A Contribution to Constitutive Relation of Cohesive Soil Based on Elasto-Plasticity Theory

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Abstract

Through a series of drained shear tests for a low plasticity clay, normally consolidated either isotropically or anisotropically, it is found that there exists a large dependency of distorsional strain on the stress path, and that this gives an influence on the shear behavior of cohesive soils. A set of constitutive relations is proposed, based on the situation of adopting the non-associated flow rule and the double functions for the consolidation and shear processes, separately. The analytical result using the proposed equations can satisfactorily explain the deformation and strength behaviors of soil.

1. Introduction

It is well known that normally consolidated clay which has been formed under unequal vertical and horizontal pressures indicates shear characteristics different from isotropically consolidated clay. The explanation of such behaviors is attributed to the anisotropic structure of soil formed by anisotropic consolidation. The present paper considers this problem in terms of stress and strain during the shearing of an anisotropically consolidated clay. Especially, in order to clarify that the large dependency of distorsional strain on the stress path influences the shear behavior of cohesive soils, the difference of distorsional strain due to the stress ratio is shown for samples consolidated isotropically and anisotropically, respectively.

2. State Boundary Surface and Drained Stress Path

The state boundary surface (SBS) defined by Roscoe *et al.*[1]) is the existence boundary of soil-like material based on the experimental fact that the combination of pore amount with stress can exist only within a finite boundary. They

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have shown a state boundary surface in the pore-stress space which was constructed by the mean effective principal stress, $p=(\sigma_1'+\sigma_2'+\sigma_3')/3$, the principal stress difference, $q=\sigma_1'-\sigma_3'$, and the void ratio, e.

The drained stress path in e–p–q space (*i.e.*, the state space, Fig. 1) during the shearing of an isotropically consolidated sample is the intersecting line IF of SBS and p $(=\sigma_m')$-const plane. The slope of this line corresponds to the dilatancy coefficient. For an anisotropically consolidated sample, on the other hand, the state path during consolidation is situated on SBS. Projecting this on a p–q plane, one obtains a set of straight lines A_1B_1, A_1C_1, ⋯, starting from the origin, which indicate constant stress ratios, $\eta=q/p$. The point corresponding to the end of consolidation (namely, the beginning of shear) B or C lies on the abovementioned state path during the shearing of the isotropically consolidated sample (IF). This means that the path during the shearing of the anisotropically consolidated sample BF or CF accumulates on the latter half of that for the isotropically consolidated sample.

In spite of the above result indicating that the state paths during drained shear coincide with each other as long as expressed in the state space, it does not mean that the two kinds of samples are in quite the same situation through their shearing processes. In other words, the state space is inadequate to express the state of soil thoroughly. The reason is due to an insufficiency of expressing strain. As long as one uses e-axis (the void ratio) alone, one cannot obtain any informa-

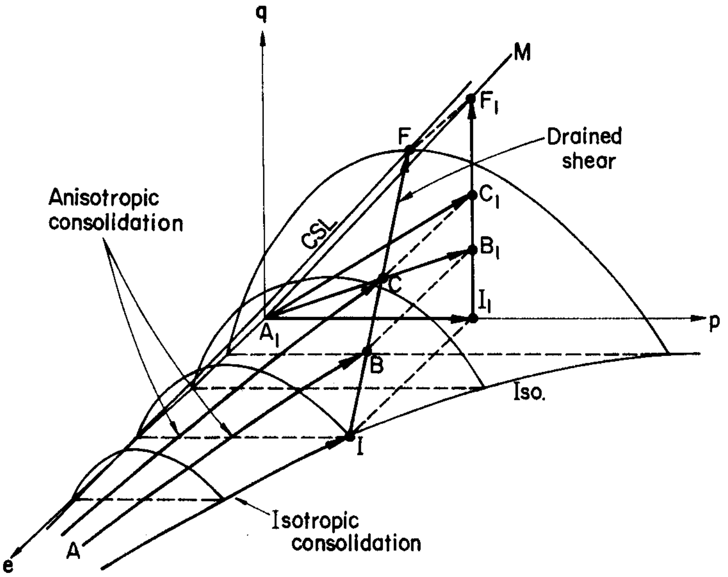

Fig. 1. State boundary surface in the state space.

tion about the deviatoric strain ε, even though the volumetric strain ι can be defined in this space.

3. Dependency of Deviatoric Strain on Stress Path

A series of p-const drained shear tests has been performed for Fukakusa clay ($G_s=2.725$, LL$=46.0\%$, PI$=27.0\%$). The clay sample was made under the precompression load of $\sigma'_{v,0}=100$ kPa from the original liquid state. The size of specimens was 5 cm in diameter and 10 cm in height. They were consolidated isotropically or anisotropically, respectively, in a manner of lateral drainage using the filter paper wrapped around the surface of the specimens under the back pressure of 50 kPa. It took one week for each loading ($\Delta q=48$ kPa) in the drained shear. Fig. 2 shows the stress paths during the test and the strain increment

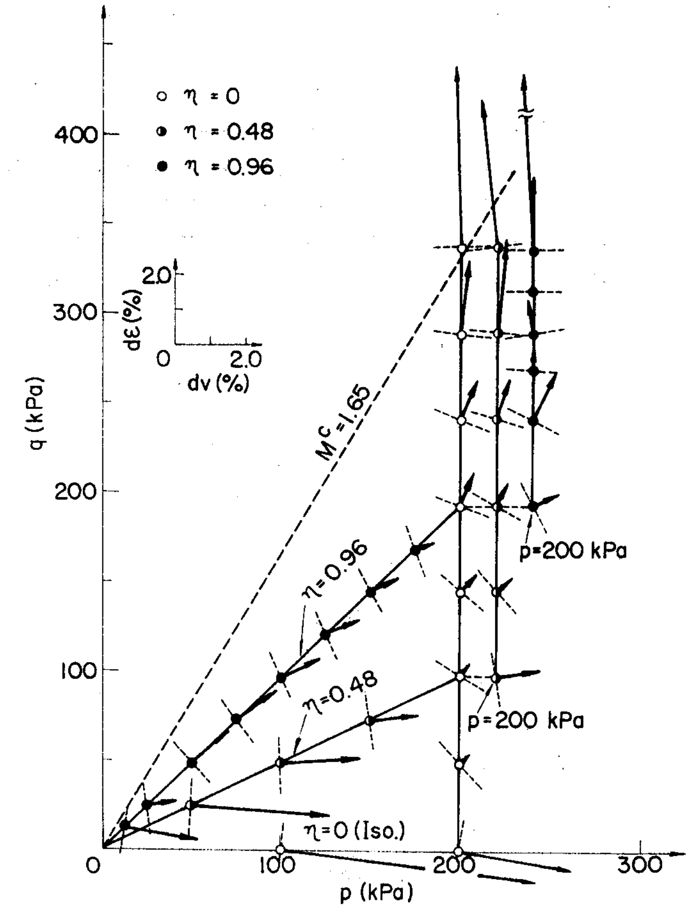

Fig. 2. Stress paths during a series of tests and strain increment vectors.

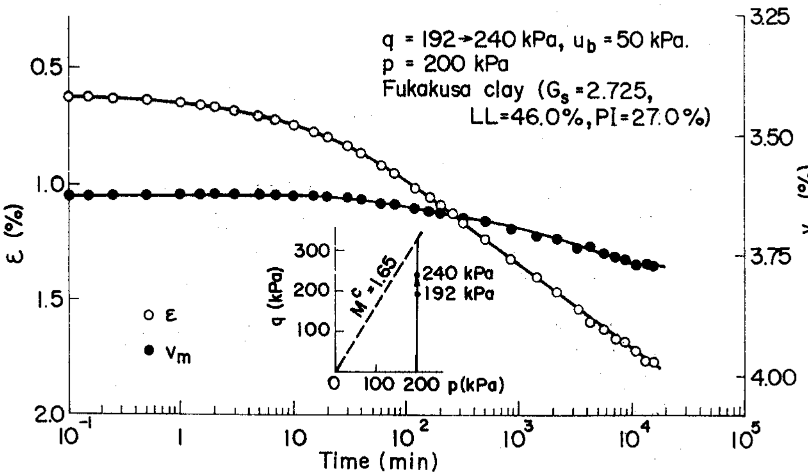

Fig. 3. Variations of the mean strain v_m and the deviatoric strain ε with time.

vectors for each loading stage, depicted for the compression side only.

From the measured values of vertical (axial) strain ε_v and the volumetric strain v, one can calculate the mean strain $v_m = v/3$ and the deviatoric strain $\varepsilon = \varepsilon_v - v/3$, as shown in Fig. 3. Comparing their time-dependencies with each other, it is known that v_m converges to the final value when the time reaches $t \fallingdotseq 10^4$ min. On the other hand, ε continues to increase almost linearly to the logarithm of time. Thus, it can be concluded that the time lag in the deformation of Fukakusa clay is superior in ε than in v_m, without regard to isotropic or anisotropic consolidation.

Fig. 4 (a) gives the stress-strain curves in the drained shear tests keeping the mean effective stress $p=200$ kPa. From this, one can compare the behaviors of the specimens anisotropically consolidated under two kinds of stress ratio $\eta = q/p$ with that of the isotropically consolidated specimen. The shape of the curves during anisotropic consolidation is concave upward, unlike that during shear. The occurrence of the initial deviatoric strain $\varepsilon_0 = -1.2\%$ during the isotropic consolidation is attributed to the memory of the soil structure caused by the preceding 1-D compression. The deviatoric strain at the end of the anisotropic consolidation with $\eta = 0.96$, namely, at point C in the key sketch, is $\varepsilon_c^* = 4.1\%$ after the shift of origin for correction. On the other hand, the deviatoric strains for the specimens with $\eta = 0$ (isotropic consolidation) and $\eta = 0.48$ (anisotropic consolidation) at the same point C show the value of 1.8% and 2.4%, respectively. This means that the deviatoric strain differs corresponding to the stress ratio during consolidation, even for the same point on the effective stress

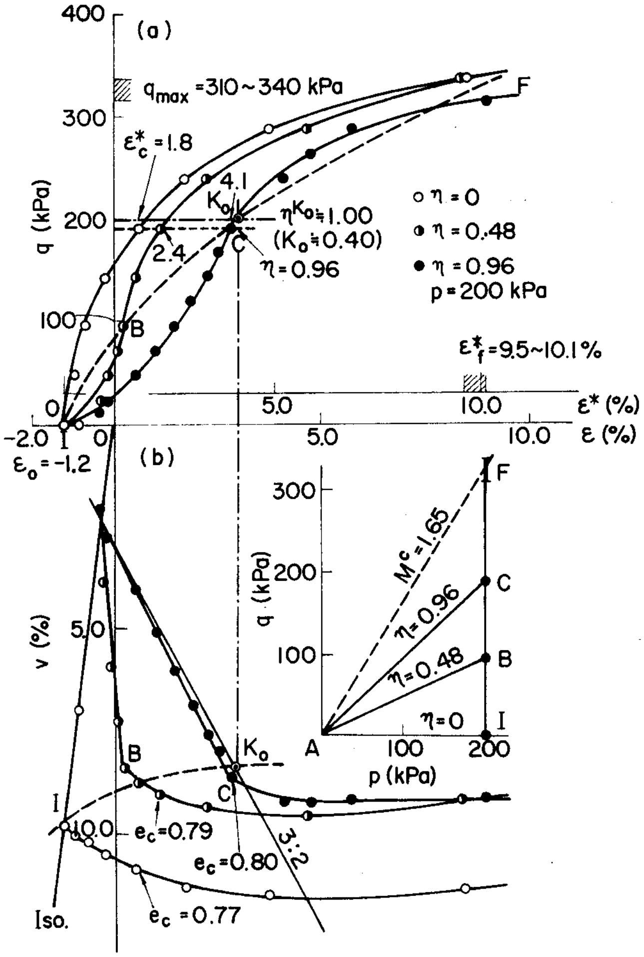

Fig. 4. Stress-strain curves and dilatancy behaviors during all stages of p-const drained shear test.

path. The peak strengths and the strains at failure for the three kinds of specimens having different consolidation histories are within a small range of $q_{max}=310$–340 kPa and $\varepsilon_f^*=9.5$–10.1%, respectively. From this, therefore, it can be seen that the clay soil subjected to anisotropic consolidation under a relatively large stress ratio η reaches its failure point F with a small increase in the deviatoric strain during shear. The dashed line in Fig. 4 (a) indicates the manner of movement of the starting point of shear (*i.e.*, the end point of consolidation) due to the stress ratio. Fig. 4 (b) illustrates the relationship between v and ε, where the volumetric strain of isotropically consolidated soil at failure is somewhat larger than that of the anisotropically consolidated soil. This indicates that the volumetric strain is less dependent on the stress path than on the deviatoric strain. Using the restrictive condition, ι: $\varepsilon=3:2$, for 1-D compression[2], one obtains $K_0 \fallingdotseq 0.40$ ($\eta^{K_0} \fallingdotseq 1.00$) as the coefficient of earth pressure at rest for Fukakusa clay, by the fitting procedure shown in Fig. 4.

4. Characteristics of Plastic Strain Increment Vector

Subtracting the elastic component from the total strain increment obtained at each point on the stress paths in Fig. 2, the relationship between the plastic strain increment ratio $dv^P/d\varepsilon^P$ and the stress ratio $\eta=q/p$ is known with respect to consolidation and shear as in Fig. 5 (a) and (b), respectively.

From these figures, for consolidation of Fukakusa clay:

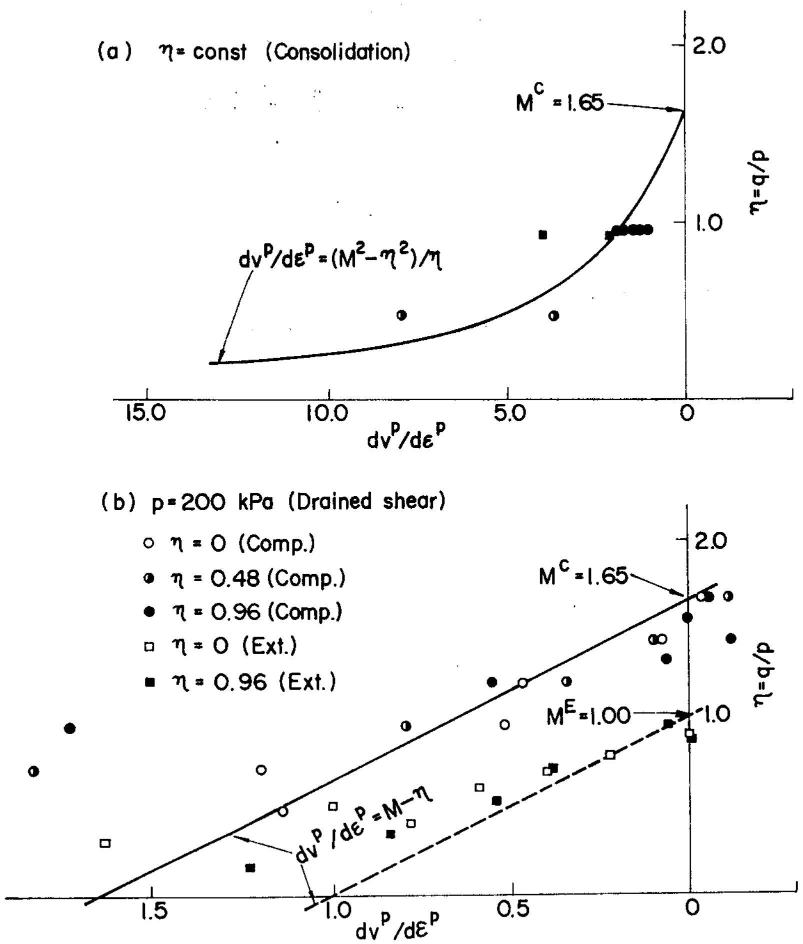

Fig. 5. Correlations between the plastic strain increment ratio $dv^P/d\varepsilon^P$ and the stress ratio $\eta=q/p$.

$$\frac{dv^P}{d\varepsilon^P}=\frac{M^2-\eta^2}{\eta} \tag{1}$$

and for shear, irrespective of the consolidation history:

$$\frac{dv^P}{d\varepsilon^P}=M-\eta \tag{2}$$

The slope of the critical state line (CSL) is $M^C=1.65$ for the compression side, while $M^E=1.00$ for the extension side. Eq. (1) has been proposed for Fukakusa clay by Omaki[3], whereas Eq. (2) is the same as the well-known original Cam-clay model[4]. Using these correlations, it is possible to establish the plastic potentials for consolidation and shear of Fukakusa clay separately, as shown below.

5. Constitutive Relations

The authors' basic assumption for the deduction of the constitutive relations for normally consolidated Fukakusa clay is the adoption of the non-associated flow rule, in which the yield function f and the plastic potential g do not coincide with each other. Moreover, the separate functions are used for the consolidation and shear processes. The functions needed to establish the constitutive relations are summarized as follows[5]:

A. For consolidation (suffix c)

1) Yield function (adopting the Modified Cam-clay model[6])

$$f_c=\frac{p}{p_y}-\frac{M^2}{M^2+\eta^2}=0 \tag{3}$$

where p_y denotes the yield stress for consolidation as an integral constant.

2) Plastic potential (from Eq. (1))

$$g_c=\eta^2-2M^2\ln\frac{p_y}{p}=0 \tag{4}$$

3) Hardening function (from the linearity of $e-\ln p$ correlation)

$$h_c=\frac{\lambda-\kappa}{1+e_0}\,\frac{M^2+\eta^2}{2M^2(M^2-\eta^2)}\,\frac{p_y}{p} \tag{5}$$

where λ and κ denote the compression index and the swelling index, respectively, and e_0 the initial void ratio.

B. For shear (suffix s)

1) Yield function (after Pender[7])

$$f_s=A(\eta-\eta_0)=0 \tag{6}$$

where $A=1$ for $d\eta>0$ (compression) and $A=-1$ for $d\eta<0$ (extension), and η_0 denotes the η-value during consolidation.

2) Plastic potential (from Eq. (2))

$$g_s = \eta - M \ln \frac{p_y}{p} = 0 \tag{7}$$

3) Hardening function (assuming the exponential stress-strain correlation)

$$h_s = \frac{1}{AG^*} \frac{M-\eta_0}{M-\eta} \tag{8}$$

where $G^* = \left(\frac{d\eta}{d\varepsilon^P}\right)_{\eta=\eta_0}$.

The non-associated flow rules giving the plastic strain increment vector are expressed as follows for the plastic volumetric strain increment dv^P and the plastic deviatoric strain increment $d\varepsilon^P$, respectively[8].

$$\left.\begin{aligned} dv^P &= C_1 h_c \frac{\partial g_c}{\partial p} df_c^* + h_s \frac{\partial g_s}{\partial p} df_s^* \\ d\varepsilon^P &= C_1 h_c \frac{\partial g_c}{\partial q} df_c^* + h_s \frac{\partial g_s}{\partial q} df_s^* \end{aligned}\right\} \tag{9}$$

where $df^* = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij}$, $C_1=0$ for $f^*<0$ and $C_1=1$ for $f^*>0$.

Some short manipulations give:

$$\left.\begin{aligned} df_c^* &= \frac{M^2}{M^2+\eta^2}\left(\frac{dp}{p} + \frac{2\eta d\eta}{M^2+\eta^2}\right) \\ df_s^* &= A d\eta \\ \frac{\partial g_c}{\partial p} &= 2(M^2-\eta^2)\frac{p}{p_y} \\ \frac{\partial g_c}{\partial q} &= 2\frac{q}{p_y} \\ \frac{\partial g_s}{\partial p} &= M-\eta \\ \frac{\partial g_s}{\partial q} &= 1 \end{aligned}\right\} \tag{10}$$

Putting these results into Eq. (9) with the hardening functions, Eqs. (5) and (8), one obtains the constitutive relations for Fukakusa clay at loading in the compression side ($A=1$, $C_1=1$) as follows.

$$\left.\begin{aligned} dv^P &= dv_c^P + dv_s^P \\ &= \frac{\lambda-\kappa}{1+e_0}\left(\frac{dp}{p}+\frac{2\eta\, d\eta}{M^2+\eta^2}\right)+\frac{1}{G^*}(M-\eta_0)\, d\eta \\ d\varepsilon^P &= d\varepsilon_c^P + d\varepsilon_s^P \\ &= \frac{\lambda-\kappa}{1+e_0}\frac{\eta}{M^2-\eta^2}\left(\frac{dp}{p}+\frac{2\eta\, d\eta}{M^2+\eta^2}\right)+\frac{1}{G^*}\frac{M-\eta_0}{M-\eta}\, d\eta \end{aligned}\right\} \quad (11)$$

Finally, the total strain, as the elasto-plastic material, is obtained by adding the elastic strain components expressed as:

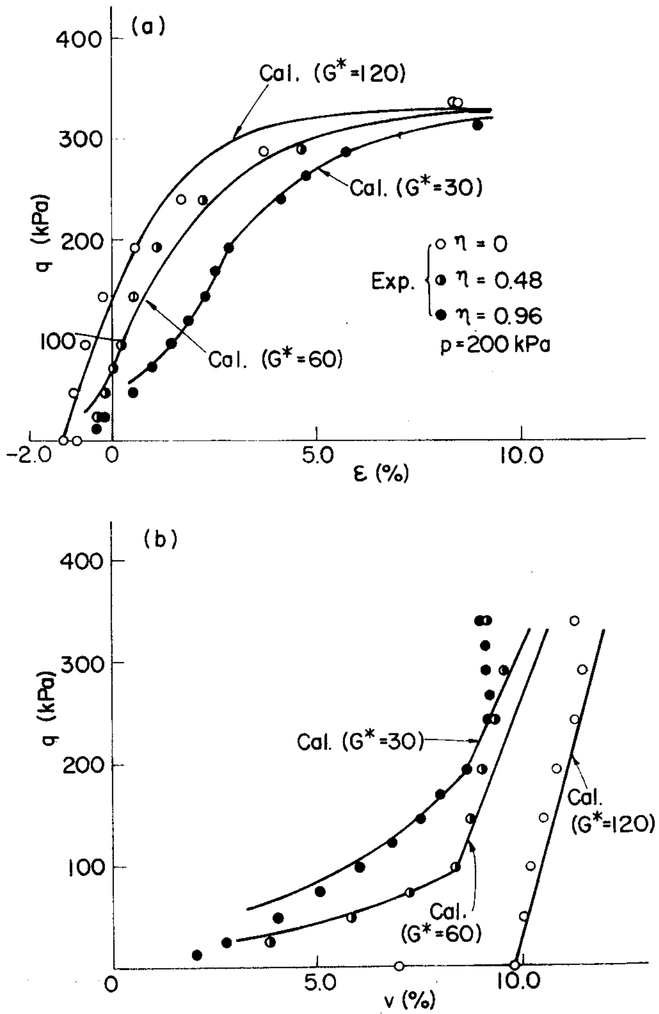

Fig. 6. Comparison of the analytical computation of the proposed constitutive relation with the test result.

$$d\varepsilon_{ij}^{E} = \frac{\kappa}{1+e_0}\frac{dp}{p}\frac{\delta_{ij}}{3} + \frac{d\sigma_{ij}}{2G} \tag{12}$$

where δ_{ij} denotes Kronecker's delta and G the elastic shear modulus, expressed by the following equation:

$$G = \frac{3(1-2\nu)(1+e_0)p}{2(1+\nu)} \tag{13}$$

Fig. 6 shows the comparison of the analytical computation using Eqs. (11) and (12) with the result of the p-const drained shear test (Fig. 4) for Fukakusa clay. It is known from this figure that, in spite of small deviations in the correspondance between q and v, when the anisotropically consolidated soil approaches its critical state during shearing, the analysis can satisfactorily explain the test results. Input data in the analysis are selected as follows (ν: Poisson's ratio):

$$\lambda = 0.092,\ \kappa = 0.019,\ \nu = K_0/(1+K_0) = 0.286\ (K_0 = 0.40)\ ,$$
$$M = 1.65,\ e_0 = 1.05,\ p_y = (1+2K_0)\sigma'_{v,0}/3 = 60\ \text{kPa}\ ,$$
$$G^* = \begin{cases} 120\ (\eta_0 = 0) \\ 60\ (\eta_0 = 0.48) \\ 30\ (\eta_0 = 0.96) \end{cases}$$

6. Conclusion

Through a series of drained shear tests for Fukakusa clay, normally consolidated isotropically or anisotropically, keeping the mean principal stress constant, it has been found that there exists a large dependency of distorsional strain on the stress path, and that this gives an influence on the shear behavior of cohesive soils. Namely, the clay soil subjected to anisotropic consolidation under a relatively large stress ratio η, including K_0-consolidation, reaches its failure point with a small increase in the deviatoric strain.

Based on the situation of adopting the non-associated flow rule and the double functions for the consolidation and shear processes separately, a set of constitutive relations has been proposed for normally consolidated Fukakusa clay. The analytical result using the proposed equations can satisfactorily explain the deformation and strength behaviors of soil during all stages of testing.

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