

## An Application of Self-Tuning Regulator to GLA system of an Aircraft

By

Makoto KOBAYAKAWA, Kiyoshi SAKURA and Hiroyuki IMAI\*

(Received September 4, 1985)

### Abstract

In this paper, a Self-Tuning Regulator (STR), one of the adaptive control strategies, is applied to the GLA system of a large civil aircraft. Two types of STR are introduced. For Algorithm 1, the restriction that the number of the inputs must be equal to that of the outputs is imposed. On the contrary, for Algorithm 2, this restriction is released. These two algorithms are applied to alleviate the gust response of aircrafts, firstly a rigid aircraft and secondly a flexible one. In consequence of simulations, it is found that the STR can alleviate the gust response of aircrafts favorably. A STR is more effective to alleviate response to a discrete gust than to a continuous gust. In spite of the inaccurate estimation of system parameters, the STR works well. Generally, the proposed STR's show better performance for a rigid aircraft than for a flexible one. However, even for the latter, by choosing initial values of parameters appropriately, we can get good results.

### Nomenclature

- $A, B, C$ =polynomial matrices of plant
- $d$ =steady state output to zero input
- $E$ =expected value, disturbance matrix, Young's modulus
- $J_1, J_2$ =cost functions of Algorithms 1 and 2
- $M_y, M_x$ =bending moment, bending moment at the root of wing
- $P, Q, R$ =weighting polynomial matrices
- $q$ =pitch rate
- $q^{-1}$ =backward shift operator
- $Q_1, Q_2$ =weighting matrices
- $t$ =time
- $u$ =control vector
- $U_0$ =flight velocity
- $w$ =reference signal, plunge velocity, displacement

---

\* Department of Mechanical Engineering, Setsunan University.

- $w_g$  = gust velocity  
 $x_1, x_2$  = data vectors of Algorithms 1 and 2  
 $X$  = state vector of flexible aircraft  
 $y$  = output vector  
 $Z_w, M_w$  = stability derivatives  
 $\beta$  = forgetting factor  
 $\gamma$  = Eq. (10)  
 $\delta_e, \delta_a, \delta_r$  = deflections of control surfaces  
 $\varepsilon$  = prediction error, disturbance vector  
 $\theta$  = component of  $\theta_1$  and  $\theta_2$ , pitch angle  
 $\theta_1, \theta_2$  = parameter matrices of Algorithms 1 and 2  
 $\lambda$  = tuning parameter  
 $\xi$  = disturbance vector, generalized displacement  
 $\phi$  = Eq. (5)  
 $\Phi$  = power spectral density of the gust  
subscripts  
1 = Algorithm 1  
2 = Algorithm 2

## 1. Introduction

Advanced flight control technology is continuing to arouse its importance for the design of future aircraft. Particularly, for a recent large-scale civil aircraft, the flight control systems (FCS), such as GLA (gust load alleviation), RSS (relaxed static stability), MLC (maneuver load control), have been studied because of their economical efficiency<sup>1)</sup>. Usually, multi-variable linear control theories are applied for such systems.

On the other hand, in the field of modern control theory, the adaptive control theory has been developed significantly during the last decade<sup>2,3)</sup>. In almost all papers in the past about the adaptive control theory, the systems to be controlled were single-input single-output (SISO) systems. However, multi-input multi-output (MIMO) systems have been treated recently<sup>4,5)</sup>. It is apparent that the MIMO adaptive control system is more applicable for a real problem, particularly for aircraft. There are several papers discussing the adaptive flight control system (AFCS). However, for real aircrafts, the gain scheduling with the information about flight conditions is taken as one of the AFCS<sup>6,7,8)</sup>, and genuine adaptive control systems have rarely been applied.

In this paper, the MIMO adaptive control theory is applied to the aircraft control system. Among several adaptive control strategies, a Self-Tuning Regulator (STR)

is applied to the GLA system for civil aircraft. Two kinds of algorithms for this purpose are developed, and numerical digital simulations are performed so as to examine the effect of the proposed control strategy.

## 2. Self-Tuning Regulator

The control of systems with unknown parameters is one of the difficult problems even for modern control theory. Once the PID-controller structure was invented, but recently, a self-tuning controller has been developed to become one of an important class of controllers which are simple to implement, and have proved to be useful in a number of practical applications. Particularly, for scalar cases, the methodology is well discussed, but for multi-variable cases, the tuning of controllers has not been extensively considered. Recently, several papers have been presented about this interesting technical field<sup>9-12</sup>.

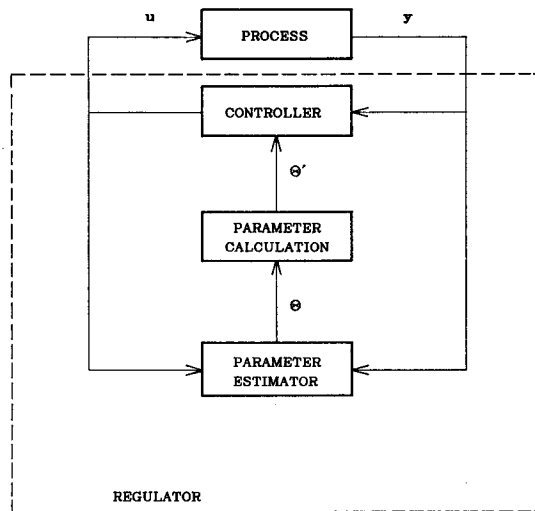


Fig. 1. Block Diagram of Self-Tuning Regulator.

The purpose of STR is to control systems with unknown but constant parameters, and, moreover, this regulator can also be applied to systems with slowly varying parameters. There are many possible STR's depending on the system to be controlled and the design and parameter estimation techniques to be used. Generally, a regulator is described by the block diagram shown by Fig.1. The regulator can be thought of as being composed of three parts, i. e. a parameter estimator, a controller and a part which relates the controller parameters to the estimated parameters.

The parameter estimations are based on recursive parameter estimation schemes, such as least squares (LS), extended least squares (ELS) and recursive maximum

likelihood (RML). The controllers are linear systems whose parameters are obtained from the estimated parameters. Two different types of controllers are frequently utilized, i.e. minimum variance (MV) and linear quadratic (LQ). The case LQ includes MV as a special case. In this paper, two types of STR algorithms are introduced.

**2.1. Algorithm 1<sup>10)</sup>**

It is assumed that the multi-variable system is described by a linear difference equation as follows:

$$A(q^{-1})y(t) = B(q^{-1})u(t-k) + C(q^{-1})\xi(t) + d \tag{1}$$

where the output vector  $y$  and the control vector  $u$  have the dimension  $m$ . The disturbance vector  $\xi$  has the dimension  $m$ , and is a sequence of independent equally distributed random vectors with a zero mean and a covariance  $E\{\xi\xi^T\} = r_t$ .  $d$  is a constant, steady state output response for a zero input signal.  $k$  is the time delay and  $q^{-1}$  is the backward shift operator, i.e.  $q^{-1}y(t) = y(t-1)$ . The polynomial matrices  $A$ ,  $B$  and  $C$  ( $m \times m$ ) are given by

$$\left. \begin{aligned} A(q^{-1}) &= I + A_1q^{-1} + \dots + A_{n_a}q^{-n_a} \\ B(q^{-1}) &= B_0 + B_1q^{-1} + \dots + B_{n_b}q^{-n_b} \quad B_0 : \text{nonsingular} \\ C(q^{-1}) &= I + C_1q^{-1} + \dots + C_{n_c}q^{-n_c} \end{aligned} \right\} \tag{2}$$

Assumptions for the system (1) are given as follows:

- (1) Number of outputs is equal to number of inputs.
- (2)  $B_0$  is non-singular.
- (3) Roots of  $\det C(\alpha)$  lie outside unit circle.

The cost function is given by the following equation:

$$J_1 = E\{\|P(q^{-1})y(t+k) - R(q^{-1})w(q^{-1})\|^2 + \|Q'(q^{-1})u(t)\|^2\} \tag{3}$$

where  $w(t)$  is the known reference signal, having the dimension  $m$ , and  $P$ ,  $Q'$  and  $R$  are  $m \times m$  matrices.  $E\{ \}$  denotes the expected value. At time  $t$ , the measurements  $y(t)$ ,  $y(t-1)$ ,  $\dots$  and the controls  $u(t-1)$ ,  $u(t-2)$ ,  $\dots$  in the past are known. The control  $u(t)$  is expressed by  $y(t)$ ,  $y(t-1)$ ,  $\dots$  and  $u(t-1)$ ,  $u(t-2)$ ,  $\dots$ . Consequently, optimal control can be obtained by minimizing the criterion (3). The minimization problem for the cost function (3) is replaced by the problem to minimize

$$J_1 = E\{\phi^2(t+k)\} = \phi^*(t+k|t)^2 + E\{\varepsilon^2(t+k)\} \tag{4}$$

where

$$\phi(t+k) = P(q^{-1})y(t+k|t) - R(q^{-1})w(t) + Q'(q^{-1})u(t) \tag{5}$$

$$\phi^*(t+k|t) = P(q^{-1})y^*(t+k|t) - R(q^{-1})w(t) + Q'(q^{-1})u(t) \tag{6}$$

and  $y^*(t+k|t)$  is the optimal predictor of  $k$ -step ahead, and  $\varepsilon$  is the prediction error.

Now, the data vector  $x_1(t)$  is defined as follows:

$$x_1(t) = [y^T(t) \cdots y^T(t-n_a+1); u^T(t) \cdots u^T(t-n_b); w^T(t) \cdots w^T(t-n_c); 1] \quad (7)$$

The parameter matrix  $\Theta_1$  is also defined by

$$\Theta_1 = [\theta_{1_1} \cdots \theta_{1_m}] = [F_0 \cdots F_{n_a-1}; G_0 \cdots G_{n_b}; H_0 \cdots H_{n_c}; \gamma]^T \quad (8)$$

where the dimension of  $x_1(t)$  is  $\rho_1$  and  $\Theta_1$  is the  $\rho_1 \times m$  matrix with integer  $\rho_1$  denoting  $\rho_1 = m(n_a + n_b + n_c + 2) + 1$ . Note that  $\sim$  is dropped here and in the following equations. Using  $x_1(t)$ ,  $\Theta_1$  and  $\phi(t+k)$  can be written component-wise as follows:

$$\phi_i(t+k) = x_1(t)\theta_i + \varepsilon_i(t+k) \quad (9)$$

where the components of  $x_1(t)$  are uncorrelated with  $\varepsilon_i(t+k)$ . The control  $u(t)$  is computed as

$$G_0 u(t) = -[\sum_i F_i y(t-i) + \sum_i G_i u(t-i) + \sum_i H_i w(t-i) + \gamma] \quad (10)$$

where  $\gamma$  is constant in the predictor  $y^*(t+k|t)$ . The control parameters,  $\Theta_1$ , are estimated by a standard recursive least squares algorithm summarized as follows:

$$\hat{\theta}_{1_t}(t+1) = \hat{\theta}_{1_t}(t) + K(t)[\phi_i(t) - x_1(t-k)\hat{\theta}_{1_t}(t)] \quad (11)$$

$$K(t) = P(t)x_1^T(t-k)[1 + x_1(t-k)P(t)x_1^T(t-k)]^{-1} \quad (12)$$

$$P(t+1) = \{P(t) - K(t)[1 + x_1(t-k)P(t)x_1^T(t-k)]K^T(t)\} / \beta \quad (13)$$

where  $\hat{\cdot}$  denotes the estimated value, and  $K(t)$  is the gain vector,  $P(t)$  is the covariance matrix and  $\beta$  is the exponential forgetting factor which is selected appropriately according to the system to be controlled. Since  $P=1$ ,  $Q=\lambda I$  ( $\lambda$ : tuning parameter),  $\phi(t)$  is written by

$$\phi(t) = y(t) - R w(t-k) + \lambda u(t-k) \quad (14)$$

Furthermore, for the case in which the reference values are constant, Eq. (14) is rearranged as follows:

$$\phi(t) = y(t) - R w(t-k) + \lambda(u(t-k) - u(t-k-1)) \quad (15)$$

Finally, Algorithm 1 is summarized as follows:

(Algorithm 1)

- (1) Read new output  $y(t)$  and setpoint  $w(t)$ .
- (2) Compute  $\phi(t)$  from Eq. (14) or Eq. (15).
- (3) Make data vector  $x_1(t-k)$  as Eq. (7).
- (4) Update  $\hat{\theta}_{1_t}$  by recursive least squares algorithm using Eqs. (11), (12) and (13).
- (5) Generate new control  $u(t)$  from Eq. (10).

## 2.2. Algorithm 2<sup>13)</sup>

It is assumed that the multi-variable system can be described by a vector difference equation<sup>13)</sup>.

$$y(t) = A(q^{-1})y(t) + B(q^{-1})u(t-1) + \varepsilon(t) \quad (16)$$

where the control vector  $u$  has the dimension  $n_u$ , the output vector  $y$  and the disturbance vector  $\varepsilon$ , have the dimension  $n_y$ . It is assumed that  $y$  and  $u$  can be measured, and the disturbance term  $\varepsilon$  is a sequence of zero mean uncorrelated random vectors.

The system's polynomial matrices are given by

$$\left. \begin{aligned} A(q^{-1}) &= A_1 q^{-1} + A_2 q^{-2} + \dots + A_{n_a} q^{-n_a} \\ B(q^{-1}) &= B_0 + B_1 q^{-1} + \dots + B_{n_b} q^{-n_b} \end{aligned} \right\} \quad (17)$$

and the coefficient matrices  $A_i$ ,  $i=1, n_a$  and  $B_j$ ,  $j=1, n_b$  are considered unknown but constant. A time delay is not introduced in this case. The significant difference of this algorithm from the preceding one is that there is no assumption of  $n_u=n_y$ .

The plant parameters in Eq. (16) are updated by means of a recursive least squares algorithm, as shown in the preceding section. At every sampling instant, the updated plant is used to derive a control strategy which minimizes the cost function

$$J_2 = E \{ y^T(t+1) Q_1 y(t+1) + u^T(t) Q_2 u(t) \} \quad (18)$$

where  $Q_1$  and  $Q_2$  are symmetric weighting matrices. The time series vector  $x_2(t)$  is defined by the data vector  $x_2(t)$  and the parameter matrices  $\Theta_2$  are defined as follows:

$$x_2(t) = [y^T(t-1) \dots y^T(t-n_a); u^T(t-2) \dots u^T(t-n_b-1)] \quad (19)$$

$$x_2(t) = [x_2(t); u^T(t-1)] \quad (20)$$

$$\begin{aligned} \Theta_2 &= [\theta_{2_1} \dots \theta_{2_m}] \\ &= [A_1 \ A_2 \ \dots \ A_{n_a} \ B_1 \ B_2 \ \dots \ B_{n_b} \ B_0]^T \end{aligned} \quad (21)$$

where the dimension of  $x_2(t)$  is  $\rho_2$  and  $\Theta_2$  is  $\rho_2 \times m$  matrix with integer  $\rho_2$  denoting  $\rho_2 = n_y \cdot n_a + n_u(n_b + 1)$ . Furthermore, the predicted value of  $y(t)$  is denoted by  $y_0(t)$  for  $u(t-1) = 0$ . The parameter matrices  $\Theta_2$  are estimated by the recursive least squares algorithm similar to Algorithm 1 (Eqs. (11), (12) and (13)). Then, the new time series vector is generated as

$$x_2(t+1) = [y^T(t) \dots y^T(t-n_a+1); u^T(t-1) \dots u^T(t-n_b)] \quad (22)$$

and the new data vector as

$$x_2(t+1) = [x_2(t+1); 0] \quad (23)$$

Consequently, the predicted value  $y_0(t+1)$  is computed from

$$y_0(t+1) = \Theta_2^T x_2^T(t+1) \quad (24)$$

The control signal is computed using the solution of the quadratic optimal control problem given as follows:

$$u(t) = -[B_0^T(t+1) Q_1 B_0(t+1) + Q_2]^{-1} B_0^T(t+1) Q_1 y_0(t+1) \quad (25)$$

Finally, Algorithm 2 is summarized as follows:

(Algorithm 2)

- (1) Read new output  $y(t)$ .
- (2) Make data vector  $x_2(t)$  as in Eq. (20).
- (3) Update  $\hat{\theta}_{2_i}$  by recursive least square algorithm using Eqs. (11), (12) and (13).
- (4) Generate new time series vector  $x_2(t+1)$  and new data vector  $x_2(t+1)$  as in Eqs. (22) and (23).

- (5) Compute predicted value  $y_0(t+1)$  from Eq. (24).  
 (6) Generate new control  $u(t)$  from Eq. (25).

### 3. Equations of Aircraft and Gust Descriptions

The theory of the STR in the preceding chapter would be applied to aircraft longitudinal motion in cruising flight. The aircraft to be controlled is a large-scale civil transport. The effect of the proposed STR is examined through a numerical simulation of gust response. First, the aircraft is assumed to be rigid, so that the plant equation of motion has only rigid state variables. Next, the flexibility is included in these considerations.

#### 3.1. Equations of Rigid Aircraft

The linearized equation of rigid aircraft longitudinal motion with a short period approximation is written by

$$\dot{x} = Ax + Bu + Ew_g \quad (26)$$

where

$$x = [w \ \theta \ q]^T, \quad u = [\delta_e \ \delta_a \ \delta_r]^T, \quad (27)$$

$$A = \begin{bmatrix} Z_w & 0 & U_0 \\ 0 & 0 & 1 \\ M_w & 0 & M_q \end{bmatrix}, \quad B = \begin{bmatrix} Z_{\delta_e} & Z_{\delta_a} & Z_{\delta_r} \\ 0 & 0 & 0 \\ M_{\delta_e} & M_{\delta_a} & M_{\delta_r} \end{bmatrix}, \quad E = \begin{bmatrix} Z_{w_g} \\ 0 \\ M_{w_g} \end{bmatrix}$$

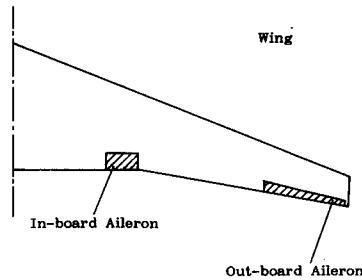
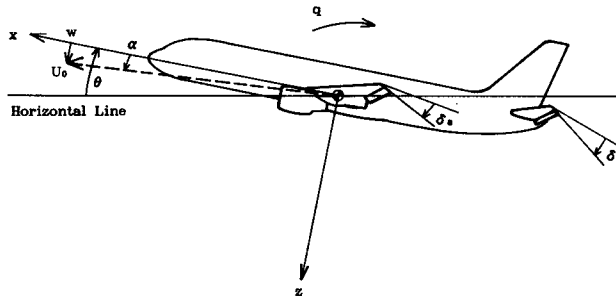


Fig. 2. Coordinate Axes of Aircraft and Location of Control Surfaces.

and  $U_0$  is the trim flight velocity and  $Z_w$ ,  $M_w$  etc. are the stability derivatives defined as

$$\left. \begin{aligned} Z_w &= -\rho U_0 S C_{L_w} / 2M & M_w &= \rho U_0 S \bar{c} C_{m_w} / 2I_y \\ Z_{\delta_e} &= -\rho U_0^2 S C_{L_{\delta_e}} / 2M & M_q &= \rho U_0 S \bar{c}^2 C_{m_q} / 4I_y \\ Z_{\delta_a} &= -\rho U_0^2 S C_{L_{\delta_a}} / 2M & M_{\delta_s} &= \rho U_0^2 S \bar{c} C_{m_{\delta_s}} / 2I_y \quad \text{etc.} \end{aligned} \right\} \quad (28)$$

The system of coordinate axes and the location of control surfaces are illustrated in Fig. 2.

### 3.2. Equations of Elastic Aircraft

For an elastic aircraft, the following equations are given (Fig. 3)<sup>14</sup>.

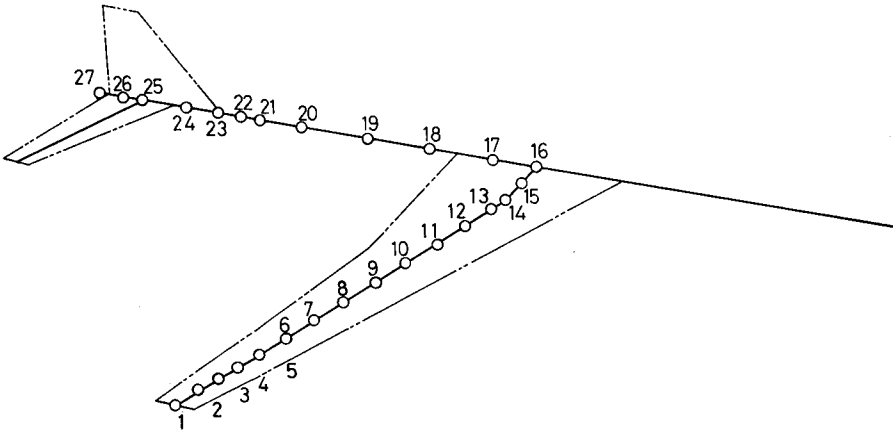


Fig. 3. Mass Distribution of Elastic Aircraft.

$$\left. \begin{aligned} \dot{w}(t) &= Z_w w(t) + U_0 \dot{\theta}(t) + \sum_{i=1}^n (Z_{\xi_i} \xi_i(t) + Z_{\dot{\xi}_i} \dot{\xi}_i(t)) \\ &\quad + Z_{\delta_x} \delta_x(t) + Z_{w_g} w_g(t) \\ \ddot{\theta}(t) &= M_w w(t) + M_q \dot{\theta}(t) + \sum_{i=1}^n (M_{\xi_i} \xi_i(t) + M_{\dot{\xi}_i} \dot{\xi}_i(t)) \\ &\quad + M_{\delta_x} \delta_x(t) + M_{w_g} w_g(t) \\ \ddot{\xi}_i(t) &= -\mathcal{L}_i \omega_i \dot{\xi}_i(t) - \omega_i^2 \xi_i(t) + F_{i_w} w(t) + \sum_{j=1}^n (F_{i_{\xi_j}} \xi_j(t) + F_{i_{\dot{\xi}_j}} \dot{\xi}_j(t)) \\ &\quad + F_{i_{\delta_x}} \delta_x(t) + F_{i_{w_g}} w_g(t) \end{aligned} \right\} \quad (29)$$

The state vector and control input vector are defined as follows:

$$\begin{aligned} X &= [w \ \theta \ \xi_1 \cdots \xi_n \ q \ \dot{\xi}_1 \cdots \dot{\xi}_n]^T \\ u &= [\delta_e \ \delta_a \ \delta_r]^T. \end{aligned}$$

Then, the equation for the elastic aircraft in state space form are expressed as

$$\dot{X} = AX + Bu + Ew_g \quad (30)$$

The bending moment  $M_y$  is related to the forced displacement  $w(y, t)$  by the following equation



$$\frac{d^2 w(y, t)}{dy^2} = -\frac{M_y}{EI_z} \quad (31)$$

The displacement  $w(y, t)$  is expressed by the generalized displacement  $\xi_i$  and the normalized mode shape  $\phi_i(y)$  as follows:

$$w(y, t) = \sum_{i=1}^n \phi_i(y) \xi_i(t) \quad (32)$$

Thus,

$$\left. \begin{aligned} M_y &= -EI_z \sum_{i=1}^n \frac{d^2 \phi_i(y)}{dy^2} \xi_i(t) \\ M_f &= -EI_z \sum_{i=1}^n \left[ \left( \frac{d\phi_i(y)}{dy} \right) \Big|_{y=l+d} - \left( \frac{d\phi_i(y)}{dy} \right) \Big|_{y=l} \right] / \Delta l \xi_i(t) \end{aligned} \right\} \quad (33)$$

### 3.3. Gust Models<sup>15)</sup>

#### (1) Discrete Gust

An isolated sharp-edged step-function was adopted as the discrete gust over the years. However, the (1-cos) gust is now considered favorable. As a first type of the gust, (1-cos) gust is utilized as the discrete gust shown in Fig. 4, in which  $w_g$  is the gust velocity and  $d$  is the distance along the flight path.

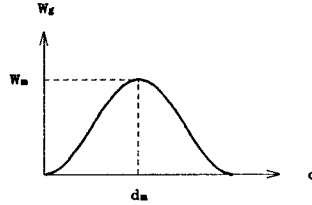


Fig. 4. (1-COS) Gust.

$$w_g(t) = \begin{cases} \frac{W_m}{2} \left[ 1 - \cos \left( \frac{2\pi U_0 t}{25\bar{c}} \right) \right] & 0 \leq t \leq \frac{25\bar{c}}{U_0} \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

The severity of the gust is determined by  $W_m$  and  $d_m$ . The specification for vertical gusts of FAR (USA) fixed  $2d_m$  as  $25\bar{c}$ .

#### (2) Continuous Gust

The continuous gust is a chaotic motion of air that is described by its statistical properties. The power spectral density of the gust is given by

$$\Phi(\Omega) = \sigma_w^2 \frac{L_w}{\pi} \frac{1 + 8/3(1.339L_w\Omega)^2}{[1 + (1.339L_w\Omega)^2]^{11/6}} \quad (35)$$

where  $\Omega$  is the reduced frequency,  $L_w$  the scale length and  $\sigma_w$  intensity of the gust. This is called the Kármán model. The following numerical data are used for the simulations.

$$L_w = 762 \text{ m} \quad \sigma_w = 1 \text{ m/s}$$

### 4. Numerical Simulations

Two kinds of self-tuning regulators (STR) are applied to an aircraft. So as to examine the effect of control, computational digital simulations are performed. The aircraft is a hypothetical large-scale transport with two engines. The dimensions and stability derivatives are abbreviated here. The elastic data are also omitted. Before the simulations, the continuous system equations (Eqs. (26) and (30)) should be written in discrete forms.

First, Algorithm 1 is applied. There is a restriction that the dimension of the inputs is equal to that of the outputs. Therefore, for a rigid aircraft, three control inputs  $\delta_a, \delta_e, \delta_f$  are applied, while it has three outputs, i. e.  $w, \theta, q$ . In the following simulations, setting  $w=0$  throughout one simulation interval, the parameter matrix  $H$  is equal to zero matrix and  $\phi$  is used as a vector function. Before the simulation starts, initial values should be set. Let  $P(0)$  be  $1000I$ , where  $I$  is unit matrix, and  $G_0$ , the element of  $\Theta_2$  be  $I$ . Finally, the exponential forgetting factors  $\beta$  and  $R$  should be given as 0.9 and 1.0 respectively. An example of simulations for a rigid aircraft is shown in Fig.5. For this case, it is very important to choose the tuning parameter in Eq. (12), because  $\lambda$  changes the profile of simulation significantly. In this case,  $\lambda=2.0$ . From Fig.5, it is found that just after the

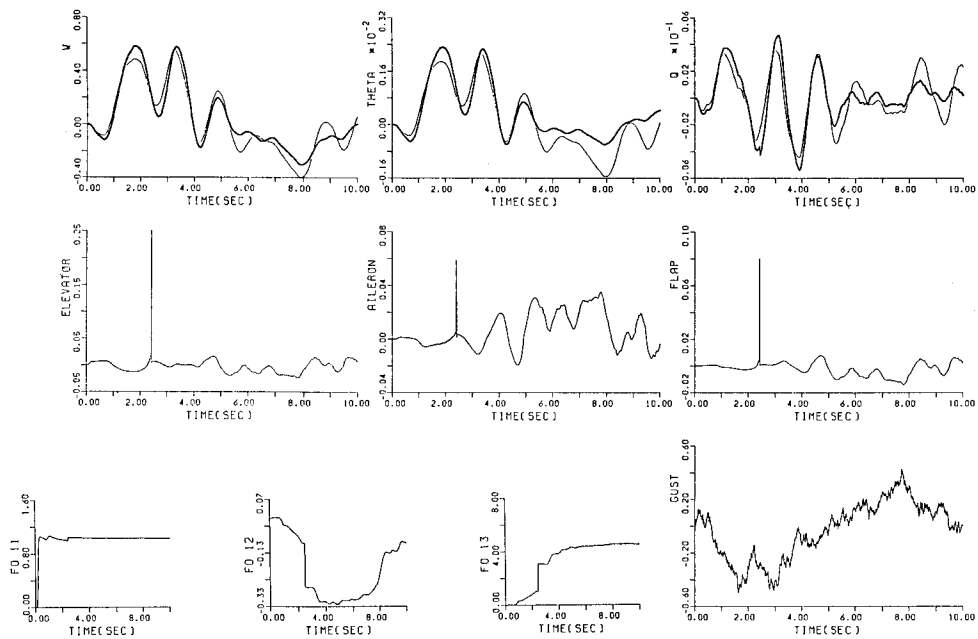


Fig. 5. Simulations of Rigid Aircraft by Algorithm 1 (Continuous Gust).

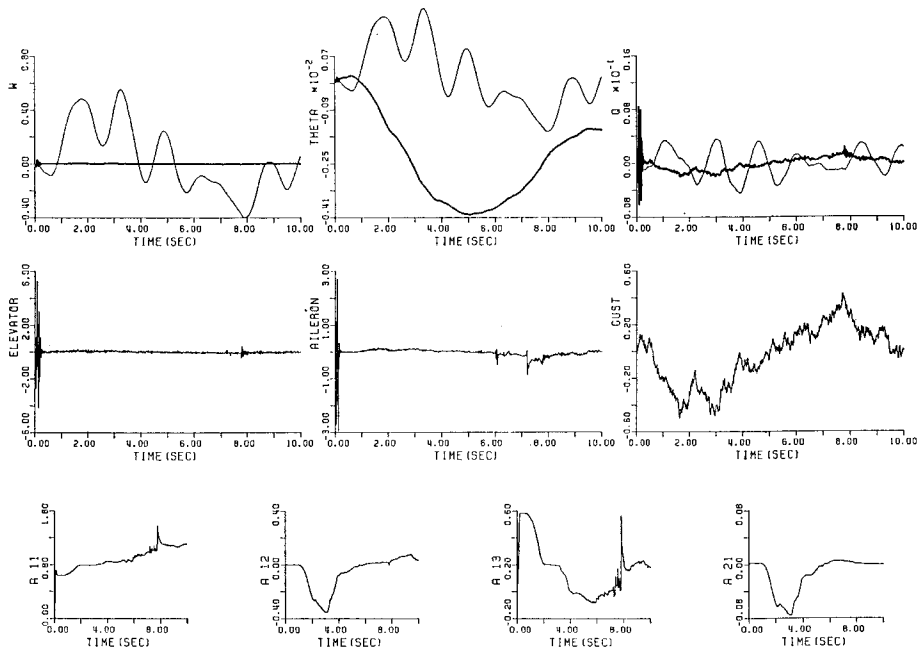


Fig. 6. Simulations of Rigid Aircraft by Algorithm 2 (Continuous Gust).

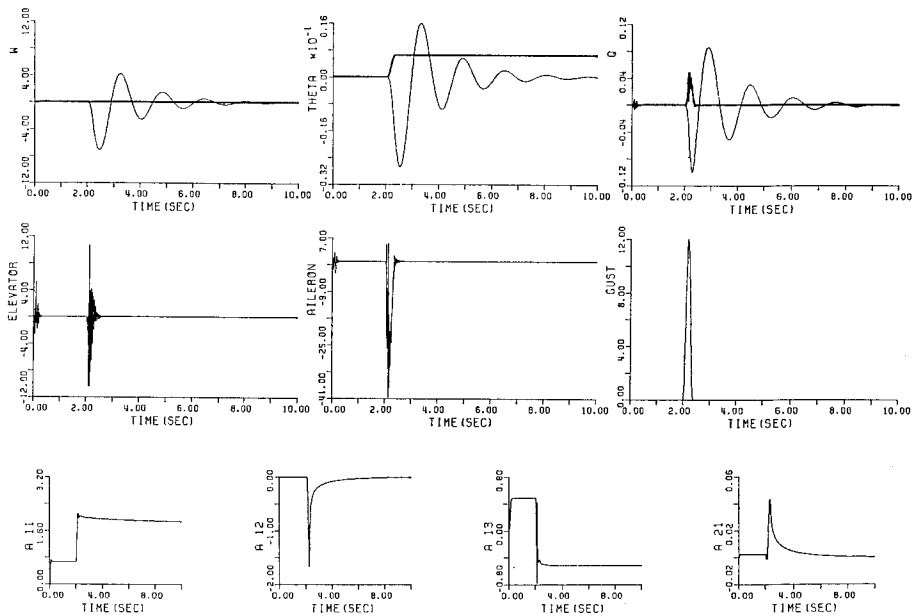


Fig. 7. Simulations of Rigid Aircraft by Algorithm 2 (1-COS Gust).

beginning of simulation the aircraft shows an unfavorable performance for a while, and then the responses are alleviated mildly. The reason for this fact seems to be that the STR concentrates on the parameter estimation at an early stage of control. In addition, the estimated values of the parameter do not converge to the accurate values.

Algorithm 2 has no restriction on the dimension of the inputs and outputs. Thus,  $\delta_\delta$  and  $\delta_a$  are used as control inputs since they are the most practical combination as GLA inputs. For the initial value of the covariance matrix  $P(t)$ ,  $P(0) = 1000I$  is also chosen and  $\beta = 0.99$  for this case. For this algorithm, the values of the weighting matrices  $Q_1$  and  $Q_2$  (Eq. (18)) are very important.  $Q_1 = I$  is chosen for all simulations.  $Q_2$  effects more on the control property than  $Q_1$ . The following values are used for  $Q_2$ .

$$Q_2 = 0.1 \times 10^{-05} I \text{ (Rigid Model Aircraft)}$$

$$Q_2 = 0.1 \times 10^{+00} I \text{ (Elastic Model Aircraft)}$$

Finally, the initial value of  $\theta_2$  (Eq. (21)) should be determined. Since there is no a priori information of a previous estimation, the initial value of  $\theta_2$  is zero inevitably. In this case, certain non-zero values have to be enforced as the initial values of the control input  $u(t)$ . Otherwise, the estimation procedure cannot progress, i.e. the system becomes uncontrolled. The results for the rigid aircraft are shown in Figs. 6 and 7. Comparing Fig. 6 (results of Algorithm 2 to the continuous gust) with Fig. 5 (results of Algorithm 1 to the continuous gust), it is clear that the former is superior to the latter. For (1-cos) gust, variations of a controlled aircraft are hardly seen, but the control surfaces are moving considerably because of the gust. As far as the parameter estimation is concerned, Algorithm 2 does not estimate true values. However, there can be seen no transient part like the output responses.

Figs. 8 and 9 show the simulation results for the elastic aircraft. In both cases, the output responses are not so favorable as those of the rigid aircraft. For the continuous gust (Fig. 8), the output responses vary violently in the early stage of the simulation, while they become mild in the later stage. The reason for these facts seems to be, like Algorithm 1, that the STR first concentrates on the parameter estimation. For (1-cos) gust (Fig. 9), the responses of a controlled aircraft are almost the same as those of an uncontrolled aircraft, i.e. the STR is ineffective for this case. The reason is considered to be that the zero initial values are a burden to the STR. Therefore, as the initial values, the estimated values in one-step before the control are appropriate. In this case, the approximate values to the true ones are applied as initial values. The results for (1-cos) gust are shown in Fig. 10. The responses of a controlled aircraft are improved significantly after the parameter estimation process. Similar results are seen for the continuous gust case.

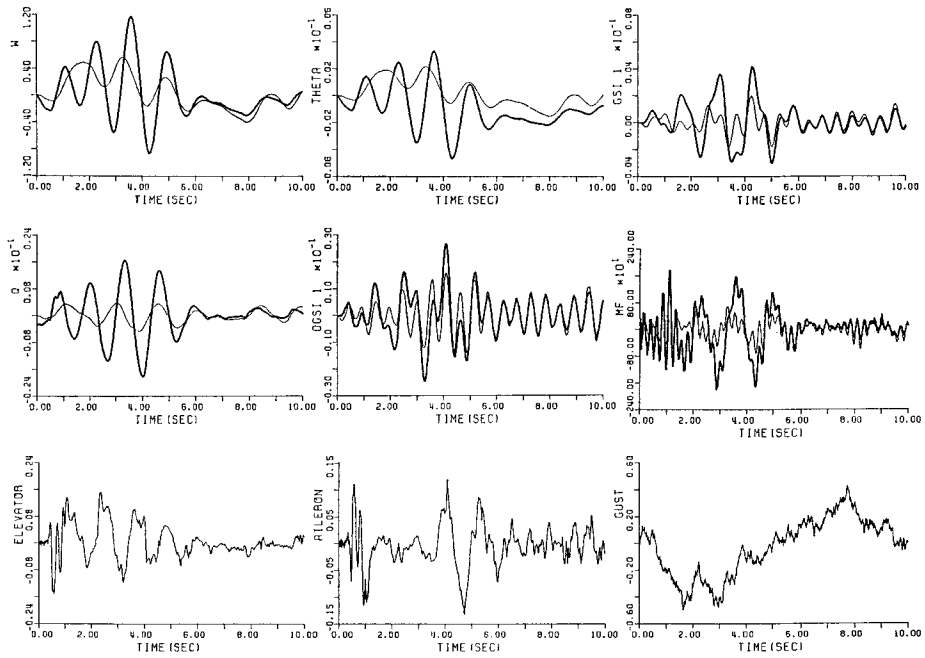


Fig. 8. Simulations of Flexible Aircraft by Algorithm 2 (Continuous Gust).

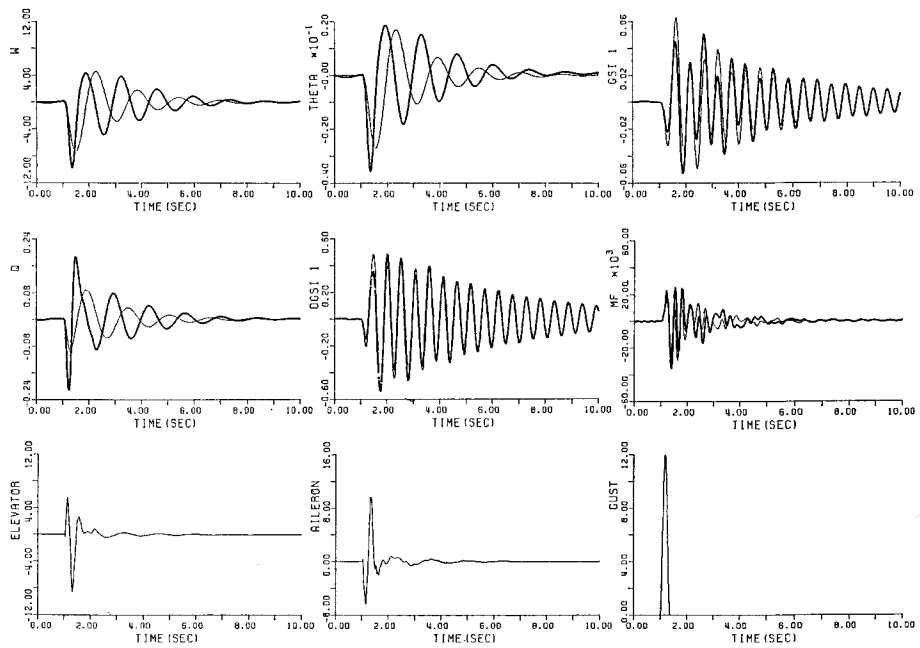


Fig. 9. Simulations of Flexible Aircraft by Algorithm 2 (1-COS Gust).

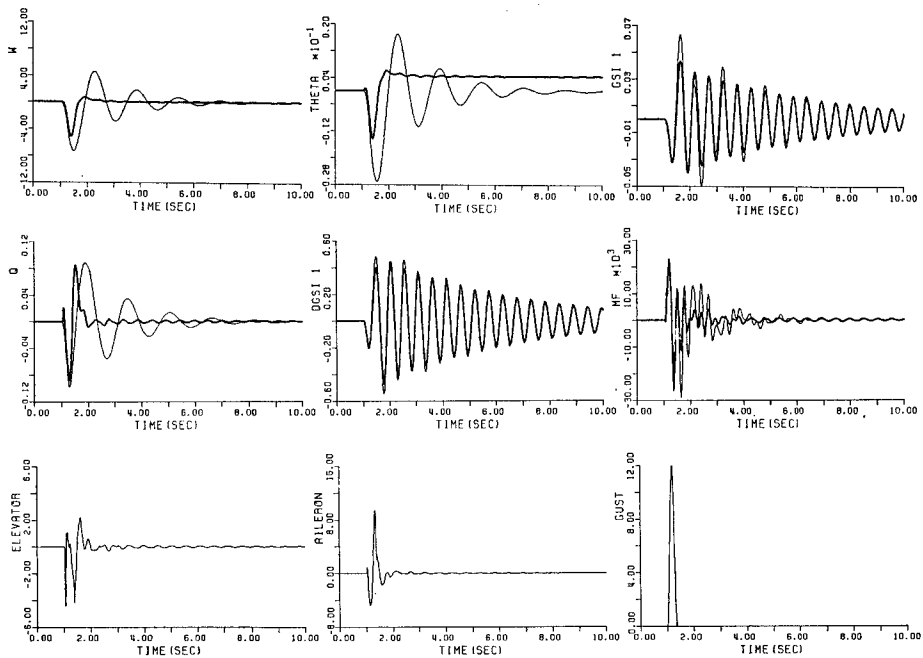


Fig. 10. Simulations of Flexible Aircraft by Algorithm 2 (1-COS Gust).

## 5. Conclusions

In this paper, two types of multi-variable self-tuning regulators (STR) are introduced and applied to the GLA system of a large-scale civil aircraft. First, two algorithms of STR are derived. For Algorithm 1, the restriction that the number of the inputs should be equal to that of the outputs is imposed. On the contrary, for Algorithm 2, this restriction is released. In consequence of the numerical simulations, some favorable results in utilizing the STR for the GLA system are obtained. So far as the two STR strategies are concerned, Algorithm 2 indicates better properties than Algorithm 1. Furthermore, it is easier to alleviate the response to the discrete (1-cos) gust than to the continuous Kármán type. In spite of an inaccurate estimation of parameters, the STR can work well and the output responses show favorable behaviors. It is for the rigid aircraft case more than the elastic case that the proposed STR shows a better performance. The overall STR ability depends on whether the initial values of the parameter matrices are given appropriately or not, particularly in the case of the elastic aircraft. For this purpose, utilization of a priori information about the system is necessary. For example, start-up experiments or previous flight tests are very important for the STR control system.

**References**

- 1) H.P.Y. Hitch, *Aeronautical Journal*, Oct., 389 (1979).
- 2) B. Wittenmark, *International Journal of Control*, Vol.21, 705 (1975).
- 3) K.J. Astrom et al., *Automatica*, Vol.13, 457 (1977).
- 4) L.W. Bezanson and S.L. Harris, *Int. Jour. Control*, Vol.39, 395 (1984).
- 5) M.M. Bayoumi et al., *Automatica*, Vol.17, 575 (1981).
- 6) I.D. Landau and B. Courtiol, *Automatica*, Vol.10, 483 (1974).
- 7) T.L. Johnson et al., *IEEE*, Vol.AC-27, 1014 (1982).
- 8) G. Stein et al., *IEEE*, Vol.AC-22, 758 (1977).
- 9) U. Borison, *Automatica*, Vol.15, 209 (1979).
- 10) H.K. Koivo, *Automatica*, Vol.16, 351 (1980).
- 11) F. Buchholt and M. Kummel, *Automatica*, Vol.17, 737 (1981).
- 12) J. Penttinen and H.N. Koivo, *Automatica*, Vol.16, 393 (1980).
- 13) K.J. Astrom and B. Wittenmark, *Automatica*, Vol.9, 185 (1973).
- 14) R.L. Swaim and D.G. Fullman, *AIAA Paper 77-403* (1977).
- 15) B. Etkin, *Journal of Aircraft*, Vol.18, 327 (1981).