

Optimal Operating Condition of Substrate Removal in Oxidation Ditch Process with Intermittent Aeration

By

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Abstract

An oxidation ditch can be used for the simultaneous removal of organic carbon and nitrogen compounds, as a relatively small scale wastewater treatment process. In this study, substrate removal and its optimal operating conditions in an oxidation ditch process with intermittent aeration was investigated through the treatment tests of synthetic sewage, and the theoretical model system combining the kinetic models of substrates removal with the tank-in series model of mixing.

In the treatment tests, over 93% of T-N could be removed under the optimal condition of the intermittent aeration without decreasing the BOD removal efficiency compared with continuous aeration. Even under the worst condition of intermittent aeration, 72.5% removal of T-N obtained, whereas T-N removal under continuous aeration was less than 60%. Therefore, it was concluded that an efficient and stable simultaneous removal of organic carbon and nitrogen compounds is possible by selecting and keeping the best condition of intermittent aeration. Also, the theoretical model gave good agreement with the experimental data, and then proved to be applicable for predicting the quality of effluent from an oxidation ditch with intermittent aeration. By another computation using the model, the simultaneous removal of over 95% of BOD and over 90% of T-N was expected under the operating conditions which gave the value of 2 to 4 (mg/l) to the index OCM proposed to evaluate the total amount of oxygen supplied in a retention time.

1. Introduction

An oxidation ditch as a relatively small scale wastewater treatment process has shown the following advantages; 1) lower initial and operating cost, 2) easy maintenance without any technical expert, 3) less production of excess sludge, 4) stability of the efficiency for the change of loading rate, 5) nitrogen removal as well as carbonaceous substances. Among these, nitrogen removal

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efficiency has been re-evaluated from the standpoint of preventing eutrophication.

In a continuously aerated oxidation ditch, it is possible to gradually decrease the concentration of mixed liquor DO in a flow direction by controlling mainly the oxygen supply and flow speed, depending on the process conditions such as loading rate, length of the ditch, and circulating flow rate. Under such a state, not only carbonaceous substances and ammonia nitrogen are oxidized in an aerobic zone, but also the generated nitrate is removed by denitrification in an anoxic zone or part.

Our previous experimental work¹⁾ and modelling study²⁾ on the simultaneous removal of BOD and nitrogen in a continuously aerated oxidation ditch indicated that the removal of total nitrogen is much influenced by hydraulic retention time (HRT). In a region of small HRT, the concentration of ammonia nitrogen increases a little and nitrate nitrogen is little produced for lack of DO. In a region of large HRT, the produced nitrate nitrogen of high concentration is little denitrified because of the high DO concentration. Since the HRT, to give the minimum concentration of total nitrogen, changes with a volumetric loading rate, it may be difficult to find and keep an optimal HRT in some practical cases.

From these findings, it is expected that any anoxic stage given by intermittent aeration in the oxidation ditch process may become useful and practical to obtain a higher and more stable efficiency of substrate removal. The effect of intermittent aeration, therefore, and desirable conditions of intermittent operation were studied by laboratory experiments, and the kinetic model developed for a continuously aerated ditch was adopted to the result and discussed.

2. Experimental Methods

As shown in Fig. 1, the experimental set-up consists of seven two-liter perspex vessels arranged in a cascade fashion, a feed tank, and a clarifier.

Synthetic sewage of high concentration (Table 1) and tap water for dilution were fed into the 1st vessel at the rate of 2.5 ml/min and 7.5 ml/min respectively. The mixed liquor of the feeding substrate (Table 2) and cultured activated sludge was circulated through the system. Air was introduced into the 4th vessel through a perspex sparger. These positions of feed and aeration had proved to be efficient for denitrification to utilize a portion of carbonaceous substances through the vessels 1st to 3rd. Additionally, this set-up, notwithstanding tanks-in-series type, had proved to be enough to simulate the change of water quality in a loop type oxidation ditch, judging especially from the

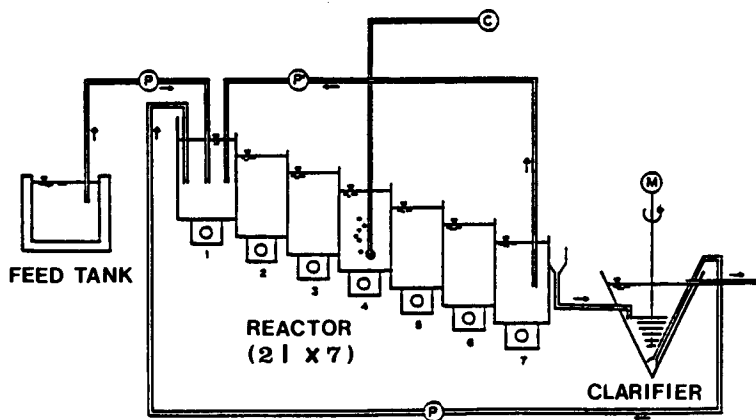


Fig. 1 Experimental apparatus

Table 1. Composition of synthetic sewage

Composition	Concentration (mg/1)
Soluble starch	400
Pepton	400
Meat Extract	300
NH ₄ Cl	267.5
NaHCO ₃	250
CaCl ₂	27.5
MgSO ₄ · 7H ₂ O	22.5
Na ₂ HPO ₄ · 12H ₂ O	133.8
K ₂ HPO ₄	65.3
KH ₂ PO ₄	25.5
FeCl ₃ · 6H ₂ O	0.2

Table 2. Quality of diluted synthetic Sewage fed

Constituent	Concentration (mg/1)
BOD	167.5-212.5
NH ₄ -N	20.2- 25.0
Org-N	12.7- 19.9
NO ₂ -N	0.0
NO ₃ -N	2.2- 2.8
T-N	37.1- 47.7
PO ₄ -P	3.4- 6.2
Alkalinity	192 -226

Table 3. Running conditions

Run No	Aeration Time (min)	Non-Aeration Time (min)	$K_L a$ During Aeration (1/hr)	MLSS (mg/l)
1	15	45	8.7	2340
2	15	45	13.0	1935
3	30	30	8.7	2300
4	30	30	13.0	2305
5	45	15	8.7	2390
6	45	15	13.0	1645
7	Continuous Aeration		8.7	1674

characteristic of an almost complete mixing in both systems except for DO.

Table 3 shows the running conditions of Runs No. 1 to 7. In Runs No. 1 to 6, the ratio of aeration time (AT) to non-aeration time (NAT) was changed to be 15 / 45, 30 / 30, and 45 / 15 within one hour of one cycle. Also, the oxygen supply was changed at two strengths. In Run No. 7, aeration was continuously done for comparison with the intermittent aeration. Hydraulic retention time (HRT) of 24 hr, circulating flow ratio of 50, return sludge ratio of 1 were set, and value of pH was kept at 7.2 by a pH controller, in all of the runs. Experiments were conducted in a temperature-controlled room, at 20° C.

3. Theoretical Analysis

3.1 Mixing model²⁾

Overall mixing in normal oxidation ditches is known to be almost complete mixing. A tank-in-series model (Fig. 2) including circulating and back flows

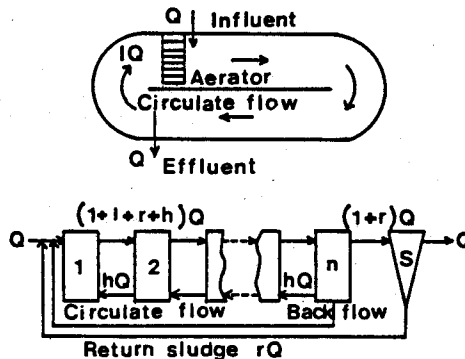


Fig. 2 Schematic mixing model of tank-in-series for an oxidation ditch

was adopted to describe the mixing in this study. The material balance equations for n tanks are as follows:

The 1st tank

$$\frac{v}{n} \cdot \frac{dC_1}{dt} = Q \{ C_0 + (I+r)C_n + hC_2 - (1+h+I+r)C_1 \} + \frac{V}{n} R_1 X \quad (1)$$

The i -th tank

$$\frac{V}{n} \cdot \frac{dC_i}{dt} = Q \{ (1+h+I+r)C_{i-1} + hC_{i+1} - (1+2h+I+r)C_i \} + \frac{V}{n} R_i X \quad (2)$$

the n -th tank

$$\frac{V}{n} \cdot \frac{dC_n}{dt} = Q(1+h+I+r)(C_{n-1} - C_n) + \frac{V}{n} R_n X \quad (3)$$

When the circulating flow ratio I is larger than 50, this model expresses an almost complete mixing without back flow.

3.2 Kinetic model²⁾

Rate equations for describing the simultaneous removal of organic substances and nitrogen compounds are obtained as follows by taking account of the kinetic, mainly monod-type, and stoichiometric relationships among BOD oxidation, ammonification, nitrification, denitrification, DO and alkalinity change, and sludge growth. The framework of the relationships is shown in Fig. 3.

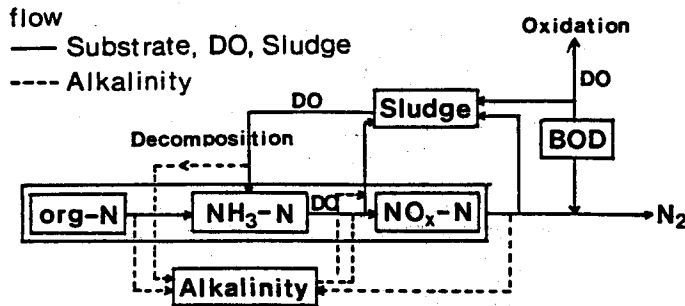


Fig. 3 Schematic relationship among parameters of water quality for simultaneous removal of BOD and nitrogen

Removal of BOD

$$\frac{1}{X} \cdot \frac{dS}{dt} = \frac{1}{X} \left(\frac{dS}{dt} \right)_B - \alpha \frac{1}{X} \left(\frac{dC_3}{dt} \right)_B \quad (4)$$

$$\frac{1}{X} \left(\frac{dS}{dt} \right)_B = - \frac{U_S S}{K_S + S} \cdot \frac{DO}{K_O + DO} \quad (5)$$

Decrease of Organic Nitrogen (Org-N)

$$\frac{1}{X} \cdot \frac{dC_{or}}{dt} = -K_{or} \cdot C_{or} \quad (6)$$

Overall change of NH₄-N

$$\begin{aligned} \frac{1}{X} \cdot \frac{dC_1}{dt} = & \frac{1}{X} \left(\frac{dC_1}{dt} \right)_B + H \cdot d + \frac{J}{X} \left\{ a \cdot \left(\frac{dS}{dt} \right)_B + b \left(\frac{dC_1}{dt} \right)_B \right. \\ & \left. - c \left(\frac{dC_3}{dt} \right)_B \right\} - \frac{1}{X} \cdot \left(\frac{dC_{or}}{dt} \right) \end{aligned} \quad (7)$$

$$\frac{1}{X} \left(\frac{dC_1}{dt} \right)_B = - \frac{U_1 C_1}{K_1 + C_1} \cdot \frac{DO}{K_{on} + DO} \cdot \frac{A}{K_a + A} \quad (8)$$

Overall change of NO_x (NO₂+NO₃)

$$\frac{1}{X} \cdot \frac{dC_2}{dt} = - \frac{1}{X} \left(\frac{dC_1}{dt} \right)_B - \frac{1}{X} \left(\frac{dC_3}{dt} \right)_B \quad (9)$$

Denitrification

$$\frac{1}{X} \left(\frac{dC_3}{dt} \right)_B = \frac{U_2 C_2}{K_2 + C_2} \cdot \frac{S}{K_S + S} \cdot \left(1 - \frac{DO}{K_o + DO} \right) \quad (10)$$

Growth of sludge

$$\frac{dX}{dt} = -a \left(\frac{dS}{dt} \right)_B - b \left(\frac{dC_1}{dt} \right)_B + c \left(\frac{dC_3}{dt} \right)_B - d \cdot X \quad (11)$$

Change of DO

$$\frac{dDO}{dt} = a \left(\frac{dS}{dt} \right)_B + b \left(\frac{dC_1}{dt} \right)_B - d'X + K_L a \cdot (DO_S - DO) \quad (\text{aeration}) \quad (12)$$

$$\frac{dDO}{dt} = a \left(\frac{dS}{dt} \right)_B + b \left(\frac{dC_1}{dt} \right)_B - d'X \quad (\text{non aeration}) \quad (12)'$$

Change of alkalinity

$$\frac{dA}{dt} = e \left(\frac{dC_1}{dt} \right)_B + f \left(\frac{dC_3}{dt} \right) - g \left(\frac{dC_{\sigma}}{dt} \right) + \left[k \cdot H \cdot d \cdot X + J \left\{ a \left(\frac{dS}{dt} \right)_B + b \left(\frac{dC_1}{dt} \right)_B - c \left(\frac{dC_3}{dt} \right)_B \right\} \right] \quad (13)$$

3.3 Computer simulation

The water quality of each of the vessels (No. 1 to 7) was estimated by solving numerically the system of equations (1) to (13) using the Runge-Kutta Verner method, under each of the experimental conditions (Table 3). Table 4 presents the kinetic and stoichiometric coefficients used, which are obtained from literatures and our previous work. As for K_{La} , two values, which were 1.2 times larger than those obtained from the experiments aerating tap water at the two strengths in one vessel, were adopted in order to get a better estimation.

Table 4. Kinetic and stoichiometric coefficients

Symbol	Value	Symbol	Value
Us(mgBOD/mgX/hr)	0.2	c(mgX/mgN)	—
U_1 (mgN/mgX/hr)	0.02	d(hr ⁻¹)	0.002
U_2 (mgN/mgX/hr)	0.015	a'(mgDO/mgBOD)	0.34
Ks, Kns(mgBOD/l)	50	b'(mgDO/mgN)	4.57
K_1 (mgN/l)	0.5	d'(mgDO/mgX/hr)	0.0008
K_2 (mgN/l)	0.1	DOs(mgDO/l)	8.1
Ko, Kon(mgN/l)	0.5	e(mgA/mgN)	7.14
Ka(mgA/l)	100	f(mgA/mgN)	3.57
Kor(1/mgX/hr)	0.000958	g, k(mgA/mgN)	3.57
α (mgBOD/mgN)	1.90	H(mgN/mgX)	0.1
a(mgX/mgBOD)	0.70	J(mgN/mgX)	0.1
b(mgX/mgN)	0.17		

4. Result and Discussion

4.1 Experimental result

In the improved oxidation ditch process, proposed by Terashima and Ishikawa,²⁾ to remove nitrogen as well as BOD, concentration of DO needs to be controlled to decrease in a flow direction. Other qualities of the water can be kept almost uniform through the ditch because of the large circulating flow with a relatively large HRT. In the circulating flow system of this study, the qualities of the water, except for DO during aeration, were little different among all of the vessels.

Figure 4 shows an example of the change of DO concentration among the vessels, obtained by Run No. 4. Concentration of DO was the highest of 3 mg /l in the 4th vessel which was aerated, and decreased in the following vessels. Vessels of the 7th, 1st to which the substrate was being introduced, 2nd, and 3rd were in anoxic state. In the 4th vessel, the value of DO increased rapidly to the maximum after beginning aeration, and decreased to zero within 5 minutes after stopping aeration, as shown in Fig. 5.

Figures 6 and 7 indicate the BOD concentration and removal percentage,

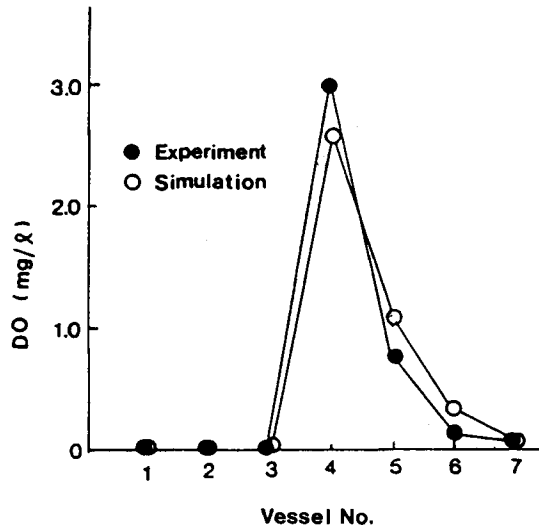


Fig. 4 Change of DO among the vessels in series during aeration (Run No. 4)

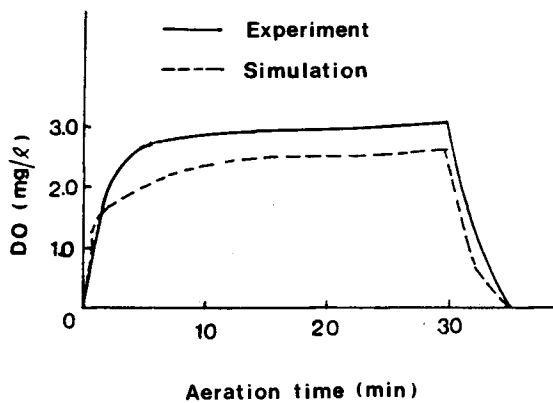


Fig. 5 Change of DO concentration in the 4th vessel during aeration (Run No. 4)

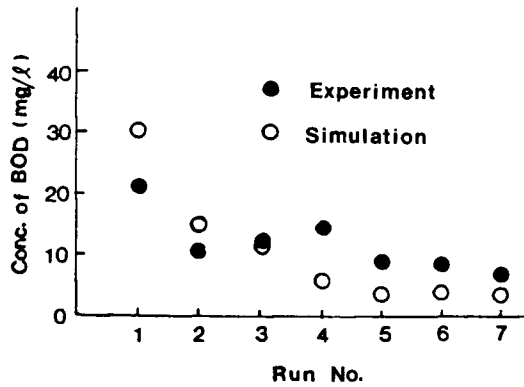


Fig. 6 Concentration of BOD in the 3rd vessel in each run

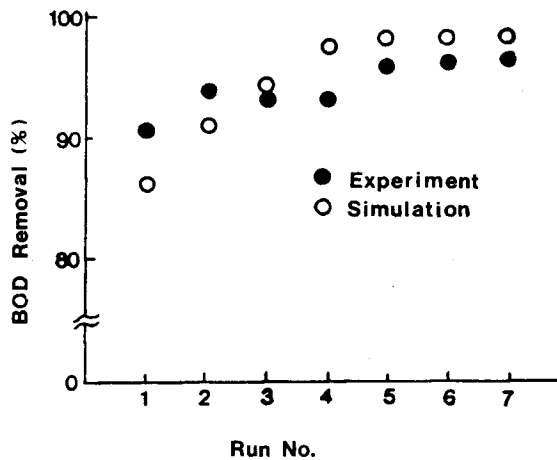


Fig. 7 Removal percentage of BOD in the 3rd vessel in each run

which were obtained in the 3rd vessel at the middle of the non-aeration period of each of the runs. In those figures, the increase of the run number on the abscissa corresponds approximately to the increase of the substantial time ratio of aeration to non-aeration, which can be evaluated by taking account of both of the values of experimental AT/NAT and K_{1a} . Although the concentration of BOD seems to increase with a decrease of substantial AT/NAT , a removal percentage over 90% was obtained in each of the runs. Excepting the result of Run No. 1 performed at a small value of AT/NAT , removal percentages obtained at larger substantial values of AT/NAT ranged from 93% to 96%, showing no significant differences from the result of continuous aeration. Considering that

the period and part of aerobic states in the experimental system under intermittent aeration were smaller than those under continuous aeration, it is estimated that a drop of BOD removal efficiency due to the relative reduction of the aerobic period and part, might be almost compensated for by an increase of the BOD consumption through the denitrification under the relatively enlarged anoxic state.

Figure 8 shows the concentration of Kjeldahl nitrogen, NO_x-nitrogen and alkalinity in each of the runs. Compared with the quality of feed water (Table 2), the reduction rate of Kjeldahl nitrogen, namely the nitrification rate, was fairly good in each of the runs. The concentration of Kjeldahl nitrogen decreased gradually with an increase of substantial AT/NAT, because the rates of ammonification of organic nitrogen and nitrification of ammonia increased with the period and extent of aerobic states in this experimental system, especially in the 4th, 5th and 6th vessels as shown in Fig. 4. Consequently, no significant difference of Kjeldahl nitrogen removal was observed between Runs No. 4 to 6 under intermittent aeration, and Run No. 7 under continuous aeration. The concentration of alkalinity, which is consumed in the course of nitrification, changed according to the removal of Kjeldahl nitrogen. On the other hand, the concentration of NO_x-N increased much at a relatively large substantial AT/NAT, namely in the cases of Runs No. 6 and 7, because the increase of aerobic state limited denitrification.

In Runs No. 1 to 5, each of the concentration of NO_x-N was very low in

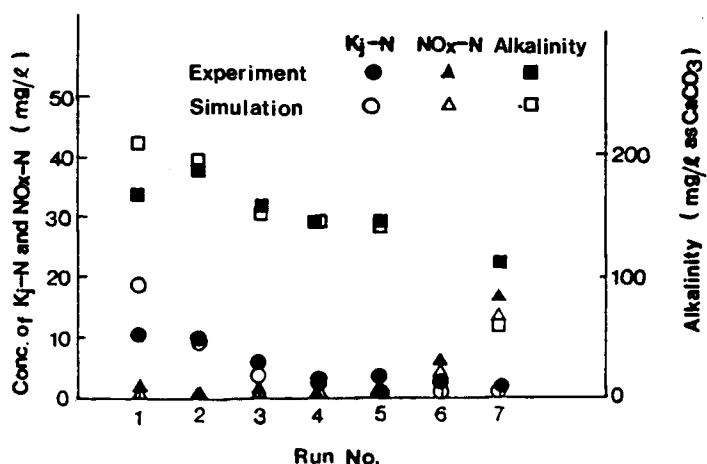


Fig. 8 Concentrations of Kj-N, NO_x-N and alkalinity in the 3rd vessel in each run

spite of the fact that nitrification occurred not a little in each of them. From this fact, it is sure that almost all of the $\text{NO}_x\text{-N}$ produced by nitrification was successively and efficiently denitrified in the anoxic part of the system.

From the results above, the concentration and removal percentages of T-N in each of the runs change as shown in Figs. 9 and 10, respectively. The most efficient removal of T-N, about 93%, was obtained in Runs No. 4 and 5 under the following conditions: AT/NAT of 1.0 in one hour cycle and K_{La} value of 13.0 (1/hr) in one vessel, and AT/NAT of 3.0 and K_{La} of 8.7 (1/hr), respectively. The values of K_{La} for the whole experimental system are calculated to be 1.86 (1/hr) and 1.24 (1/hr) by dividing the above values by the total number of vessels, seven.

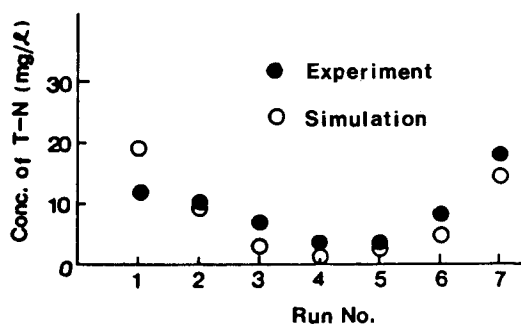


Fig. 9 Concentration of T-N in the 3rd vessel in each run

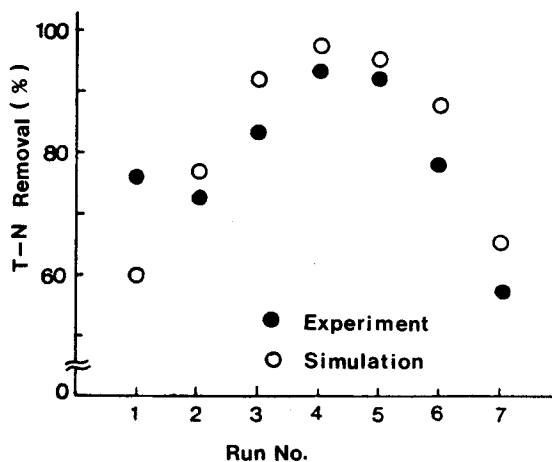


Fig. 10 Removal percentage of T-N in the 3rd vessel in each run

Compared with the result of T-N removal by continuous aeration (57%, Run No. 7), the above result by intermittent aeration is much better. Even under the worst condition of intermittent aeration (Run No. 1), a larger removal efficiency of 72% was obtained. This results from the fact that denitrification under continuous aeration was poor in spite of the occurrence of effective nitrification. It is a matter of course that a better condition of continuous aeration may result in a more efficient removal of T-N. In our previous work of continuous aeration under similar experimental conditions using the same experimental apparatus, the maximum T-N removal efficiency of 83% was obtained at HRT of 30 hours. The removal efficiency, however, was much influenced by HRT, decreasing to 71% at the same HRT of 24 hours as that of the present experiment. This is due to the fact explained in the first section of this paper. Based on the present and previous results above on T-N removal, it is highly possible that removal of T-N in an intermittently aerated oxidation ditch will be much more efficient and stable than in a continuously aerated ditch.

Finally, from the findings on the removal of BOD and nitrogen compounds, it is concluded that an efficient and stable simultaneous removal of them is possible by selecting and keeping the best condition of intermittent aeration for an oxidation ditch. It will be easier to stabilize the efficient removal by an intermittent aeration than by a continuous one.

4. 2 Result of simulation

The results of computer simulation of water quality for the experimental conditions are shown in Figs. 4 to 10, together with the experimental data.

The experimental change of DO concentration in the flow direction, namely among the vessels in series, was fairly well simulated by the proposed model, in spite of depending on the rates of BOD oxidation, nitrification, endogenous respiration and oxygen supply (Fig. 4). Other experimental water quality was also well simulated, showing little change in the flow direction.

Several of the experimental concentrations of BOD and removal percentages in the 3rd vessel in the runs were not in close agreement with the results by simulation, probably due to some analytical error (Fig. 6, 7). However, characteristic of the experimental change among the runs was simulated on the whole.

The experimental concentrations of Kjeldahl nitrogen, NO_x-N, alkalinity and T-N, and removal percentages of T-N in each of the runs were well simulated, except for a few runs, in spite of relatively complex relations among the substances as shown in Fig. 3. For instance, a change of Kjeldahl nitrogen depends on the rates of ammonification, nitrification, denitrification and sludge

growth, and the limiting effects of DO and alkalinity.

Based on a series of the results above, the proposed model proved to be applicable for predicting the quality of the effluent from an oxidation ditch intermittently aerated.

5. Optimal Operating Conditions

Figure 11 shows the results of computer simulation for T-N removal by intermittent aeration and continuous aeration, which were computed using the average quality of feed water of Runs 1 to 7. In each aeration pattern, the volume of oxygen supplied in one cycle was the same. In this figure, the region of HRT where a high removal efficiency of T-N could be obtained was wider in the intermittent aeration than in the continuous one. This supports the indication described in above section, that is, the superiority of intermittent aeration in the stable removal of T-N.

To discuss the optimal operating conditions of simultaneous removal of organic carbon and nitrogen compounds by intermittent aeration, the following index (OCM) was introduced:

$$OCM = (AT / (AT + NAT)) K_L a \cdot DO_s \cdot V \cdot HRT \quad (14)$$

where DO_s is the saturation concentration of DO, V is tank volume, and AT and NAT represent the aeration time and the non-aeration time in one hour of one cycle, respectively. The index (OCM) can be regarded as a total amount of oxygen which comes into contact with the substrates fed during the retention

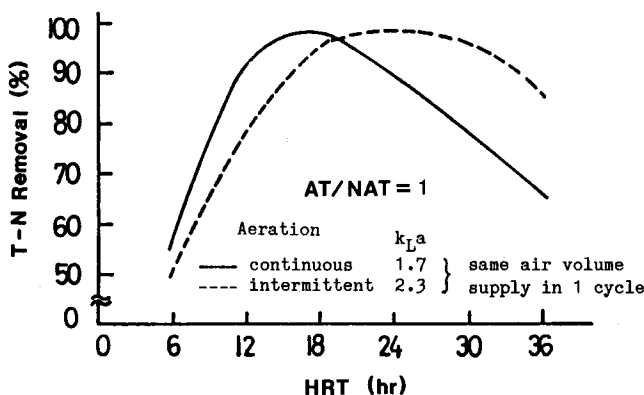


Fig. 11 Effect of HRT on T-N removal in intermittent aeration and continuous one

time. The relationships between OCM and the removal percentages of BOD and T-N were obtained, as shown in Figs.12 and 13, by computing their removal percentages under the operating condition combining HRT of 6,12,18,24,30 or 36 (hr) and K_{La} of 12 or 16 (1/hr) with the aeration time ratios, i.e. AT/ (AT

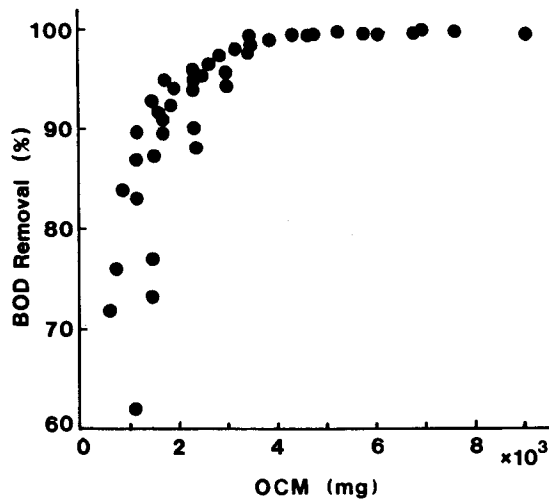


Fig. 12 Relationship between OCM and BOD removal

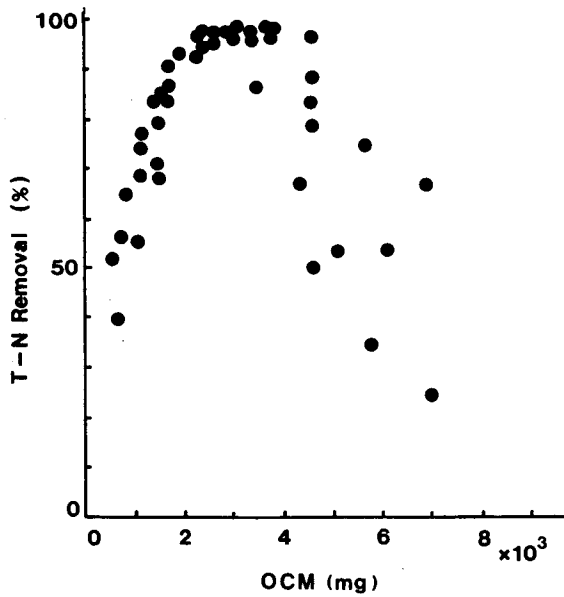


Fig. 13 Relationship between OCM and BOD removal

+NAT) of 0.25, 0.5, 0.75 or 1.0 (continuous aeration). These figures indicate that over 95% of BOD and over 90% of T-N can be simultaneously removed in the range of about 2 to 4 (mg/l) of OCM. Consequently, efficient and simultaneous removal of organic carbon and nitrogen compounds will be accomplished by selecting aerating conditions and/or HRT giving the value within the above optimal range of OCM.

6. Conclusion

1. Carbonaceous substances as BOD were removed over 90% up to 96% also by intermittent aeration as well as continuous aeration.

2. With an increase of the substantial time ratio of aeration to non-aeration (AT/NAT), the concentration of Kjeldahl nitrogen decreased. On the other hand, the concentration of NO_x-nitrogen increased. Consequently, the removal of nitrogen depends on superficial AT/NAT and aerating strength, K_{La} .

3. Total nitrogen could be removed up to 93% under the following conditions; superficial AT/NAT of 1.0 in one hour of one cycle and K_{La} value of 1.86 (1/hr), for the whole experimental system, and AT/NAT of 3.0 and K_{La} of 1.24 (1/hr). Even under the worst conditions of this study, 72.5% removal of T-N, larger than by the continuous aeration, was obtained.

4. Based on these results, an efficient and stable simultaneous removal of BOD and T-N is possible by selecting and keeping the best condition of intermittent aeration for an oxidation ditch. It will be easier to stabilize the efficient removal by an intermittent aeration than by a continuous aeration.

5. The theoretical model developed for describing the simultaneous removal of carbonaceous substances and nitrogen compounds in a continuously aerated oxidation ditch, combining the kinetic models of substrates removal with the tank-in-series model of mixing, also had good agreement with the experimental data of intermittent aeration.

6. From this result, the proposed model proved to be applicable to designing an oxidation ditch to be aerated intermittently.

7. By computer simulations with the proposed model, the simultaneous removal of over 95% of BOD and over 90% of T-N was expected under the operating conditions which gave the value of 2 to 4 (mg/l) to the index OCM proposed to evaluate the total amount of oxygen supplied in a retention time.

Acknowledgement

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Symbols

A	; Alkalinity (mg A/1)
AT	; aeration time (min)
NAT	; Non-aeration time (min)
C_o	; Water quality of inflow (mg/1)
C_i	; Water quality in i-th tank (mg/1)
C_{or}	; Concentration of Org-N (mg N/1)
C_1	; Concentration of NH_4 -N (mg N/1)
C_2	; Concentration of NOx-N (mg N/1)
C_3	; Concentration of N_2 (mg N/1 as liquid)
DO	; Dissolved oxygen (mg DO/1)
DO_s	; Saturation concentration of DO (mg DO/1)
H	; Exchange coefficient for NH_4 -N produced with endogenous decay of sludge (mg N/mg X)
I	; Circulation flow ratio (-)
J	; Exchange coefficient for NH_4 -N consumed with sludge growth (mg N/mg X)
K_1	; Saturation constant for NH_4 -N (mg N/1)
K_2	; Saturation constant for NOx-N (mg N/1)
K_a	; Saturation constant for Alkalinity (mg A/1)
K_{La}	; Overall oxygen transfer (1/hr)
K_o	; Saturation constant for DO in BOD oxidation (mg DO/1)
K_{od}	; Saturation constant for DO in denitrification (mg DO/1)
K_{on}	; Saturation constant for DO in nitrification (mg DO/1)
K_{or}	; Decay coefficient for Org-N (1/mg X/hr)
K_s	; Saturation constant for BOD (mg BOD/1)
K_{sn}	; Saturation constant for BOD in denitrification (mg BOD/1)

- R_i ; Reaction rate in i-th tank (1/hr)
 S ; Concentration of BOD (mg BOD/l)
 U_1 ; Maximum rate of nitrification (1/hr)
 U_2 ; Maximum rate of denitrification (1/hr)
 U_3 ; Maximum rate of BOD oxidation (1/hr)
 V ; Tank volume (m^3)
 X ; Biomass of MLVSS (mg X/l)
 a ; Yield coefficient for BOD removal (mg X/mg BOD)
 b ; Yield coefficient for nitrification (mg X/mg N)
 c ; Yield coefficient for denitrification (mg X/mg N)
 d ; Endogenous decay rate (1/hr)
 a' ; Oxygen consumption coefficient for BOD removal (mg DO/mg BOD)
 b' ; Oxygen consumption coefficient for nitrification (mg DO/mg N)
 d' ; Oxygen consumption rate for endogenous decay of biomass (mg DO/mg X/hr)
 e ; Alkalinity consumption coefficient for nitrification (mg A/mg N)
 f ; Alkalinity production coefficient for denitrification
 g ; Alkalinity production coefficient for Org-N decay
 h ; Back flow ratio (-)
 n ; Tank number
 k ; Alkalinity consumption or production coefficient for endogenous decay or growth of biomass (mg A/mg N)
 α ; BOD consumption coefficient for denitrification (mg BOD/mg N)
 $\left(\frac{ds}{dt}\right)_B$; BOD oxidation rate (mg BOD/l hr)
 $\left(\frac{dC_1}{dt}\right)_B$; Nitrification rate (mg N/l hr)
 $\left(\frac{dC_2}{dt}\right)_B$; Denitrification rate (mg N/l hr)