On the Modulation Characteristics of an Electrically-biased Branched-waveguide Light Intensity Modulator

by
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Abstract

This paper presents a simple but elegant analysis of an electrically-biased branched-waveguide light intensity modulator. The analysis considers a perfectly symmetrical structure of the modulator, and subsequently takes into account the effect of fabricational asymmetry of the branching region on the modulation process. It brings out the important role of the optical bias in the linearity of modulation.

1. Introduction

Various methods of light intensity modulation have been proposed in literature[1-7] from time to time. Kaminow and Turner[1] have described the possibility of light intensity modulation by the interference of two phase-modulated polarizations under the application of a quarter wave optical bias to a waveguide fabricated on an anisotropic crystal exhibiting a linear electro-optic effect. The bias is applied here by an external optical compensator, such as a quarter wave retardation plate. The disadvantage is that the compensator provides the proper bias for a given wavelength for which it is designed. So, to modulate a different optical carrier, a different compensator should be used. In this method, linearity in the intensity modulation is obtained only for low indices of modulation. Also, other physical parameters remaining constant, the half-wave voltage is larger than the present one to be discussed.

The latest trend[6,7] in the design of a light intensity modulator may be seen to be the fabrication of the interferometric branched-waveguide electro-optic modulator. This is perhaps with a view to obtaining integration and to achieving a higher depth of modulation and a lower half-wave voltage. To facilitate digital switching, it is important to obtain perfect extinction. The branched type interferometric modulator studied in 6) yielded a maximum modulation depth

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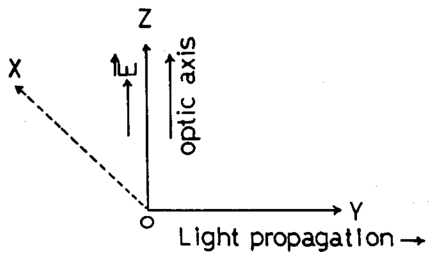
of 80% . Moreover, the detected output contains an appreciable nonlinear distortion under all operationg conditions. This seems to be due to the improper optical bias applied in the experiment. Unless we have an apriori knowledge about the proper bias, it is not possible to obtain a faithful modulation.

In this paper, we develop a simple analysis of a branched-waveguide electro-optic light intensity modulator fabricated on a $LiNbO_3$ crystal, where the optical bias is provided electrically by applying a dc voltage across the waveguide in series with the modulating signal. The analysis reflects the essentiality of an appropriate optical bias in order to obtain a distortionless intensity modulation. The analysis reveals some important features of this modulation process.

2. Analysis

We consider the optical waveguide fabricated on an X-cut $LiNbO_3$ crystal with the lengths of the arms parallel to the Y-axis of the coordinate system shown in Fig. 1. Let there be a plane polarized optical carrier with its electric field vector E along the optical axis (i. e., the Z-axis in Fig. 1 (a)) of the $LiNbO_3$ crystal plate. Then, assuming a perfectly symmetrical design of the waveguide branches, the incident light wave splits into two equi-amplitude waves which are guided through the waveguide branches along the y-direction of the coordinate system chosen. A schematic diagram of this integrated modulator is shown in Fig. 1 (b).



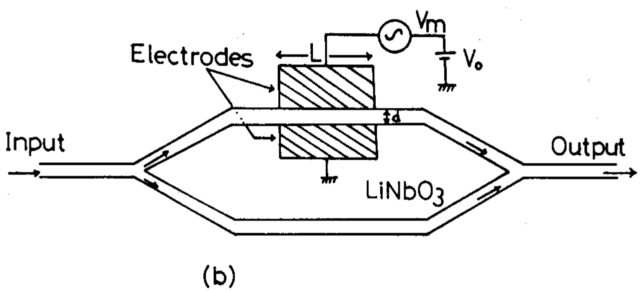


Fig. 1 : (a) Coordinate system;
(b) Schematic circuit diagram of the light modulator.

Now, let an electric field $E(t)=E_O+E_m(t)$ be applied along the z-direction across one branch, say, the upper branch, of the waveguide with the help of planar electrodes fabricated on the crystal surface. Here, E_0 is the dc electric field which provides the optical bias to the modulator, and $E_m(t)$ is the modulating signal field. If d be the effective width of each waveguide arm, then the externally applied voltage is $V(t)=E(t)\,d$. The applied electric field produces a change in the refractive index of the guiding medium in the z-direction as a result of the electro-optic effect, which is given by

$$\Delta n_z=-n_e^3 r_{33} E(t)/2 \quad \cdots\cdots (1)$$

where n_e is the principal index along the z-direction, and r_{33} is the (3,3) element of the matrix of the electro-optic coefficients of the crystal. The phase change of the z-polarized optical carrier produced by the electro-optic effect is

$$\phi(t)=2\pi L\Delta n_z/\lambda_0$$

$$=-(\phi_{0z}+\phi_m(t)) \quad \cdots\cdots (2)$$

where

$$\phi_{0z}=\pi L n_e^3 r_{33} E_0/\lambda_0,$$

$$\phi_m(t)=\pi L n_e^3 r_{33} E_m(t)/\lambda_0,$$

λ_0 is the vacuum wavelength of light and L is the length of the electrodes. Using a complex representation, the optical carrier at the input of the modulator can be written as

$$A_c(t)=A\ exp[j(\omega t-ky)] \quad \cdots\cdots (3)$$

where A is the amplitude, ω is the angular frequency and k is the magnitude of the wave vector of the light wave propagating along the y-direction. The actual fields are, however, the real parts of the complex representation. The waves coming out from the upper and lower branches of the waveguide are

$$A_u(t)=(A/\sqrt{2})exp[j(\omega t-ky-\phi_z(t)-\phi_{0u})] \quad \cdots\cdots (4a)$$

and

$$A_1(t)=(A/\sqrt{2})exp[j(\omega t-ky-\phi_{01})] \quad \cdots\cdots (4b)$$

respectively. Here, $\phi_{0u}$and ϕ_{01} are the phase changes along the upper and lower branches due to the constant path lengths. The resultant wave at the output of this modulator is

$A_0(t)=A_u(t)+A_1(t).$

The intensity of the light wave at the output of this modulator is

$$I\propto<A_0(t)A_0^*(t)>$$
$$=Constant.\ A^2[1+\cos(\phi_{01}-\phi_{0u}+\phi_{0\pi}+\phi_m(t))] \quad (5)$$

We provide the optical bias such that

$$\phi_{01}-\phi_{0u}+\phi_{0\pi}=(2n-1)\pi/2 \quad (6)$$

where n= 1, 2, 3, ……etc. In the case of a perfectly symmetrical design, $\phi_{01}=\phi_{0u}$. Even if there is an asymmetry in the lengths of the upper and lower branches, the above condition can be realized simply by adjusting the dc voltage applied. The intensity of the optical carrier (I) is proportional to A^2. Equation (5) can then be rewritten as

$$I=I_0[1\pm\sin\phi_m(t)] \quad (7)$$

where the '+' sign is for an even n and the '−' sign is for an odd n. Now, the half-wave voltage for the modulator is defined to be that voltage which produces a phase shift of the π radian of the wave by the electro-optic effect. Then,

$$V_\pi=\lambda_0 d/(Ln_e^3 r_{33}). \quad (8)$$

Equation (7) can be recast in the form

$$I=I_0[1\pm\sin(\pi V_m/V_\pi)] \quad (9)$$

where $V_m=E_m(t)\ d$. Thus, for perfect extinction, we must have

$$V_{m0}=(2k-1)V_\pi/2 \quad (10\text{ a})$$

for $k=1, 2, 3, \ldots$ etc. V_{m0} is the peak voltage of the modulating signal. The smallest value of V_{m0} corresponding to $k=1$ is

$$V_{m0}=V_\pi/2 \quad (10\text{ b})$$

We will see later that an increase of the modulating signal voltage corresponding to $k>1$ reduces the amplitude of the fundamental component and enhances the harmonic distortion of the output modulation.

Again, assumimg identical geometrical lengths of the waveguide arms, the optical bias voltage (V_0) is given by

$$V_0=(2n-1)V_\pi/2 \quad (11)$$

where n is any non-zero positive integer. Thus, the proper optical bias for a modulator operation can be provided by any half integral multiple of the half-wave voltage. 'n' may be termed as the order of the optical bias. The lowest bias voltage is obtained for $n=1$, and the corresponding first order bias is given by $V_0=V_\pi/2$. Hence, under the first order biasing we must have

$$V_{m0}=V_0 \qquad (12)$$

for perfect extinction.

In order to plot the characteristic of this modulator, we recast Eq. (5), by making use of Eq. (8), as

$$I=I_0[1+\cos(\pi V(t)/V_\pi)] \qquad (13)$$

where it is assumed that $\phi_{01}=\phi_{0u}$. The characteristic is shown in Fig. 2, which indicates that there exist proper values of the optical bias and modulating signal amplitude for the most linear operation of the modulator with the possibility of perfect extinction.

Let us consider, as an example, the modulating signal in the form $V_m(t)=V_{m0}\sin\omega_m t$, where ω_m is the angular frequency of the modulating signal. Equation (9) can now be written as

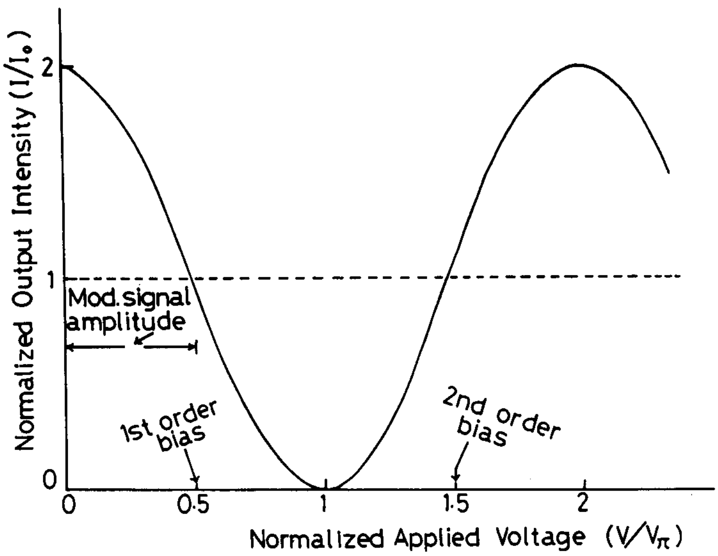


Fig. 2 : Characteristics of the electrically-biased branched-waveguide light modulator.

$$I=I_0[1\pm2\sum_{r=1}^{\infty}J_{2r-1}(\pi V_{m0}/V_\pi)\sin(2r-1)\omega_m t] \quad (14\,a)$$

$$=I_0\pm\sum_{r=1}^{\infty}I_{2r-1}\sin(2r-1)\omega_m t \quad (14\,b)$$

Here, $J_{2r-1}(.)$ is Bessel's function of order $(2r-1)$, and $I_{2r-1}=2I_0J_{2r-1}(\pi V_{m0}/V_\pi)$ is the amplitude of the intensity modulation for the $(2r-1)$th harmonic of the modulating signal. The amplitude of the fundamental intensity modulation is I_1 which is proportional to the first order Bessel's function $J_1(\pi V_{m0}/V_\pi)$. The variation of the normalized amplitude (I_1/I_0) of the fundamental intensity modulation as a function of the modulating signal amplitude (V_{m0}) is shown in Fig. 3. Since the value of $J_1(x)$ as a function of its argument varies in an oscillatory manner with a decreasing amplitude, the value of V_{m0} should be such that the amplitude of the fundamental modulation is maximum. The maximum peak value of $J_1(x)$ is 0.586 which occurs at $x=1.84$. Under a perfect extinction condition, $J_1(\pi V_{m0}/V_\pi)=J_1(\pi/2)=0.5668$, which incidentally happens to be nearly equal to the desired maximum value.

It is seen from Eq. (14 a) that the intensity modulation contains only odd harmonic distortions. The amplitude of the n-th odd harmonic is proportional to

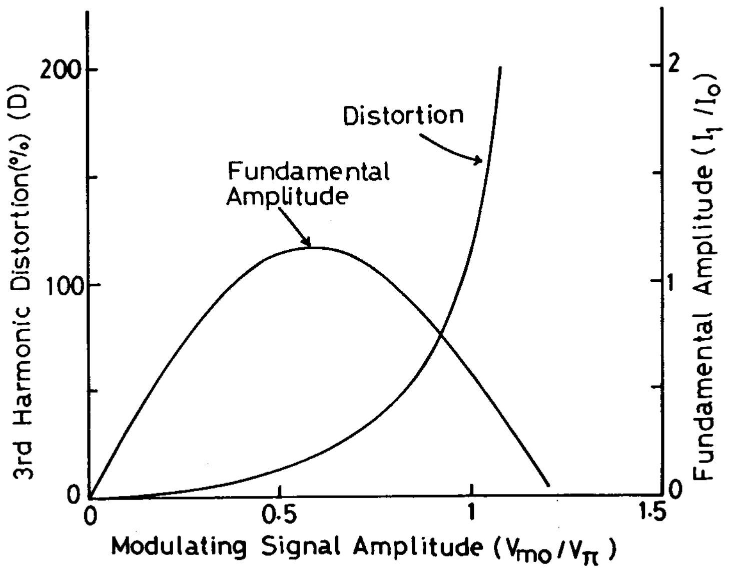


Fig. 3 : Variations of the amplitude of fundamental intensity modulation and the third harmonic modulation distortion with the modulating signal amplitude.

$J_n(\pi V_{m0}/V_\pi)$. Accordingly, the percentage of the third harmonic distortion is given by

$$D=[J_3(\pi V_{m0}/V_\pi)/J_1(\pi V_{m0}/V_\pi)].\ 100\% \cdots\cdots (15)$$

The third harmonic distortion as a function of the modulating signal amplitude is plotted Fig. 3, which indicates that the distortion increases rapidly as the amplitude of the intensity modulating signal is increased beyond its value corresponding to perfect extinction. Fig. 3. Under the condition of achieving perfect extinction of the modulated light, the percentage of the third harmonic distortion is given by

$$[J_3(\pi/2)/J_1(\pi/2)]=12.17\%.$$

In terms of demodulated signal power this corresponds to 1.48% . This is the minimum third harmonic distortion of the output modulation corresponding to the smallest possible modulating voltage determined by (10 a). If a smaller depth of intensity modulation is desired, the harmonic distortion can be reduced considerably. If the bias is selected so that it does not satisfy (11), we obtain

$$I=I_0[1\pm\sin\{\theta+(\pi V_m(t)/V_\pi)\}] \cdots\cdots (16a)$$

which when expanded in terms of Bessel's functions indicates that the fundamental modulation is decreased and the harmonic distortion of the modulation increases. Particularly, if the bias is selected according to the condition

$$\phi_{01}-\phi_{0u}+\phi_{0z}=m\pi \cdots\cdots (16b)$$

where m is any integer including zero, Eq. (5) indicates that no intensity modulation can be produced at the fundamental modulating frequency.

If the modulating field and the dc bias field is applied also to the second branch of the interferometric waveguide in the negative z-direction, then we get

$$I=I_0[1\pm\sin(2\pi V_m/V_\pi)] \cdots\cdots (17)$$

assuming $(\phi_{01}-\phi_{0u}+2\phi_{0z})=(2n-1)\pi/2$. In this case, for perfect extinction we have $V_{m0}=V_\pi/4$, and under the first order biasing $V_0=v_\pi/4$. Thus, in this configuration, the modulating signal voltage and the bias voltage can be reduced to half of their values required in the former case, where we considered the field to be applied across one branch only. Taking $\lambda_0=6328$ Å, $d=4.5\,\mu$m and $L=6$ mm, we get $V_\pi=1.45$ V for a $LiNbO_3$ crystal having $n_e=2.2$ and $r_{33}=30.8\times10^{-12}$ m/V.

3. Asymmetry of the Fork Region

The function of the input fork region of the branched waveguide is to distribute the incident light energy equally in two branches of the waveguide. This, in turn, requires a perfectly symmetrical structure of the branching section. But, if there occurs some fabricational asymmetry in the structure of the fork region, the optical carrier energy is not distributed equally between the arms of the waveguide. In this section, we attempt to study the effect of unequal energy splitting between the two branches of the waveguide on the modulation performance of the device.

Let the input light energy be divided into fractions of $(0.5-\delta)$ and $(0.5+\delta)$ along the two branches of the waveguide modulator due to this fabricational asymmetry. The guided waves appearing at the output of the upper and lower branches of the waveguide are

$$x_1(y,\ t)=(A/\sqrt{2})(1-2\delta)^{0.5}\mathrm{Exp}[j(\omega t-ky-\phi_z(t)-\phi_{0u})]$$

and

$$x_2(y,\ t)=(A/\sqrt{2})(1+2\delta)^{0.5}\mathrm{Exp}[j(\omega t-ky-\phi_{01})]$$

respectively. The output intensity is

$$I'\propto<(x_1+x_2)(x_1+x_2)^*>.$$

Following the same procedure involved in the derivation of Eq. (9), we obtain

$$I'=I_0[1+(1-4\delta^2)^{0.5}\sin(\pi V_m/V_\pi)] \quad \cdots\cdots \quad (18)$$

The minimum intensity is

$$I'_{min.}=I_0[1-(1-4\delta^2)^{0.5}]\simeq 2\delta^2$$

assuming $\delta<<0.5$. Thus, a first order asymmetry in the energy division between the branches appers as a second order effect in the resultant intensity modulation. So, the effect of this asymmetry is small, although perfect extinction is not achieved due to this effect.

4. Discussion and Conclusion

The electrical biasing of the interferometric optical intensity modulator makes it convenient to vary the optical bias simply by varying the applied dc voltage. Moreover, the small value of the half-wave voltage makes it useful for integrated circuits. The analysis has revealed the critical role of the optical bias in the design of this modulator.

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