Analysis of Optimal Machining Conditions for Flow-type Automated Manufacturing Systems under the Maximum Profit

by

Katsundo HITOMI*

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Abstract

The optimal machining speeds to be utilized on the multiple stages of a flow-type automated manufacturing system were analyzed to achieve the maximum profit. Based on the analytical results, the optimizing algorithm was developed to determine the optimal machining speeds on the multiple stages of the manufacturing system. A numerical example was given to show the effectiveness of the algorithm.

1. Introduction

These days automated manufacture by machining/turning centers is prevalent. In such a case, the worn cutting tool can be replaced with a new edge in a short time within the setup time of a workpiece. Then the production time required to manufacture a piece of product is not expressed as a unimodal convex function with respect to the machining speed, as is the usual case where the operator manually replaces the worn cutting tool with a new edge. Hence, the regular maximum production-rate machining speed no longer exists in the case of automated manufacture. But the minimum cost machining speed exists and it was proved that the maximum profit-rate machining speed also exists above the minimum cost machining speed [1].

A flow-type multi-stage automated manufacturing system consisting of multiple machining/turning centers was also analyzed to determine the optimal machining speeds under the maximum production rate [2].

This paper aims to analyze a flow-type multi-stage automated manufacturing system for determining the optimal machining speeds to be utilized on the

* Department of Precision Mechanics

multiple stages under the maximum profit.

2. Preconditions

The following preconditions are placed on the analysis of the maximum profit for producing a single product on an automated manufacturing system.

(1) The automated manufacturing system is of a flow type of N stages which are connected by an automated material-handling equipment (such as a conveyor). The stage index is denoted by $j : j= 1, 2, \ldots, N$. $\{N\}$ signifies a set of all stages.

(2) The loading and unloading of workparts on all stages are done automatically during the same cycle (or tact) time. Each setup time is denoted by a (min/pc).

(3) The production time and production cost on stage $j(S_j)$ are dependent on the machining speed, v_j(m/min), utilized on that stage.

(4) The tool life, T_j(min/edge), on S_j follows the Taylor tool life equation [3]:

$$v_j T_j^{n_j} = C_j \ (j=1, 2, \ldots, N) \tag{1}$$

where n_j and C_j are constants (slope constant and 1-minute tool-life machining speed).

(5) The replacement of a worn cutting edge with a new one is done automatically within the setup time, a(min/pc).

3. Production Model

3. 1. Stage production time

The machining time (min/pc) on S_j is b_j/v_j, where b_j is a machining constant. Hence, the stage production time (min/pc) on S_j to manufacture a workpiece is:

$$t_j(v_j) = a + b_j/v_j \ (j=1, 2, \ldots, N) \tag{2}$$

since the setup time is a.

3. 2. Cycle time

The most important factor regarding a multi-stage manufacturing system is a cycle time which is a time interval of the successive flowout of products completed by passing through the system.

The cycle (or tact) time is the maximum among stage production times given by equation (2):

$$t=\max_{1\le j\le N} t_j(v_j)\equiv a+b_K/v_K \qquad (3)$$

where K signifies a bottleneck stage, which has the largest stage production time.

3. 3. Stage production cost

The production cost of a workpart on S_j consists of the machining cost and the tool cost. Denoting the machining overhead on S_j such as electricity, cutting fluid, and others by c_j(¥/min), the machining cost required to manufacture a piece of workpart on S_j is:

$$u_{jc}(v_j)=b_jc_j/v_j \; (j=1, 2, \ldots, N) \qquad (4)$$

Each tool can produce an amount, $T_j/(b_j/v_j)$, of products during its life time. Hence, by denoting the cost of a new tool utilized on S_j by c_{tj}(¥/edge) and by using equation (1), the tool cost required to manufacture a piece of workpart is:

$$u_{jt}(v_j)=b_jc_{tj}v_j^{1/n_j-1}/C_j^{1/n_j} \; (j=1, 2, \ldots, N) \qquad (5)$$

Then the stage production cost required to manufacture a piece of workpart on S_j is given by the sum of equations (4) and (5):

$$u_j(v_j)=b_jc_j/v_j+b_jc_{tj}v_j^{1/n_j-1}/C_j^{1/n_j} \; (j=1, 2, \ldots, N) \qquad (6)$$

This is a convex function with respect to v_j. Hence, there exists a minimum cost machining speed, v_{jc}, to minimize the stage production cost as follows:

$$v_{jc}=C_j[c_j/c_{tj}(1/n_j-1)]^{n_j} \; (j=1, 2, \ldots, N) \qquad (7)$$

3. 4. Total production cost

The invested capital for an automated manufacturing system and the direct and indirect costs for workers on this system are denoted by k(¥/min). Since a piece of product is produced in each cycle time, the overhead required to manufacture a piece of product is:

$$u_0(v)=kt \qquad (8)$$

where $v=(v_1, v_2, \ldots, v_N)$.

Then the total production cost required to manufacture a unit of product is given by:

$$u(\boldsymbol{v})=u_o(\boldsymbol{v})+\Sigma_{j=1}^{N}u_j(v_j)=kt+\Sigma_{j=1}^{N}(b_jc_j/v_j+b_jc_{tj}v_j^{1/n_j-1}/C_j^{1/n_j}) \tag{9}$$

3.5. Profit

Denoting the revenue (selling price minus material cost) obtained through a piece of product by r, the profit obtained is:

$$p(\boldsymbol{v})=r-u(\boldsymbol{v})=r-kt-\Sigma_{j=1}^{N}(b_jc_j/v_j+b_jc_{tj}v_j^{1/n_j-1}/C_j^{1/n_j}) \tag{10}$$

3.6. Constraints

The machining speed, v_j, to be utilized is technologically limited within a suitable "allowable speed range":

$$\underline{v}_j\leq v_j\leq\bar{v}_j \ (j=1, 2, \ldots, N) \tag{11}$$

where $\underline{v}_j$ and $\bar{v}_j$ are the lower and upper limits of the machining speed on S_j.

If $v_{jc}<\bar{v}_j$, a speed range E_j between v_{jc} and $\bar{v}_j$ is called "efficiency speed range" [4]:

$$E_j=[v_{jc}, \bar{v}_j] \ (j=1, 2, \ldots, N) \tag{12}$$

Any speed in this range is called the "efficiency speed".

4. Determining the Optimal Machining Speeds under the Maximum Profit

4.1. Analysis

The maximum profit is attained by solving the following problem.

(P) Maximize equation (10) subject to equation (11) and

$$a+b_j/v_j-t\leq 0 \ (j\in\{N\}-\{K\}) \tag{13}$$

Relaxing the constraint given by equation (11) (this is considered in developing the algorithm) and applying the Kuhn-Tuker condition [5], there exist $\lambda_j\geq 0$, $j\in\{N\}-\{K\}$, for the optimal v_j^* such that

$$c_{tK}(1/n_K-1)v_K^{*1/n_K-2}/C_K^{1/n_K}-(c_K+k-\lambda_K)/v_K^{*2}=0, \ \lambda_K=\Sigma_{j\in\{N\}-\{K\}}\lambda_j \tag{14}$$

$$c_{tj}(1/n_j-1)v_j^{*1/n_j-2}/C_j^{1/n_j}-(c_j+\lambda_j)/v_j^{*2}=0 \tag{15}$$

$$\lambda_js_j^*=0, \ s_j^*=b_K/v_K^*-b_j/v_j^*\geq 0 \tag{16}$$

Hence,

$$v_K^* = C_K[(c_K + k - \lambda_K)/c_{tK}(1/n_K - 1)]^{n_K} \quad (17)$$

$$v_j^* = C_j[(c_j + \lambda_j)/c_{tj}(1/n_j - 1)]^{n_j} \quad (18)$$

Then the optimal cycle time is:

$$t^* = a + b_K/v_K^* \quad (19)$$

As shown above, the optimal machining speeds vary with Lagrange multipliers, λ_j, $j = 1, 2, \ldots, K-1, K+1, \ldots, N$. We have the following results.

(i) The optimal machining speed on the slack stage, v_j^*, is the minimum cost speed, v_{jc}, if $\lambda_j = 0$.

(ii) The optimal machining speed on the bottleneck stage, v_K^*, is the efficiency speed:

$$v_{Ke} = C_K[(c_K + k)/c_{tK}(1/n_K - 1)]^{n_K} \quad (20)$$

if λ_j's are all zero.

Appropriate values for λ_j's are set such that for the optimal machining speeds given by equations (17) and (18) and the optimal cycle time given by equation (19), equation (13) holds for stages having $\lambda_j = 0$ and the equality of this equation holds for stages having $\lambda_j > 0$.

4. 2. Algorithm

Step 1: Calculate v_{jc} for all $j \in \{N\}$ by equation (7). If $v_{jc} < \underline{v}_j$, set $v_{jc} \leftarrow \underline{v}_j$. If $v_{jc} > \bar{v}_j$, set $v_{jc} \leftarrow \bar{v}_j$.

Step 2: Calculate $t_{jc} = b_j/v_{jc}$ for all $j \in \{N\}$.

Step 3: Calculate v_{Ke} for all $K \in \{N\}$ by equation (20). If $v_{Ke} < \underline{v}_K$, set $v_{Ke} \leftarrow \underline{v}_K$. If $v_{Kc} > \bar{v}_K$, set $v_{Ke} \leftarrow \bar{v}_K$.

Step 4: Calculate $t_{Ke} = b_K/v_{Ke}$ for all $K \in \{N\}$.

Step 5: Proceed to the following by sequentially setting $K = 1, 2, \ldots, N$.

(5-1) If $t_{Ke} \geq t_{jc}$ for all $j \in \{N\} - \{K\}$, set $t_K \leftarrow t_{Ke}$ and go to step 6.

(5-2) Denote by $\{J\}$ a set of j such that $t_{jc} > t_{Ke}$.

(5-3) Set appropriate positive values for λ_j's for $j \in \{J\}$ such that $t_K = b_K/v_K^* \geq b_j/v_j^*$ by using equations (17) and (18), and set $v_{Ke} \leftarrow v_K^*$ and $v_{jc} \leftarrow v_j^*$. Go to step 6.

Step 6: Calculate $u_K = u(\boldsymbol{v})$ by equation (9) for $K = 1, 2, \ldots, N$.

Step 7: (Determining the bottleneck stage) Set K for $\min_{K \in \{N\}} u_K$ as K^* (the bottleneck stage).

Step 8 : (Determining the optimal cycle time) $t^* = t_{K^*} + a$.

Step 9 : (Determining the optimal machining speeds) $v_{K^*}{}^* = v_{K^*e}$ and $v_j{}^* = v_{jc}$ for $j \in \{N\} - \{K^*\}$. Stop.

5. Numerical Example

The basic data for manufacturing a product on a 3-stage manufacturing system are shown in Table 1.

Table 1 Basic data for the numerical example.

Stage	Machining parameters					Tool parameters			Setup time	Cost parameters		
j	s_j	D_j	L_j	L_j'	$\bar{v}_j$	Z_j	C_j	n_j	a	c_j	c_{tj}	r
1	.20	104	200	–	400	1	450	.25	.5	10	750	—
2	.05	100	300	100	350	8	500	.33	.5	15	1500	—
3	.15	10	95	–	250	2	400	.33	.5	15	600	5000

〈Legend〉 $j=1$: turning
$j=2$: milling
$j=3$: drilling

Following the algorithm mentioned in section 4.2, the result obtained is indicated in Table 2. During the procedure b_j is calculated by [6]:

Table 2 The optimal solution.

	Stage 1	Stage 2	Stage 3
Optimal machining speed (m/min)	172 e*	166 e*	94 c
Cycle time (min/pc)		2.40	
Maximum profit (¥/pc)		4506	

〈Legend〉 c minimum cost speed
e efficiency speed
* bottleneck stage

$$b_j = \begin{cases} D_j L_j Z_j / 1000 s_j & \text{for turning and drilling} \\ D_j (L_j + L'_j) / 1000 s_j Z_j & \text{for milling} \end{cases} \qquad (j = 1, 2, \ldots, N) \qquad (21)$$

where D_j is the machining diameter (mm) for turning, or the tool diameter (mm) for milling and drilling; L_j is the machining length (mm) for turning and milling, or the length of the hole to be machined (mm) for drilling; L_j' is the additional feed length (mm) for milling; s_j is the feed rate per tooth (mm/tooth) for milling, or the feed rate per revolution (mm/rev) for the other operations; Z_j is the number of teeth of the cutter or the drill ($j = 1, 2, \ldots, N$).

6. Conclusions

(1) The optimal machining speeds to be utilized on the multiple stages of a flow-type automated manufacturing system were analyzed under the maximum profit criterion for single-product production.
(2) Based on the optimization analysis the algorithm determining the bottleneck stage, the optimal cycle time, and the optimal machining speeds on all stages of the manufacturing system was developed.
(3) A numerical example was demonstrated to show the effectiveness of the algorithm developed.

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