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Vibration Isolation of a Compound Mounting System on an Elastic Floor

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Abstract

This paper describes a theoretical investigation of a vibration isolation of a mounting system which consists of three masses and two isolators on an elastic floor. The formulation of the transmissibility of the system is derived in a plain expression by the use of the receptance method.

The following conclusions are obtained from the results of numerical calculations. The effect of the two mounting frequencies on the transmissibility is not changed, though the combination of them is exchanged. The loss factor of one of the isolators whose mounting frequency is lower than that of the other has a great effect on the first resonance. The loss factor of the other isolator has a great effect on the second resonance. When the top mass becomes lighter and the middle mass becomes heavier, the transmissibility is reduced above the second resonant frequency. The heavy bottom mass reduces the transmissibility above the third resonant frequency, but it makes the third resonance level rise.

1. Introduction

Many buildings have machihes which generate vibrating forces such as engines on their elastic floors. The transmitted vibration is often a cause of annoyance in buildings, and vibration isolation is therefore a topic of continuing study. In most cases, the mounting system has a machine base with accessory equipment, a concrete bed built up on the floor and two isolators, as shown in Fig. 1. However, in past investigations of vibration isolation, the models of the mounting system were usually simplified by omitting some components from the system, or by assuming the floor to be rigid. Therefore, the real performance of vibration isolation has been only inferred from the results obtained from the simplified models.

This paper describes a theoretical investigation of a vibration isolation of a mounting system which consists of three masses and two isolators on an elastic floor. An effective vibration isolation satisfactorily reduces the transmissibility, which is a quantity

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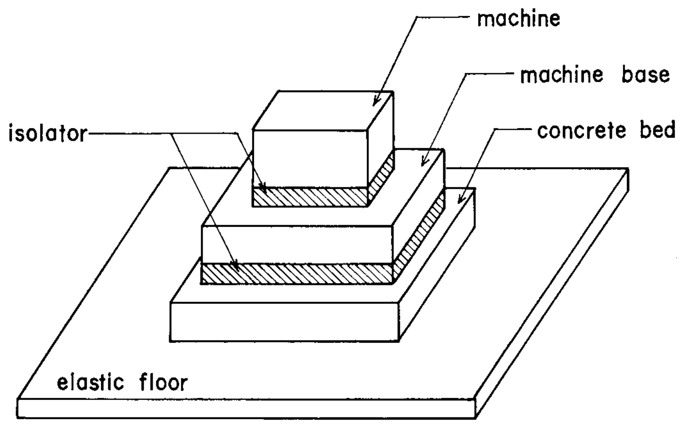

Fig. 1. A mounting system which consists of three masses and two isolators on an elastic floor.

of basic interest in this problem. The formulation of the transmissibility, which is expressed in a plain mathematical form, is derived by the use of the receptance method. The effects on the isolation of the parameters such as the stiffness and the loss factors of the isolators, and the masses are discussed.

2. Formulation

In this chapter, the transmissibility of the mounting system shown schematically in Fig. 2 is formulated by the receptance method. The receptance, or the compliance, is defined as the complex ratio of displacement to force. Here, a sinusoidal varying force $Fe^{i\omega t}$ is applied to, or generated within a machine, and the force is transmitted to the floor through the machine base, the upper isolator, the concrete bed and the lower isolator. It is assumed that the machine, the machine base and the concrete bed are rigid bodies which vibrate only in a vertical direction. The stiffness k_j^* of an isolater is taken to be complex so to account for structural damping:

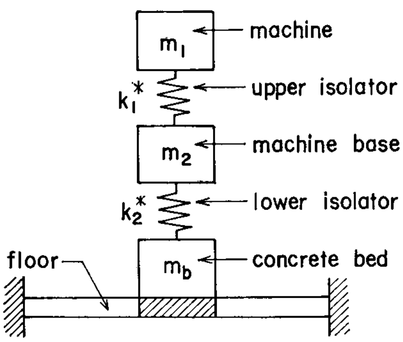

Fig. 2. Schematic view of the mounting system.

$$k_j^*=k_j(1+i\eta_j) \quad (j=1, 2), \qquad (1)$$

where $i=\sqrt{-1}$, and k_j and η_j are the real part of the stiffness and the loss factor of the isolater, respectively. The index j is 1 for the upper isolator, and 2 for the lower isolator.

The transmissibility τ is defined as the

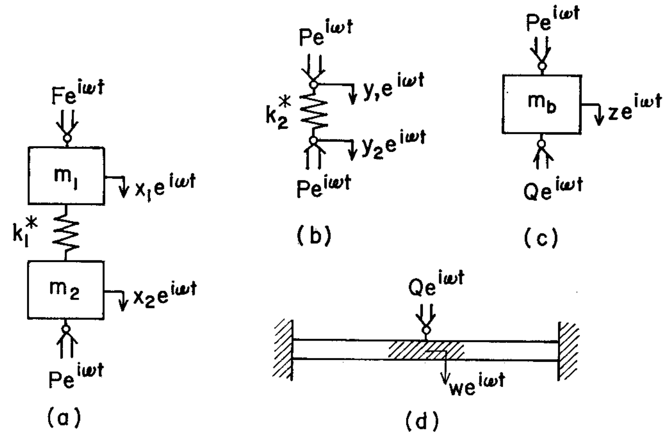

Fig. 3. Divided sub-systems of the mounting system, and notation of displacements and forces.

absolute value of the force ratio Q/F, where Q is the complex amplitude of the transmitted force to the floor; thus

$$\tau = |Q/F|. \tag{2}$$

The system is divided into several connected sub-systems, as shown in Fig. 3, for the application of the receptance method. Fig. 3(a) shows the upper sub-system which consists of the machine, the machine base and the upper isolator. Fig. 3(b), (c), and (d) show the sub-systems of the lower isolator, the concrete bed, and the floor, respectivelly.

The relations of the amplitudes of displacements and forces are given by the following equations by the use of the terms of the receptances.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \alpha_{11} \alpha_{12} \\ \alpha_{21} \alpha_{22} \end{pmatrix} \begin{pmatrix} F \\ -P \end{pmatrix} \tag{3}$$

$$y_1 - y_2 = \beta P, \tag{4}$$

$$z = \gamma_b (P - Q), \tag{5}$$

$$w = \gamma_f Q, \tag{6}$$

where the notation of the complex amplitudes of the displacements and the forces are shown in Fig. 3. The conditions of the connection between the sub-systems are written as

$$x_2 = y_1, \quad y_2 = z = w. \tag{7}$$

The matrix α_{ij} is the receptance matrix of the upper sub-system whose components are given by

$$\alpha_{11}=(k_1^*-\omega^2 m_2)/\Delta,$$
$$\alpha_{22}=(k_1^*-\omega^2 m_1)/\Delta, \tag{8}$$
$$\alpha_{12}=\alpha_{21}=k_1^*/\Delta,$$

where m_1 and m_2 are the masses of the machine and the machine base, respectively, and

$$\Delta=\omega^4 m_1 m_2-\omega^2(m_1+m_2)k_1^*. \tag{9}$$

The receptance of the lower isolator β and the receptance of the concrete base γ_b are given by

$$\beta=1/k_2^*,\quad \gamma_b=-1/(\omega^2 m_b), \tag{10}$$

where m_b is the mass of the concrete bed. The receptance γ_f of the floor will be obtained, for example, by the FEM technique or by the impact measurement.

The transmissibility τ is derived from Eqs. (2)-(7), that is

$$\tau=|\alpha_{21}\gamma/\{(\alpha_{22}+\beta+\gamma)\gamma_f\}|, \tag{11}$$

where γ is the compound receptance of the concrete bed and the floor, which is written as

$$\gamma=\gamma_b\gamma_f/(\gamma_b+\gamma_f). \tag{12}$$

3. Numerical Results and Discussion

The transmissibility of a mounting system which consists of three masses and two isolators on an elastic floor derived above has been numerically calculated by using the computing system of the Data Processing Center, Kyoto University. The floor is treated as a thin clamped plate by the FEM technique, and is assumed to be made of reinforced concrete. The floor has an area 4m×6m, a thickness 0.15m and a loss factor 0.01. Its Young's modulus is 2.0×10^{10}N/m^2, its Poisson's ratio is 0.15, and its density is 2.3×10^3kg/m^3. The concrete bed which has an area 2m×3m is built up on the center of the floor. The part of the floor where the concrete bed is built up is assumed to be rigid. It is also assumed that k_j^* is not dependent on a frequency, and that the wave effects of the isolators are negligible.

From past fundamental investigations of transmissibility, it is apparent that the low stiffness of isolators is effective in suppressing the transmissibility. However, the extent to which the stiffness may be reduced is limited, because the lateral stability of the mounting system must be maintained. This limitation may depend on the material or the type of the isolator, and may relate to the ratio of the stiffness to a static load. Consequently, it is appropriate to define the following mounting frequencies as

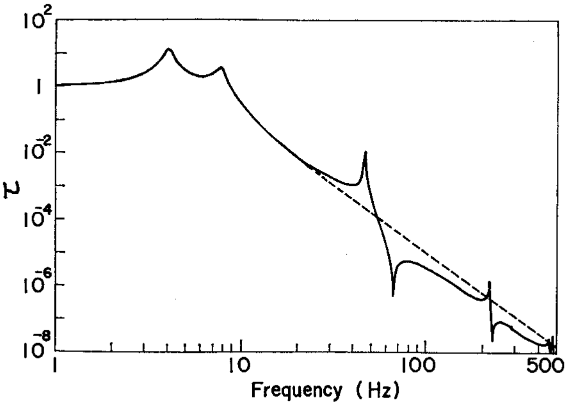

Fig. 4. The transmissibility of the mounting system ($\mu_1=0.2$, $\mu_2=0.4$, $\mu_b=0.5$, $f_1=f_2=5$Hz, $\eta_1=\eta_2=0.1$).

$$f_1=1/(2\pi)(k_1/m_1)^{\frac{1}{2}}, \quad f_2=1/(2\pi)\{k_2/(m_1+m_2)\}^{\frac{1}{2}}. \tag{13}$$

In the following, f_1 and f_2 are referred to as the upper mounting frequency and the lower mounting frequency, respectively.

It is convenient to define the following mass ratios as

$$\mu_1=m_1/m_f, \quad \mu_2=m_2/m_f, \quad \mu_b=m_b/m_f, \tag{14}$$

where m_f is the mass of the floor (8280kg).

The typical results of the calculations of the transmissibility are shown in Fig. 4. These calculations have been made from Eq. (11) for values of $\mu_1=0.2$, $\mu_2=0.4$, $\mu_b=0.5$, $f_1=f_2=5$Hz, and $\eta_1=\eta_2=0.1$ as a function of frequency through the frequency

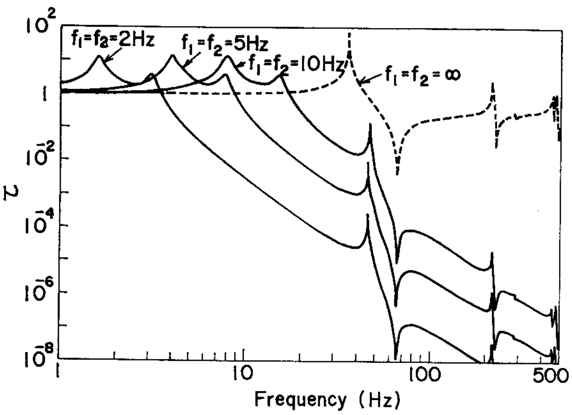

Fig. 5. The transmissibility of the mounting system for various f_1 and f_2 ($\mu_1=0.2$, $\mu_2=0.4$, $\mu_b=0.5$, $\eta_1=\eta_2=0.1$).

range 1 to 500Hz. The dashed-line curve of this figure refers to the transmissibility to the rigid floor ($\gamma_f=0$). The transmissibility to the elastic floor is almost equal to that to the rigid floor ($\gamma_f=0$) in the low frequency region below 20Hz. In the low frequency region, the curve of the transmissibility has two peaks at about 4Hz and 8Hz, which correspond to the resonances of the upper sub-system with the lower isolator. In the high frequency region above 20Hz, the curve of the transmissibility to the elastic floor has a series of upward and downward peaks. The upward peaks correspond to the resonance of the floor with the concrete bed, and the downward peaks correspond to the resonance of the floor without the concrete bed. The sharpness of these peaks is dependent on the loss factor of the floor. In the high fre-

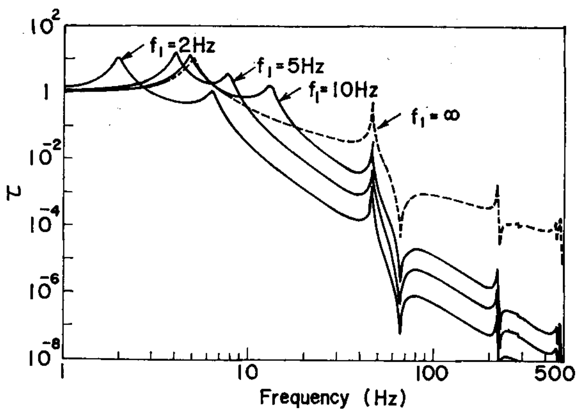

Fig. 6. The transmissibility of the mounting system for various f_1 ($\mu_1=0.2$, $\mu_2=0.4$, $\mu_b=0.5$, $f_2=5$Hz, $\eta_1=\eta_2=0.1$).

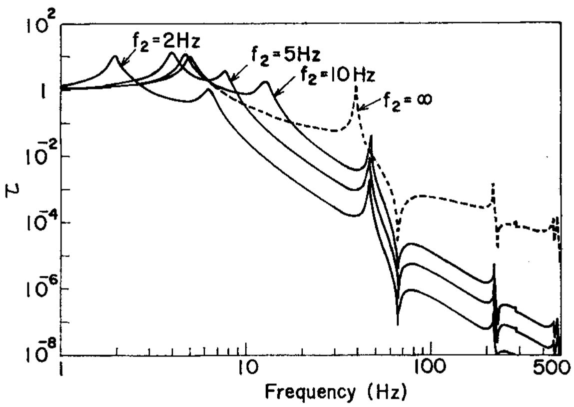

Fig. 7. The transmissibility of the mounting system for various f_2 ($\mu_1=0.2$, $\mu_2=0.4$, $\mu_b=0.5$, $f_1=5$Hz, $\eta_1=\eta_2=0.1$).

quency region, the mean level of the transmissibility falls along that to the rigid floor which decreases in proportion to the fourth power of frequency, that is, at 24dB/ octave.

Fig. 5 shows the effect of the stiffness of the isolators, for which $f_1=f_2=2$, 5, and 10Hz. The dashed-line curve of this figure refers to the transmissibility of the system with no isolator ($f_1=f_2=\infty$). Figs. 6 and 7 show the effect of the stiffness of the upper isolator, for which $f_1=2$, 5, and 10Hz, and $f_2=5$Hz, and the effect of the stiffness of the lower isolator, for which $f_1=5$Hz, and $f_2=2$, 5, and 10Hz, respectively. The dashed-line curves of these figures refer to the transmissibility of the system without the upper or lower isolator ($f_1=\infty$, or $f_2=\infty$). It is apparent that the lower

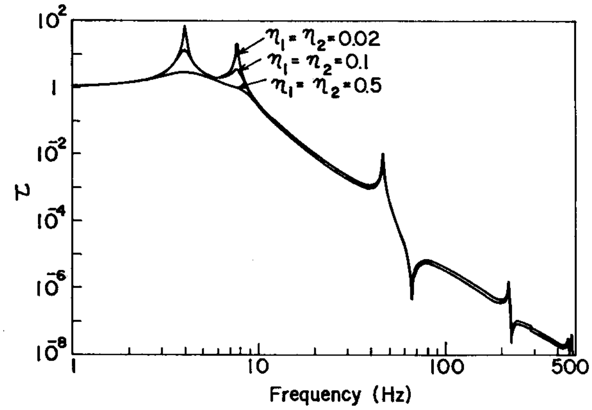

Fig. 8. The transmissiblity of the mounting system for various η_1 aind η_2 ($\mu_1=0.2$, $\mu_2=0.4$, $\mu_b=0.5$, $f_1=f_2=5$Hz).

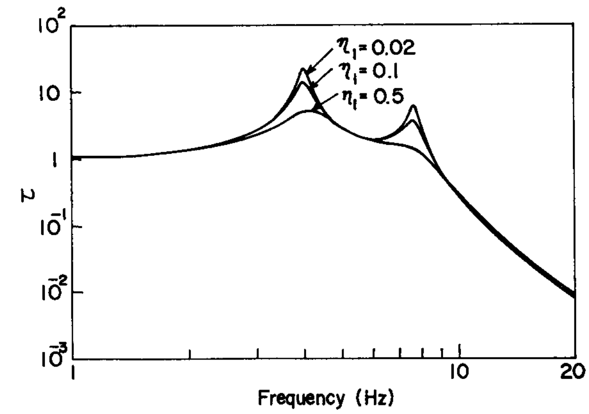

Fig. 9. The transmissibility of the mounting system for various η_1 in the case when $f_1=f_2$ ($\mu_1=0.2$, $\mu_2=0.4$, $\mu_b=0.5$, $f_1=f_2=5$Hz, $\eta_2=0.1$).

the mounting frequencies are, the more effective is the mounting system for vibration isolation through the frequencies above the second resonant frequency. From Figs. 6 and 7, it is found that the effect of the combination of the mounting frequencies on the transmissibility is almost equal to that of the exchanged combination throughout the all frequencies. For example, there is no visible difference between the curves for the combinations $(f_1, f_2) = (2\text{Hz}, 5\text{Hz})$ and $(5\text{Hz}, 2\text{Hz})$. Therefore, in the case of utilizing a costly set of isolators having a low mounting frequency for only one of the two isolators, it is economical to employ it as the upper isolator, since the requisite amount of the isolator for the lower mount is $(1+m_2/m_1)$ times as large as the one for the upper mount.

Fig. 8 shows the effect of the loss factors of the isolators, for which $\eta_1=\eta_2=0.02$, 0.1, and 0.5. The loss factors of the isolators have a great effect on the transmissibility near the two resonant frequencies of the upper sub-system with the lower isolator. However, they have little effect on the transmissibility above them. Fig. 9, where the tramsmissibility is plotted through the frequency range 1 to 20Hz, shows the effect of the loss factor of the upper isolator, for which $\eta_1=0.02$, 0.1, and 0.5, when the two mounting frequencies are equal $(f_1=f_2=5\text{Hz})$. In this case, the effect of the loss factor of each isolator at the first resonant frequency is the same as its effect at the second resonant frequency. Figs. 10 and 11 show the effect of the loss factors when the two mounting frequencies are different. From Fig. 10, when f_1 is lower than f_2, it is found that the level of the transmissibility near the first resonant frequency is mainly dominated by the loss factor of the upper isolator. Also, its level near the second resonant frequency is mainly dominated by the loss factor of the lower isolator. On the contrary, from Fig. 11, when f_1 is higher than f_2, it is found that the dominant loss factors for the two resonances are exchanged. Consequently, the loss factor of one of the isolators whose mounting frequency is lower than that of the other has a great effect on the first resonance level. The loss factor of the other has a great effect on the second resonance level.

The effect of the mass of the upper sub-system, that is, the machine and the machine base, is shown in Fig. 12. It is found that the mass of the upper sub-system suppresses the transmissibility through the frequency region from 10 to 100Hz, especially near the third resonance, when the upper sub-system is much heavier than the floor. However, when the upper sub-system is as heavy as, or lighter than, the floor, the mass of the upper sub-system hardly has an effect on the transmissibility.

Fig. 13. shows the effect of the mass allotment of the upper sub-system on the machine and the machine base. When the machine becomes lighter and the machine base becomes heavier, the transmissibility is reduced in the frequency range above the second resonant frequency, since this frequency falls. Therefore, for vibration isola-

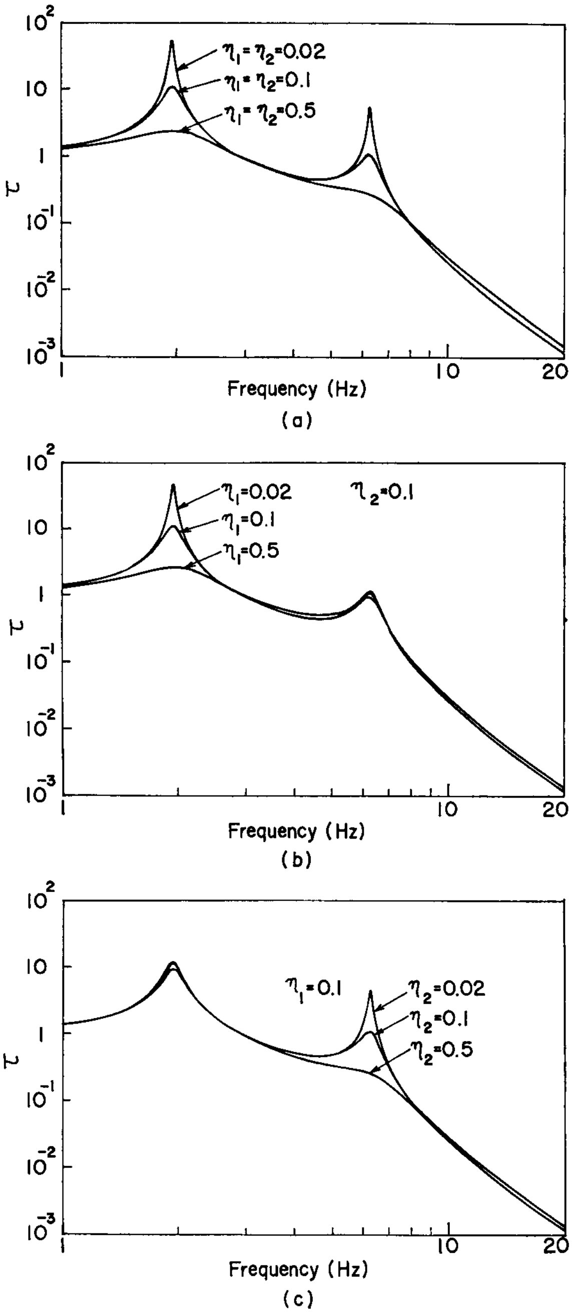

Fig. 10. The transmissibility of the mounting system for various η_1 and η_2 in the case when $f_1 < f_2$ ($\mu_1 = 0.2$, $\mu_2 = 0.4$, $\mu_b = 0.5$, $f_1 = 2\text{Hz}$, $f_2 = 5\text{Hz}$).

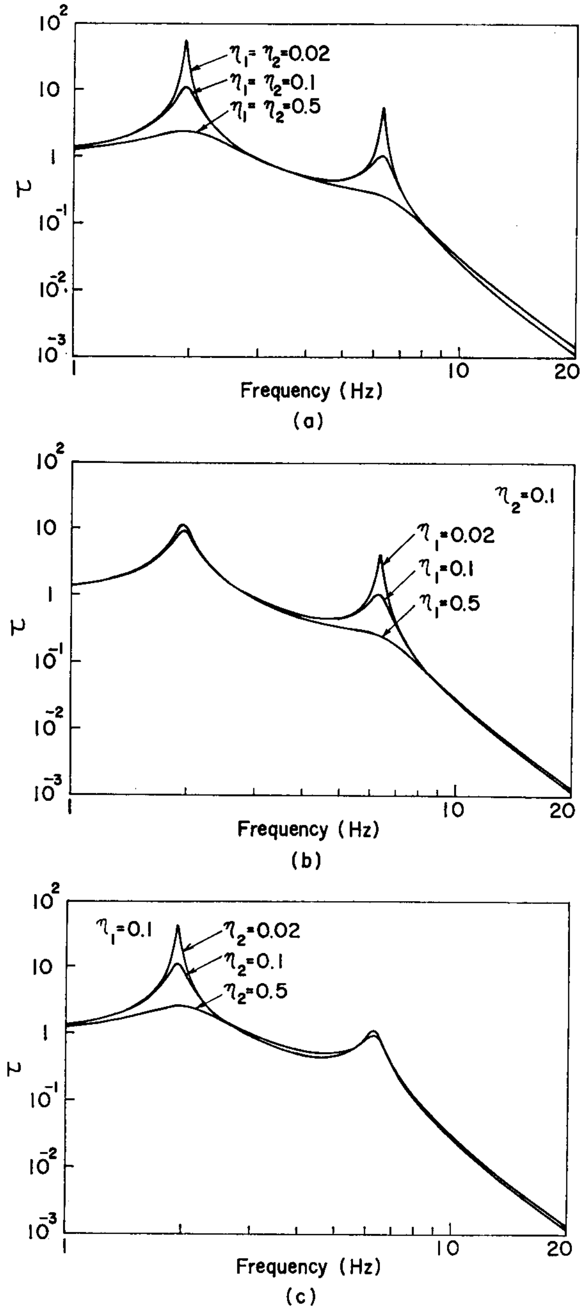

Fig. 11. The transmissibility of the mounting system for various η_1 and η_2 in the case when $f_1>f_2$ ($\mu_1=0.2$, $\mu_2=0.4$, $\mu_s=0.5$, $f_1=5\text{Hz}$, $f_2=2\text{Hz}$).

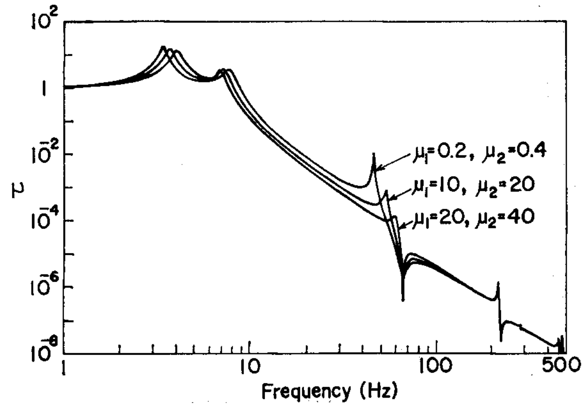

Fig. 12. The transmissibility of the mounting system for various μ_1 and μ_2 when the ratio of μ_1 and μ_2 is constant ($\mu_b=0.5$, $f_1=f_2=5$Hz, $\eta_1=\eta_2=0.1$).

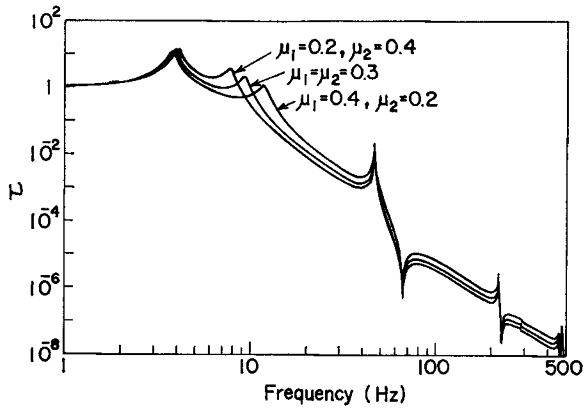

Fig. 13. The transmissibility of the mounting system for various μ_1 and μ_2 when $\mu_1+\mu_2$ is constant ($\mu_1+\mu_2=0.6$, $\mu_b=0.5$, $f_1=f_2=5$Hz, $\eta_1=\eta_2=0.1$).

tion, the accessory equipment should be not fixed on the machine, but on the machine base.

The effect of the mass of a concrete bed is shown in Fig. 14. The dashed-line curve of this figure refers to the transmissibility of the system without a concrete bed ($\mu_b=0$), which has neither upward nor downward evident peaks. It is found from this figure that the mass of a concrete bed reduces the transmissibility in the frequency region above the third resonant frequency. Therefore, a heavy concrete bed has an effect on the vibration isolation in the high frequency region. However, it is neccessary to pay attention to the fact that a heavy concrete bed makes the third resonant frequency fall, and makes the third resonance level rise.

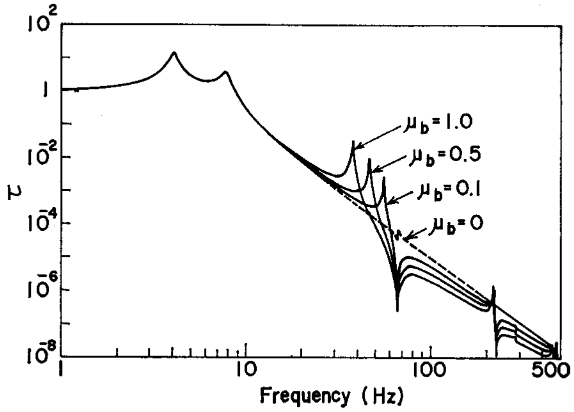

Fig. 14. The transmissibility of the mounting system for various μ_b ($\mu_1=0.2$, $\mu_2=0.4$, $f_1=f_2=5$Hz, $\eta_1=\eta_2=0.1$).