Analysis of Competition between Electricity and Town-Gas Suppliers under Time-of-Use Pricing

by

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Abstract

Time-of-use pricing attracts attention in energy utilities such as electricity and town-gas, as a method of load management. These energy utilities are supplied by separate private companies, and hence there may be a certain competition between these suppliers by strategic use of time-of-use pricing for substitutable demand. In the pesent paper, strategic price setting by these competitive energy suppliers is analyzed based on a game model. Theoretical analysis and a numerical case study of the model show that the competitive suppliers discount prices for peak demand, and that the competition makes the market inefficient.

1. Introduction

In recent years, time-of-use pricing (TOUP), a novel pricing scheme in which the price is altered depending on season and time of day, attracts attention in energy utilities such as electricity and town-gas. In electric power systems, the load fluctuation is getting larger due to the sophistication of industrial production and human life style, and the necessity for load management [1], [2] is increasing to keep the supply systems stable and efficient. TOUP is expected to play an important role as a method of load management in electric power systems facing the aforesaid difficulties, and it has been introduced to the systems as an optional contract for industrial customers since 1988, and for residential customers since 1990, in Japan.

The authors have studied the economic effects of load management of electricity and town-gas with TOUP [3]. In these two energy systems, their peak loads appear in different seasons and times of day, i.e., the peak load of electricity appears in summer afternoons, and that of town-gas in winter evenings (Fig. 1). The peak loads on these energy systems are brought about by the demand for air conditioning, space heating and water heating. Nowadays, these demands are

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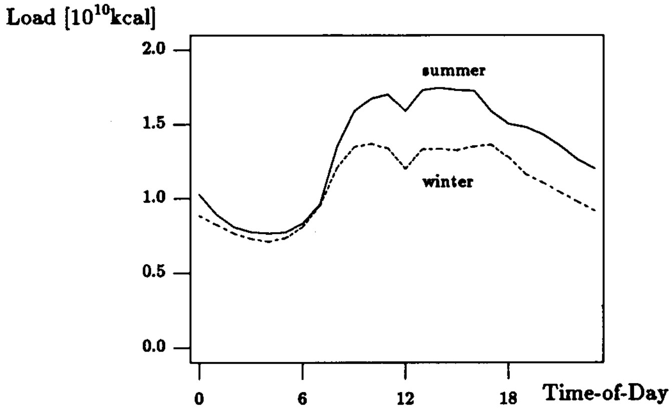
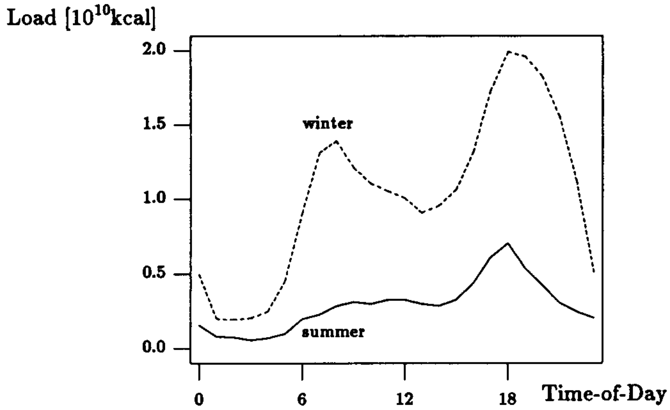


(a) Electricity Load



(b) Town-Gas Load

Fig. 1. Typical daily load curves of the electricity and town-gas systems.

efficiently substitutable between these sorts of energy by means of, for example, electric heat pump and gas-engine heat pump. Considering this situation, the economic effects of a cooperative supply of these energy utilities have been studied in the aforesaid paper.

However, actually in Japan, electricity and town-gas are supplied by separate private companies having their own goals, e.g., profits and sales. Hence there exists a certain competition for the mutually substitutable demands between these companies with strategic time-of-use pricing. Since the supply system needs a huge amount of investment, these energy utility companies are allowed to supply the energy monopolistically in their service areas. At the same time, some regulations are imposed on their princing policies in order to prevent the suppliers from getting monopolistic profits and to protect the public welfare. The most typical regulation is to require the profit of the supplier to keep a fair ratio to its investment.

In the present paper, the competition between an electricity and a town-gas supplier under TOUP is studied. The situation is formulated into a competition problem between two regulated companies which supply utilities partially substitutable. For this, a static type game model is proposed in Section 2. In Section 3, the characteristics of the equilibrium prices are discussed analytically. In Section 4, some numerical examples are presented to illustrate the results.

2. Game Model of Inter-Energy Competition

2.1 Formulation of the model

Let us consider the supply/demand of electricity and town-gas in N periods. The electricity supplier decides the time-of-use price of electricity $\boldsymbol{p}_E = (p_{E1}, \ldots, p_{EN})^T$ in these N periods, and the town-gas supplier decides the time-of-use price of town-gas $\boldsymbol{p}_G = (p_{G1}, \ldots, p_{GN})^T$ as well. Responding to these prices, the time-of-use demand of electricity $\boldsymbol{q}_E = (q_{E1}, \ldots, q_{EN})^T$ and that of town-gas $\boldsymbol{q}_G = (q_{G1}, \ldots, q_{GN})^T$ in the N periods are determined according to demand functions D_E and D_G, respectively:

$$\begin{aligned} \boldsymbol{q}_E &= D_E(\boldsymbol{p}_E, \boldsymbol{p}_G) \\ \boldsymbol{q}_G &= D_G(\boldsymbol{p}_E, \boldsymbol{p}_G). \end{aligned} \tag{1}$$

As a matter of course, the vectors $\boldsymbol{p}_E$, $\boldsymbol{p}_G$, $\boldsymbol{q}_E$ and $\boldsymbol{q}_G$ are constrained to be non-negative.

The goal of each supplier is assumed to be maximization of its total sales. Namely, the payoff function of the electricity supplier f_E and that of the town-gas supplier f_G are, respectively:

$$\begin{aligned} f_E &\equiv \boldsymbol{p}_E^T \boldsymbol{q}_E \\ f_G &\equiv \boldsymbol{p}_G^T \boldsymbol{q}_G. \end{aligned} \tag{2}$$

We consider a regulation that each supplier is required to keep a fair ratio of its profit to the total supply cost:

$$\begin{aligned} f_E - C_E(\boldsymbol{q}_E) &\leq \varepsilon C_E(\boldsymbol{q}_E) \\ f_G - C_G(\boldsymbol{q}_G) &\leq \varepsilon C_G(\boldsymbol{q}_G) \end{aligned} \tag{3}$$

where ε is an upper limit of the profit ratio to the total supply cost, and $C_E(\boldsymbol{q}_E)$ and

$C_G(\boldsymbol{q}_E)$ are the total supply costs of the electricity and the town-gas, respectively.

The problem of the competition between the suppliers is formulated as the following game problem:

$$\begin{aligned} &\max_{\boldsymbol{p}_E} f_E \quad \text{for the Electricity Supplier} \\ &\max_{\boldsymbol{p}_G} f_G \quad \text{for the Town-Gas Supplier} \\ &\text{sub. to} \quad \boldsymbol{g}(\boldsymbol{p}_E, \boldsymbol{p}_G) \leq 0 \end{aligned} \tag{4}$$

where a vector function $\boldsymbol{g}(\boldsymbol{p}_E, \boldsymbol{p}_G) = (g_1, \dots, g_{4N+2})^T$ is defined as follows:

$$g_1 \equiv f_E - (1 + \varepsilon)C_E \tag{5a}$$

$$g_2 \equiv f_G - (1 + \varepsilon)C_G \tag{5b}$$

$$(g_3, \dots, g_{2+N})^T \equiv -\boldsymbol{p}_E \tag{5c}$$

$$(g_{3+N}, \dots, g_{2+2N})^T \equiv -\boldsymbol{p}_G \tag{5d}$$

$$(g_{3+2N}, \dots, g_{2+3N})^T \equiv -\boldsymbol{q}_E = -\boldsymbol{D}_E(\boldsymbol{p}_E, \boldsymbol{p}_G) \tag{5e}$$

$$(g_{3+3N}, \dots, g_{2+4N})^T \equiv -\boldsymbol{q}_G = -\boldsymbol{D}_G(\boldsymbol{p}_E, \boldsymbol{p}_G). \tag{5f}$$

Equations (5a) and (5b) correspond to the regulatory constraints (3), and Eqs. (5c) through (5f) are for nonnegativity of the prices and the demands.

2.2 Concept of the solution

Let us consider the problem (4) with an assumption that both suppliers behave noncooperatively, and confine our discussion only to pure strategies. In noncooperative game problems, the most acceptable concept of the solution is the Nash equilibrium [4]. For the problem (4), the Nash equilibrium $(\boldsymbol{p}_E^N, \boldsymbol{p}_G^N)$ is defined as follows:

$$\begin{aligned} &f_E(\boldsymbol{p}_E^N, \boldsymbol{p}_G^N) \geq f_E(\boldsymbol{p}_E, \boldsymbol{p}_G^N), \quad \text{for all } \boldsymbol{p}_E \text{ such that } \boldsymbol{g}(\boldsymbol{p}_E, \boldsymbol{p}_G^N) \leq 0 \\ &f_G(\boldsymbol{p}_E^N, \boldsymbol{p}_G^N) \geq f_G(\boldsymbol{p}_E^N, \boldsymbol{p}_G), \quad \text{for all } \boldsymbol{p}_G \text{ such that } \boldsymbol{g}(\boldsymbol{p}_E^N, \boldsymbol{p}_G) \leq 0. \end{aligned} \tag{6}$$

However, in a problem with constraints such as problem (4), it is known that there can exist an infinite number of the Nash equilibria on the boundary of its feasible region [4]. In order to avoid this difficulty, we introduce a more strict concept of equilibrium, 'the normalized Nash equilibrium' which is proposed by Rosen [5]. The normalized Nash equilibrium $(\boldsymbol{p}_E^* \boldsymbol{p}_G^*)$ for the problem (4) is defined as follows [4]:

$$f_E(p_E^*, p_G^*) + f_G(p_E^*, p_G^*) =$$

$$\max_{p_E, p_G} [f_E(p_E, p_G^*) + f_G(p_E^*, p_G)] \qquad (7)$$

$$\text{sub. to} \quad g(p_E, p_G) \le 0.$$

Rosen has shown that the normalized Nash equilibrium is unique under certain conditions [5]. He has also shown that equilibrium is achieved by a process in which both players adjust their decision variables according to the projected gradients of their payoff functions to the feasible region [5]. This process is quite similar to a famous process proposed by Cournot as a model of duopoly [6].

3. Analytical Study

In order to discuss the natures of the equilibrium prices of the problem (4), let us make some assumptions as follows:

1) There is no inter-period cross price elasticity of demand, while there are some nonzero inter-energy cross price elasticities of simultaneous demands.
2) The equilibrium prices (and the associated demands) are positive.
3) The fair profit ratio to the cost, ε, is assumed to be 0 for simplicity. This assumption can be easily relaxed.
4) The functions f_E, f_G, C_E, C_G, D_E and D_G are continuously differentiable with respect to both the price and the demand.
5) There exists at least one equilibrium point, and at that point, Kuhn-Tucker constraint qualification [4], [7] holds.

With the above assumptions, the necessary conditions for the normalized Nash equilibrium* in the problem (4) are given by:

$$\begin{aligned} &-\nabla_{pE} f_E + \lambda_E \nabla_{pE} g_1 + \lambda_G \nabla_{pE} g_2 = 0 \\ &-\nabla_{pG} f_G + \lambda_E \nabla_{pG} g_1 + \lambda_G \nabla_{pG} g_2 = 0 \\ &g_1 \le 0, \ g_2 \le 0 \\ &\lambda_E g_1 = \lambda_G g_2 = 0 \\ &\lambda_E, \lambda_G \ge 0 \end{aligned} \qquad (8)$$

where λ_E and λ_G are Lagrange multipliers for the regulatory constraints (3).

Now, let us consider the problem without the regulatory constraints. Then, the

* This set of necessary conditions is derived from a theorem which applies the Kuhn-Tucker condition for a nonlinear programming problem to a game problem [4].

necessary conditions (8) are simplified as follows:

$$MR_E = 0$$
$$MR_G = 0 \tag{9}$$

where MR_E and MR_G stand for $\nabla_{pE} f_E$ and $\nabla_{pE} f_G$, respectively, i.e., the marginal change of the revenues according to the marginal change of the *prices*. Here we call these quantities 'marginal revenues' for simplicity, although the term marginal revenue usually indicates a marginal change of revenue according to a marginal change in the *supply*.

Eqs. (9) mean that, at an equilibrium point, each supplier could not change his revenue by unilateral change of his offering price.

Then, let us consider the regulatory constraints. From the first equation of Eqs. (8), the following is readily derived:

$$MR_{Ei} = \frac{\lambda_E}{\lambda_E - 1} MC_{Ei} \frac{\partial q_{Ei}}{\partial p_{Ei}} + \frac{\lambda_G}{\lambda_E - 1} (MC_{Ei} - p_{Gi}) \frac{\partial q_{Gi}}{\partial p_{Ei}} \tag{10}$$

where MR_{Ei} is the i-th element of MR_E. $MC_{Ei} \equiv \partial C_E/\partial q_{Ei}$ and $MC_{Gi} \equiv \partial C_G/\partial q_{Gi}$, i.e., the marginal supply costs of electricity and town-gas, respectively. Taking the symmetry of the model into consideration, a similar equation for the marginal revenue of the town-gas is obtained as well.

The first term of the RHS of Eq. (10) means that without regulatory constraints the marginal revenue of the supplier deviates from equilibrium according to his marginal supply cost. At the same time, the second term of the RHS implies that the equlibrium marginal revenue is also influenced by the deviation of the price offered by the competitor from his marginal supply cost.

It depends on the sign of factor of each term in the RHS of Eq. (10) whether the term raises or reduces the equilibrium marginal revenue from that without regulatory constraints. Let us examine the signs of these terms. The partial derivatives of the demands with respect to the price, i.e., $\partial q_{Ei}/\partial p_{Ei}$ or $\partial q_{Gi}/\partial p_{Ei}$ are determined according to the characteristics of the demand functions. Ordinarily, the partial derivative $\partial q_{Ei}/\partial p_{Ei}$ is negative. Further, if both sorts of energy are substitutable, the partial derivative $\partial q_{Gi}/\partial p_{Ei}$ will be positive. Hence, to decide the signs of the factors in the RHS of Eq. (10), we must know the ranges of Lagrange multipliers λ_E and λ_G. The following proposition shows that the values of the Lagrange multipliers are between null and unity under some conditions.

Proposition

Under the following conditions, the values of the Lagrange multipliers which appear in Eq. (10) are between null and unity, i.e.:

$$0 < \lambda_E,\ \lambda_G < 1. \tag{11}$$

Conditions

1) At the equilibrium point, both regulatory constraints are active.
2) Considering a problem which maximizes C_E with respect to $\boldsymbol{p}_E$ under constraints $g_1(\boldsymbol{p}_E, \boldsymbol{p}_G^*) \geq 0$ and $g_2(\boldsymbol{p}_E, \boldsymbol{p}_G^*) \leq 0$ and with a fixed price of town-gas $\boldsymbol{p}_s^*$, the former constraint becomes active, and the Lagrange multiplier associated with the constraint is not degenerated. Similar conditions hold for the symmetric problem with respect to C_G and $\boldsymbol{p}_G$.
3) With an adequate selection of the periods i and j, the following condition holds at the equilibrium:

$$(MR_{Ei} - \overline{MC}_{Ei})(\overline{MR}_{Gj} - \overline{MC}_{Gj}) \neq (MR_{Ej} - \overline{MC}_{Ej})(\overline{MR}_{Gi} - \overline{MC}_{Gi}) \tag{12}$$

where $\overline{MC}_{Ei}$ and $\overline{MC}_{Gi}$ are $\partial C_E/\partial p_{Ei}$ and $\partial C_G/\partial p_{Ei}$, respectively. $\overline{MR}_{Gi}$ stands for $\partial f_G/\partial p_{Ei}$.

Proof

Due to the Kuhn-Tucker condition (8), the following equation is derived:

$$\{(MR_{Ei} - \overline{MC}_{Ei})(\overline{MR}_{Gj} - \overline{MC}_{Gj}) - (MR_{Ej} - \overline{MC}_{Ej})(\overline{MR}_{Gi} - \overline{MC}_{Gi})\}\lambda_E =$$
$$MR_{Ei}(\overline{MR}_{Gj} - \overline{MC}_{Gj}) - MR_{Ej}(\overline{MR}_{Gi} - \overline{MC}_{Gi}). \tag{13}$$

Now, let us consider two unilateral optimization problems in choosing $\boldsymbol{p}_E$ with a fixed town-gas price $\boldsymbol{p}_G^*$:

$$\max_{\boldsymbol{p}_E} f_E(\boldsymbol{p}_E, \boldsymbol{p}_G^*) \tag{14}$$
$$\text{sub. to } g_1(\boldsymbol{p}_E, \boldsymbol{p}_G^*) = f_E - C_E|_{\boldsymbol{p}_G = \boldsymbol{p}_G^*} \leq 0,$$
$$g_2(\boldsymbol{p}_E, \boldsymbol{p}_G^*) = f_G - C_G|_{\boldsymbol{p}_G = \boldsymbol{p}_G^*} \leq 0$$

and

$$\max_{\boldsymbol{p}_E} C_E(\boldsymbol{p}_E, \boldsymbol{p}_G^*) \tag{15}$$
$$\text{sub. to } g_1(\boldsymbol{p}_E, \boldsymbol{p}_G^*) = f_E - C_E|_{\boldsymbol{p}_G = \boldsymbol{p}_G^*} \geq 0,$$
$$g_2(\boldsymbol{p}_E, \boldsymbol{p}_G^*) = f_G - C_G|_{\boldsymbol{p}_G = \boldsymbol{p}_G^*} \leq 0.$$

The first constraints in both problems are active due to the required conditions 1) and 2). Then the optimal $\boldsymbol{p}_E$ of problems (14) and (15) coincide with each other. The equilibrium price p_E^* of the original problem (10) is the optimal price of problem (14) because a normalized Nash equilibrium is a solution to the unilateral optimization problem (14), and consequently it is also a solution to problem (15). Following the Kuhn-Tucker condition for problem (15), the following equation is derived:

$$\{(MR_{Ei} - \overline{MC}_{Ei})(\overline{MR}_{Gj} - \overline{MC}_{Gj}) - (MR_{Ej} - \overline{MC}_{Gi})(\overline{MR}_{Gi} - \overline{MC}_{Gi})\}\mu_E = - MR_{Ei}(\overline{MR}_{Gj} - \overline{MC}_{Gj}) + MR_{Ej}(\overline{MR}_{Gi} - \overline{MC}_{Gi}) \quad (16)$$

where μ_E is a Lagrange multiplier associated with the first constraint of the problem (15).

According to Eqs. (13), (16) and Condition 3), the Lagrange multipliers λ_E and μ_E satisfy the relation:

$$\lambda_E + \mu_E = 1. \quad (17)$$

Considering that the Lagrange multipliers are positive under Condition 2), the range of λ_E,

$$0 < \lambda_E < 1 \quad (18)$$

is obtained. Because of the symmetry of the model, the range of λ_G is also derived similarly.

QED.

If the aforesaid proposition holds, the sign of each factor appearing in the RHS of Eq. (10) is determined. The factor of the first term, $(\lambda_E/(\lambda_E - 1))\cdot\partial q_{Ei}/\partial p_{Ei}$ is nonnegative because $\lambda_E > 0$, $\lambda_E - 1 < 0$ and $\partial q_{Ei}/\partial p_{Ei} \leq 0$. Consequently, it raises the equilibrium marginal revenue MR_{Ei} above that in the case without the regulatory constraints. If the marginal revenue MR_{Ei} is decreasing with respect to the price p_{Ei} (See footnote†), and if the influence of the price offered by the competitor is small, raising the marginal revenue means that the price is discounted according to the marginal supply cost at that period.

The factor of the second term $(\lambda_G/(\lambda_E - 1))\cdot\partial q_{Gi}/\partial p_{Ei}$ is nonpositive because $\lambda_G > 0$, $\lambda_E - 1 < 0$ and $\partial q_{Gi}/\partial p_{Ei} \geq 0$. Table 1 shows the influences of the price offered by the competitor and his marginal supply cost on the equilibrium marginal revenue and the corresponding equilibrium price.

Let us examine the natures of the equilibrium prices from the viewpoint of

† The partial derivative of MR_{Ei} with respect to p_{Ei} is:

$$\partial MR_{Ei}/\partial q_{Ei} = 2\partial q_{Ei}/\partial p_{Ei} + p_{Ei}\partial^2 q_{Ei}/\partial p_{Ei}^2$$

If the second term, i.e., the second order partial derivative of q_{Ei} with respect to p_{Ei} is negligible, and if $\partial q_{Ei}/\partial p_{Ei}$ is negative, which holds ordinarily, the marginal revenue MR_{Ei} decreases with respect ot p_{Ei}.

Table 1. Influence of the Competitor's Price

$MC_{Gi}-p_{Gi}$	MR_{Ei}	p_{Ei}
+	↓	↑
−	↑	↓

It is assumed that the marginal revenue $MR_{Ei}=\partial p_E^T q_E/\partial p_{Ei}$ is decreasing with respect to p_{Ei}.

supply/demand efficiency. The first term of the RHS of Eq. (10) has an effect of discounting the price according to the marginal supply cost. If the partial derivative of the demand with respect to the price does not vary remarkably by the time-of-use, it implies that the price in the peak period is reduced much more than those in the off-peak periods. It is because the marginal supply cost in the peak period is usually higher than those in the off-peak periods. Consequently it makes the supply-demand more inefficient by expanding the peak load. This fact has been pointed out by Bailey and White [8] while their model does not take the competition between the regulated companies into consideration, and it is called the B-W effect.

The effect of the competition appears through the second term of the RHS of Eq. (10). Let us suppose a situation in which a price lower than the marginal price is offered at the peak period in one energy market by the associated supplier. As shown in Table 1, it has the effect of raising the price of the competitor, and consequently more demand will shift to the market of present concern from the other energy market. Namely, the substitutable structure of the energy markets does not make the supply/demand efficient by itself if both sorts of energy are supplied by regulated monopolistic suppliers

4. Numerical Examples

4.1 Study area and time division

The Kinki district in 2000 is taken as the study area. Supply and demand of electricity and town-gas in a year is considered. One year is divided into 5 periods according to the characteristics of electricity and town-gas loads (Table 2).

Table 2. Time Division in the Model

Period	Duration [hour]	Season	Time-of-Day	Comments
1	456.25	Summer	12:00–17:00	Peak Load of Electricity
2	365	Summer	17:00–21:00	Middle Load of Electricity
3	365	Winter	17:00–21:00	Peak Load of Town-Gas
4	1186.25	Winter	12:00–17:00 & 21:00–24:00	Middle Load of Town-Gas
5	6387.5	the others		Base Loads

4.2 Demand and cost functions

Let us consider the electricity demand $\boldsymbol{q}_E = (q_{E1}, \dots, q_{E5})^T$ consisting of the heat demand $\boldsymbol{q}_{HE} = (q_{HE1}, \dots, q_{HE5})^T$, i.e., demand for air conditioning, space heating and water heating, and the other demand (nonheat demand) $\boldsymbol{q}_{NE} = (q_{NE1}, \dots, q_{NE5})^T$ of a linear form. Likewise, the town-gas demand $\boldsymbol{q}_G = (q_{G1}, \dots, q_{G5})^T$ consists of the heat demand $\boldsymbol{q}_{HG} = (q_{HG1}, \dots, q_{HG5})^T$ and nonheat demand $\boldsymbol{q}_{NG} = (q_{NG1}, \dots, q_{NG5})^T$:

$$q_{Ei} = q_{NEi} + q_{HEi} \tag{19}$$

$$q_{Gi} = q_{NGi} + q_{HGi} \tag{20}$$

$$q_{NEi} = \alpha_{NEEi} p_{Ei} + \bar{q}_{NEi} \tag{21}$$

$$q_{NGi} = \alpha_{NGGi} p_{Gi} + \bar{q}_{NGi} \tag{22}$$

$$q_{HEi} = \alpha_{HEEi} p_{Ei} + \alpha_{HEGi} p_{Gi} + \bar{q}_{HEi} \tag{23}$$

$$q_{HGi} = \alpha_{HGEi} p_{Ei} + \alpha_{HGGi} p_{Gi} + \bar{q}_{HGi}, \quad i = 1, \dots, 5 \tag{24}$$

where α_*, $\bar{q}_*$ are constants. These parameters are estimated with several assumptions on the demand in 2000, the price elasticity of demand and mutual substitutability of electricity and town-gas. As the parameters of the demand functions, three cases which differ in inter-energy substitutability are assumed. The estimated parameters of the demand functions in each case are listed in Table 3. The constraints of the nonnegativity of the demands are considered on both the nonheat and the heat demand, separately:

$$q_{NEi} \geq 0, \; q_{NGi} \geq 0, \; q_{HEi} \geq 0, \quad q_{HGi} \geq 0, \quad i = 1, \dots, 5. \tag{25}$$

As the supply costs of electricity and town-gas, the following function forms are used:

$$C_E = k_{EO} \sum_{i=1}^{5} T_i q_{Ei} + k_{EC} \Big(\sum_{i=1}^{5} q_{Ei}^{10}\Big)^{1/10} + k_{EF} \tag{26}$$

$$C_G = k_{GO} \sum_{i=1}^{5} T_i q_{Gi} + k_{GC} \Big(\sum_{i=1}^{5} q_{Gi}^{10}\Big)^{1/10} + k_{GF} \tag{27}$$

where T_i is the duration of the period i, and k_* are constants. The first terms of the RHSs of Eqs. (26) and (27), being proportional to the total energy demands, stand for the operating costs. The second terms stand for the capacity costs which are decided mainly by the peak demand, and the final terms are the fixed costs. The values of these parameters are listed in Table 4.

Table 3. Parameters of the Demand Functions

Case 1: High inter-energy substitutability

Period	α_{HEE}	α_{HEG}	$\bar{q}_{HE}$	α_{NEE}	$\bar{q}_E$
1	−24.979	29.363	0.723	−26.011	2.294
2	−17.573	19.763	0.554	−22.837	2.014
3	−43.131	66.762	0.439	−22.837	2.014
4	−19.830	30.302	0.222	−23.832	2.102
5	—	—	—	−20.494	1.808
Period	α_{HGE}	α_{HGG}	$\bar{q}_{HG}$	α_{NGG}	$\bar{q}_G$
1	64.514	−117.450	0.229	−16.724	0.843
2	40.385	−79.054	0.422	−15.811	0.797
3	53.913	−126.809	1.636	−15.811	0.797
4	24.787	−58.792	0.777	−16.098	0.811
5	—	—	—	−15.138	0.763

Case 2: Middle inter-energy substitutability

Period	α_{HEE}	α_{HEG}	$\bar{q}_{HE}$	α_{NEE}	$\bar{q}_E$
1	−21.045	22.479	0.723	−26.011	2.294
2	−14.250	13.948	0.554	−22.837	2.014
3	−25.788	36.412	0.439	−22.837	2.014
4	−11.464	15.662	0.222	−23.832	2.102
5	—	—	—	−20.494	1.808
Period	α_{HGE}	α_{HGG}	$\bar{q}_{HG}$	α_{NGG}	$\bar{q}_G$
1	48.779	−89.915	0.229	−16.724	0.843
2	27.093	−55.794	0.422	−15.811	0.797
3	32.235	−88.872	1.636	−15.811	0.797
4	14.330	−40.492	0.777	−16.098	0.811
5	—	—	—	−15.138	0.763

Case 3: Low inter-energy substitutability

Period	α_{HEE}	α_{HEG}	$\bar{q}_{HE}$	α_{NEE}	$\bar{q}_E$
1	−17.112	15.595	0.723	−26.011	2.294
2	−10.927	8.133	0.554	−22.837	2.014
3	−8.445	6.062	0.439	−22.837	2.014
4	−3.098	1.022	0.222	−23.832	2.102
5	—	—	—	−20.494	1.808
Period	α_{HGE}	α_{HGG}	$\bar{q}_{HG}$	α_{NGG}	$\bar{q}_G$
1	33.045	−62.380	0.229	−16.724	0.843
2	13.802	−32.534	0.422	−15.811	0.797
3	10.556	−50.935	1.636	−15.811	0.797
4	3.873	−22.192	0.777	−16.098	0.811
5	—	—	—	−15.138	0.763

Units of α_* and $\bar{q}_*$ are respectively [10^{10} kcal^2/hour·yen] and [10^{10} kcal/hour].

Table 4. Parameters of the Supply Cost Functions

Parameter	Value	Unit
k_{OE}	8.82	[10^{10} yen/10^{13} kcal]
k_{OG}	9.48	[10^{10} yen/10^{13} kcal]
k_{CE}	5.1×8760	[10^{10} yen·hour/10^{13} kcal]
k_{CG}	2.1×8760	[10^{10} yen·hour/10^{13} kcal]
k_{FE}	76.61	[10^{10} yen]
k_{FG}	−9.89	[10^{10} yen]

4.3 Algorithm for numerical calculation

To obtain a normalized Nash equilibrium numerically, we use an algorithm which applies penalty functions [7] to the pseudogradient method [5]. As a penalty function method to obtain Nash equilibrium, Shimizu [4] has proposed a method which uses interior penalty functions. In the present study, his method is used with some modifications, i.e. exterior penalty functions are used instead of the interior penalty functions. The superiority of the exterior penalty functions over the interior penalty functions is flexibility in choosing the initial values. Justification for using exterior penalty functions in game problems is given by Kawano [9]. Since the model is not convex, the normalized Nash equilibrium point may not be unique. In the following simulation, an equilibrium point obtained with an initial value equal to the present prices is regarded as the solution.

4.4 Results of simulation

Simulation results without and with the regulatory constraints are shown in Fig. 2 and Fig. 3, respectively. Comparing Fig. 3 with Fig. 2, the price of each energy is reduced remarkably at its peak period, i.e., Period 1 in electricity supply and Period 3 in town-gas supply, in the case with regulatory constraints. Then the peak demand is magnified much more than in the case without regulation. Consequently, the B-W effect is observed even under a competitive situation.

The variations of the prices and demands according to the change of the substitutability of the demand are not clear. However, even in the case of the highest substitutability, the peak prices are still discounted remarkably, and therefore the peak demands are still large. The effect of competition summarized in Table 1 is not clearly observed. It should be noted that the town-gas heat demand in Period 1 is null, and therefore the second assumption made in the beginning of the previous section does no hold in the simulation.

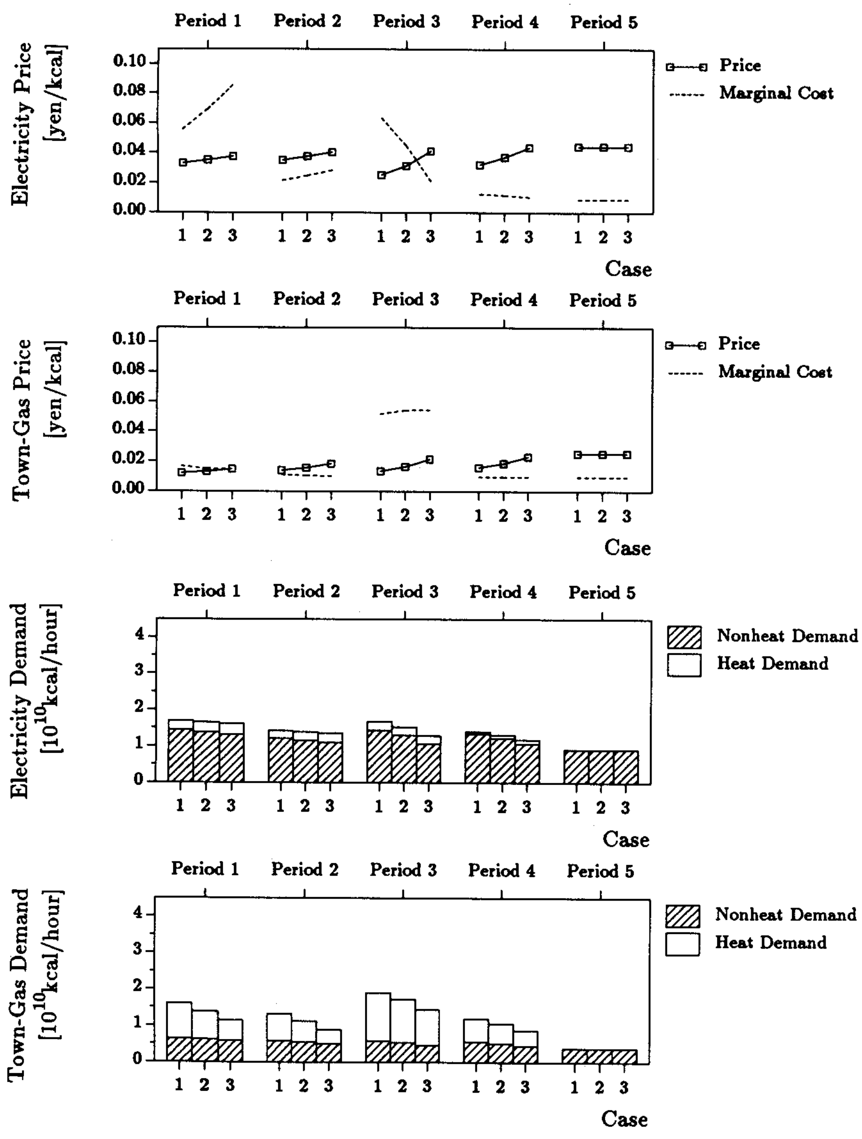


Fig. 2 Results of the simulations without regulatory constraints.
Case 1: High inter-energy substitutability.
Case 2: Middle inter-energy substitutability.
Case 3: Low inter-energy substitutability.

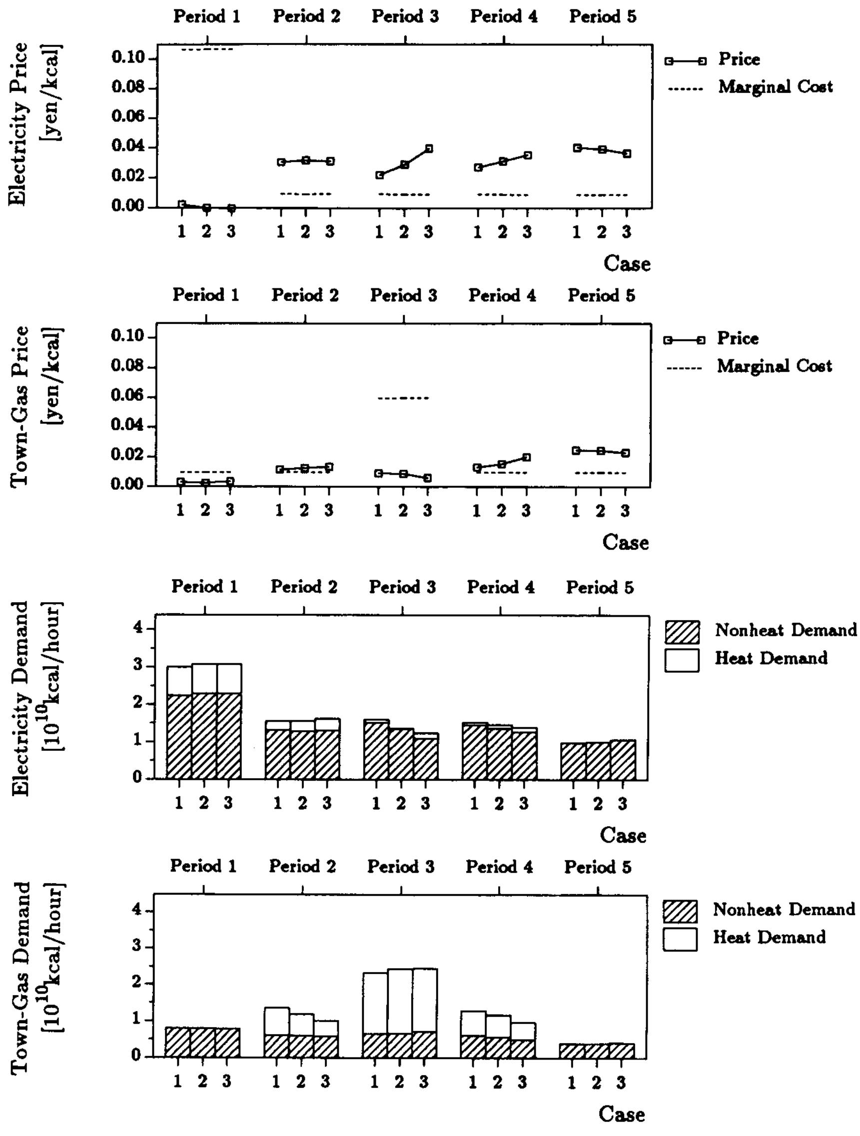


Fig. 3 Results of the simulations with regulatory constraints.
Case 1: High inter-energy substitutability.
Case 2: Middle inter-energy substitutability.
Case 3: Low inter-energy substitutability.

5. Concluding Remarks

This paper is concerned with a competitive supply of electricity and town-gas under time-of-use pricing. This situation of competition is modeled as a game problem between the regulated companies which supply the partially substitutable utilities. The analytical study on the model and the numerical simulations are presented. The main findings of the study are as follows:

(1) If both energy suppliers adopt time-of-use pricing aiming at maximization of their own sales, regulation of the profit rate causes reduction of the peak price, and consequently makes the demand/supply still more inefficient. Namely, the B-W effect is observed even in a competitive situation.

(2) Under regulatory constraints, while the relation between inter-energy substitutability and supply/demand efficiency is not clear, there is a case observed where the supply/demand still remains inefficient even under high inter-energy substiutability.

The implication of the above findings is that, when the time-of-use price is offered by a monopolistic energy utility company, regulation of the profit ratio is not sufficient to make the supply and demand efficient even if there exists a certain competition between the regulated companies. Hence, some other regulations on the time-of-use pricing, e.g., a regulation that the time-of-use price must be decided based on the time-of-use marginal supply cost, are needed to achieve an efficient supply and demand.

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