A Road Network Design Model Considering Node Capacity

by

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(Received September 2, 1986)

ABSTRACT

Optimal road network design models have been investigated in order to generate alternatives of road network planning. Most of the previous works assume that the decision variables of the planners are the attributes of the links. The actual road network, however, consists of links and nodes, and it is useful to distinguish links and nodes explicitly. Planning models should involve the attributes of nodes in decision variables as well as the link attributes. We formulate a road network design model considering the nodal capacity. The frame of the proposed model is interpreted as a two level optimal problem, which is a system optimizing problem including an optimal problem as its constraint. Before the formulation of a two level problem, the structure of a node is simplified, and a nodal passing time function is introduced, which represents the performance of a node. As a user equilibrium is the most relevant assumption for the description of road network flow, the lower problem is formulated as a fixed demand user equilibrium problem, using nodal passing time functions as well as link travel time functions. Given the total investment cost constraint, the higher problem decides the link and nodal capacities in order to optimize a measure of the whole network performance. The lower problem describes the traffic flow on a network for given capacities. The structure of the model is explained by using the frame of the Stackelberg differential game, whose players are planner (leader) and aggregate term of network user (follower). In order to solve the formulated problem, a heuristic algorithm is proposed. This is the input/outputiterativemethod, and it is expected to be effective for a normal size problem. The convergence of the algorithm is numerically confirmed through an example for simple and hypothetical datum. A sensitivity analysis for the value of the cost constraint is executed to determine the effective value of an investment. Remaining problems are extensively discussed for further research.

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1. Introduction

Optimal road network design models have been investigated in order to support the decision making process of road network planning. They can be used as effective tools for the generation of network alternatives. Almost all of the previous researches on optimal network design assume that the decision variable of the planner is the link attribute of the concerned network, for example, the capacity of a link or the number of lanes.

The actual road network, however, consists of links and nodes, and the lack of capacity of the nodes usually causes traffic congestion on a network. When the planner tries to find the set of alternatives which will improve the network performance, it is useful to classify links and nodes explicitly. Optimal network design models should include the nodal attributes in the decision variables as well as the link attributes.

Here, we use the term of node for the road section or the place where one directed traffic flow meets the others. For example, a junction or an intersection can be considered as a node if we treat the street network in a city. For a regional network, a node may denote a town or a city.

An optimal network design model considering the nodal attributes is expected to give the optimal combinations of both link and nodal capacity improvements. Therefore, the optimal amounts and timings of lane widening and signal setting are useful information for the construction and management planning of a street network.

Two important assumptions should be examined before the mathematical formulation of an optimization problem. They are employed as the basic assumption throughout this paper. One assumption is that the continuous variables are used as the decision variables of the netwok planner or the transportation service supplier.

The traffic capacity is the decision variable for a link ; the road section of an uninterrupted flow is simply called a link. The traffic capacity is regarded to be a continuous variable, while the number of lanes is a discrete variable. Similarly, the decision variable for a node is the traffic capacity. In a few words, we classify these two kinds of capacities as the planner's decision variables. They are simply referred to as the link capacity and the nodal capacity respectively.

The other key issue lies in the description of the road network flow for a given value of the decision variables of the network planner. It can be assumed that the actual network flow is brought by the route choice behavior of the individual network user. The transportation service supplier can decide the link and nodal capacities. However, he cannot directly intervene in the individual route choice behavior.

The most appropriate assumption on the description for the road network flow is the user equilibrium (UE) concept, in other words, Wardrop's first principle. It is reasonable to assume that every motorist will try to minimize his or her own travel time from the origin to the destination. A stable condition is reached only when no traveler can improve his or her travel time by unilaterally changing routes. This stable condition is defined as the user equilibrium. At the user equilibrium, the travel time on all used paths is equal, and less than or equal to the travel time that would be experienced by a single vehicle on any unused path.

Then, the optimal network design problem discussed in this paper concerns the planner's decision making process considering the network user's aggregated behavior. Later, this situation will be explained in detail.

Here, we review several previous works which discussed optimal network design problems (NDP) and optimal traffic signal setting problems (TSSP). The former problems relate to the link design and the latter to the nodal design for our problem. We consider only the works which try to concern both the planner's decision variables and the UE traffic flow in the same framework. We assume the continuous decision variable for the transportation service supplier.

One of the buds which tries to include the UE traffic flow in NDP can be found in the work by Abdulaal and LeBlanc (1979)[1] . They define the traffic flow as the function of link capacities and formulate NDP. This problem, however, is difficult to handle for numerical calculations, because the objective function is non-convex and non-differentiable.

Using the idea of a variational inequality, Marcotte (1983)[8] formulates the NDP constrained by the variational inequalities in the flow variables. He proposes an exact (relaxation and accumulation) algorithm and a heuristic input-output algorithm.

Harker and Freisz (1984)[7] apply the idea of the two level optimizing problem in order to explain the structure of the NDP with a UE flow. The formulated NDP can be interpreted as the Stackelberg Game problem with two players, the network planner and the user. They show that the solution of the NDP with a system optimal flow gives the lower bound, and the heuristic input-output solution gives the upper bound of the exact solution respectively.

Independently of this work, Asakura (1985)[2] formulates an NDP with a UE flow, and introduces the Stackelberg Game framework for the explanation of the problem. He also applies the problem to the actual case of a road widening in the suburbs of Kyoto City.

These works show that the NDP considering the network user's behavior can be formulated mathematically in some forms, and the exact solution procedure might be less attractive than the heuristics in the actual size of the network design problem.

On the other hand, Charlesworth (1977)[3] proposes a method which can simultaneously determine both the timing of traffic signals and the network assignment flows, based on Wardrop's 1 st principle. Although his calculation remains at a heuristic stage, it should be noted that he analyzed the simultaneous determination problem which had been previously pointed.

Marcotte (1983)[8] also formulates the TSSP using the same idea of the NDP constrained by variational inequalities. The exact algorithm for the NDP, however, might be modified for the TSSP, or the problem should be reformulated.

Fisk (1984)[5] considers the relation between the Game theory and the transportation system models. The TSSP with a UE flow is shown as an example of the Stackelberg Game whose players are the control organization and the network user. She also formulates the TSSP, using variational inequalities and proposes an exact solution algorithm by a penalty function method. This procedure can be applied if the size of the problem is not too large.

Besides these works, problems assuming the system optimal network flow have been investigated. For example, Gartner (1977)[6] formulates the TSSP and shows the solution procedure by decomposition. The idea is methodologically equivalent to the works on the NDP by Dantzig et al. (1979)[4] . The NDP and the TSSP with a system optimal flow are comparatively easier than the problem with a user equilibrium flow. They are useful for both understanding the problem and considering solution methods.

Taking care of the founding of the previous works, this research takes the following procedures. At first, it is necessary to make a model of a road network. Especially, the most important point is the description of the road section which corresponds to a node. We bring the idea suggested by Sasaki and Inouye (1974)[9] and present a simple model of a node. Then the UE traffic assignment problem can be formulated. This problem describes the network flow which comes from the user's route choice behavior for the given value of the decision variable, and network capacities, of the planner. The description of a node and the formulation of the UE problem are discussed in Section 2 of this paper.

In the next part of this paper, the decision making process for a road network design is formulated as an optimizing problem, using the framework of the Stackelberg Game problem. The decision variables of the network planner are the node capacities as well as the link capacities. The UE network flow is explicitly concerned in the problem. This optimal problem can be interpreted as a two level optimizing problem which involves the other optimal problem, the UE problem, as its constraint. A heuristic solution algorithm is proposed for the formulated NDP, and a numerical example is executed for the hypothetical data.

2. UE Traffic Assignment Problem with Nodal Passing Time

2.1. Nodal Passing Time Function

There are a few theoretical works on a traffic assignment which explicitly take into consideration the nodal passing time. The nodal passing time corresponds to the delay time spent at a node. Almost all of the previous works assume that the nodal passing time is involved in the link travel time. It is necessary to define the nodal passing time function, separated from the link travel time function, when we formulate a traffic assignment problem with the nodal passing time.

We can derive several types of the nodal passing time function, based on the model of a node. Fig. 1 shows a case in which the node and the nodal passing time are not explicitly considered, and this case has been used for the usual traffic assignment. If the nodal passing time is assumed to be separated by the directions of the traffic flow, both inflow and outflow directions, the node can be described as shown in Fig. 4. In this case, left turn, right turn and straight directions are distinguished from each other if the node corresponds to the intersection of a street network. The cases shown in Fig. 2 and Fig. 3 stand

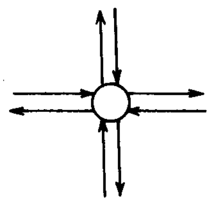

Fig 1. Usual Case without Node Passing Time

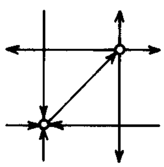

Fig 2. Simplest Model with Node Passing Time

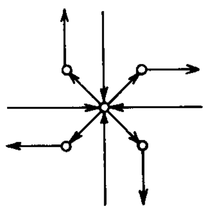

Fig 3. Outflow Directions Distinguished

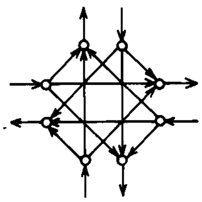

Fig 4. All Directions Distinguished

between the above tow cases. If the outflow direction is considered, while the inflow direction is not, Fig. 4 yields to Fig. 3. Further, if the outflow direction is not distinguished, Fig. 3 yields to Fig. 2. Consequently, Fig. 2 presents the simplest model of a node which includes the nodal passing time.

In this paper, we consider the case shown in Fig. 2 for two reasons. One reason is that this simplest case will give the most basic founding for the network design including a nodal capacity. The other reason is that the nodal capacity may not be easily divided into several directions. It means that the capacity for each direction is not independent of each other, and it is difficult to put the capacities to separate handling. It is noticed, however, that the case of Fig. 2 is not enough for the detailed analysis of a node, and the most appropriate case should be studied according to the problem of the network.

Here, we refer to the work by Sasaki and Inouye (1974)[9] , which makes a simple model of a node and introduces the nodal passing time function. They formulate the UE traffic assignment by the analogy of an electric circuit. The assumptions for the model of a node and the nodal passing time function are stated as follows.

1. The nodal passing time T_n of node n is not distinguished by its inflow direction and outflow direction.
2. T_n is defined as the function of the total outflow of traffic Q_n (or total inflow Q_n').
3. The relation between T_n and Q_n (Q_n') is assumed to be linear.
4. If T_n is a function of Q_n, T_n is loaded at the link where the outflow will

meet.

Assumptions 1 and 4 are equivalent in assuming a node as the one shown in Fig. 2. Assumption 3 is necessary because Sasaki and Inouye formulate the UE traffic assignment by the analogy of an electric circuit. However, this should be relaxed: the value of T_n increases monotonously, not necessarily linearly, with Q_n.

If we formulate a traffic assignment as an optimizing problem, it is convenient to assume that a link travel time is the function of its link flow, and never affected by the value of the other links. This assumption is similarly applied for the nodal passing time and corresponds to assumption 2.

Moreover, the considering assignment problem makes no sense if the nodal capacity is not involved in the problem. In other words, the nodal passing time function must include the nodal capacity. The following types of functions will satisfy the requested conditions.

$$T_n = t_{no}\ (1 + r\ (Q_n/C_n)^k\ ) \qquad (1.\ a)$$

$$T_n = t_{no} /\ (1 - (Q_n/C_n)^m\ ) \qquad (1.\ b)$$

t_{no}; nodal passing time for $Q_n = 0$

C_n; nodal capacity

r, k, m; parameters

The capacity C_n for (1.a) is not the limit prohibiting an excess, but the maximum value of the flow which is in a rather smooth condition. The function (1.a) may be practical because the nodal passing time rarely increases to an infinite value if the flow increases closely to the capacity. Further, it is found that the function (1.a) approximates Webster's delay function at the signalized intersection. Then, we employ the same type of function (1.a) as the nodal passing time function.

2.2. Formulation of Assignment Problem

Based on the above mentioned assumptions on the description of a node and the nodal passing time function, we can formulate the traffic assignment problem ruled by the user equilibrium for the given capacities of links and nodes. In addition to the assumption on the travel time function of the network, the travel demand is assumed to be given and fixed at the level of the origin and destination flow. The formulation of the UE traffic assignment problem including the nodal capacity is similar to the usual fixed demand UE problem, and written as the following optimization problem (P1):

(P1)

$$\min \sum_a \int_0^{V_a} S_a(x) \cdot dx + \sum_n \int_0^{Q_n} T_n(y)\, dy \qquad (2.a)$$

s. t.

$$\sum_m h_{mij} = X_{ij} \quad \text{for all } i, j \qquad (2.b)$$

$$V_a = \sum_{m,}\sum_{i,}\sum_{j} d_{amij}\, h_{mij} \quad \text{for all } a \qquad (2.c)$$

$$Q_n = \sum_{m,}\sum_{i,}\sum_{j} e_{nmij}\, h_{mij} \quad \text{for all } n \qquad (2.d)$$

$$h_{mij} \geqq 0 \quad \text{for all } m, i, j \qquad (2.e)$$

Here the notations are:

V_a ; traffic flow on link a

Q_n ; traffic flow on node n

$S_a(x)$; travel time function of link a

$T_n(y)$; nodal passing time function of node n

X_{ij} ; travel demand between origin i and destination j

h_{mij} ; traffic flow of path m between i and j

d_{amij} (e_{nmij}) ; $=1$ if path m between i, j includes link a (node n)
$=0$ otherwise

The decision variables of this problem are the path flows h_{mij}. The constraint (2.b) means the condition of the origin and destination flow conservation. Eq. (2.c) and (2.d) denote the relation between the path flow and link flow or nodal flow. If we put these equations into the objective function, the link flow and the nodal flow can be eliminated from the formulation.

It is easily proved that the solution of the problem (P1) gives the UE condition including nodes. That is derived in the same way as for the usual UE problem. If we show that the optimal condition of (P1) corresponds to the equal travel time condition, (P1) is equivalent to the UE problem.

The constraints of (P 1) are linear equality equations and non-negative conditions for the flow variables. The Kuhn-Tucker conditions of (P 1) are written as follows.

$$\sum_a S_a\ (V_a^*)\ d_{amij}+\sum_n T_n\ (Q_n^*)\ e_{nmij}-W_{ij}^*=0,\ \text{if } h_{mij}^*>0 \qquad (3.a)$$

$$\geqq\ 0,\ \text{if } h_{mij}^*=0 \qquad (3.b)$$

$$\sum_m h_{mij}^*=X_{ij} \quad \text{for all } i,\ j \qquad (3.c)$$

$$h_{mij}^*\geqq 0 \quad \text{for all } m,\ i,\ j \qquad (3.d)$$

Here

h_{mij}^*; optimal path flow

$V_a^*=\sum_m\sum_i\sum_j d_{amij}\,h_{mij}^*$

$Q_n^*=\sum_m\sum_i\sum_j e_{nmij}\,h_{mij}^*$

W_{ij}^*; optimal Lagrange multiplier

The Kuhn-Tucker conditions show Wardrop's first principle which is the UE condition.

3. Network Design Model

3. 1. Assumption

The policy decision making process considering the user equilibrium of the road netwok flow can be formulated as the Stackelberg Game model with two players: the network planner and the aggregated means of the network user. Several assumptions for both planner and user are necessary in advance of the formulation of a road netwok design problem. The assumptions for the behavior of the network planner are stated as follows.

At first, the objective of the netwok planner is mainly focused on the minimization of the total travel time of the whole network. The total travel time is one of the available and important measures which can evaluate the direct cost of the network performance. (The total investment/operational cost is discussed later.) The planner does not explicitly consider the other social costs which are brought on by the network improvement. These factors could be included in the constraints of the problem if they must be discussed.

Secondarily, the traffic capacity, a continuous variable, is chosen as the decision variable of the planner. It is needless to say that the capacity of a node and that of a link are separately handled. Actually, it is a rare case that the new network is constructed for a place without existing roads. Therefore, the planner considers the extension and improvement of the existing network. Further, the set of nodes and links for the improvement is given by the plan of the higher level.

Thirdly, every improvement cost of the link and the node is additive and the upper limit of the total improvement cost is given. The planner must find the value of the capacity which minimizes the total travel time within the limit of the total investment cost.

The fourth assumption is the type of the improvement cost function of each link and node. The simplest form, the linear function, is used for the cost function. The improvement cost linearly increases according to the amount of the capacity

$$G_a\ (Z_a)\ = g_a Z_a \qquad (4.a)$$

$$G_n\ (Z_n)\ = g_n Z_n \qquad (4.b)$$

Here,

$G_a\ (Z_a)$, $G_n\ (Z_n)$; the improvement cost function of link a (node n)
Z_a, Z_n ; the capacity of link a (node n) which increases by the improvement
g_a, g_n ; the unit cost of the improvement on link a (node n)

The length of link, land price and geographical features should be considered in the value of g_a, and the circumstances of nodal and signal control cost might be reflected in the value of g_n.

The fifth assmuption is the type of the performance functions: the link travel time function and the nodal passing time function. These functions are necessary to describe the user's behavior as well as to compose the planner's objective function. We employ the form of Eq. (1.a) for both the link travel time function and the nodal passing time function. The performance functions are:

$$S_a\ (V_a,\ Z_a)\ = t_a\ (1 + r_1\ (V_a /\ (C_a + Z_a))^{k_1}\ ) \qquad (5.a)$$

$$T_n\ (Q_n,\ Z_n)\ = t_n\ (1 + r_2\ (Q_n/\ (C_n+Z_n))^{k_2}\ ) \tag{5.b}$$

The notations are;

C_a, C_n; the existing value of the capacity of link a (node n)

r_1, k_1; parameters for link, common for all links

r_2, k_2; parameters for node, common for all nodes

The other notations have the same meaning as in the previous section. For the case of a new construction of node and link, the existing capacities are set to be zero.

Besides these assumptions for the planner, the following three assumptions are introduced to the user's behavior for given network capacities. At first, the travel demand is given and fixed at the level of the trip distribution between the origin and the destination. It means that the trip table is externally estimated, and the origin–destination flows are not varied if the network capacities are improved.

Secondarily, the available travel mode for the user is limited to the automobile. Other travel modes, such as buses and bikes are not used.

Thirdly, the road traffic flow is assumed to be the user equilibrium flow that is explained and formulated in the previous section. It means that the every user can choose his or her own route (travel path) from the origin to the destination.

3.2. Formulation

Based on the above-mentioned assumptions, a network design model is formulated as follows.

(P2)

$$\min \sum_a S_a\ (V_a,\ Z_a)\ V_a + \sum_n T_n\ (Q_n,\ Z_n)\ Q_n \tag{6.a}$$

s. t.

$$\sum_a G_a\ (Z_a)\ + \sum_n G_n\ (Z_n)\ \leqq G \tag{6.b}$$

$$L_a \leqq Z_a \leqq H_a \quad \text{for all } a \tag{6.c}$$

$$L_n \leqq Z_n \leqq H_n \quad \text{for all } n \tag{6.d}$$

$$\min \sum_a \int_0^{V_a} S_a(x, Z_a)\, dx + \sum_n \int_0^{Q_n} T_n(y, Z_n)\, \mathrm{dy} \qquad (6.e)$$

s. t.

$$\sum_m h_{mij} = X_{ij} \quad \text{for all } i, j \qquad (6.f)$$

$$V_a = \sum_{m,} \sum_{i,} \sum_{j} d_{amij}\, h_{mij} \quad \text{for all } a \qquad (6.g)$$

$$Q_n = \sum_{m,} \sum_{i,} \sum_{j} e_{nmij}\, h_{mij} \quad \text{for all } n \qquad (6.h)$$

$$h_{mij} \geqq 0 \quad \text{for all } m, i, j \qquad (6.i)$$

The objective function of the planner, the total travel time, is written as Eq. (6. a). The first and the second terms of the equation denote the total travel time of links and that of nodes respectively. Eq. (6.b) means the total improvement cost constraint, and G denotes the upper limit of the investment. The upper and lower bounds for both link and nodal capacities are introduced and written as Eq. (6.c) and (6.d). The most relaxed case of the bounds are;

$$L_a = L_n = 0 \qquad (7.a)$$

$$H_a = H_n = +\infty \qquad (7.b)$$

The UE traffic flow is obtained as the solution of the optimizing problem shown in Eq. (6.e), (6.f), (6.g), (6.h) and (6.i). This optimizing problem constraints the whole optimizing problem. We refer to this problem as the lower or sub puoblem, while the problem shown in Eq. (6.a), (6.b), (6.c) and (6.e) is referred to as the upper or master problem. Therefore, the structure of the whole problem is interpreted as a two level optimization problem, which has the one optimization problem, lower problem, as its constraints.

The framework of the Stackelberg Game with two players is employed in order to explain the formulated network design problem. In the Stackelberg Game, one of the players is called the leader who has complete information on the behavior of the other player, who is called the follower, The leader has the advantage to decide his strategy, considering the behavior of the follower. In contrast with the leader, the follower knows the strategy given from the leader, and he has no information on the decision process of the leader.

The typical Stackelberg Game is formulated as the two level optimization problem as follows. Player 1 and the player 2 correspond to the leader and the follower respectively.

(P 3)

$$\min_{X} F_1 (\boldsymbol{X}, \ \boldsymbol{Y'} (\boldsymbol{X})) \tag{8.a}$$

s. t.

$$\boldsymbol{G}_1 (\boldsymbol{X}, \ \boldsymbol{Y'} (\boldsymbol{X})) \leqq 0 \tag{8.b}$$

$$F_2 (\boldsymbol{X}, \ \boldsymbol{Y'} (\boldsymbol{X})) = \min_{Y} F_2 (\boldsymbol{X}, \ \boldsymbol{Y}) \tag{8.c}$$

s. t.

$$\boldsymbol{G}_2 (\boldsymbol{X}, \ \boldsymbol{Y}) \leqq 0 \tag{8.d}$$

The notations are;
$\boldsymbol{X}$, $\boldsymbol{Y}$; vector of the decision variables of players 1 and 2
$\boldsymbol{Y'} (\boldsymbol{X})$; parametric optimum solutions of the sub problem for given $\boldsymbol{X}$
$F_1 (\boldsymbol{X}, \boldsymbol{Y})$, $F_2 (\boldsymbol{X}, \boldsymbol{Y})$; objective function of players 1 and 2
$\boldsymbol{G}_1 (\boldsymbol{X}, \boldsymbol{Y})$, $\boldsymbol{G}_2 (\boldsymbol{X}, \boldsymbol{Y})$; constraints vector of players 1 and 2

The relations between the formulated NDP (P 2) and the typical Stackelberg Game (P 3) are summarized in Table 1.

Table 1. The Relations NDP with Stacklberg Game

	leader	follower
player	network planner	network user
decision variable	capacity of link and node	path flow
behavioral principle	total travel time minimization	individual travel time minimization (UE)
objective function	F_1 = (6. a)	F_2 = (6. b)
constraints	(6.b), (6.c), (6.d)	(6.f), (6.g), (6.h), (6.i)

3.3. Solution Algorithm

Shimizu (1982)[10] proposed a general solution algorithm for the Stackelberg problems. His algorithm is expected to give the exact solution for the NDP. If the sub problem is replaced by its necessary and sufficient conditions, the problem could be reduced to the usual non-linear problem, and solved by some appropriate programming algorithms. This procedure, however, may not be so effective for a case with a large number of variables and constraints as the formulated NDP.

Therfore, we employ a heuristic iterative optimization algorithm which is composed of the input/output phases, the UE assignment phase for the given capacity and the optimization phase of the network capacity up-dating for the given traffic flow. The solution procedure can be summarized as follows.

step 0 Set $m=1$ (m; iteration number)
Start with the initial feasible capacities Z_n^1, Z_a^1, and feasible network flow V_a^1, Q_n^1 satisfying the constraints.

Step 1 Solve the upper problem for the given flow variables.
$\min \sum_a S_a\ (V_a^m,\ Z_a)\ V_a^m + \sum_n T_n\ (Q_n^m,\ Z_n)\ Q_n^m$
s. t. (6.b), (6.c) and (6.d)
Set the new capacities Z_a^{m+1}, Z_n^{m+1}.

Step 2 Check the convergence of the value of capacities.
If the following criteria are satisfied, stop the calculation. Otherwise, set $m=m+1$ and go to step 3.

$|Z^{m+1}-Z^m| \leqq e$ $\quad e$; appropriate small positive

Step 3 Solve the UE assignment problem for given capacities Z_a^m, $Z_n^m$and obtain link flow V_a^m and nodal flow Q_n^m.

Still more explanations should be added to step 1 and step 3. For step 3, in order to solve the UE assignment including the nodal passing time, the Frank-Wolfe algorithm for a usual assignment problem can be used.

For step 1, the following decomposition can be applied. If we choose the appropriate value of the multiplier β, it is possible to include the cost constraint in the objective function.

$$\sum_a S_a\ (V_a^m,\ Z_a)\ V_a^m+\sum_n S_n\ (Q_n^m,\ Z_n)\ Q_n^m+\beta\ (\sum_a G_a\ (Z_a)\ +\sum_n G_n\ (Z_n)) \quad (9)$$

The reformulated master problem could be decomposed to the optimization problem for each node and link. They are:

$$\min\ \ S_a\ (V_a^m,\ Z_a)\ V_a^m+\beta G_a\ (Z_a) \quad (10.\text{a})$$

$$\text{s.t.}\ \ L_a \leqq Z_a \leqq H_a \quad (10.\text{b})$$

and

$$\min\ \ S_n\ (Q_n^m,\ Z_n)\ Q_n^m+\beta G_n\ (Z_n) \quad (11.\text{a})$$

$$\text{s.t.}\ \ L_n \leqq Z_n \leqq H_n \quad (11.\text{b})$$

When the improvement cost functions, $G_a\ (Z_a)$ and $G_n\ (Z_n)$, are given by Eq. (4.a) and the performance functions, $S_a\ (V_a,\ Z_a)$ and $S_n\ (Q_n,\ Z_n)$, are given by Eq. (5.a) and (5.b), the unique solution of Z_a and Z_n can be obtained as follows.

$$Z_a^m = \begin{cases} La & \text{if}\ \ V_a^m \leqq\ (L_a+C_a)\ f_a & (12.\text{a}) \\ f_a^{-1}\ V_a^m - C_a & \text{if}\ \ (L_a+C_a)\ f_a < V_a^m \leqq\ (H_a+C_a)\ f_a & (12.\text{b}) \\ H_a & \text{if}\ \ (H_a+C_a)\ f_a < V_a^m & (12.\text{c}) \end{cases}$$

and

$$Z_n^m = \begin{cases} L_n & \text{if}\ \ Q_n^m \leqq\ (L_n+C_n)\ f_n & (13.\text{a}) \\ f_n^{-1} Q_n^m - C_n & \text{if}\ \ (L_n+C_n)\ f_n < Q_n^m \leqq\ (H_n+C_n^m)\ f_n & (13.\text{b}) \\ H_n & \text{if}\ \ (H_n C_n)\ f_n < Q_n^m & (13.\text{c}) \end{cases}$$

Here, the values of f_a and f_n are composed by the constant variables as follows.

$$f_a=\ (k_1 r_1 t_a/\beta\ g_a)^{1/(k_1+1)} \quad (14.\text{a})$$

$$f_n=\ (k_2 r_2 t_n/\beta\ g_n)^{1/(k_2+1)} \quad (14.\text{b})$$

The values of Z_a^m and Z_n^m may not satisfy the total investment constraint. Then, the value of β must be arranged so that the capacity satisfies the following eqution.

$$\sum_a G_a\ (Z_a^m)\ +\sum_n G_n\ (Z_n^m)\ =G \quad (15)$$

Instead of the inequality cost constraint shown in Eq. (6.b), the equality constraint Eq. (15) is employed, because the optimal solutions are expected to be obtained at the maximum level of the investment to the network.

The exact algorithm should be developed instead of this heuristic solution procedure, for example, applying the idea proposed by Fisk (1984)[5] for TSSP. She presented the solution algorithm for TSSP with a UE traffic flow, which is also applicable for NDP with slight modifications. A discussion on this point will appear in forthcoming studies.

3.4. Numerical Example

A numerical example is executed for hypothetical data set in order to find the numerical features of the formulated problem. The figure of the existing network is shown in Fig. 5, and the values of the input variables are listed in Tables 2, 3 and 4. The value of the parameters of the performance functions are set to:

$r_1 = r_2 = 2.62$

$k_1 = k_2 = 5$

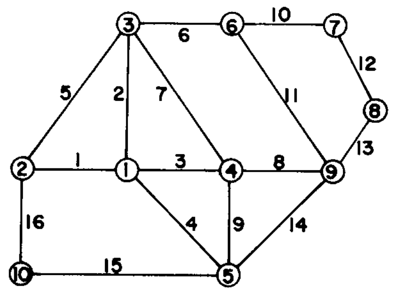

Fig 5. Existing Network For Improvement

Table 2. OD Matrix

OD	1	2	3	4	5	6	7	8	9	10	sum
1	0	3000	5394	3709	1418	776	139	624	300	348	15708
2		0	1413	201	11	104	45	144	54	42	5114
3			0	931	39	2716	47	85	115	4	10744
4				0	235	60	12	28	240	3	5419
5					0	27	13	39	841	3	2716
6						0	255	124	192	1	4255
7							0	6664	295	1	7471
8								0	2682	2	10392
9									0	0	4719
10										0	404

Table 3. Input Variables for Link

a	1	2	3	4	5	6	7	8
t_a	21	22	36	66	65	19	40	35
C_a	4800	4800	1200	1200	4800	1200	1200	1200
g_a	2280	2312	3840	2648	2584	2024	1584	4256
a	9	10	11	12	13	14	15	16
t_a	50	11	12	23	48	38	49	72
C_a	1200	1200	1200	4800	4800	1200	1200	1200
g_a	3980	4396	4772	2456	5128	15036	1968	2860

$H_a = 12000$ and $L_a = 0$ for all links
t_a; min. C_a, H_a, L_a; veh./day
g_a; $\times 10^3$ yen/veh./day

Table 4. Input Variables for Node

n	1	2	3	4	5	6	7	8	9	10
C_n	22800	14400	12000	8400	4800	7200	6000	9600	8400	2400
g_n	3200	1600	1600	1600	800	1600	1600	3200	1600	800

$t_n = 5$, $H_n = 50000$ and $L_n = 0$ for all nodes
t_n; min. C_n, H_n, L_n; veh./day
g_n; yen/veh./day

Table 5. Convergency for $\beta = 3 \times 10^{-5}$

iteration	F_1	Time	Cost	F_2
1	6.70	4.18	84.0	2.67
2	6.45	4.14	76.7	2.89
3	6.36	4.15	73.5	2.92
4	6.31	4.17	71.6	2.93
5	6.30	4.16	70.9	2.94
6	6.29	4.18	70.6	2.95
7	6.29	4.18	70.3	2.95

F_1 = Time + Cost ($\times 10^6$ min.)
Time; total travel time ($\times 10^6$ min.)
Cost; total inverstment cost ($\times 10^9$ yen)
F_2; value of objective function of UE problem

In order to avoid the complexities of resetting the value of β, the original NDP is slightly modified: the total investment cost is included in the objective function using the externally given value of β. The first calculation is executed for the fixed value of β and the second one for the parametric changing β. This case is equivalent to solving the original problem.

For a case where the value of β is set to 3.0×10^{-5}, the convergence of the value of the total travel time, the total investment cost and the value of the objective function of the sub problem are shown in Table. 5. Although some of these values are not converged, the value of the temporal objective function, the sum of the total travel time and the total investment cost, tend to converge. Therefore, the number of iterations seems to be enough for this example.

The values of the link capacity and the nodal capacity are shown in Fig. 6. It is found that the optimum capacity, which is the sum of existing and extended values, and the traffic flow, is balanced at every node, while the optimum capacity is 1.5 times the traffic flow at every link. This result means that the planner should keep the better performance for the part of the nodal section. In particular, he should invest more money for the improvement of the nodes numbered 1, 2 and 3 and the links numbered 1, 2 and 5.

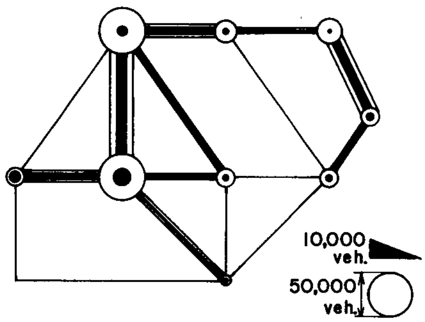

Fig 6. Current and Extended Capacity

If we sequently solve the NDP, including the total improvement cost in the objective function for the parametric varied β, it is equivalent to solving the original NDP for the parametrically changing total cost constraint. The results of the sensitivity analysis, the calculation for the value of β changing from 3.0×10^{-8} to 3.0×10^{-1}, are shown in Tab. 6.

For the increasing value of β, the total travel time increases, while the total investment cost decreases. We can assume the case for $\beta = 3 \times 10^{-1}$ as the current situation in which the network is not improved at all. If the planner decides to invest at the level for the case of $\beta = 3 \times 10^{-8}$, the total travel time could be reduced to 1 / 6 the level of the current network. Between the cases of $\beta = 3.0 \times 10^{-8}$ and $\beta = 3.0 \times 10^{-7}$, the difference of the total investment cost accounts to twice the amount, while the total travel time could be saved slightly. It means that any further effect by the investment may not be expected. These findings are useful for finding the appropriate level of the investment on the road network, as well as the effective combination of the improvement of the links and the nodes.

Table 6. Sensitivity for the value of β

β	F_1	Time	Cost	Lcost	Ncost
3×10^{-8}	2.73	2.72	531.0	530.0	1.0
3×10^{-7}	2.85	2.74	359.0	358.0	1.0
3×10^{-6}	3.53	2.92	203.0	202.5	0.55
3×10^{-5}	6.29	4.18	70.3	69.9	0.34
3×10^{-4}	13.30	9.60	12.5	12.3	0.18
3×10^{-3}	18.00	17.70	0.07	0	0.07
3×10^{-2}	18.50	18.20	0.01	0	0.01
3×10^{-1}	19.00	19.00	0	0	0

Lcost; total investment cost for link ($\times 10^9$ yen)
Ncost; total investment cost for node ($\times 10^9$ yen)
Cost = Lcost + Ncost

4. DISCUSSIONS

The optimal road network design problem considering the nodal capacity as well as the link capacity is shown in this paper, and the following points should be discussed in any future research.

Instead of the simple representation of a node used in this study, the alternative model of a node should be accepted when the proposed network design problem is applied for the microscopic case, for example, the traffic signal settings. If the nodal capacity is divided into several directions, and the nodal passing time function is defined independently for each direction, a formulation and solution procedure similar to this study may be used with a slight modification. It may be suitable to relax the assumption on the independence for some actual cases. However, the mutual dependence of the passing time functions in the same node may cause the problem to have a complicated form. In relation to the modeling of a node, a suitable form and a parameter value of the node passing time function should be considered.

The solution procedure remains at the heuristic stage, and it is noted that the Stackelberg problem can not be solved exactly in this procedure. The heuristic procedure is accepted because it may be applicable for the actual size of the road network design. An exact algorithm, however, should be developed to know the detailed numerical features of the problem.

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