Mem. Fac. Eng., Kyoto Univ. Vol. 53, No. 4 (1991)

Estimating Time-Varying Origin-Destination Flows from Link Traffic Counts

by

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(Received June 4, 1991)

Abstract

A dynamic model for estimating real-time origin-destination flows from time-series of traffic counts is presented. The time variation of flows is explicitly treated as a dynamic process. The model is formulated based on minimizing the integrated squared error between predicted and observed output traffic counts over the period of observation. An efficient solution method is developed by using Fourier transformation and illustrated with numerical examples. The numerical simulation experiment shows that the system dynamic approach may be particularly suitable for on-line traffic management and control in urban transportation systems.

1. Introduction

The estimation of origin-destination (O-D) trip matrices from link traffic counts has been considered by many researchers during the past decade. One main advantage of this approach is that the traffic counts are available at a relatively low cost in relation to traditional methods for estimating O-D matrices, e.g., home interviews and roadside interviews. Most of the previous studies have focused on using accumulated traffic counts and some priori information to estimate or update a long-term O-D matrix which represents the basic information for transport planning and design purposes. However, travel demand for on-line traffic management and control on heavily congested roads has become increasingly important in recent years. For this purpose, the real-time O-D flows become necessary and must be estimated by considering the variation of trip matrices over time.

Recent work by Cremer and Keller (1987), Nihan and Davis (1987, 1989) proposed a new family of dynamic methods for the estimation of O-D matrices based on error minimization, recursive least squares, Kalman filtering approaches etc.. These approaches aim at estimating turning movements in real-time from exit and entry measurements of traffic flows for a simple intersection. A

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fundamental assumption is that the time taken by vehicles to traverse the intersection is small in relation to the chosen time interval. The complicated problems concerning the changes in travel time lags and travelers' route choice behavior are avoided. These dynamic methods can be applied efficiently to traffic-responsive signal control for an intersection but are unsuitable for efficient real-time traffic management in general urban networks.

However, real-time traffic monitoring and route guidance systems in urban areas often require the use of time-dependent origin-destination flows estimated by continuous on-line measurement of the traffic flows for a subset of links of the general networks. The related estimation problem is likely to be large-scale and have nonlinearities introduced by the route choice process. Namely, we have to consider the variation in traffic flow over time not only with respect to its volume but also with respect to its structural origin-destination relationships.

To deal with this general estimation problem, dynamic estimation models based on an entropy-maximizing approach have been proposed recently by several authors. Willumsen (1984) describes an extension of an entropy maximizing model for estimating time-dependent O-D matrices. This model was implemented and tested in combination with CONTRAM: a traffic management simulation model. Janson and Southworth (1989) describe the use of traffic count data with a dynamic assignment algorithm to estimate the distribution of trip departure times based on the entropy maximizing principle. Furthermore, Nguyen et al. (1988) considered the discrete time dynamic estimation problem for passenger origin-destination matrices on transit networks, and described an entropy-based optimization formulation where the traveler's route choices and travel times are assumed to be not affected by congestion. The link choice proportions for each time interval are determined by a proportional assignment prior to the estimation process.

All these methods have the following two aspects in common: (1) They divide the time period of interest into discrete time intervals and introduce 0–1 variables to describe the fundamental relationship between a link count for a given time interval and trips associated with different time intervals. (2) They use additional priori information on the time distribution of O-D matrices and derive a model by the entropy-maximizing approach constrained by different time interval counts.

This paper presents a new dynamic method for the estimation of origin-destination flows, different from the dynamic methods proposed previously. Traffic flows are treated as continuous time-variant variables and the model is formulated based on minimizing the integrated squared error between predicted and observed output counts. The remaining part of the paper is organized as follows: The model is formulated in section 2, and some discussion about the estimation problem is given in section 3. In section 4, a numerical method based on the discrete Fourier transform is described. In section 5, a simulation experiment with hypothetical data for a simple network is used to evaluate the estimator and the numerical algorithm. Finally some conclusions are drawn and the potential for further improvement is given in section 6.

2. Model Formulation

In order to consider explicitly the variation of traffic flow over time, the origin-destination flows and flows through counting points are assumed to be continuous functions of time. The following notation is employed in the paper.

A: the set of observation links
R, S: the sets of origin and destination nodes respectively
Q: the set of origin-destination pairs
K_{rs}: the set of available paths between origin r and destination s
δ_{akrs}: 1 if the k^{th} path between origin r and destination s passes along link a, 0 otherwise.
t_{kra}: the time for traveling from origin node r to traffic counting point a along path k, if link a is not on path k, then $t_{kra} = \infty$
$v_a(t)$: traffic flow measured by the detector at a counting site on link a (here it is referred to as the flow rate function measured as the traffic flow per unit time)
$f_{rs}(t)$: traffic flow leaving r destined for s at time t (origin-destination flow rate function measured as O-D traffic flow per unit time)
p_{krs}: the proportion of trips between origin r and destination s using path k.

For the moment we assume that the traffic congestion does not significantly affect traveling time and route choice. Thus, the route choice proportions p_{krs} between every O-D pair $k \in K_{rs}$ can be determined using some form of proportional assignment. For example, the multinomial logit assignment model (Sheffi, 1985) may be employed.

$$p_{krs} = \frac{\exp(-\theta t_{krs})}{\sum_{l \in K_{rs}} \exp(-\theta t_{lrs})} \qquad k \in K_{rs},\ rs \in Q \tag{1}$$

where θ is a positive parameter, t_{krs} is the travel time on path $k \in K_{rs}$, $rs \in Q$.

Since the time incurred by a vehicle traveling from origin r to traffic counting point a along path k is assumed to be t_{kra}, the driver should have departed at time $(t - t_{kra})$ from origin r if he (or she) is observed at counting point a at time t. Thus the predicted traffic flow rate $\bar{v}_a(t)$ passing over point a at time t can be expressed as

$$\bar{v}_a(t) = \sum_{rs \in Q} \sum_{k \in K_{rs}} f_{rs}(t - t_{kra}) p_{krs} \delta_{akrs} \qquad a \in A,\ t_0 \leqq t \leqq t_1 \tag{2}$$

where $[t_0, t_1]$ is the time period of observation.

Because of the variation of travel times t_{kra} and measurement errors, the predicted traffic flow rates do not necessarily equal the observed flow rates. Let $\zeta_a(t)$, $a \in A$ denote the difference or error between observed and predicted traffic flow through counting point a. $\zeta_a(t)$ is, in general, a random error function of time t. The following stochastic dynamic system is then posed.

$$\zeta_a(t) = \begin{bmatrix} \bar{v}_a(t) - v_a(t) & t_0 \leqq t \leqq t_1 \\ 0 & t < t_0 \text{ or } t > t_1 \end{bmatrix} \quad a \in A \tag{3}$$

Here it is assumed that $\zeta_a(t) = 0$ for $t < t_0$ and $t > t_1$ since the traffic flow out of the analysis period is not taken into account.

Therefore the time-varying O-D traffic flows can be estimated by minimizing integrated squared error over the period of observation.

$$\begin{aligned} \text{Min } E[f_{rs}(t)] &= \sum_{a \in A} \int_{t_0}^{t_1} \{\zeta_a(t)\}^2 \, dt \\ &= \sum_{a \in A} \int_{t_0}^{t_1} \{ \sum_{rs \in Q} \sum_{k \in K_{rs}} f_{rs}(t - t_{kra}) p_{krs} \delta_{akrs} - v_a(t)\}^2 \, dt \end{aligned} \tag{4}$$

Because the time lags t_{kra} $(r \in R, a \in A)$ appear in the above formulation, it is extremely difficult to calculate O-D flow rate $f_{rs}(t)$ $(r \in R, s \in S)$ in a direct manner. Hence we employ the Fourier transform (Robillard, 1974; Bracewell, 1978) on the integrated function to derive the solution method of the model.

Let $F_{rs}(t)$ and $V_a(t)$ be the Fourier transforms of the flow rate functions $f_{rs}(t)$ and $v_a(t)$, respectively:

$$F_{rs}(x) = \int_{-\infty}^{\infty} e^{-i2\pi xt} f_{rs}(t) \, dt \tag{5}$$

$$V_a(x) = \int_{-\infty}^{\infty} e^{-i2\pi xt} v_a(t) \, dt \tag{6}$$

where $i = \sqrt{-1}$.

Let the Fourier transform of error function $\zeta_a(t)$ be $\eta_a(x)$. Applying transitional theorem on the Fourier transform, we obtain

$$\eta_a(x) = \sum_{rs\in Q} \sum_{k\in K_{rs}} p_{krs}\delta_{akrs}e^{-i2\pi x t_{kra}} F_{rs}(x) - V_a(x) \tag{7}$$

Since the error function $\zeta_a(t)$ shown in (3) is a real function and since $\zeta_a(t) = 0$ for $t < t_0$ and $t > t_1$, by extending the integral time domain $[t_0, t_1]$ of function $\zeta_a(t)$ to $(-\infty, +\infty)$, we have

$$\int_{t_0}^{t_1} \{\zeta_a(t)\}^2 \, dt = \int_{-\infty}^{\infty} |\zeta_a(t)|^2 \, dt \tag{8}$$

Furthermore, employing the Parseval's theorem on the Fourier transform (Whitfield and Williams, 1988), we obtain

$$\int_{-\infty}^{\infty} |\zeta_a(t)|^2 \, dt = \int_{-\infty}^{\infty} |\eta_a(x)|^2 \, dx \tag{9}$$

According to the above relations, formulation (4) can now be transformed into the following problem P1.

P1: Estimating real-time origin-destination flows

$$\text{Min } E[f_{rs}(t)] = \sum_{a\in A} \int_{t_0}^{t_1} \{\zeta_a(t)\}^2 \, dt$$

$$= \sum_{a\in A} \int_{-\infty}^{\infty} \Big| \sum_{rs\in Q} \sum_{k\in K_{rs}} p_{krs}\delta_{akrs}e^{-i2\pi x t_{kra}} F_{rs}(x) - V_a(x)\Big|^2 \, dx \tag{10}$$

Consequently, using the function $F_{rs}(x)$ $(rs \in Q)$ in the above error minimization problem, the time-varying O-D traffic flow $f_{rs}(t)$ during the analysis period can be obtained by means of the following inverse Fourier transform:

$$f_{rs}(t) = \int_{-\infty}^{\infty} e^{i2\pi t x} F_{rs}(x) \, dx \tag{11}$$

In order to guarantee the uniqueness of the solution of P1, sufficient traffic flow observations at different counting sites are needed, and hence the estimation problem is computationally demanding. If the destination choice probabilities q_{rs} $(\sum_s q_{rs} = 1.0)$ are known somehow (e.g., by sample survey or based upon an existing O-D matrix), the model can be formulated in terms of the trip generation rate $f_r(t)$ (trip generation by origin per unit time). The estimation variables are thus substantially reduced.

For simplicity, we assume that the destination choice probabilities q_{rs}

($r \in R$, $s \in S$) are fixed and known. Using function $f_r(t)$ of the trip generation rate, the time-varying O-D traffic flow $f_{rs}(t)$ can be expressed as

$$f_{rs}(t) = f_r(t) q_{rs} \qquad t_0 \leqq t \leqq t_1,\ r \in R,\ s \in S \tag{12}$$

By employing relation (12), the estimation of time-varying trip generations or trip departure rates can be formulated as follows:

P2: Estimating real-time trip generations

$$\text{Min } E(f_r(t)] = \sum_{a \in A} \int_{t_0}^{t_1} \{\zeta_a(t)\}^2 \, dt$$

$$= \sum_{a \in A} \int_{-\infty}^{\infty} |\sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} q_{rs} p_{krs} \delta_{akrs} e^{-i2\pi x t_{kra}} F_r(x) - V_a(x)|^2 \, dx \tag{13}$$

where functions $F_r(x)$ ($r \in R$) represent the Fourier transform of function $f_r(t)$.

Therefore the time distribution of trip generations from each origin can be obtained by implementing an inverse Fourier transform on the function $F_r(x)$ obtained from P2.

3. Discussion

(1) The destination and route choice probabilities in the formulation P1 and P2 are assumed to be fixed. If the time variation of the O-D distribution pattern and route choice between origin and destination is of minor importance, the proposed method is applicable to the identification of traffic flow variation in real-time from time-series of traffic counts. If the influence of the time variation of the O-D distribution pattern and route choice is significant and cannot be omitted, the destination and route choice probabilities can be treated as continuous or discrete variables of time or time intervals and can be incorporated into the estimation process. But how to estimate time-dependent destination and route choice probabilities needs to be further studied. In this regard some fundamental results have been reported by Iida and Takayama (1987); the problem is, however, still far from being solved.

(2) The traffic flows passing traffic counting points are assumed to be a continuous function of time in the model formulation. However, only the discrete values of traffic flow rates at a limited number of sampling points are needed in the numerical procedure which is based upon the discrete Fourier transform. Namely, the traffic count data for a limited number of time intervals (for example, traffic counts per 10-minute time interval) can cover the requirement of the estimation problem. At

present, because of the popularity of varieties of vehicle detector, traffic count data throughout each day in $5 \sim 15$ minute intervals are available, so the dynamic estimation of time-dependent O-D traffic flows can be realized in practice.

(3) A large number of estimation methods proposed previously have been primarily limited to determining a static matrix from limited counts, i.e., they dealt with a static underdetermined system of static equations concerned with the average traffic flow pattern over a whole day. For this reason, they can be referred to as static estimators in comparison with a time-varying flow situation. The estimates obtained by these static methods can be viewed as the results of a stationary process throughout an infinite time period $(-\infty, +\infty)$. In contrast to these approaches, the present dynamic method explicitly considers traffic flows as a dynamic process. The O-D flows are treated as time-dependent variables and the model is based on minimizing the integrated squared error between predicted and observed link flows. If the traffic flows through the network do not vary with time, i.e., for the static situation, the model formulation P1 or P2 will result in one of the types of least squares models proposed by lida and Takayama (1986).

(4) For the practical application of the method, we must first consider how to determine the time domain $[t_0, t_1]$. Here we define maximum lag time τ_{Max} and minimum lag time τ_{min} for the observation system as follows:

$$\tau_{Max} = \max_r \max_k \max_a t_{kra}, \qquad \tau_{Min} = \min_r \min_k \min_a t_{kra} \tag{14}$$

Furthermore, we let

$$T_0 = t_0 - \tau_{Max}, \qquad T_1 = t_1 - \tau_{Min} \tag{15}$$

Obviously, the trips departing before T_0 do not pass over, or are not observed at any counting site during time period $[t_0, t_1]$, thus we may set the origin of the time axis at T_0. On the other hand, the O-D traffic flows throughout the time period $[t_0, t_1]$ are calculated in the Fourier transforms. However, due to time lag, the trips departing in the interval $[T_1, t_1]$ will not be observed during the period $[t_0, t_1]$, thus the estimated accuracy of the O-D flow within interval $[T_1, t_1]$ contained in time period $[t_0, t_1]$ may possibly become poor. Therefore, in order to obtain highly reliable estimates of time-dependent O-D traffic flows, the traffic counting time should take longer than that of the O-D flow estimation.

4. Numerical Solution Method

The model (10) for estimating origin-destination flows and the model (13) for estimating trip generations are unconstrained integral minimization problems in terms of imaginary functions. The flow rate functions $f_{rs}(t)$ or $f_r(t)$ ($r \in R$, $s \in S$) cannot be solved analytically in general. Hence we adopt a numerical method to evaluate the transformed functions $F_{rs}(t)$ and $F_r(t)$, and then go back to compute the distribution of flow rate functions over time using the inverse Fourier transform.

There is no conceptual difference in the numerical solution methods between formulation (10) and (13). When the destination choice probabilities q_{rs} ($\sum_s q_{rs} = 1.0$) are known, the time distribution of the O-D traffic flow can be directly calculated after the trip generation rates are obtained. Here, we only present the numerical solution method for formulation P2.

Letting imaginary function

$$G_{ar}(x) = \sum_{s \in S} \sum_{k \in K_{rs}} q_{rs} p_{krs} \delta_{akrs} e^{-i2\pi x t_{kra}} \qquad a \in A,\ r \in R \tag{16}$$

and substituting (16) into (13), we have

$$\text{Min } E[f_r(t)] = \sum_{a \in A} \int_{-\infty}^{\infty} |\sum_{r \in R} G_{ar}(x) F_r(x) - V_a(x)|^2 \, dx \tag{17}$$

Seemingly, Eq. (17) is concerned with integral over $(-\infty, \infty)$, but it does not imply that the phenomena continues infinitely. The numerical procedures are performed using a finite Fourier transform and hence it is only necessary to evaluate imaginary function $F_r(x)$ at a certain number of points over an equivalent frequency domain $[a, b]$ corresponding to time domain $[t_0, t_1]$.

We now divide the closed interval $[a, b]$ into M sub-intervals. The central coordinates of the intervals are denoted by $x_0, x_1, \cdots, x_m, \cdots, x_{M-1}$ and the corresponding values of functions $G_{ar}(x)$, $F_r(x)$, $V_a(x)$ at these points by $G_{ar}(x_m)$, $F_r(x_m)$, $V_a(x_m)$ $(m = 0, 1, \cdots, M-1)$ respectively. The squared error integral (17) can now be approximately expressed as the following numerical integral.

$$E[f_{rs}(t)] = \sum_{a=1}^{P} \sum_{m=0}^{M-1} w_m | \sum_{r=1}^{N} G_{ar}(x_m) F_r(x_m) - V_a(x_m)|^2 \tag{18}$$

where P and N denote the numbers of observation links and origin nodes respectively, the weighting coefficients w_m depend upon the specific quadrature rule chosen to approximate the integral in (17). For example, w_m equals $(x_{m+1} - x_m)/2 (m = 0, 1, \cdots\cdots, M-1)$ when the trapezoidal rule is employed.

Furthermore, letting $Re[\cdot]$ and $Im[\cdot]$ denote the real and imaginary parts of an imaginary number, we then have the following relationship.

$$
\begin{aligned}
&|\sum_{r=1}^{N} G_{ar}(x_m)F_r(x_m) - V_a(x_m)|^2 \\
&= Re[\sum_{r=1}^{N} G_{ar}(x_m)F_r(x_m) - V_a(x_m)]^2 + Im[\sum_{r=1}^{N} G_{ar}(x_m)F_r(x_m) - V_a(x_m)]^2
\end{aligned} \tag{19}
$$

Consequently, the error minimization of (17) in terms of $F_r(x_m)$ ($r \in R$, $m = 0, 1, \cdots, M-1$) constitutes the following linear least-squares problem.

$$
\underset{u_j}{\text{Min}} \sum_{k=1}^{2PM} [\sum_{j=1}^{2NM} c_{kj}u_j - d_k]^2 \tag{20}
$$

Here, the imaginary functions $G_{ar}(x)$, $F_r(x)$, $V_a(x)$ are sampled with equal intervals, the weighting coefficients w_m are thus excluded from the objective function. Moreover, c_{kj} and d_k are real constants to be prescribed; u_j are real optimizable parameters to be evaluated.

$$
u_j = \begin{bmatrix} Re[F_r(x_{m-1})]: & j = 2M(r-1) + 2m - 1 \\ Im[F_r(x_{m-1})]: & j = 2M(r-1) + 2m \end{bmatrix}
$$
$$
m = 1, 2, \cdots, M, \quad r = 1, 2, \cdots, N
$$

$$
d_k = \begin{bmatrix} Re[V_a(x_{m-1})]: & k = 2M(a-1) + 2m - 1 \\ Im[V_a(x_{m-1})]: & k = 2M(a-1) + 2m \end{bmatrix}
$$
$$
m = 1, 2, \cdots, M, \quad a = 1, 2, \cdots, P
$$

The coefficients c_{kj} ($k = 1, 2, \cdots, 2PM$, $j = 1, 2, \cdots, 2NM$) are determined by the real and imaginary parts of imaginary function $G_{ar}(x_{m-1})$ ($a \in A$, $r \in R$, $m = 1, 2, \cdots, M$) as follows:

$$
c_{kj} = \begin{bmatrix} Re[G_{ar}(x_{m-1})]: & k = 2M(a-1) + 2m - 1, & j = 2M(r-1) + 2m - 1 \\ -Im[G_{ar}(x_{m-1})]: & k = 2M(a-1) + 2m - 1, & j = 2M(r-1) + 2m \\ Im[G_{ar}(x_{m-1})]: & k = 2M(a-1) + 2m, & j = 2M(r-1) + 2m - 1 \\ Re[G_{ar}(x_{m-1})]: & k = 2M(a-1) + 2m, & j = 2M(r-1) + 2m \end{bmatrix}
$$
$$
m = 1, 2, \cdots, M, \quad r = 1, 2, \cdots, N, \quad a = 1, 2, \cdots, P
$$

Let C denote the (2PM $\times$ 2NM) matrix with elements (c_{kj}: $k = 1, 2, \cdots, 2\text{PM}$, $j = 1, 2, \cdots, 2\text{NM}$) and let vector $U = (u_1, u_2, \cdots, u_{2NM})^T$ and $D = (d_1, d_2, \cdots, d_{2PM})^T$. From the necessary conditions of non-linear programming, we obtain the following normal equations with respect to vector U which minimizes (20).

$$
(C^TC)U = C^TD \tag{21}
$$

where "^{T}" denotes transposition of a vector or matrix.

If the traffic counts are independent from each other, matrix $C^T C$ is generally non-singular and its inversion $(C^T C)^{-1}$ exists. Hence the explicit solution U^* for (21) can be expressed as

$$U^* = (C^T C)^{-1} C^T D \tag{22}$$

Therefore, after the M-values of function $F_r(x_m)$ $(r \in R)$ at sample points $x_0, x_1, \cdots, x_m, \cdots, x_{M-1}$ are evaluated, the trip generation rate $f_r(t)$ $(r \in R)$ can be obtained by employing the inverse Fourier transform.

According to the above idea, the numerical algorithms based upon the discrete Fourier transforms may be described as the following step 1 $\sim$ step 4. Here it should be noted that since the traffic volume counts are given as discrete time-series data, the discrete Fourier transform corresponding to the continuous Fourier transform shown in (5), (6) and (11) is employed in the numerical procedure. In place of t_m and x_m, k and m are used to indicate the data number. Moreover, formula (16) is rewritten in the corresponding discrete form.

$$G_{ar}(m) = \sum_{s \in S} \sum_{k \in K_{rs}} q_{rs} p_{krs} \delta_{akrs} e^{-i2\pi m(t_{kra}/T)} \qquad a \in A,\ r \in R \tag{23}$$

where $T = t_1 - t_0$.

Solution Algorithm

Step 1: Partition the observation period $[t_0, t_1]$ into sequential M-intervals with equal length, and calculate the average traffic flow rate $v_a(m)$ $(m = 0, 1, \cdots, M-1,\ a \in A)$ during each interval using the time sequences of traffic counts.

Step 2: Use the following discrete Fourier transform to calculate the real and imaginary parts of $V_a(k)$ and hence obtain vector $D = (d_1, d_2, \cdots, d_{2PM})$. Determine the coefficient matrix C according to formula (23).

$$V_a(k) = \frac{1}{M} \sum_{m=0}^{M-1} v_a(m) e^{-i2\pi km/M} \qquad k = 0, 1, \cdots, M-1, \quad a \in A$$

Step 3: Solve the normal equation (21) to calculate vector U^*, and hence obtain the real and imaginary parts of imaginary number $F_r(m)$ $(m = 0, 1, \cdots, M-1,\ r \in R)$.

Step 4: Use the following inverse Fourier transform to evaluate the trip generation rates for each time interval.

$$f_r(m) = \sum_{k=0}^{M-1} F_r(k) e^{i2\pi mk/M} \qquad m = 0, 1, \cdots, M-1, \quad r \in R$$

5. A Numerical Example

In order to verify and test the model and its numerical algorithm proposed above, we present a numerical simulation for a simple network consisting of 8-links, 6-nodes and 4 O-D pairs as shown in Fig. 1. The link travel times are indicated on each link and the destination choice probabilities are given in Table 1. Further, the utilized routes and route choice probabilities are displayed in Table 2.

In the numerical example, we try to estimate the trip generation rates from origin 1 and 2, assuming that the destination and route choice probabilities are known and are constants across time. Two traffic counting points *a* and *b* are assumed to be located at exit points on links 35 and 46.

The analysis time period is assumed to be the peak period 7:40 AM ~ 9:20 AM, the 1-minute trip generation profiles at origin 1 and 2 during the peak period are shown in Fig. 2. The trips departing from each origin are then assigned to the network according to the assumed destination and route choice probabilities. As shown in Fig 3, the trip arrival profiles at counting point *a* and *b* can be obtained based on the traveling time from origin to traffic counting point. In a practical situation, continuous measurements of traffic flows may be obtained by a variety of types of vehicle detectors. Here we assume that the traffic count data are collected throughout the period 8:00 AM ~ 9:20 AM ($t_0 = 8:00$, $t_1 = 9:20$) at

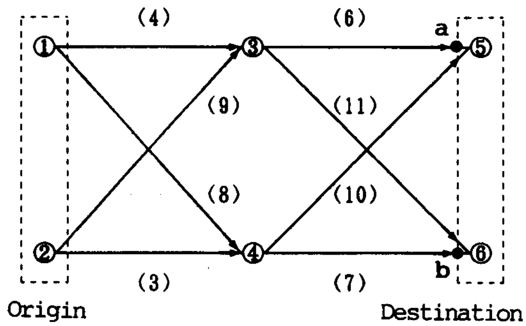

Fig. 1 Test Network

Table 1. Destination Choice Probability

Origin	Destination 5	Destination 6
1	0.60	0.40
2	0.30	0.70

Table 2. Route Choice Probability

Path	O→D			
	1→2	1→6	2→5	2→6
1−3−5	0.80			
1−3−6		0.50		
1−4−5	0.20			
1−4−6		0.50		
2−3−5			0.40	
2−3−6				0.10
2−4−5			0.60	
2−4−6				0.90

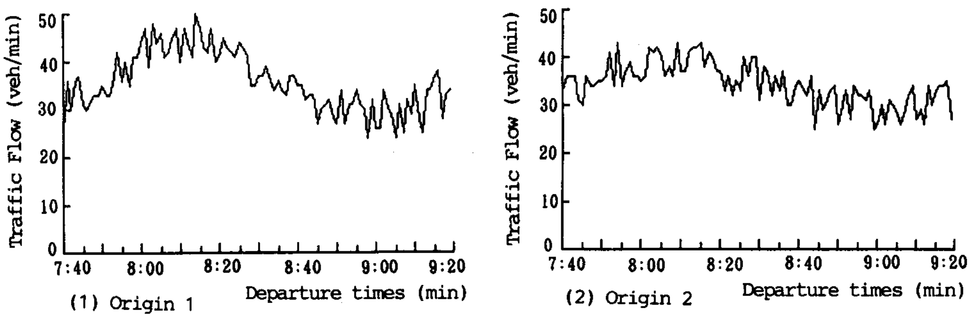

Fig. 2 Generation Rate Profiles

5-minute intervals. The transitional period 7:40 AM ~ 8:00 AM is therefore left out of account.

Fig. 3 shows 1-minute trip arrival profiles and 5-minute traffic counts (input data) at counting sites *a* and *b*. Obviously, the shorter the time intervals between data collection, the more precise traffic count data consistent with the time fluctuation of traffic flow obtained correspondingly. The resulting estimation accuracy of trip generation rates may possibly be improved.

The actual and estimated time distributions of trip generation rates at origin 1 and 2 are shown in Fig. 4. The relative estimation errors for 16 intervel periods are displayed in Table 3. It can be observed that the model can track the time fluctuation of trip generation rates and maintain sufficient accuracy with respect to the estimates except for the less accurate estimates for the last two intervals. The estimation error arises mainly from the cut off error associated with the finite Fourier transform. The poor estimates for the last two intervals are due to the fact that the trip departures during these two intervals are not measured at any traffic counting point during the observation period as stated

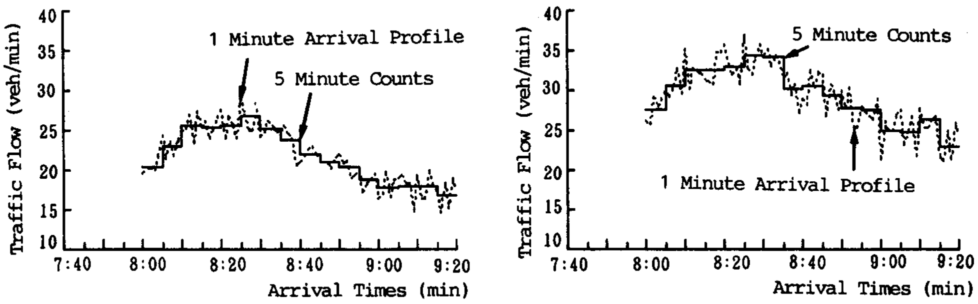

Fig. 3 Arrival Time Profiles at Traffic Counting Points

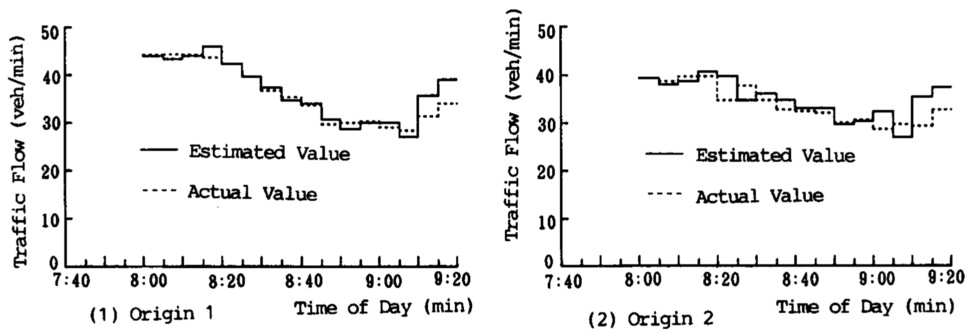

Fig. 4 Estimated and Actual Time Distributions of Trip Generation

Table 3. Estimated and Actual Distribution of Trip Departture Times

Departure Time Interval	Actual Flow (veh/min)		Estimated Flow (veh/min)		Relative Error (%)	
	Origin 1	Origin 2	Origin 1	Origin 2	Origin 1	Origin 2
8:00~8:05	44.40	39.20	43.95	39.31	1.00	−0.29
:05~ :10	44.20	38.60	43.27	37.89	2.10	1.85
:10~ :15	44.40	39.80	43.93	38.57	1.06	3.09
:15~ :20	43.80	39.80	46.16	40.59	−5.38	−1.99
:20~ :25	42.40	34.60	42.25	39.54	0.35	−14.28
:25~ :30	39.60	37.80	39.66	34.54	−0.16	8.63
:30~ :35	36.60	34.80	37.37	35.89	−2.09	−3.13
:35~ :40	35.40	32.80	34.73	34.79	1.90	−6.05
:40~ :45	33.60	32.20	34.00	33.10	−1.20	−2.78
:45~ :50	29.60	32.00	30.83	32.92	−4.14	−2.87
:50~ :55	30.00	30.00	28.72	29.75	4.27	0.84
8:55~9:00	30.20	30.60	30.06	30.41	0.45	0.63
9:00~9:05	29.00	28.60	30.10	32.25	−3.79	−12.77
:05~ :10	28.20	29.80	27.04	27.00	4.12	9.40
:10~ :15	31.20	29.20	35.70	35.27	−14.43	−20.80
:15~ :20	34.00	32.60	39.03	37.18	−14.80	−14.06

earlier. The estimation error for the last two intervals can be reduced by prolonging the counting time.

6. Summary and Conclusions

A dynamic method for estimating time-varying origin-destination flows using time-series of traffic count data is presented. Traffic flow through the network is treated as a dynamic process using time-continuous variables. The essential idea is to minimize the integrated squared error between predicted and observed link flows, and the finite Fourier transform solves the problem numerically. The dynamic model is mainly characterized by the following aspects:
(1) If time variation of traffic flows is explicitly incorporated into the model, not only the time distribution of O-D traffic flow can be estimated continuously by on-line measurements, but also the information on the daily average O-D traffic demand can be obtained simultaneously based on the real-time estimated results.
(2) The numerical procedure of the dynamic model is only concerned with the discrete Fourier transform and solution of a system of linear equations. From the viewpoint of on-line applications, the suggested non-iterative method may be particularly suitable for real-time traffic management and control where the time distribution of origin-destination flows has to be estimated using information contained in the time sequence of the traffic counts.
(3) Our continuous time dynamic approach may be contrasted with the discrete method of Nguyen et al. (1988) for the dynamic estimation of passenger origin-destination matrices on transit networks. The adoption of continuous time variables avoids the complicated interface relationship between a link count for a given time interval and trips associated with different time intervals. Our method is particularly applicable for the case of transit networks where the congestion effects are of minor importance.

Future work will concentrate on the generalization of the assumptions introduced in the model formulation for simplicity. Namely, it is expected to extend the model to congested cases where flow-dependent route choices and travel times are taken into account. Furthermore, the approach of "Maximum Possible Relative Error", which is proposed by the authors (1991) for the reliability analysis of static matrix estimation problems, can also be applied to the estimation problems of real-time origin-destination flows.

Acknowledgment

The authors would like to thank Dr. M. G. H. Bell of the University of Newcastle upon Tyne, U. K. for his valuable comments and corrections concerning an earlier draft of the paper.

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