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A Generalization of Diagnosability Analysis

by

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Abstract

A new concept of generalized diagnosability is proposed for a formal diagnosis model which incorporates most diagnosis models so far proposed. A self-diagnosis model consists of a set of units which can test other units and be tested by other units. Generalized diagnosability is a new measure of diagnosability in system diagnosis problems which is extensively studied with respect to self-diagnosis models. This diagnosability expresses explicitly such information as (1) the maximum number of units to be identified as faulty, (2) the maximum number of units to be identified as fault-free, and (3) the maximum number of units whose states are definitely identified when the upper bound on the number of faulty units is assumed.

Conditions for generalized diagnosability are expressed by certain relations between the power sets of a set of faulty units. Since these conditions are of the form that they must be checked all over the possible syndrome, it is generally difficult to investigate generalized diagnosability. However, these conditions are meaningful in cases in which the graph of a diagnosis model has symmetricity, or a diagnosis model has a constrained structure about the relation between fault patterns and syndromes. Some examples of these cases are presented.

Furthermore, we discuss the problem of finding the minimal fault pattern consistent with a given syndrome. This problem is formulated as a mathematical programming problem with the same relations both in constraints and objective functions as those used to express conditions for generalized diagnosability.

1. Introduction

The concept of system diagnosis is becoming important with the development of highly integrated digital systems and complicated computer networks [1]–[4]. Especially, self-diagnosis models (SDM) have been studied extensively [5]–[8]. An SDM consists of n units, each of which can test and be tested by other units. This SDM can be expressed by a graph $G(V, E)$ where V is a set of vertices $\{v_i\}$ corresponding to a set of units of SDM, and E is a set of arcs $\{a_{ij}\}$ such that :

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$$\begin{cases} a_{ij} \in E: \text{if unit i tests unit } j. \\ a_{ij} \notin E: \text{otherwise} \end{cases}$$

Each test (arc a_{ij}) of SDM of PMC type [5] produces binary test outcomes t_{ij}:

$$t_{ij} = \begin{cases} 0 & : \text{if both units } i \text{ and } j \text{ are fault–free.} \\ 1 & : \text{if unit } i \text{ is fault–free and unit } j \text{ is faulty.} \\ 1/0 & : \text{if unit } i \text{ is faulty, where 1/0 indicates the test outcome can be either 1 or 0.} \end{cases}$$

For this SDM, many diagnosabilities such as *t*–fault diagnosability (*t*–fd), *t*–fault diagnosability with repair (*t*–fdwr), and t out of s diagnosability (*t/s–d*) have been proposed [8]–[13]. These diagnosabilities are defined under the common assumption that the number of faulty units does not exceed t. Under this assumption, a system is called *t*–fd if and only if all the faulty units are identified exactly, *t*–fdwr if and only if at least one faulty unit is identified, and *t/s–d* if and only if all the faulty units are specified within a subset of units whose cardinality is less than s. Generalized diagnosability is a new concept which totally expresses parameters appeared one by one in the above mentioned diagnosabilities.

Definition 1. (Generalized diagnosability)

A system is called t/s/r/w–d if and only if the following conditions are satisfied, provided the number of faulty units present does not exceed t.

(1) All the faulty units are specified within a set of at most s units,

(2) all the fault–free units are specified within a set of at most r units whenever the number of fault–free units is less than r, and

(3) the states of at least w units can be identified.

When some of these parameters can not be specified, they are denoted by a dot. With this generalized diagnosability, t–fd, t–fdwr, and t/s–d are termed t/. /. /n–d, t/. /(n–1)/1–d, and t/s/. /(n–s)–d respectively where n is the number of units.

In section 2, we define a formal diagnosis model which incorporates almost all existing diagnosis models. Certain relations between the power sets of a set of units are defined on this diagnosis model. Conditions of generalized diagnosability are expressed with these relations. In section 3, we discuss generalized diagnosability on a diagnosis model with constrained structure of association between fault patterns and syndromes. Conditions of generalized diagnosability are reduced to a form easy to check with this constrained structure. In section 4, we consider the diagnosis problem of finding the minimal fault patterns consistent with a given syndrome without assuming an upper bound on the number of faulty units. This problem is formulated as a mathematical programming problem with constraints and objective functions expressed by the relations defined in section 2.

2. Conditions of generalized diagnosability

We define a formal diagnosis model which includes most diagnosabilities so far proposed. And we discuss conditions for generalized diagnosability of the model. Then, the generalized diagnosability of SDM is investigated as an example. We use the following notations.

1) $x=\{x_j\}$ $(i=1 \dots n)$ denotes a set of units each of which can take binary state : faulty and fault–free, and $Y=\{y_j\}$ $(j=1 \dots m)$ denotes a set of measurements each of which can take a binary state : abnormal and normal.

2) $F(\subseteq X)$ denotes a fault pattern : the subses of all units which are faulty, and σ denotes a syndrome : the subset of all measuresents which are abnormal. We use $\sigma(F)$ to denote a syndrome which is consistent with fault pattern F: That all $y_j \in \sigma(F)$ become abnormal and all $y_j \notin \sigma(F)$ become normal when fault pattern F exists in a system.

3) We use S to denote a subset of a set X, and $P(X)$ to denote the power set of a set X.

Definition 2. (Formal Diagnosis Model)

A the formal diagnosis model is a triple (X, Y, M) where $X=\{x_i\}$ $i=1 \dots n$ is a set of units, $Y=\{y_j\}$ $j=1 \dots m$ is a set of measurements, and $M=\{\mu\}$ is a binary relation of $P(X)xP(Y)$. Binary relation μ is defined as below :

$$\begin{cases}(F, \sigma) \in \mu : \text{if a fault pattern } F \in P(X) \text{ is consistent with a syndrome } \sigma \in P(Y). \\ (F, \sigma) \notin \mu : \text{otherwise}\end{cases}$$

With this formal diagnosis model, the next two types of set relations and two other types of set functions are defined. This formal diagnosis model is the generalized version of the failure diagnosis model discussed later in Example 2.

Definition 3.

Two types of set relations, $G_1^\sigma(S_i)$ and $G_0^\sigma(S_i)$, and two other types of set functions, $S^1(t, \sigma)$ and $S^0(t, \sigma)$ are defined as follows :

$$G^\sigma_{1(0)}(S_i)=\{F_0 : |F_0| \leqq |F_i|_{(i=1 \dots q)}, F_j \supseteq S_i (\bar{F}_j \supseteq S_i) \text{ and } \underset{(j=0 \dots q)}{(F_j, \sigma)} \in \mu\}$$

$$S^{1(0)}(t, \sigma)=\{x_i \in X : |G^\sigma_{1(0)}(x)| \geqq t+1\}$$

where $\bar{F}=X-F$ and the minimum is taken with respect to the car dinality of fault pattern F. Since there may be many minimal fault patterns which satisfy the constraints of $G^{\sigma}_{1(0)}(S_i)$, these are relations. And $|G^{\sigma}_{1(0)}(S_i)|$ denotes the cardinality of one of the minimal fault patterns, whereas it is interpreted as ∞ if there is no fault pattern consistent with σ. $G^{\sigma}_{1}(S_i)$, for example, indicates minimal fault patterns consistent with a syndrome σ under constraints in which units in S_i are all faulty. Among these relations and functions, $G^{\sigma}_{1(0)}(S_i)$ with S_i restricted to a unit x_i has already been proposed to express the condition for t-fdwr [10]. Properties of these relations and functions are summarized in Appendix. With these relations and functions, conditions of generalized diagnosability are obtained in the next Theorem 1.

Theorem 1. (Conditions of generalized diagnosability)

The formal diagnosis model is t/s/r/w-d if and only if

$$\min_{\sigma\in\Sigma_A}|S^1(t,\sigma)|\geqq n-s \tag{1}$$

$$\min_{\sigma\in\Sigma_{n-r}}|S^0(t,\sigma)|\geqq n-r \tag{2}$$

$$\min_{\sigma\in\Sigma_A}(|S^0(t,\sigma)|+|S^1(t,\sigma)|)\geqq w \tag{3}$$

where Σ_A is a set of all possible syndromes: $\Sigma_A=\{\sigma: (F,\sigma)\in\mu \text{ for some } F\subseteq X\}$, Σ_{n-r} is a set of syndromes which are consistent only with fault patterns whose cardinality is less than $n-r$ where $n=|X|$; $\Sigma_{n-r}=\{\sigma: (F,\sigma)\in\mu \text{ for some } F\subseteq X \text{ such that } |F|\leqq n-r\}$.

Proof

Sufficiency: It should be noted that $S^{1(0)}(t,\sigma)$ denotes a set of all units which are known to be fault-free (faulty) given a syndrome σ, under the assumption that the number of faulty units does not exceed t. Therefore, if condition (1) is satisfied, there exists a set of units $S_i\in P(X)$ such that each unit in it is fault-free and whose cardinality is larger than $n-s$. Thus, faulty units are specified within a set whose cardinality is less than s.

In the same manner, condition (2) implies that all fault-free units are specified within a set whose cardinality is less than r. But in this case the minimum is taken over Σ_{n-r}, for $\sigma\in\Sigma_A-\Sigma_{n-r}$ implies that the ca dinality of all the fault patterns consistent with σ is larger than $n-r$ and hence all fault-free units are specified within a set whose ca dinality is less than r.

Further, since $S^1(t,\sigma)$ is a set of units identified as fault-free and $S^0(t,\sigma)$ is a set of units identified as faulty, $S^1(t,\sigma)\cup S^0(t,\sigma)$ is a set of units whose states are identified and $S^1(t,\sigma)\cap S^0(t,\sigma)=\phi$. Thus $|S^1(t,\sigma)\cup S^0(t,\sigma)|=|S^1(t,\sigma)|+|S^0(t,\sigma)|$ and condition

(3) implies that the state of at least w units can be identified.

Necessity : If a formal diagnosis model is t/s/. /.−d, then for all the set of fault patterns $\{F_i\}$ $i=1 \ldots q$ such that (a) $|F_i| \leqq t$ $i=1 \ldots q$ (b) $\sigma=\sigma(F_1)=\sigma(F_2)=\ldots=\sigma(E_q)$ and (c) $\sigma \neq \sigma(F_j)$ for all $F_j \notin \{F_i\}$, it must be satisfied that $|\bigcup_{i=1}^{q} F_i| <$ s by the definition of *t/s/. /.−d*. This fact implies that $|\overline{\bigcup_{i=1}^{q} F_i}| \geq n-s$. And further, $|G_1^{\sigma}(x_i)| \geqq t+1$ for all x_i in $(\overline{\bigcup_{i=1}^{q} F_i})$ by Lemma 5, (2)−(i) in the Appendix. Therefore condition (1) holds.

If a formal diagnosis model is *t/. /r/.−d*, then it must be satisfied that $|\overline{\bigcap_{i=1}^{q} F_i}| \geqq n-r$ for all the set of fault patterns $\{F_i\}$ such that (a) and (b) mentioned above hold for all $\sigma \in \Sigma_{n-r}$ by the definition of *t/. /r/.-d* and Σ_{n-r}. Further, $|G_0^{\sigma}(x_i)| \geqq t+1$ for all x_i in $(\bigcap_{i=1}^{q} F_i)$ by Lemma 5 (2)−(ii) in the Appendix, and henece condition (2) holds. Finally, if a formal diagnosis model is *t/. /. /w −d*, then the above mentioned sets of fault patterns $\{F_i\}$ satisfy that $|(\overline{\bigcup_{i=1}^{q} F_i}) \cup (\bigcap_{i=1}^{q} F_i)| \geq w$ for all $\sigma \in \Sigma_A$ where $(\overline{\bigcup_{i=1}^{q} F_i})$ and $(\bigcap_{i=1}^{q} F_i)$ correspond to $S^1(t, \sigma)$ and $S^0(t, \sigma)$ respectively with σ which realizes the minimum of condition (3). *Q. E. D.*

As shown by the proof of Theorem 1, conditions (1), (2), and (3) correspond to the conditions to be *t/s/. /. −d*, *t/. /r/. −d*, and *t/. /. /w −d* respectively. Among these conditions, conditions (1) and (2) are expressed in a different form. Next Theorem 2 is obtained by replacing (1) and (2) of Theorem 1 with (1)′ and (2)′, each of which is equivalent to (1) and (2) by Lemma 6 in the Appendix.

Theorem 2

A formal diagnosis model is *t/s/r/w −d* if and only if

$$\min_{\sigma \in \Sigma_A} (\max_{|S_i|=n-s} (\min_{x_i \in S_i} |G_1^{\sigma}(x_i)|)) \geqq t+1 \qquad (1)'$$

$$\min_{\sigma \in \Sigma_{n-r}} (\max_{|S_i|=n-r} (\min_{x_i \in S_i} |G_0^{\sigma}(x_i)|)) \geqq t+1 \qquad (2)'$$

$$\min_{\sigma \in \Sigma_A} (|S^0(t, \sigma)| + |S^1(t, \sigma)|) \geq w \qquad (3)$$

Conditions (1) and (2) of Theorem 1 are convenient forms to obtain parameters *s* and *r* of the generalized diagnosability for a certain *t*. On the other hand, conditions (1)′and (2)′of Theorem 2 are used to investigate the permissible upper bound of *t* for given parameters *s* and *r* of the generalized diagnosability. About the relations among these parameters, the following Lemma 1 is easily understood.

Lemma 1

If a formal diagnosis model is *t/s/r/w-d*, then $w \geqq 2n-s-r$.

Proof

If a formal diagnosis model is $t/s/r/w$-d, then at least $n-s$ units are known to be fault-free and at least $n-r$ units are known to be faulty. Thus altogether, the state of at least $2n-s-r$ units are known. *Q. E. D.*

$t/s/.\ /.\ $-$d$ is the same concept as t/s-d which was first proposed by Friedman about SDM [11]. Necessary and sufficient conditions for t/s-d have not yet been obtained except for some special cases [13]. $t/.\ /r/.$-d is a new concept. However, special case, when $r=n-1$ is known as t-*fdwr* about SDM. The condition for t-*fdwr* will be obtained by substituting $r=n-1$ in condition (2) of Theorem 1, and this condition is the same as that proposed in [10].

The concept of $t/.\ /r/.$-d also plays an important role in the situation of sequential diagnosis which proceeds by replacing faulty units with fault-free units and diagnosing for renewed syndromes iteratively. The diagnosis terminates in this way as long as the graph of SDM is strongly connected [6]. The number of exchanged units in a step increases as the number r of the generalized diagnosability decreases, and the number of steps of the sequential diagnosis will decrease as well.

These are discussions about diagnosability. As for detectability, a similar discussion to that of diagnosability can be done with $G^{\sigma}_{1(0)}(x_i)$. A formal diagnosis model is said to be t-fault detectable (t-*fdt*) if and only if all the syndromes consistent with the null fault pattern $\phi\in P(X)$ (all units are fault-free) are distinguishable from those consistent with the fault pattern $F\in P(X)$ where $F\neq\phi$. The condition of t-*fdt* can also be characterized by $G^{\sigma}_1(x_i)$.

Theorem 3 (Condition for t-*fdt*)

A formal diagnosis model is t-*fdt* if and only if

$$\min_{x_i\in X}|G_1^{\sigma_0}(x_i)|\geqq t+1 \text{ where } \sigma_0=\sigma(\phi).$$

Proof

$|G_1^{\sigma_0}(x_i)|\geqq t+1$ implies that $\sigma(F_i)\neq\sigma_0$ for all $F_i\neq\phi$, and thus the formal diagnosis model is $t-fdt$. On the other hand, $|G_1^{\sigma_0}(x_i)|<t$ implies that the syndrome σ_0 is consistent with the fault pattern F_i such that $F_i\ni x$, hence $F_i\neq\phi$. *Q. B. D.*

Furthermore, the condition in which the state of at least one unit can be identified is characterized with respect to the syndrome σ_1 which is consistent with the fault pattern $X.\in P(X)$.

Theorem (Theorem 7 in [6])

The state of at least one unit can be identified in the formal diagnosis model under the assumption that the number of faulty units present does not exceed t if and only if

(1) $\max_{x_i \in X}(|G_0^{\sigma_1}(x_i)|, |G_1^{\sigma_1}(x_i)|) \geqq t+1$ where $\sigma_1 = \sigma(X)$.

(2) σ_1 is the only syndrome with which all units can be faulty or fault-free under the assumption.

Proof See Theorem 7 in [6]

Condition (2) of the Theorem is automatically satisfied for SDM of BGM type [6], or FDM with inclusive structure discussed later in this paper. And for these models, only condition (1) is necessary and sufficient so that the state of at least one unit can be identified.

Although Theorem 1 and 2 present a necessary and sufficient condition for generalized diagnosability, it is difficult to check them. This is because the domain of syndromes over which the minimum is taken is sizable for large-scale systems. Even in such cases, it often occurs in fault diagnosis that fragmentary knowledge useful in diagnosis is known, such as that some unit is faulty with syndrome σ or a certain set of units must not contain faulty units under syndrome σ. This fragmentary knowledge will be used to roughly evaluate parameters t, s, r, and w of the generalized diagnosability.

Furthermore, these conditions in Theores 1 and 2 become chackable by reducing the domain of syndrome patterns. This is done by imposing restrictions on the relation between fault patterns and syndromes as studied in section 3 or by using the symmetricity of the graph as discussed in the next Example 1. The domain of syndromes over which the minimum of conditions of Theorem 1 and 2 is considered might be reduced to a great extent when the graph of SDM has symmetricity. Assign distict integers of $\{1, 2, .. j\}$ to each arc (test) of the grph of SDM. Permutation $\binom{1, 2, \ldots, j}{i_1, i_2, \ldots, i_j}$ is called possible permutation if some rotation (around axis or point) results in the same rearrangement in the integers associated with arcs as in this permutation. Syndrome σ_i is regarded as equivalent to σ_j under possible permutation p if $p(\sigma_i) = \sigma_j$ where $p(\sigma_i) = p(y_{i_1} y_{i_2} \ldots y_{i_k}) = j \in p(i_1, \ldots i_k) \cup (y_j)$.

The minimum of conditions of Theorem 1 and 2 is taken over the domain which consists of all the syndromes that are not equivalent with each other. The cardinality of this set of syndromes is obtained by Burnside's Theorem [14].

Example 1 (An Example of SDM)

Fig. 1 shows the graph of an example of SDM of the PMC type. The number of the cardinality of a set of syndromes over which conditions of Theorem 1 and 2 are checked is reduced to twenty four. This reduced set of syndromes is denoted by Σ'. $\min_{\sigma \in \Sigma'} |S^k(t,)|$ $k=$ 0, 1 are shown for several t in Table 1. It is shown by Theorem 1 that this SDM is 1/3/6/3-d, 2/4/6/3-d, and 3/6/6/0-d. It is also shown that this SDM is not even 1-fdt by Theorem 3. If we suppose the syndrome σ_0 never occurs, then the parameter r of general-

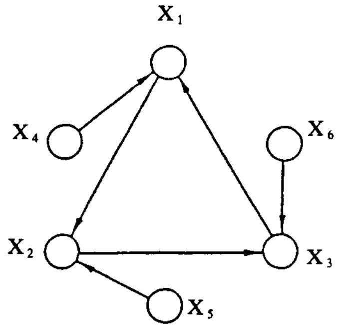

Fig. 1 An Exanple of SDM of the PMC type.

Table 1 The Cardinality of the Syndromes

t	min $(\|s^1(t, \sigma)\|)$ $\sigma \in \Sigma'$	min $(\|s^0(t, \sigma)\|)$ $\sigma \in \Sigma'$	min $(\|s^1(t, \sigma)\| + \|s^0(t, \sigma)\|)$ $\sigma \in \Sigma'$
1	3	0	3
2	2	0	3
3	6	6	0

ized diagnosability become 5 when $t=1, 2$.

3. Generalized diagnosability of diagnosis models with constrained structure

The last section presents conditions of generalized diagnosability without any restriction on the structure of association between fault patterns and syndromes. It is almost impossible to evaluate parameters t, s, r, and w of the generalized diagnosability, since the domain of syndromes over which the minimum is taken is enormous. In this section, we study corresponding theorems for diagnosis models with constrained structure. Using the constrained structure, conditions for generalized diagnosability become easy to check. We present some illustrative examples of diagnosis models with constrained structure.

Definition 4 (Constrained Structure)

A formal diagnosis model is said to have inclusive structure if and only if

$$\sigma(F_i \cup F_j) \supseteq \sigma(F_i) \cup \sigma(F_j) \text{ for all } F_i, F_j \in P(X)$$

and it is said to have additive structure if and only if equality holds for all F_i, $F_j \in P(X)$.

A characteristic property of this inclusive structure is that some fault-free units can be known from the relation between syndromes.

Lemma 2 (A Property of Inclusive Structure)

If a formal diagnosis model has an inclusive structure, then unit x_i is fault-free under the syndrome $\sigma(F)$ such that $\sigma(F) \not\supseteq \sigma(x_i)$.

Proof

Suppose unit x_i is faulty, then the fault pattern present F includes x_i and hence $\sigma(F) \supseteq \sigma(x_i)$ by the inclusive structure. This contradicts the hypothesis $\sigma(F) \not\supseteq \sigma(x_i)$. *Q. E. D.*

Thus Lemma 2 does not hold for diagnosis in general. By this Lemma 2, the next Theorem 4 follows from condition (1) of Theorem 1.

Theorem 4 ($t/s/./(n-s)-d$ of Models with Inclusive Structure)

A formal diagnosis model with inclusive structure $t/s/. /(n-s)\text{-}d$ if and only if

$$\min_{\sigma \in \Sigma_s} |S^1(t, \sigma)| \geqq n-s$$

where Σ_S is the set of all syndromes such that $\sigma \in \Sigma_S$ implies that $|\{x_i : \sigma(x_i) \subseteq \sigma\}| \geqq s+1$.

Proof

Divide Σ_A into Σ_S and $\Sigma_A - \Sigma_S$ where Σ_A is the set of all possible syndromes. Since $\sigma \in \Sigma_A - \Sigma_S$ implies that $|\{x_i : \sigma(x_i) \subseteq \sigma\}| < s$. And thus it can be observed by Lemma 2 (A property of inclusive structure) that all faulty units are specified within a set S_i such that $|S_i| \geqq s$ for all $\sigma \in \Sigma_A - \Sigma_S$. Therefore, it is sufficient that condition (1) of Theorem 1 holds only for $\sigma \in \Sigma_S$. *Q. E. D.*

This Theorem 4 means that the inclusive structure reduces the domain of syndrome over which the minimum of condition (1) is taken. About $t/(n-1)/. /1-d$, a simpler result can be obtained, since Σ_S become only one syndrome $\Sigma_{n-1} = \sigma_1 (= Y)$.

Corollary 1 ($t/(n-1)/. /1$-. of Models with Inclusive Structure)

A formal diagnosis model with inclusive structure is $t/(n-1)/. /1\text{-}d$ if and only if

$$\max_{x_i \in X} |G_1^{\sigma_1}(x_i)| \geqq t+1.$$

As for $t/. /r/(n-r)\text{-}d$, the problem is more difficult than that of $t/s/. /(n-s)\text{-}d$. This is because, even if $y_j \in \sigma$, it does not mean unit x_i such that $y_j \in \sigma(x_i)$ is faulty. Whereas, $y_j \notin \sigma$ means unit x_i such that $y_j \in \sigma(x_i)$ is fault-free under the syndrome σ by inclusive structure. However, some faulty units become explicit with an additive structure. The next Lemma 3 shows a propery which makes additive structure different from inclusive structure.

Lemma 3 (A Property of Additive Structure)

If a formal diagnosis model has additive structure, then unit x_i is faulty under syndrome $\sigma(F)$ such that $|\{x_i : y_j \in \sigma(x_i)\}| = 1$ for some $y_j \in \sigma(F)$.

Proof

Suppose x_i is fault-free, then $y_j \in Y$ such that $|\{x_i: y_j \in \sigma(x_i)\}| = 1$ is normal; and hence the syndrome present $\sigma(F)$ $(\ni Y_j)$ never occurs, for $\sigma(F') \not\ni y_j$ for all fault patterns $\sigma(F') \not\ni y_j$ for all fault patterns F'such that $F' \not\ni x_i$. *Q. E. D.*

It should be noted that this Lemma 3 does not always hold for formal diagnosis models with inclusive structure, because, even if $|\{x_i | y_j \in \sigma(x_i)\}| = 1$, it may occur $\sigma(x_{i_1} \ldots x_{i_q}) - \{\sigma(x_{i_1}) \cup \ldots \cup \sigma(x_{i_q})\} \in y_j$, for some $F_i = \{x_{i_1}, \ldots, x_{i_q}\} \not\ni x_i$ by the definition of inclusive structure. By this Lemma 3, we obtain the next Theorem 5 about $t/. /r/(n-r)$-d which is derived from condition (2) of Theorem 1.

Theorem 5 ($t/. /r/(n-r)-d$ of Models with Additive Structure)

A formal diagnosis model with additive structure is $t/. /r/(n-r)-d$ if and only if

$\min_{\sigma \in \Sigma'} |S^0(t, \sigma)| \geqq n-r$, where Σ' is the set of syndromes such that $\bigcup_{\sigma \in \Sigma'} |\{x_i : |\{x_i : y_j \in \sigma(x_i)\}| = 1$ for some $\{y_j \in \sigma\}| \leqq n-r$.

Proof

$\sigma \in \Sigma_A - \Sigma'$ imples that the number of units identified faulty is not less than $n-r$ by the definition of Σ' and Lemma 3(A property of additive structure). Therefore condition (2) of Theorem 1 is clearly satisfied for all $\sigma \in \Sigma_A - \Sigma'$. *Q. E. D.*

Thus, if a formal diagnosis model with additive structure has relatively large number of units mentioned above, it becomes easy to evaluate parameter r of the generalized diagnosabiliy.

Example 2 (Diagnosis Model with Additive Structure)

The failure diagnosis model (FDM) is a triple (X, Y, Λ) of a set of units $X = \{x_i\}$ $i = 1 \ldots n$, $Y = \{y_j\}$ $j = 1 \ldots m_j$ both X and Y take the same binary state as the formal diagnosis model, and $\Lambda = \{\lambda\}$ which indicates a binary relation between $x_i \in X$ and $y_j \in Y$ is defined as below :

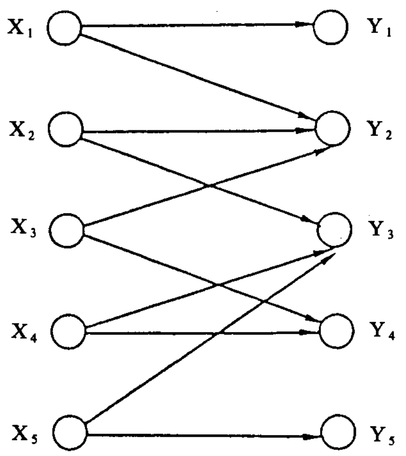

Fig. 2 An Exanple of FDM.

$$\begin{cases}(x_i, y_j) \in \lambda & \text{if the faulty state of unit } x_i \text{ always implies that the abnormal state of measurement } y_j. \\ (x_i, y_j) \notin \lambda & \text{otherwise}\end{cases}$$

Fig. 2 shows an example of FDM expressed by bipartite graph $G(X, Y, E)$ where the arc $e_{ij} \in E$ only if $(x_i, y_j) \in \lambda$. Since FDM ias additive structure, Σ_A of Theorem 1 is reduced to $\Sigma_3 = \{(y_1\, y_2\, y_3\, y_4\, y_5), (y_2\, y_3\, y_4\, y_5), (y_1\, y_2\, y_3\, y_4)\}$ when $r=3$ by Theorem 4. And $\min_{\sigma \in \Sigma_3} |S^1(3, \sigma)| \geqq 5-3$ implies that this example is 2/3/. /2$-d$. Further, $\Sigma^4 = \{(y_2\, y_3\, y_4), \phi\}$ and $\min_{\sigma \in \Sigma^4} |S^0(2, \sigma)| \geqq 5-4$ implies that this example is 1/. /4/1-d by Theorem 5. By Corollary 2, it is also easily understrrd that this FDM is 3/4/. /1-d, for $\max_{x_i \in X} |G_1^{\sigma_1}(x_i)| = \max(3, 4, 4, 3, 3) = 4$.

Example 3 (Diagnosis Model with Incluseive Structure)

The AND–node is defined in the graph of FDM in addition to the nodes which correspond to fault units and measurements. The AND–node is the node which propagates the an abnormal state to all the measurements connected to it only when all inputs into it are from faulty units. The failure diagnosis model with this AND–node has inclusive structure and not additive structure. Fig. 3 shows an example of FDM with this AND–node. The square node with a dot inside denotes the AND–node. This example actually has inclusive structure, for $\sigma(x_2 \cup x_3) \supset \sigma(x_2) \cup \sigma(x_3)$. Σ_3 of this example is $\{(y_1\, y_2\, y_3\, y_4\, y_5), (y_2\, y_3\, y_4\, y_5)\}$. Because $\min_{\sigma \in \Sigma_3} |S^1(3, \sigma)| \geqq 5-3$, this example is 2/3/. /2-d by Theorem 4. And the application of Corollary 2 shows that this example is 3/4/. /1-d, for

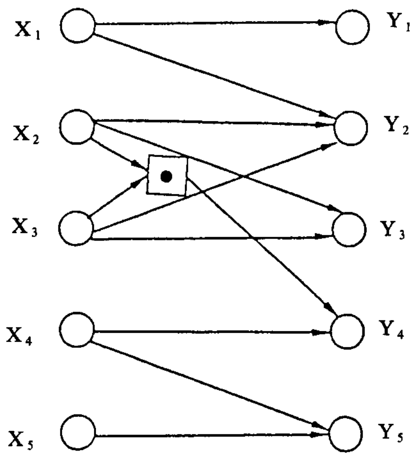

Fig. 3 An Exanple of FDM with AND-node.

$\max_{x_i \in X} | G_1^{\sigma_1}(x_i) | = \max(3, 3, 3, 3, 4) = 4.$

4. Diagnosis without the assumption of upper bound t in the number of faulty units

Most diagnosability studies have been carried out under the assumption of upper bound t on the number of faulty units. However, this assumption seems meaningless in the following cases :

In the case of (i), a probabilistic version of generalized diagnosability should be considered. The probabilistic version of generalized diagnosability is obtained by replacing the assumption about the permissible number t of faulty units with the assumption of the probability of the fault pattern. All the discussions proceed in the same manner as in section 2, interpreting $| G_{1(0)}(S) |$, $| S^{1(0)}(t, \sigma) |$ as the probability of the fault pattern.

(i) It often happens that the probability of the occurence of fault pattern F_i is greater than that of fault pattern F_j even if $|F_i| > |F_j|$.

(ii) In the case that most fault patterns F_i such that $|F_i| > t$ can be identified even though the system is not t-fd. Moreover, it is difficult to assign t adeqately beforehand.

In this section, we remove this hypothesis and study the diagnosis directly from the syndromes with $G_{1(0)}^{\sigma}(S_i)$discussed before. We present a property of $G_{1(0)}^{\sigma}(S_i)$ in order to use it as a means of diagnosis without restriction on the number of permissible faulty units.

Lemma 4 (A Property of $G_{1(0)}^{\sigma}(S_i)$)

A fault pattern $F_i \in P(X)$ is the minimal fault pattern consistent with syndrome σ if and only if

$G_0^\sigma(\bar{F}_i) = G_1^\sigma(F_i)$ where $\bar{F}_i = X - F_i$.

Proof

Sufficiency : By the properties of $G^\sigma(S_i)$ (see Lemma 5, (1) in the Appendix), it follows that $G_1^\sigma(S_i) \supseteq S_i \supseteq G_0^\sigma(\bar{S}_i)$. Thus, $G_0^\sigma(\bar{F}^i) = G_1^\sigma(F_i)$ implies that $G_0^\sigma(\bar{F}_i) = G_1^\sigma(F_i) = F_i$.

Necessity : If a fault pattern F is consistent with a syndrome σ, then $G_1^\sigma(F_i) = F_i$ and $G_0^\sigma(\bar{F}_i) \subseteq F_i$. Furthermore, since F_i is the minimum fault pattern, thare is no fault pattern F_j such that $G_0^\sigma(\bar{F}_i) = F_j \subset F_i$. *Q. E. D.*

To find the minimal fault pattern consistent with a given syndrome, the min–max form of Lemma 4 is convenient.

Theorem 6 (Min–Max Form of Lemma 4)

A fault pattern F_5 is the minimal fault pattern consistent with a given syndrome σ if and only if

the equation $\{F\colon F = G_1^\sigma(s_j)$ for some $S_j \subseteq X$ and $|F| = F1\min\} =$

$\{F\colon F = G_0^\sigma(S_k)$ for some $S_k \subseteq X$ and $|F| = F0\max\}$

has the unique solution F_s

where $F0\max = \max_{S_i \in P(X)} |G_0^\sigma(S_i)|$ and $F1\min = \min_{S_i \in P(X)} |G_1^\sigma(S_i)|$.

By this Theorem 6, the problem of finding the minimal fault pattern consistent with a syndrome is formulated ints two mathematical programming problems dual to related to each other. And the minimal fault pattern is given by solving either of these mathematical programming problems.

Problem

$\min_{S_j \in P(X)} |G_1^\sigma(S_j)|$ under constraint that $G_0^\sigma(\bar{S}_j) = S_j$.

Dual Problem

$\max_{S_k \in P(X)} |G_0^\sigma(S_k)|$ under conder constraint that $G_1^\sigma(S_k) = S_k$.

Example 4

Consider the same FDM as discussed in Example 2. Let σ_2 and σ_3 denose syndromes $(y_1\, y_2)$ and $(y_2\, y_3\, y_4)$ respectively. The only fault pattern F_i which satisfies $G_1^{\sigma_2}(S_i) = S_i$ is x_1 and this realizes the minimum of $|G_1^{\sigma_2}(S_j)|$. Therefore, the minimal fault pattern consistent with the syndrome is x_1. Next, the fault patterns which satisfy $G_1^{\sigma_3}(S_i) = S_i$ are as many as $\{x_2x_3,\ x_3x_4,\ x_2x_4\}$, and all of them realize $\min_{S_j \in P(X)} G_1^{\sigma_3}(S_j)$. Hence, if the syndrome occurs, this model is not diagnosable in the because sense that the fault pattern consistent with it is not uniquely identified.

5. Conclusion

A new concept of generalized diagnosability is proposed for a formal diagnosis model which includes most diagnosis models so far proposed as particular cases. A detailed diagnosis is possible by this generalized dianosablity, sence this diagnosability incorporates information expressed in t–fault diagnosability, t–fault diagnosablity with repair, and *t* out of *s* diagnosability.

Conditions of this generalized diagnosability are discussed with some set relations between the power sets of a set of units. A though these conditions are difficult to check, for they must be checked over all the possible syndromes, this domain may be reduced in some cases. By the same set relations as used to express conditions of generalized diagnosability, the minimal fault set consistent with a given syndrome is obtained as well. This problem is formulated as a mathematical programming problem which has these set relations in constraints and objective functions.

As a result, once these set relations are obtained, the generalized diagnosability as well as the minimal fault set consistent with a syndrome are given by these set relations.

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Appendix

Lemma 5 (Properties of $G^{\sigma}_{1(0)}(S)$)

(1) (i) If $G^{\sigma}_{1}(S_i) \neq \phi$, then $G^{\sigma}_{1}(S_i) \supseteq S$ where equality holds when $(S_i, \sigma) \in \mu$.

(ii) $G^{\sigma}_{0}(\bar{S}_i) \subseteq S_i$ where equality holds when S_i is the minimal fault pattern consistent with a syndrome σ.

(2) If $\{S_i\}$ $i=1\,..\,q$ is a set of units such that (a) $|S_i| \leqq t$ $i=1\,..\,q$ (b) $\sigma=\sigma(S_1)=\sigma(S_2)=\ldots=\sigma(S_q)$ and $\sigma(S_j) \neq \sigma$ for all $S_j \notin \underline{\{S_i\}}$. then

(i) $|G^{\sigma}_{1}(x_k)| \geqq t+1$ for all x_k in $(\overline{\bigcup_{i=1}^{q} S_i})$.

(ii) $|G^{\sigma}_{0}(x_k)| \geqq t+1$ for all x_k in $(\overline{\bigcap_{i=1}^{q} S_i})$.

Proof

(1) is straightforward from the definition of $G^{\sigma}_{1(0)}(S_i)$; and we prove only (2). For the proof of (i), suppose there exists a unit x_k such that $|G^{\sigma}_{1}(x_k)| \leqq t$ and $x_k \in (\overline{\bigcup_{i=1}^{q} S^i})$. This implies that there exists a fault pattern S which is consistent with the syndrome σ and that $S_j \notin \{S_i\}$. This fact contradicts (b). In the same manner, the existence of x_k such that $|G^{\sigma}_{0}(x_k)| < t+1$ and $x_k \in \bigcap_{i=1}^{q} S_i$ also violates the hypothesis(b). *Q. E. D.*

Lemma 6 (Relationship between $S^k(t, \sigma)$ and $G^{\sigma}_{k}(S)$)

(1) $\max_{|S_i|=n-s} (\min_{x_i \in S_i} |G^{\sigma}_{1}(x_i)|) \geqq t+1$ if and only if $|S^1(t, \sigma)| \geqq n-s$.

(2) $\max_{|S_i|=n-r} (\min_{x_i \in S_i} |G^{\sigma}_{0}(x_i)|) \geqq t+1$ if and only if $|S^0(t, \sigma)| \geqq n-r$.

Proof

$\min_{x_i \in S_i} G^{\sigma}_{1}(S_i)$ indicates all the fault patterns consistent with the syndrome σ and satisfies the constraint that at least one unit is faulty in S_i. Therefore, the condition $\max_{|S_i|=n-s} (\min_{x_i \in S_i} |G^{\sigma}_{1}(x_i)|) \geqq t+1$ is equivalent to: all the fault patterns consistent with the syndrome and whose caxdinality is less than $t+1$ must satisfy the constraint that all the subsets of units whose cardinality is greater than $n-s$ consist of only fault-free units. This fact is equivalent to the condition $|S^1(t, \sigma)| \geqq n-s$. The proof of (2) is done in the same manner as this. *Q. E. D.*