GROUP SCHEDULING WITH INTERACTIVE COMPUTER GRAPHICS

by

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Abstract

Petrov's method for the conventional flow-shop scheduling, which is an extension of Johnson's algorithm for the two-stage flow-shop scheduling, is applied to obtain a near-optimal solution for a large-scale scheduling based on group technology. A computational algorithm is developed. Input data into a mico-computer are: group setup times on all stages for all groups, and job processing times on all stages for all jobs. Then the solutions —Gantt chart, total production time, critical path, and starting and finishing times for all group setups and all job processings— are instantly displayed on an interactive mode. An example is shown, comparing the near-optimal solution with the optimal solution.

1. INTRODUCTION

A recent, important contribution to the manufacturing area is the concept of Group Technology (GT), which has now found wide acceptance in real-life workshops[1]. GT is a technique and philosophy directed toward an increase in production efficiency by grouping various parts into "families", each having similarities of shape, dimension, and/or processing route.

The following advantages are obtained by applying GT to multi-item, small-sized production:

1. The total production time is reduced by setting a group setup for a parts-family included in a group.

2. The material-flow pattern can be of the flow type by establishing a GT layout.

Hence, with GT operations scheduling can be simplified as flow-shop scheduling. It has been called "Group Scheduling"[2,3].

The optimal solutions for group scheduling, i. e., the optimal sequences for groups and for jobs included in each group under the minimum-time criterion are determined through the branch-and-bound technique. However, this procedure is not practical because of a huge consumption of computer time, especially as the number of jobs and the number of stages increase. This is recognized as an NP-hard problem.

In this paper, Petrov's method for the conventional flow-shop scheduling, which is an extension of Johnson's method for the two-stage flow-shop scheduling, is applied to obtain a near-optimal solution for group scheduling on a multi-stage manufacturing system, with much less time. Input data into a micro-computer are: group setup times on all stages for all groups and job processing times on all stages for all jobs. Then the solutions, such as Gantt chart, total production time, critical path, starting and finishing times for all group setups and all job processings, are instantly displayed on an interactive mode. The model and the solution procedure for this interactive group scheduling are explained with an example.

2. GROUP SCHEDULING PROBLEM DEFINED

In order to establish a model for group scheduling for a multi-item, multi-stage manufacturing system, the following pre-conditions are placed.

1. A multi-stage manufacturing system is composed of K stages (machines) sequenced in the specified technological order. The stage index is denoted by k $(=1, 2, \ldots, K)$.

2. Jobs (or parts) to be processed are classified into M groups. The group index is denoted by $i(=1, 2, \ldots, M)$. In group $i(G_i)$, N_i jobs are included. The job index is denoted by $j(=1, 2, \ldots, N_i)$; then, $\sum_{i=1}^{M} N_i = N$; that is, the number of jobs is N. Job j in G_i is expressed as J_{ij}.

3. Ready times for all jobs are identical (=time 0).

4. Group sequence and job sequence are identical at all stages of the manufacturing system.

5. Group processing time consists of group setup time and the sum of job processing times contained in each group.

Suppose that the setup time for G_i on stage $k(M_k)$ is S_{ik}, and the processing time (including a small amount of preparation time for loading and unloading) for J_{ij} on M_k is p_{ijk}. Let the total processing time of G_i on M_k be denoted by P_{ik}, then,

$$P_{ik} = \sum_{j=1}^{N_i} p_{ijk} \tag{1}$$

Then,

$$S_{ik}+P_{ik}=Q_{ik} \tag{2}$$

is the total production time (=group setup time+job processing times) of G_i on M_k.

6. The scheduling criterion employed is the minimum time. That is to say, the total flow time, or makespan, $F_{\max}$, which is the time elapsed from the beginning of the first job of the first group (say, at time 0) to the ending of the last job of the last group.

Thus, the problem defined as Group Scheduling (GS) for the multi-item, multi-stage manufacturing system is to determine both the optimal group sequence and the optimal job sequences, so as to minimize the makespan.

3. EXTENSION OF PETROV'S METHOD TO GROUP SCHEDULING

Petrov's Flowline Group Production Planning[4] is an extension of Johnson's method (or algorithm)[5] for obtaining an optimal job sequence on the two-stage flow-shop scheduling, minimizing the makespan, to a large-scale flow-shop scheduling. With this method optimality is not ascertained, but it is said that a fairly good solution is obtained.

In this paper, Petrov's method is extended to group scheduling, in order to obtain fairly good group and job sequences for a multi-stage flow-shop scheduling[6].

A. Determining job sequence in the group

The following values are calculated for $J_{ij}(j=1, 2, \ldots, N_i)$ included in $G_i(i=1, 2, \ldots, M)$.

$$A_{ij}=\sum_{k=1}^{h} p_{ijk} \tag{3}$$

$$B_{ij}=\sum_{k=h'}^{K} p_{ijk} \tag{4}$$

where $h=K/2$ and $h'=h+1$ if K is even; otherwise, $h=h'=(K+1)/2$.

A heuristic algorithm for determining job sequence in the group is as follows.

Step 1: Find a minimum among A_{ij}'s and B_{ij}'s. (In case of a tie, select arbitrarily.)

Step 2: If it is A_{ij}(or B_{ij}), place J_{ij} in the first (or last) position in the sequence. When all positions in the sequence are filled, stop. Otherwise, proceed.

Step 3 : Remove J_{ij} from consideration and return to Step 1.

B. Determining group sequence

The following values are calculated for $G_i (i=1, 2, \ldots, M)$.

$$A_i = \sum_{k=1}^{h} Q_{ik} \tag{5}$$

$$B_i = \sum_{k=h'}^{K} Q_{ik} \tag{6}$$

A heuristic algorithm for determining group sequence is the same as the above three steps, except that A_{ij}, B_{ij}, and J_{ij} should be replaced with A_i, B_i, and G_i.

C. Calculating the makespan

After both group and job sequences have been determined by the above procedures, the makespan, $F_{\max}$, is calculated as follows[7, 8] :

$$F_{\max} = \sum_{i=1}^{M} (Q_{(i)K} + \sum_{j=1}^{N_i} D_{(i)(j)k}) \tag{7}$$

where

$$D_{(i)(j)k} = \begin{cases} F_{(i)(1)k-1} - F_{(i-1)(N_{i-1})k} - S_{(i)k} > 0 \text{ for } j=1 \\ F_{(i)(j)k-1} - F_{(i)(j-1)k} > 0 \text{ for } j=2, 3, \ldots, N_i \\ 0, \text{ otherwise} \end{cases} \tag{8}$$

$$F_{(i)(j)k} = \sum_{h=1}^{i-1} (Q_{(h)k} + \sum_{j=1}^{N_h} D_{(h)(j)k}) + S_{(i)k} + \sum_{h=1}^{j} (D_{(i)(h)k} + p_{(i)(h)k}) \tag{9}$$

The symbol ( ) in the subscript is used to signify the order in the sequence. $G_{(i)} = G_\xi$ means that G_ξ(group ξ) is processed in the ith position in the group sequence, and $J_{(i)(j)} = J_{\xi\eta}$ means that $J_{\xi\eta}$(job η in G_ξ) is processed in the jth position in G_ξ.

In place of Equation (7), $F_{\max}$ can be easily calculated by using Table 1. In this table, groups and jobs are sequenced as determined by the procedures mentioned in A and B. In each group, group setup is firstly done, and then jobs in the group are processed in the order as determined. In each cell, group setup time or job processing time is entered in the upper half. The left and the right sides of the lower half indicate starting and finishing times of group setup or job processing; e.g., $\underline{ik}$ and $\overline{ik}$ mean starting and finishing times of the setup of $G_{(i)}$

Table 1 Calculating the makespan for Group Scheduling

Group Sequence	Job Sequence	Stage									
		1		2		…	k		…	K	
$G_{(1)}$	$S_{(1)k}$	$S_{(1)1}$		$S_{(1)2}$			$S_{(1)k}$			$S_{(1)K}$	
		$\underline{11}$	$\overline{11}$	$\underline{12}$	$\overline{12}$		$\underline{1k}$	$\overline{1k}$		$\underline{1K}$	$\overline{1K}$
	$J_{(1)(1)}$	$P_{(1)(1)1}$		$P_{(1)(1)2}$			$P_{(1)(1)k}$			$P_{(1)(1)K}$	
		$\underline{111}$	$\overline{111}$	$\underline{112}$	$\overline{112}$		$\underline{11k}$	$\overline{11k}$		$\underline{11K}$	$\overline{11K}$
	$J_{(1)(2)}$	$P_{(1)(2)1}$		$P_{(1)(2)2}$			$P_{(1)(2)k}$				
		$\underline{121}$	$\overline{121}$	$\underline{122}$	$\overline{122}$		$\underline{12k}$	$\overline{12k}$			
	⋮										
	$J_{(1)(N_1)}$	$P_{(1)(N_1)1}$		$P_{(1)(N_1)2}$							
		$\underline{1N_1 1}$	$\overline{1N_1 1}$	$\underline{1N_1 2}$	$\overline{1N_1 2}$						
$G_{(2)}$	$S_{(2)k}$	$S_{(2)1}$		$S_{(2)2}$							
		$\underline{21}$	$\overline{21}$	$\underline{22}$	$\overline{22}$						
	$J_{(2)(1)}$	$P_{(2)(1)1}$		$P_{(2)(1)2}$							
		$\underline{211}$	$\overline{211}$	$\underline{212}$	$\overline{212}$						
⋮	⋮										
$G_{(i)}$	$S_{(i)k}$						$S_{(i)k}$				
							$\underline{ik}$	$\overline{ik}$			
	$J_{(i)(1)}$						$P_{(i)(1)k}$				
							$\underline{i1k}$	$\overline{i1k}$			
	⋮										
	$J_{(i)(j)}$						$P_{(i)(j)k}$				
							$\underline{ijk}$	$\overline{ijk}$			
	⋮										
	$J_{(i)(N_i)}$						$P_{(i)(N_i)k}$				
							$\underline{iN_i k}$	$\overline{iN_i k}$			
⋮	⋮										
$G_{(M)}$	$J_{(M)(N_M)}$	$P_{(M)(N_M)1}$					$P_{(M)(N_M)k}$			$P_{(M)(N_M)K}$	
		$\underline{MN_M 1}$	$\overline{MN_M 1}$				$\underline{MN_M k}$	$\overline{MN_M k}$		$\underline{MN_M K}$	$\overline{MN_M K}$

on M_k (stage k), and $\underline{ijk}$ and $\overline{ijk}$ mean starting and finishing times for processing $J_{(i)(j)}$ on M_k.

These values are sequentially calculated as follows:

$\underline{11}=0$

$\overline{11}=\underline{11}+S_{(1)1}=\underline{111}$

$$\overline{111} = \underline{111} + p_{(1)(1)1} = \underline{121}$$
$$\overline{121} = \underline{121} + p_{(1)(2)1} = \underline{131}$$
$$\vdots$$
$$\underline{1N_1 1} = \overline{1N_1 - 11}$$
$$\overline{1N_1 1} = \underline{1N_1 1} + p_{(1)(N_1)1} = \underline{21}$$
$$\overline{21} = \underline{21} + S_{(2)1} = \underline{211}$$
$$\overline{211} = \underline{211} + p_{(2)(1)1} = \underline{221}$$
$$\vdots$$
$$\underline{MN_M 1} = \overline{MN_M - 11}$$
$$\overline{MN_M 1} = \underline{MN_M 1} + p_{(M)(N_M)1}$$
$$\overline{12} = \underline{112} = \max(\overline{111},\ S_{(1)2})$$
$$\underline{12} = \overline{12} - S_{(1)2}$$
$$\overline{112} = \underline{112} + p_{(1)(1)2}$$
$$\underline{122} = \max(\overline{112},\ \overline{121})$$
$$\overline{122} = \underline{122} + p_{(1)(2)2}$$
$$\vdots$$
$$\underline{1N_1 2} = \max(\overline{1N_1 - 12},\ \overline{1N_1 1})$$
$$\overline{1N_1 2} = \underline{1N_1 2} + p_{(1)(N_1)2}$$
$$\overline{22} = \underline{212} = \max(\overline{211},\ \overline{1N_1 2} + S_{(2)2})$$
$$\underline{22} = \overline{22} - S_{(2)2}$$
$$\overline{212} = \underline{212} + p_{(2)(1)2}$$
$$\vdots$$
$$\overline{1k} = \underline{11k} = \max(\overline{11k - 1},\ S_{(1)k})$$
$$\underline{1k} = \overline{1k} - S_{(1)k}$$
$$\overline{11k} = \underline{11k} + p_{(1)1)k}$$
$$\underline{12k} = \max(\overline{11k},\ \overline{12k - 1})$$
$$\overline{12k} = \underline{12k} + p_{(1)(2)k}$$
$$\vdots$$

$$\overline{ik} = \underline{ilk} = \max(\overline{ilk-1},\ \overline{i-1\,N_{i-1}k} + S_{(i)k})$$
$$\underline{ik} = \overline{ik} - S_{(i)k}$$
$$\overline{ilk} = \underline{ilk} + p_{(i)(l)k}$$

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$$\underline{ijk} = \max(\overline{ij-lk},\ \overline{ijk-l})$$
$$\overline{ijk} = \underline{ijk} + p_{(i)(j)k}$$

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$$\underline{iN_ik} = \max(\overline{iN_i-lk},\ \overline{iN_ik-l})$$
$$\overline{iN_ik} = \underline{iN_ik} + p_{(i(N_i)k}$$

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$$\underline{MN_Mk} = \max(\overline{MN_M-lk},\ \overline{MN_Mk-l})$$
$$\overline{MN_Mk} = \underline{MN_Mk} + p_{(M)(N_M)k}$$

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$$\overline{1K} = \underline{11K} = \max(\overline{11K-l},\ S_{(l)K})$$
$$\underline{1K} = \overline{1K} - S_{(l)K}$$
$$\overline{11K} = \underline{11K} + p_{(l)(l)K}$$

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$$\underline{MN_MK} = \max(\overline{MN_M-lK},\ \overline{MN_MK-l})$$
$$\overline{MN_MK} = \underline{MN_MK} + p_{(M)N_M)K} = F_{max}$$

4. INTERACTIVE GROUP SCHEDULING

Interactive human–computer systems have been widely used[9]. In this paper, an interactive mode is applied to Group Scheduling based on the solution procedures mentioned above.

Inputs for Interactive Group Scheduling are:

1. Group setup times, S_{ik}
2. Job processing times, p_{ijk}

for $i=1, 2, \ldots, M$, $j=1, 2, \ldots, N_i$, and $k=1, 2, \ldots, K$.

Then, the following graphical outputs are provided:

1. Gantt chart drawn according to the group and job sequences obtained.
2. Makespan (total production time), $F_{\max}$.
3. Critical path, on which there is no time slack.
4. Operation sheet representing starting and finishing times of group setups and job processings.

To begin with, the number of stages(K), the number of groups(M), the number of jobs in each group(N_i, $i=1, 2, \ldots, M$), group setup times(S_{ik}), and job processing times(p_{ijk})($j=1, 2, \ldots, N_i$; $k=1, 2, \ldots, K$) are put in the computer as basic data on an interactive mode. Fig. 1 indicates this process.

After all the basic data have been given to the computer, they are expressed on the display as a table. An example of the table is shown in Table 2.

Then the computer asks whether or not these basic data are correct. If there are any mistakes, the table is interactively modified.

If the human answers that the table is correct, then the computer determines group and job sequences, and displays the Gantt chart based on the specified group and job sequences. The Gantt chart which resulted from Group Scheduling for the basic data indicated in Table 2, is demonstrated in Fig. 2. In this

```
INPUT THE NUMBER OF THE GROUPS AND STAGES ?
4,5
INPUT THE NUMBER OF JOBS IN GROUP 1 ?
3
INPUT SETUP TIMES AT THE STAGES FOR GROUP 1 ?
30,15,25,30,10
INPUT PROCESSING TIMES AT THE STAGES FOR JOB 1 ?
41,65,39,79,52
INPUT PROCESSING TIMES AT THE STAGES FOR JOB 2 ?
75,75,68,71,61
INPUT PROCESSING TIMES AT THE STAGES FOR JOB 3 ?
32,25,62,73,54
INPUT THE NUMBER OF JOBS IN GROUP 2 ?
4
INPUT SETUP TIMES AT THE STAGES FOR GROUP 2 ?
10,20,15,30,25
INPUT PROCESSING TIMES AT THE STAGES FOR JOB 1 ?

... Ready ...
*
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Fig. 1 Interactively inputting basic data for Group Scheduling

Table 2 Basic data for Group Scheduling

GROUP	JOB	STAGE 1	STAGE 2	STAGE 3	STAGE 4	STAGE 5
G 1	S_1	30	15	25	30	10
	J_{11}	41	65	39	79	52
	J_{12}	75	75	68	71	61
	J_{13}	32	25	62	73	54
G 2	S_2	10	20	15	30	25
	J_{21}	50	41	22	41	55
	J_{22}	30	28	41	48	64
	J_{23}	70	20	56	54	62
	J_{24}	48	34	48	29	52
G 3	S_3	15	25	30	20	10
	J_{31}	29	55	46	37	31
	J_{32}	26	20	37	51	28
	J_{33}	72	66	40	47	62
G 4	S_4	25	30	10	25	35
	J_{41}	47	71	29	38	24
	J_{42}	27	69	42	75	57
	J_{43}	78	45	73	74	29
	J_{44}	22	42	35	68	17

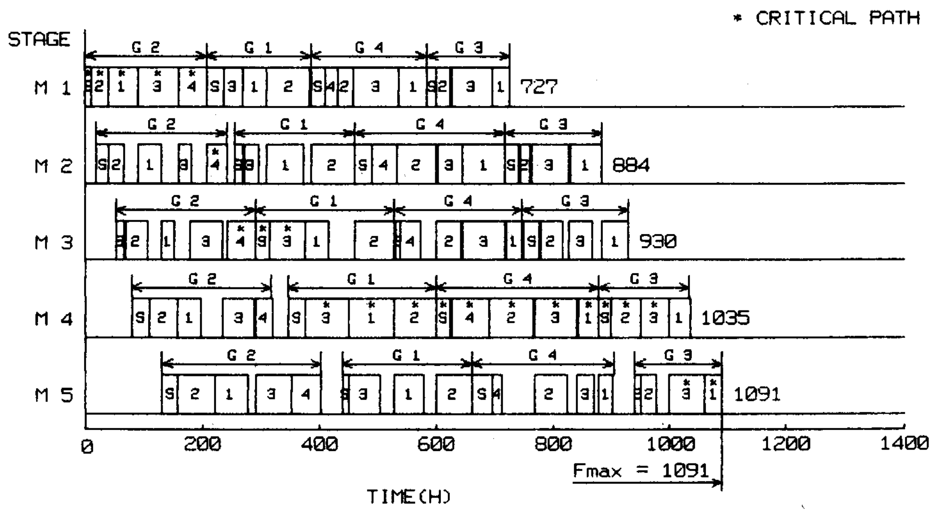

Fig. 2 The Gantt chart obtained through Group Scheduling for basic data of Table 2

figure, a series of setups and jobs marked with an asterisk is on a critical path. No delay of completing setups or jobs is allowed on this path.

The operation sheet representing the starting and finishing times of group setups and job processings can be also displayed, as demonstrated in Table 3. Setups and jobs marked with an asterisk are on the critical path. This sheet is convenient for operators in practising workshops.

The computer employed in the above example is a micro-computer, which has a CPU of Z80A (8 bits). The Interactive Group Scheduling software by BASIC for the number of stages≤ 5, the number of groups≤ 5, and the number

Table 3 The operation sheet obtained through Group Scheduling for basic data of Table 2

GROUP	JOB	STAGE 1		STAGE 2		STAGE 3		STAGE 4		STAGE 5	
		START	FINISH	START	FINISH	START	FINISH	START	FINISH	START	FINISH
G 2	S_2	* 0	10	20	40	53	68	79	109	132	157
	J_{22}	* 10	40	40	68	68	109	109	157	157	221
	J_{21}	* 40	90	90	131	131	153	157	198	221	276
	J_{23}	* 90	160	160	180	180	236	236	290	290	352
	J_{24}	* 160	208	* 208	242	* 242	290	290	319	352	404
G 1	S_1	208	233	255	270	* 290	315	347	377	440	450
	J_{13}	238	270	270	295	* 315	377	* 377	450	450	504
	J_{11}	270	311	311	376	377	416	* 450	529	529	581
	J_{12}	311	386	386	461	461	529	* 529	600	600	661
G 4	S_4	386	411	461	491	529	539	* 600	625	661	696
	J_{44}	411	433	491	533	539	574	* 625	693	696	713
	J_{42}	433	460	533	602	602	644	* 693	768	768	825
	J_{43}	460	538	602	647	647	720	* 768	842	842	871
	J_{41}	538	585	647	718	720	749	* 842	880	880	904
G 3	S_3	585	600	718	743	749	779	* 880	900	941	951
	J_{32}	600	626	743	763	779	816	* 900	951	951	979
	J_{33}	626	698	763	829	829	869	* 951	998	* 998	1060
	J_{31}	698	727	829	884	884	930	998	1035	* 1060	1091

Table 4 The optimal group and job sequences obtained for basic data of Table 2

Group Sequence	G_4	G_2	G_1	G_3
Job Sequence	$J_{44}-J_{42}-J_{43}-J_{41}$	$J_{21}-J_{22}-J_{23}-J_{24}$	$J_{13}-J_{12}-J_{11}$	$J_{33}-J_{31}-J_{32}$ $J_{33}-J_{32}-J_{31}$

of jobs in each group≤ 5, consists of 1239 steps. It took CPU time of 12 sec. to implement the above GS example.

The makespan for the example is 1091 hours, as indicated in Fig. 2 and Table 3.

The optimal group and job sequences can be determined through the branch-and-bound method, as mentioned in the Introduction. It is, however, almost impossible to process this procedure with the present micro-computer. The optimal group and job sequences obtained for the basic data (Table 2) with use of a large-scale computer, are indicated in Table 4. The makespan for this solution is 1031 hours, which is minimal.

The makespan obtained through the heuristic procedure presented in this paper is larger than the minimal value, but only by 5.8%.

The great advantage of the Interactive Group Scheduling developed in this paper is the very short computer time to obtain a near-optimal solution. This shows the advantage of this procedure in practising workshops.

The traditional flow-shop scheduling is a special case of group scheduling, in that only one job is contained in each group. Hence, by setting $N_i=1 (i=1, 2, \ldots, M)$, and by replacing $S_{ik}+p_{(i)(1)k}$ with the processing time for ith job on stage $k (=1, 2, \ldots, K)$, the heuristic algorithm and computer software developed in this paper can be used to obtain near-optimal solutions —the Gantt chart, the critical path, the makespan, and the starting and finishing times for jobs— for conventional flow-shop scheduling problems.

5. CONCLUSIONS

A heuristic, interactive group scheduling procedure was developed to obtain near-optimal solutions with a very short computer time. With this method, the Gantt chart, the critical path, the total production time, and the starting and finishing times for all group setups and all job processings are displayed on an interactive mode. An example of group scheduling was demonstrated.

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