Dynamic Stability Analysis of Large-scale Power System by the Frequency Response Method

by

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Abstract

A frequency response method has been proposed to analyze a dynamic stability on power oscillations of large interconnected power systems. However, its application to large-scale power systems is severely limited because of its long computation time. In this paper, several measures were proposed to remove this disadvantage. A method of directly calculating the determinant of an operational transfer matrix was first proposed in order to shorten the computation time at each frequency. The determinant is efficiently obtained by utilizing sparsity natures of three intermediate matrices. Multiple step sizes of frequency were next introduced instead of a fixed step size in order to reduce the number of frequency points to be scanned. These measures were applied to an example system with 107 generators. The computation time for each frequency response was reduced to 1 / 22 of the usual method. The number of frequency points was reduced to 28% of the usual method. The total computation time was reduced to 17.1 seconds, which corresponds to 1 / 75 of the usual method. These results verify the effectiveness of the proposed measures.

1. Introduction

Dynamic stability studies of power systems are widely performed in power system plannings and operations. Several methods such as simulations and eigen value analysis are used as tools for these studies according to their purposes and sizes of object systems [1-10]. The frequency response method is one of these methods, and it has been proposed to analyze the stability on power oscillations of large interconnected power systems [6].

This method deals with a matrix called operational transfer matrix. The dimension of the matrix is equal to the number of generators in an object system. Therefore, the method is not restricted by computation memories, and

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its application to large-scale power systems is expected. However, this method has the disadvantage that it needs a long computation time to obtain a whole frequency response. The whole response consists of many responses at different frequencies, and the problem is separated into two parts. One of them is concerned with the calculation of a response at each frequency. The size of the operational transfer matrix gets larger as the number of generators increases. It is a time-consuming job to calculate the matrix and its determinant. The other is that the number of frequencies to be scanned increases because a smaller step size of frequency must be used, which means that the amount of calculation increases. Because of this disadvantage, the application of the method has been limited to power systems with generators fewer than 50 [7].

In this paper, several measures are proposed to remove this disadvantage. A method of directly calculating the determinant of the operational transfer matrix is first proposed in order to shorten the computation time at each frequency. The determinant is obtainable through three intermediate matrices. Their sparsity natures make it possible to calculate their determinants with some efficient programming techniques [11]. Multiple step sizes of frequency are next introduced instead of a fixed step size in order to reduce the number of frequencies to be scanned. A fine step size is used in a frequency range where the response varies rapidly. A rough step size is used in a range where the response varies slowly. By using multiple step sizes, unnecessarily detailed calculation is omitted, and the amount of calculation is reduced.

Basic equations on the frequency response method are provided in Chapter 2. The direct method of calculating the determinant of the object matrix is described in Chapter 3. The method of selecting the step sizes is described in Chapter 4. Conclusions of this paper are summarized in Chapter 5.

2. Basic Equations

Basic equations on the frequency response method are described in this chapter. Several equations which express a power system consisting of m generators, n buses, and l transmission lines are given first. These equations are linearized next at an operating point in order to derive a fundamental equation for a dynamic stability study on power oscillations. Based on this equation, the frequency response method is explained last.

2.1 System equations

The main components of the power system are generators, transmission lines, and loads. These components are modeled as follows. Each generator is a synchronous machine, and its motion is described by

$$m_i \frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei} \tag{1}$$

$$T'_{doi} \frac{dE'_{qi}}{dt} = E_{fdi} - E'_{qi} - (x_{di} - x'_{di})\, i_{di} \tag{2}$$

$$v_{di} = x_{qi} i_{qi} \tag{3}$$

$$v_{qi} = E'_{qi} - x'_{di} i_{di} \tag{4}$$

$$P_{ei} = v_{di} i_{di} + v_{qi} i_{qi} \tag{5}$$

$$for\ i = 1, \ldots, m$$

where for each generator i,

P_{mi} : mechanical power input,

P_{ei} : electric power output,

m_i : angular momentum constant,

δ_i : rotor angle relative to a network frame rotating at synchronous speed,

E'_{qi} : electromotive force due to field flux linkage,

E_{fdi} : excitation voltage,

i_{di}, i_{qi} : d–axis, q–axis armature currents,

v_{di}, v_{qi} : d–axis, q–axis terminal voltages,

x_{di}, x_{qi} : d–axis, q–axis synchronous reactances,

x'_{di} : d–axis transient reactance,

T'_{doi} : d–axis transient open–circuit time constant.

The axes "d" and "q" are those in a generator frame fixed to each rotor. The mechanical power input P_{mi} is regulated by a speed governor, and the excitation voltage E_{fdi} is controlled by an automatic voltage regulator (AVR) and, if necessary, by a power system stabilizer (PSS). The dynamics of these regulators are not shown here for brevity's sake.

Transmission lines and buses form a network. The terminal voltages and injected currents of the buses in the network are related by

$$\begin{bmatrix} i_{Q1} \\ i_{D1} \\ \vdots \\ i_{Qn} \\ i_{Dn} \end{bmatrix} = \begin{bmatrix} b_{11} & g_{11} \cdots\cdots b_{1n} & g_{1n} \\ g_{11} & -b_{11} \cdots\cdots g_{1n} & -b_{1n} \\ \vdots & \vdots \qquad \vdots & \vdots \\ b_{n1} & g_{n1} \cdots\cdots b_{nn} & g_{nn} \\ g_{n1} & -b_{n1} \cdots\cdots g_{nn} & -b_{nn} \end{bmatrix} \begin{bmatrix} v_{D1} \\ v_{Q1} \\ \vdots \\ v_{Dn} \\ v_{Qn} \end{bmatrix} \tag{6}$$

where for each bus i,

i_{Di}, i_{Qi} : D–axis, Q–axis injected currents,

v_{Di}, v_{Qi} : D–axis, Q–axis terminal voltages,

g_{ij}, b_{ij} : conductance and suceptance components of the network admittance between the i–th and j–th buses.

The axes "D" and "Q" are those in the network frame. The first m buses correspond to the generators, and the remaining $(n–m)$ buses correspond to the loads. Eq. (6) can be expressed in matrix form as follows;

$$I=YV \tag{7}$$

where Y is a $2n \times 2n$ admittance matrix, and I, V are the current and voltage vectors of dimension $2n$, respectively. The transformation of current and voltage from each generator frame to the network frame is performed by

$$\begin{bmatrix} v_{di} \\ v_{qi} \end{bmatrix} = T_i \begin{bmatrix} v_{Di} \\ v_{Qi} \end{bmatrix} \qquad \begin{bmatrix} i_{qi} \\ i_{di} \end{bmatrix} = T_i^t \begin{bmatrix} i_{Qi} \\ i_{Di} \end{bmatrix} \tag{8}$$

for $i = 1, \ldots, m$, where the superscript "t" denotes the transposition of the matrix. T_i is a transformation matrix defined by

$$T_i = \begin{bmatrix} \sin\delta_i & -\cos\delta_i \\ \cos\delta_i & \sin\delta_i \end{bmatrix} \tag{9}$$

The loads in the system are all treated as constant admittances, and have already been incorporated to the network admittances. Hence, the injected currents to the load buses are zero, namely,

$$i_{Di} = i_{Qi} = 0 \qquad \text{for } i = m+1, \ldots, n \tag{10}$$

hold at each load bus.

2. 2 Linearization

The power system is described by Eqs. (1) - (10). Some of these equations are nonlinear, however, so it is necessary to linearize them at an operating point in order to perform dynamic stability studies by the frequency response method. Eqs. (2) - (4) and (8) are linearized to derive a relation

$$\begin{bmatrix} \Delta i_{Qi} \\ \Delta i_{Di} \end{bmatrix} = b_{1i}(p) \begin{bmatrix} \Delta v_{Di} \\ \Delta v_{Qi} \end{bmatrix} + b_{2i}(p)\ \Delta\delta_i \tag{11}$$

where

$$b_{1i}(p) = T_i^o X_i^{-1} \{T_i^o + G_i AVR_i \begin{bmatrix} 0 & 0 \\ v_{Di}^o / v_{ti}^o & v_{Qi}^o / v_{ti}^o \end{bmatrix}\}$$

$$b_{2i}(p) = T_i^o X_i^{-1} \{ \begin{bmatrix} v_{qi}^o \\ -v_{di}^o \end{bmatrix} - X_i \begin{bmatrix} -i_{di}^o \\ i_{qi}^o \end{bmatrix} - G_i AVR_i PSS_i p \begin{bmatrix} 0 \\ 1 \end{bmatrix} \}$$

p : time derivative operator $(=d/dt)$,

Δ : small variance,

o : operating point,

X_i : diag $(x_{qi}(p),\ -x_{di}(p))$,

$x_{di}(p)$: d-axis operational reactance, $=x_{di}(1+T'_{di}p)/(1+T'_{doi}p)$,

$x_{qi}(p)$: q-axis operational reactance, $= x_{qi}$,

$G_i(p)$: transfer function of field winding,

$AVR_i(p)$: transfer function of AVR,

$PSS_i(p)$: transfer function of PSS.

Eqs. (1) and (5) are linearized to obtain equation

$$(m_i p^2 + b_{4i}(p))\ \Delta\delta_i = -\ b_{3i}(p) \begin{bmatrix} \Delta v_{Di} \\ \Delta v_{Qi} \end{bmatrix} \tag{12}$$

where

$$b_{3i}(p) = [\ v_{Qi}^o\ \ v_{Di}^o\ ]\ b_{1i}(p) + [\ i_{Di}^o\ \ i_{Qi}^o],$$

$$b_{4i}(p) = [v^{\circ}_{Qi}\ v^{\circ}_{Di}]\ b_{2i}(p) + GOV_i p,$$

$GOV_i(p)$: *transfer function of speed governor.*

Linearized form of (7) is clear, namely,

$$\Delta I = Y \Delta V \tag{13}$$

If we substitute (11) into (13) and eliminate current ΔI, then a relation between ΔV and $\Delta\delta$ is obtained, i. e.,

$$(Y - B_1)\ \Delta V = B_2 \Delta\delta \tag{14}$$

where

$$B_1 = \left[\begin{array}{ccc|c} b_{11} & & & \\ & \ddots & & 0 \\ & & b_{1m} & \\ \hline & 0 & & 0 \end{array}\right] \qquad B_2 = \left[\begin{array}{ccc} b_{21} & & \\ & \ddots & \\ & & b_{2m} \\ \hline & 0 & \end{array}\right]$$

On the other hand, the matrix form of (12) is given as follows;

$$(Mp^2 + B_4)\ \Delta\delta = -B_3 \Delta V \tag{15}$$

where

$$B_3 = \left[\begin{array}{ccc|c} b_{31} & & & \\ & \ddots & & 0 \\ & & b_{3m} & \end{array}\right] \qquad B_4 = \left[\begin{array}{ccc} b_{41} & & \\ & \ddots & \\ & & b_{4m} \end{array}\right]$$

If we eliminate voltage ΔV from (14) and (15), then a relation on rotor angles is obtained, namely,

$$F(p)\ \Delta\delta = 0 \tag{16}$$

where

$$F(p) = (Mp^2 + B_4) + B_3\ (Y-B_1)^{-1}\ B_2 \tag{17}$$

This equation is a fundamental equation which describes the dynamic stability natures on power oscillations of the power system. $F(p)$ is a $m \times m$ matrix, and each element is a function of operator p. This matrix is called an operational transfer matrix.

2. 3 Frequency response method

The frequency response method is briefly introduced here. For a detailed explanation, refer to [6]. The system consists of m generators, so it has m power oscillation modes. These modes correspond to eigen values of (16). Each eigen value of p is such that

$$|\ F(p)\ | = 0 \tag{18}$$

is satisfied, where $|\quad|$ means the determinant of the matrix. If it is possible to obtain all the eigen values by solving (18), then the stability of the system is made clear. It is not easy to find all of them because the determinant is an extremely complex function of operator p [5, 7]. The frequency response method has been proposed in order to avert this difficulty [6].

In this method, operator p is replaced by an imaginary number, i. e.,

$$p = j\omega \tag{19}$$

where ω is an angular frequency (unit: rad/s). When this ω varies as follows;

$$\omega = 0 \rightarrow \infty \tag{20}$$

determinant $|\ F\ (j\omega)\ |$ moves on the complex plane as shown in Fig. 1. The stability of the system is judged from this behavior. The determinant rotates round the origin of the complex plane. If the number of rotations is equal to $m/2$, then the system is stable, where m is the number of generators. "Stable" means that all eigenvalues have negative real parts and all power oscillation modes decay with the passing of time. The number of rotations decreases one by one for each eigenvalue with a positive real part, so it is possible to know the number of unstable oscillation modes from the number of rotations. Besides, if operator p is chosen as follows;

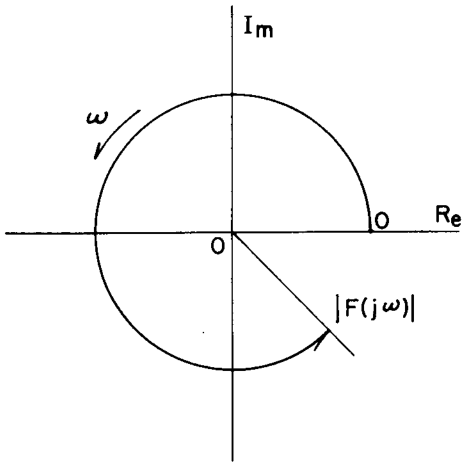

Fig 1. Movement of the frequency response.

$$p = \sigma + j\omega \tag{21}$$

where σ is a constant, then it is possible to compare the real parts of the eigenvalues with this constant instead of 0. The number "∞" in (20) is replaced by a finite number, *e. g.*, 20 rad/s, in actual calculation.

3. Calculation of Frequency Response

It is necessary to calculate the determinant of the operational transfer matrix F at each frequency in order to apply the frequency response method. Hence, it is very important to do this calculation swiftly. One method which satisfies this requirement is proposed in this chapter.

3. 1. Usual method

When obtaining a response at each frequency, the usual way is to calculate the transfer matrix F first, and then calculate its determinant. However, this is a very time-consuming way of calculation. The reason for it is shown as follows:

The transfer matrix F is expressed by (17). According to this equation, F contains a $2n \times m$ matrix, i.e.,

$$(Y - B_1)^{-1} B_2 \tag{22}$$

where $Y - B_1$ and B_2 are $2n \times 2n$, $2n \times m$ matrices, respectively. There are several methods of obtaining this matrix. One of them is to solve a set of linear equations

$$Dx = b \tag{23}$$

where $D = Y - B_1$, and x, b are 2 n-dimension vectors. Each column of B_2 is substituted to b, and (23) is solved for b, then its solution x becomes a corresponding column of the object matrix. Eq. (23) is solved as follows. Matrix D is factorized first into two unique triangular matrices L and U, that is,

$$D = LU \tag{24}$$

where

$$L = \begin{bmatrix} l_{11} & & 0 \\ \vdots & \ddots & \\ l_{N1} & \cdots & l_{NN} \end{bmatrix} \qquad U = \begin{bmatrix} 1 & u_{12} & \cdots & u_{1N} \\ & \ddots & & \vdots \\ & & \ddots & u_{\cdot\cdot} \\ 0 & & & 1 \end{bmatrix}$$

$$N = 2n$$

L and U are called the lower triangular matrix and upper triangular matrix, respectively. With these triangular matrices, (23) is replaced by two equivalent equations, i, e.,

$$Ly = b \qquad Ux = y \tag{25}$$

where y is an intermediate vector of 2 n-dimension. These equations can be easily solved by iterating substitutions forward and backward, respectively. Thus, solution x of (23) is obtained through two steps. The matrix D is structurally the same as the admittance matrix Y, as is clear from its definition. It is well known that Y is an extremely sparse matrix in general. This sparsity nature is widely utilized in load flow and other calculations. Namely, calculation is carried out only over non-zero elements of a matrix, and storage memories are allocated only for them [11]. By using this technique in the above two steps, it is possible to shorten computation time to a certain extent. However, the calculation of the matrix F still gets a heavy burden as generators increase in number because the second step shown by (25) must be repeated m times, where

m is the number of generators. Besides, the matrix F obtained in this way is a dense matrix, and accordingly, it takes considerable time to obtain its determinant, too.

As seen above, there are several factors to prolong the computation time in the usual way of calculating the determinant of the object matrix. These factors become a big obstacle to the application of the frequency response method to large-scale power systems.

3. 2 Proposed method

The problem described in the preceding section arises from calculating the operational transfer matrix. However, what is necessary is not the matrix but its determinant. If it is possible to obtain the determinant without the matrix, the problem is averted.

The matrix F in (17) can be rewritten as follows;

$$F = A - BD^{-1}C \tag{26}$$

where

$$A = Mp^2 + B_4 \quad : m \times m$$

$$B = -B_3 \quad : m \times 2n$$

$$C = B_2 \quad : 2n \times m$$

$$D = Y - B_1 \quad : 2n \times 2n$$

These matrices are combined, as in (26), to form the matrix F. There are several other ways of combining them, however. One example is a matrix defined as follows;

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{27}$$

This matrix is a large matrix containing the four matrices as its submatrices. The determinant of the matrix is closely related to that of the matrix F. There are two ways of expressing the determinant, i. e.,

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A| \ |D - CA^{-1}B| = |A - BD^{-1}C| \ |D| \tag{28}$$

By easy transformation of this equation,

$$|A - BD^{-1}C| = |A| \ |D - CA^{-1}B| \ / \ |D| \tag{29}$$

is obtained. The left hand side of this equation is clearly the determinant of the matrix F. Eq. (29) shows that the determinant can be obtained through the three determinants on the right hand side. It certainly seems inefficient to calculate three determinants for one determinant, but it is not true in this case.

There are two reasons for this. One is that it is easy to obtain three matrices A, D, and D' $(=D-CA^{-1}B)$, as is clear from their difinition. A is a diagonal matrix, and D and D' are obtained by adding small changes to the admittance matrix Y. Hence, the computation time necessary for them is short. The other is that it is also easy to calculate their determinants. Since A is a diagonal matrix, its determinant is given by taking the product of its diagonal elements, i. e.,

$$|A| = \prod_{i=1}^{m} (m_i p^2 + b_{4i}) \tag{30}$$

There is no difficulty in this calculation. On the other hand, D is a general matrix, and therefore it is not so easy as A to calculate its determinant. One method is to factorize D into two triangular matrices, as in (24). Since D is the product of two matrices L and U, its determinant is given as follows;

$$|D| = |L| \ |U| \tag{31}$$

From the forms of L and U, it is easily seen that their determinants are equal to the products of their diagonal elements, respectively. The diagonal elements of U are all 1, so its determinant is also 1. The determinant of D is accordingly given by

$$|D| = \prod_{i=1}^{N} l_{ii} \tag{32}$$

This equation is similar to (30) except that it needs a factorization of D. It has already been described that this factorization is performed efficiently with some programming technique based on the sparsity nature of the matrix. Likewise, there is no serious problem in calculating the determinant of D. The remaining part is to calculate the determinant of D'. This matrix has the same structure as the matrix D, as is easily verified by examining the forms of matrices A, B, and C. Hence, its determinant is obtainable in the same way as D. It is required only to factorize D' into two triangular matrices, as in (24). From these observations, it is concluded that all of the determinants on the right hand side of (29) can be obtained quickly, and it is not necessarily a roundabout way to calculate according to this equation.

3. 3 Example

Table 1 shows the computation times required to do dynamic stability studies of two example power systems by the usual frequency response method and the proposed method. One system consists of 107 generators, and the other consists of 9 generators. A frequency response is obtained in 53.3 ms for the 107-machine system by the proposed method, while 1178.8 ms is necessary in the case of the usual method. The ratio of time between them is 22.1. The frequency response method needs responses at many frequencies. If a frequency range 0～20 rad/s is scanned with step size 0.02 rad/s, then one thousand responses are obtained. It takes 19.6 minutes to do this scan by the usual method. On the other hand, the scan ends in 53.3 seconds if the proposed method is used. As for the 9-machine system, there is not so much difference

Table 1. Comparisons of computation time.

	107-machine system		9-machine system	
Number of responses	Usual method	Proposed method	Usual method	Proposed method
1	1179ms	53.3ms	6.47ms	3.09ms
1000	19.6min	53.3s	6.47s	3.09s
Ratio	22.1		2.09	

(Computer: FACOM M 382)

Table 2. Breakdown of computation time (ms).

	Usual method	Proposed method
Matrices	14.1 (D)	7.9 (A) 8.6 (D, D')
Factorization	14.2	14.1×2
Substitutions	816.9 (F)	–
Determinant	330.8	–
Others	2.8	8.6
Sum	1178.8	53.3

between the two methods, but the proposed method is still twice as fast as the usual method.

Table 2 shows a breakdown of the computation time to obtain a frequency response of the 107-machine system. In the case of the usual method, it does not take so long time to make the matrix D and to factorize it. However, it takes 816.9 ms and 330.8 ms to make the matrix F by the forward and backward substitutions and to calculate its determinant, respectively. The sum of these times occupies more than 97% of the total computation time. On the other hand, it is required to make two matrices D and D' and to factorize them in the case of the proposed method, but the time spent for the calculation is short. Conversely, this method does not need to make F and to calculate its determinant, thus saving much computation time.

4. Selection of Step Size

Fig. 2 shows the variation of the number of times which the frequency response of the 107-machine system rotates around the origin of the complex plane. A fixed step size was used to obtain this response, but the response does not vary uniformly. The speed of rotation varies with the frequency range. If a fixed number of responses is enough to trace each rotation, it is not the best way to use in slowly varying range a step size as small as in rapidly varying ranges. In this chapter, several step sizes are introduced in order to omit unnecessary frequency responses.

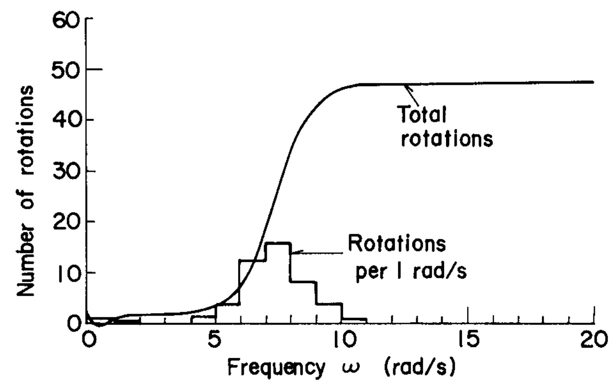

Fig 2. The number of rotations of the frequency response (107-machine system).

4. 1 Division of frequency

The frequency response varies slowly in ranges $\omega = 0 \sim 5$ and $10 \sim 20$ rad/s, and there is no big change in the number of rotations in these ranges. Most of the rotations appear in the frequency range $5 \sim 10$ rad/s, which comes from the fact that the eigenvalues of the system concentrate in this range. The rotation is conspicuous, especially in range $6 \sim 9$ rad/s, so temporarily assume that all rotations appear in this frequency range. The number of rotations is $m/2$, i. e, half the number of generators. If three sequence frequencies are necessary to see each rotation, then $3m/2$ of the frequency points must be scanned in this range. The number of points in each 1 rad/s is given by

$$(3m/2)/3 = m/2 \tag{33}$$

For example, if there are 100 generators in a system, then the 50 (= 100 / 2) frequencies must be checked every 1 rad/s in the range. The step size which should be used in this range is 0.02 (= 1 / 50) rad/s. In actual calculation, however, more than two points are enough to see each rotation, so 1.5 (= 3 / 2) times as many as $m/2$ rotations can been traced with this step size. Accordingly, $m/4$ of the rotations are safely traced every 1 rad/s, even if the rotations distribute unevenly in this range. It has been assumed until now that all rotations concentrate in $6 \sim 9$ rad/s, but this range may shift with systems. The range is extended to $5 \sim 10$ rad/s while keeping the step size at the value determined from (33), in order to take this situation into consideration. Rough step sizes are used in the other frequency ranges. A step size twice as wide as that of range $5 \sim 10$ rad/s is used in ranges $3 \sim 5$ and $10 \sim 12$ rad/s. The number of frequency points in each 1 rad/s is given by

$$m/4 \tag{34}$$

in these ranges. The step size is fixed to 0.1 rad/s in ranges $0 \sim 3$ and $12 \sim 15$ rad/s, so the number of frequency points in each 1 rad/s is given by

$$10. \tag{35}$$

The number of permissible rotations is 5 per 1 rad/s, which corresponds to 10 eigenvalues. In the remaining range $15 \sim 20$ rad/s, the step size is 1 rad/s.

The above investigation is summarized in Fig. 3. The symbol $[x]$ in the figure denotes an integer nearest but not smaller than x. This symbol is

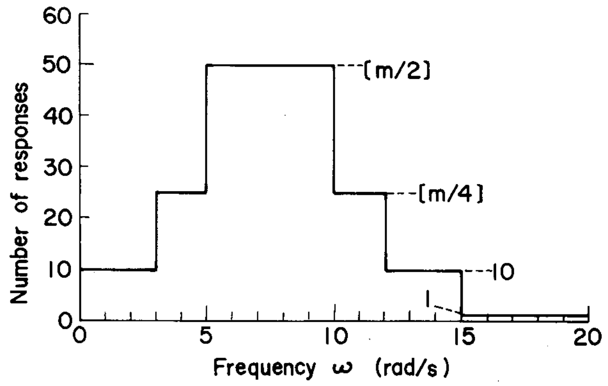

Fig 3. The number of frequency responses per 1 rad/s.

introduced to make the number of frequency points integer in each 1 rad/s. The total number of frequency points to be scanned in $0 \sim 20$ rad/s is given as follows;

$$\begin{aligned} N_p &= 14\ [m/4] + 65 && \textit{for } m \geq 40, \\ &= 10\ [m/4] + 105 && \textit{for } 40 \geq m \geq 20, \\ &= 155 && \textit{for } m \leq 20 \end{aligned} \tag{36}$$

There are three cases according to the number of generators. This classification is made so as not to make the step size larger than 0.1 rad/s. For systems with generators fewer than 20, uniformly 155 of frequency points are scanned. On the other hand, if one step size determined from (33) is used over all frequency range $0 \sim 20$ rad/s, then the total number of frequency points is given by

$$N_o = 20\ [m/2] \tag{37}$$

Ratio of N_p to N_o for several m is given as follows;

$$\begin{aligned} N_p / N_o &= 155 / 200 = 0.78 && \textit{for } m = 20, \\ &= 205 / 400 = 0.51 && \textit{for } m = 40, \\ &= 415 / 1000 = 0.42 && \textit{for } m = 100, \end{aligned}$$

$$= 765 \ / \ 2000 = 0.38 \qquad \textit{for } m = 200.$$

The ratio approaches 0.35 as m increases. In the case of the 107-machine system, the ratio becomes 0.41, which means that the number of frequency points to be scanned decreases to 41% of the usual method. The whole frequency response is obtained in 23.6 seconds by using the variable step size, while it takes 57.6 seconds to obtain the response in the case of the fixed step size. The ratio between them is 0.41, which agrees with the ratio N_p / N_o.

4. 2 Relaxation of step size for D

The determinant of the operational transfer matrix F consists of three components, as shown in (29). Therefore, its phase angle can be expressed as the sum of their phase angles as follows;

$$O = O_1 + O_2 - O_3 \tag{38}$$

where

$$O_1 = \angle \mid A \mid \qquad O_2 \angle \mid D' \mid \qquad O_3 = \angle \mid D \mid$$

Fig. 4 shows the variations of O_1, O_2, and O_3 for the 107-machine system. The angle O_1 varies rapidly in range $8 \sim 12$ rad/s. Most of its rotations appear in this range. O_2 varies in a wide range. Its variation is conspicuous especially in front and behind of a peak at 8.5 rad/s. Compared with these angles, O_3 varies

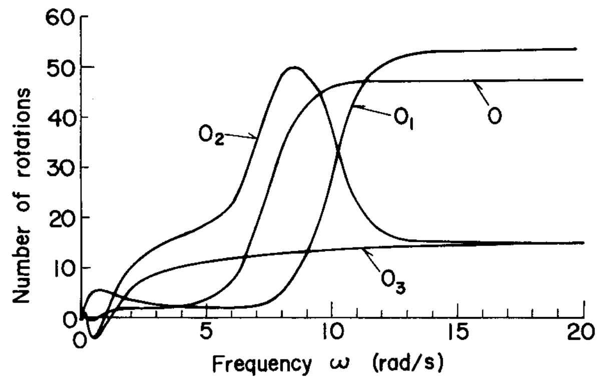

Fig 4. The number of rotations of each component (107-machine system).

slowly. O_3 completes most of its variation by 3 rad/s, and shows a monotonous and slow variation in the remaining range. The object angle O varies mostly in 5 ~ 10 rad/s. This variation is mainly due to O_1 and O_2, and is not affected so much by O_3. In order to obtain O_3, it is necessary to make D and factorize it. If a rough step size is used for this slowly varying angle, then computation time for it is saved. The step size is set to 0.1 rad/s in range 0 ~ 15 rad/s. This step size has already been used in 0 ~ 3 and 12 ~ 15 rad/s. The number of frequency points to be scanned for O_3 is reduced to 155, while it does not change for O_1 and O_2. Therefore, the total number of factorizations of D and D' is given by

$$N_q = 14\ [m/4] + 220 \qquad for\ m \geq 40 \tag{39}$$

This number corresponds to $N_q / 2N_p$ of that in the preceding section. The ratio changes with m as follows;

$$N_q / 2N_p = 360 / 410 = 0.88 \qquad for\ m = 40,$$

$$= 570 / 830 = 0.69 \qquad for\ m = 100,$$

$$= 920 / 1530 = 0.60 \qquad for\ m = 200$$

The ratio approaches 0.5 as m increases. By using this technique, the whole frequency response is scanned in 17.1 seconds for the 107-machine system. The ratio of the time to the previous result is

$$17.1 / 23.6 = 0.72$$

This value is near the ratio $N_q / 2N_p$ for $m = 100$. The calculation of O_1 and others makes the value a little bit higher than 0.69.

Lastly, some consideration is made about an error introduced by using a rough step size for O_3. As an example, consider a case where O_3 uniformly varies by 36° every 0.1 rad /s, as shown in Fig. 5, which corresponds to one rotation in 1 rad/s. Since the step size is set to 0.1 rad/s for O_3, O_3 jumps at each frequency multiple of 0.1 rad/s in actual calculation. While O_3 is kept constant after the jump, O expressed by (38) moves according to the varitions in O_1 and O_2. O_1 and O_2 are calculated with an adequately small step size, so O is traced accurately in this interval. However, O changes suddenly when a new

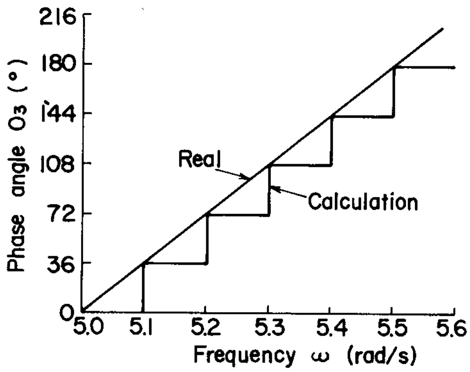

Fig 5. Variation of phase angle O_3.

value of O_3 is calculated. If the change is small enough, like 36°, O moves to a new place on the complex plane without losing any information on the number of its rotations. The sudden change causes trouble, however, if its amount is greater than 180°. In this case, at least ± 1 error is introduced in the number of rotations, because there is no way to distinguish, for example, 270° from $-90°$, and it is treated as $-90°$. This case arises when O_3 rotates more than 5 times per 1 rad/s, but it does not happen in the example of Fig. 4.

5. Conclusions

Straight application of the frequency response method to dynamic stability studies of large-scale power systems is severely limited due to its long computation time. In this paper, several measures were proposed to ease this limitation. It was tried first to reduce the computation time required to obtain a response at each frequency, and a new method of calculating the determinant of the operational transfer matrix was shown.

(a) The determinant is obtainable through three matrices.

(b) One of them is a diagonal matrix, so its determinant is easily obtained. The other matrices are of general form, but their sparsity natures make it possible to use some efficient programming techniques to calculate their determinants.

(c) By using this method, the computation time was reduced to 1 / 22 of the usual method in the case of the 107-machine power system.

It was next tried to reduce the number of frequency points to be scanned. Several step sizes were introduced instead of a fixed step size.

(d) The number of frequency points was reduced to 14 [m/ 4] + 65 from 20 [m / 2] by using the new step sizes.

(e) The number of factorizations of D and D' was reduced to 14 [m/ 4] + 220 from 28 [m / 4] + 130 by using a rough step size for *D*.

(f) The amount of calculation was reduced to some 28% of the usual method in the case of the 107-machine system.

By combining these measures, the total computation time was reduced to 17.1 seconds, which corresponds to 1 / 75 of the usual method. These results verify the effectiveness of the proposed measures.

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