Budyko (Ref. 4). The straight line is the best linear fit for T and $\delta^2 T$.

Watt $m^{-2}C^{\circ -1}$. On equating the two, one finds that the diffusive transfer coefficient is given by

$$D=0.697$$

in the same units.

Now in the model discussed by North, the diffusion coefficient was adjusted so that the calculated model might agree with the present global climate. The numerical value of D/B obtained by North, where B is an empirical quantiiy in Budyko's empirical formula for infra-red out going radiation, is 0.310. Adopting the same value for B, our estimate of D gives $D/B=0.48$. This difference in the numerical value of D gives rise to a very large difference in the Q-x_s relationship, where x_s stands for the since of the latitude for the edge of the polar ice cap. This problem will be discussed in the next section.

3. $Q-x_s$ relationship

Budyko analyzed the amount of outgoing radiation in the infra-red and obtained the following empirical formula :

$$\mathrm{I}=a-a_1n+(b-b_1n)T=A+BT \tag{3.1}$$

where n is the cloudiness and T the temperature in degrees Celsius. Budyko and other workers assumed that n remains constant at 0.5. Then,

$$A=201.4\ Wm^{-2},\quad B=1.45\ Wm^{-2}\,{}^{\circ}K^{-1}.$$

Again, the albedo of the earth is such that

$$\alpha(x)=\begin{cases} 1-0.38 & x>x_s \\ 1-0.68 & x<x_s \end{cases} \tag{3.2}$$

where x is the since of the latitude. North found that the incident solar radiation is closely represented by $QS(x)$ where

$$S(x)=1+S_2P_2(x),\quad S_2=-0.482$$

and where $P_2(x)$ is a Legendre polynomial. He then equated the incoming solar radiation to the out going radiation $I(x)$ and to the heat transfer, obtaining the relationship between Q and x_s, namely

$$Q=I_s(P_\nu f'_{1\nu}-f_{1\nu}P'_\nu)\,[I_{p0}P_\nu f'_\nu-I_{p1}P'_\nu f_{1\nu}+(I'_{p1}-I'_{p0})f_{1\nu}P_\nu]^{-1} \tag{3.3}$$

where

$$P_\nu=P_\nu(x),\quad \nu(\nu+1)=\frac{D}{B}$$

$$f_{1\nu}=F\left(-\frac{\nu}{2},\ \frac{1}{2}+\frac{1}{2}\nu,\ \frac{1}{2},\ x^2\right)^*$$

and where I_s is the value of I at the ice boundary (which corresponds to the temperature, $T=-10°C$). I_{p0} and I_{p1} are particular solutions of the basic equation of the heat balance for $x>x_s$ and $x<x_s$, respectively. All functions are evaluated at $x=x_s$, which corresponds to the ice boundary.

The $Q-x_s$ curve is given in Fig. 2. It can be seen that with $D/B=0.48$ and $S_2=-0.482$, the present solar constant ($Q=334.5\ Wm^{-2}$) corresponds to $x_s=0.39$ ($\theta=23°$), which is far different from the present edge of the polar ice cap ($x_s=0.95$, $\theta=72°$). North adjusted D/B so that the present global condition might be obtained. The numerical value of D/B so obtained is 0.31.

It seems obvious that the diffusion coefficient should not be adjusted so that the present climate may be represented. The coefficient is a measurable quantity so that the actually measured value should be adopted. On the Budyko model, or its modification by North, the other parameter which may be varied is S_2. This parameter gives the variation of the solar heating with the latitude. In fact, North noted that with $S_2=-1$, which is realized at equinox conditions, a global climate which is more stable than the $S_2=-0.482$ climate can be obtained. Therefore, instead of adjusting the D/B value, we vary S_2 so that the present climate may be obtained.

In order to obtain the planetary temperature T_p, consistent with the present global value (14.4°C), it is necessary to modify the numerical value of S_2. By trial and error, it has been found that $S_2=-0.785$ gives the present value of T_p with the

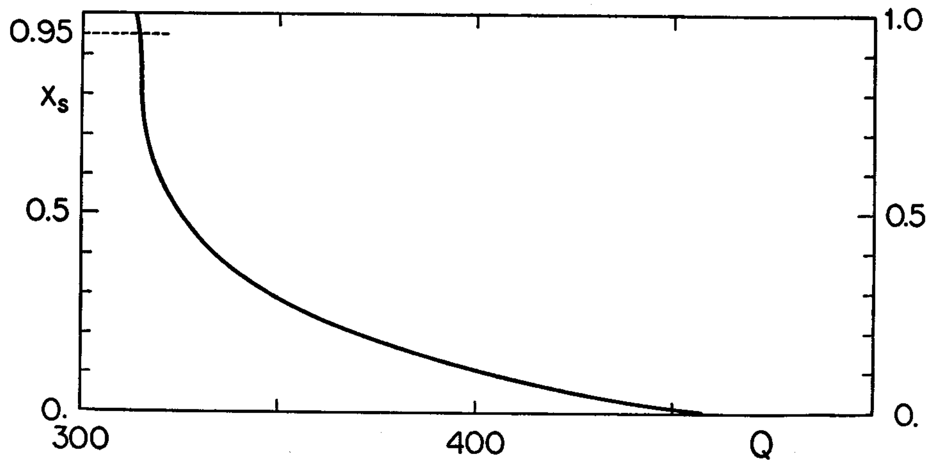

Fig. 2. The relation between the so ar constant and the edge of the polar ice cap ($x=\sin\theta$) based on the diffusive global model with $D/B=0.48$ and $S_2=-0.482$.

* The function $F(a, b, c, z)$ denotes the hypergeometric function.

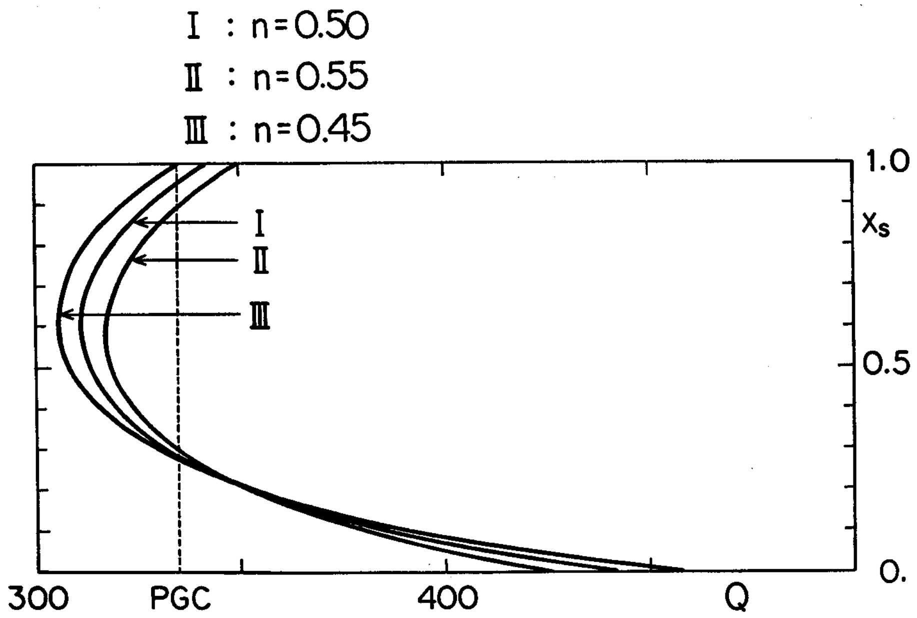

Fig. 3. Global climatic models such that $D/B=0.48$ and $S_2=-0.785$. This value of S_2 gives the present planetary temperature. The $Q-x_s$ relation is plotted for the various cloudiness, n. PGC stands for the present global climate ($Q=334.5\ Wm^{-2}$).

adopted value of $D/B=0.48$. The $Q-x_s$ curve for this value of S_2 is plotted in Fig. 3.

We note that the minimum possible value of the solar constant is 7.3% less than the present value. On the other hand in the North and Budyko models, the minimum solar constant is only 1.6% less than the present value. The result suggests that the $Q-x_s$ relationship is highly sensitive to the diffusion coefficient, and a global climatic change might be brought about by changes in atmospheric and oceanic circulations, such that the diffusion coefficient is thereby increased.

4. Effect of cloudiness

In the present section, we propose to investigate the effect of cloudiness on the global climate, using the model presented in the previous section. The effect of cloudiness has been investigated by many authors[5,6], but they have all neglected the ice-feed back process. In this section, we vary the cloudiness factor, n, independent of the solar constant, Q, and calculate the global climate. There should be a physical relationship between the two. However, apart from the work of Paltridge[7], such a relationship has not been discussed. A possible relationship between the solar constant and the global climate via a cloudiness variation will be discussed in the next section.

A change in cloudiness has two effects. First the albedo of the earth will be altered, and second, the outgoing radiation is modified.

Henderson-Sellers[6] calculated the albedo under varying cloudiness. Her results are given below.

albedo (%)	cloud cover (%)
24.6	40
30.1	50 present day
35.7	60

These values correspond to the earth uncovered by ice. On the other hand, Budyko's value of albedo is 0.32. So we adopt the albedo-cloudiness relation of the form :

$$\text{albedo}=0.32+0.55(n-0.5)$$

which is readily obtainable from the simple consideration that

$$\text{albedo}=A_c\cdot n+A_s(1-\alpha)$$

were $A_c(=0.5)$ and A_s are the average reffectivities of the cloud and the surface respectively. For outgoing radiation, we adopt the empirical formula obtained by Budyko.

The $Q-x_s$ relation for coudiness different from the present value ($n=0.5$) is shown in Fig. 3. The general trend of the curve remains unaltered. One can easily note that an increase in cloudiness, while keepng Q constant, results in lowering the value of x_s (the position of the polar ice).

Table 2
Planetary temperature and cloudiness factor calculated by the ice-feedback model.

n	T_p
0.46	17.9°C
0.48	16.4
0.5	14.8
0.52	13.1
0.54	11.3

The effect of cloudiness may be seen from a different point of view. Keepng the solar constant unaltered, it is possible to obtain a relation between n and T_p. This is shown in Table 2.

From Table, 2, we may infer that the rate of change of T_p with cloudiness is given by

$$\left.\frac{\partial T_p}{\partial n}\right|_Q=-0.8°\text{C} \quad \text{for } 1\% \text{ change in } n.$$

This is the value of $\partial T/\partial n$ through the ice-feed back mechanism.

We may compare the above value with the result obtained by Schneider[11]. By discussing energy balance in the atmosphere but neglecting the ice-feed back mechanism, he obtained $\partial T_p/\partial n=-0.25° K$ for a 1 % change in Q. The ice-feed back will enhance any climatic variation, so that it is resonable that the numerical value for $\partial T_p/\partial n$, obtained in the present paper, is greater than Schneider's value.

5. Discussions and Conclusions

We have obtained a numerical value of the diffusion coefficientfrom the empirical rate of heat transfer obtained by Budyko. We also constructed a global climatic model by adjusting the numerical value of S_2 which determines the latitude dependence of solar radiation. It has been found that the modfied cilmatic model is more stable than the models of North and of Budyko. By making use of the albedo under a cloudiness different from the present, the planetary temperature has been calculated, while keeping Q constant. The lowering of the temperature with cloudiness is consistent with the work of Henderson-Sellers.

It may be argued that the solar constant should be related to the cloudiness factor, n. That may be true, bot there is no satisfactory theory. However, Paldridge[7] argued that if the equation which describes the energy balance of the atmosphere could be split into two, a relation could be found which connects n to Q. Using this process, he was able to calculate the present average climatic parameters, such as temperature, cloudiness, latent heat and sensible heat, which are close to the observed values. Therefore, there may be some validity in his treatment. He found that

$$\frac{dn}{dQ}=0.009 \qquad \text{in } n \text{ for a } 1\% \text{ increase in } Q,$$

$$\frac{dT}{dQ}=0.35°K \qquad \text{for } a \text{ a } 1\% \text{ increase in } Q.$$

These calculations do not include the effect of an ice-feed back mechanism, but include the effect of a cloudiness change induced by the solar constant variation.

On the other hand, our calculation presented in section 3 shows

$$\frac{dT_p}{dQ}=0.56°K \qquad \text{for a } 1\% \text{ change in } Q.$$

The net effect which incorporates the cloudiness variation should be smaller, so that it would be closer to the value calculated by Paltridge.

It is commonly accepted[8,9,10] that ice ages during the Quarternaly Period are due to small fluctuations in the solar constant, which in turn is due to small perturbations of the Earth's orbit. During glaciations, the planetary temperature drops by a few degrees. Variations in the solar constant are $2 \sim 3\ \%$. Then, the drop in temperature would be a degree or so. If the result of the present section is reliable at all, it will be difficult to ascribe the ice ages to the small variations of the solar constant, and other mechanisms can not be ruled out.

Finally, we wish to remark that if a value of B different from $1.45\ Wm^{-2}\,°C^{-1}$ were adopted, a somewhat greater value for dT/dQ might be obtained. For example, Budyko (Ref. 4, p. 94) gives $B=1.67$ from the data obtained by satellite observations. For this value of B a greater value for dT/dQ can be calculated. If this were the case, the temperature drops during ice ages may just be due to the variations in the solar constant.

References

1) M. I. Budyko ; Tellus, **21**, 611 (1969).
2) G. R. North ; J. atmosph. Sci., **32**, 1301 (1975).
3) M. J. Suarez & I. M. Held ; Nature, **263**, 46 (1976).
4) M. I. Budyko ; The Earth's Climate, Academic Press (1982).
5) R. I. Temkin & F. M. Snell ; J. atmosph. Sci., **33**, 1671 (1976).
6) A. Henderson-Sellers ; Nature, **279**, 786 (1979).
7) G. W. Palddridge ; J. atmosph. Sci., **31**, 1571 (1974).
8) M. Milankovitch ; Handbuch der Geophysik, **9**, 593 (1938).
9) B. J. Mason ; Quart. J. R. meter. Soc., **102**, 473 (1976).
10) D. Pollard ; Nature, **272**, 233(1978).
11) S. H. Schneider ; J. atmosph. Sci. **29**, 1413 (1972).