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Three-Dimensional Response Spectra for Multiple-Support Input Motions

By

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Abstract

New response spectra for multiple-support input motions are presented to study the seismic responses of long-span bridges. Three-dimensional response spectra are defined using the response of simple beams with various natural periods. These spectra clearly verify the dominant vibration mode and the critical location of the maximum response. Numerical examples using the earthquake records observed by a newly-installed array at the Akashi Kaikyo bridge construction site in Japan show that the different input motions to each support produce different predominant vibration modes compared with the identical excitation case. It has also been observed that response usually decreases when multiple-support input motions with phase lags are considered. The ordinary response spectra method is also examined and it has been found that the approximated values using the RMS method overestimated the maximum response.

1. Introduction

An array observation system for earthquakes has been increasing throughout the world making it possible to obtain records observed by several stations simultaneously. One of the important usages of the simultaneously obtained records is for the multiple-support excitation problem. Abdel-Ghaffar, et al.1),2) used the array observed records of the 1979 Imperial earthquake for the multiple-support excitation problem of suspension bridges and cable-stayed bridges. The response under multiple-support excitation is an important problem especially for long-span bridges. However, these array observed records are usually expressed in various ways such as time histories, Fourier spectra, response spectra, and so on, but not in a suitable way for the multiple-support excitation problem. One reason is that the response subjected to multiple-support excitation depends on the many characteristics of the structure, so that it is difficult to show a general expression on the effect of multi-input ground motions.

The general equations of motions are described in the literature (e.g. Clough

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and Penzien[3]), however, the multiple-support excitation problem has been studied in several different ways. Bogdanoff, et al.[4] evaluated random responses considering the transmission time of a seismic disturbance in terms of extreme value statistics. Werner, et al.[5] analyzed the steady-state response of a simple bridge supported on an elastic half-space subjected to traveling waves. Abdel-Ghaffar, et al.[1],[2] and Toki[6] used time-domain analyses for the response of bridges using the finite element method (FEM). Sakurai, et al.[7] applied the ordinary root mean square method of the response spectra approach to the multiple-support excitation problem. This method can solve any type of structure. However, the response spectra method gives only an approximate estimation and overestimates the maximum values for cases of low correlation between the input motions[7]. These methods tackle specific structural models, making it a case by case study for each problem. Hence, these methods are inadequate to process the preliminary information on simultaneously obtained array records for the multiple-support excitation problem.

In this paper, the most basic multiple-support excitation problem, in which only 2 input motions are applied to the structure, is studied to see the effect of the multi-input excitations graphically. The first part of this research introduces a three-dimensional response spectra using simple beams instead of the SDOF system in the ordinary response spectra. The second part of this paper demonstrates numerical examples using the array observed earthquake records. And the last part compares the maximum values of the proposed multi-input response spectra with the approximated values obtained from the root mean square method using the ordinary response spectra.

2. Response of Simple Beam under Multiple-Support Excitations

Consider a Bernoulli-Euler type simple beam with multiple-support input excitations as in Figure 1. The continuum equation of motion of this beam at

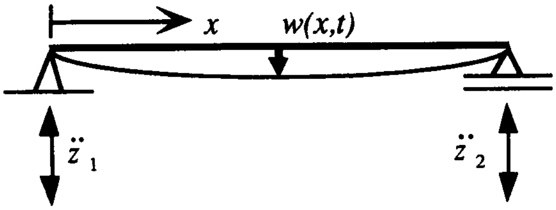


Fig. 1 Simple beam model with multiple-support input excitations

time t with position x is expressed as

$$m\frac{\partial^2 w(x,t)}{\partial t^2}+EI\frac{\partial^4 w(x,t)}{\partial x^4}=-m\frac{L-x}{L}\ddot{z}_1(t)-m\frac{x}{L}\ddot{z}_2(t) \tag{1}$$

in which m is the mass per unit length, EI is the flexural stiffness, L is the length of the beam, $\ddot{z}_1(t)$ and $\ddot{z}_2(t)$ are input accelerations, and $w(x,t)$ is the deflection response of the beam. Constant values for m and EI are assumed with position x along the span in Equation (1).

Using modal analysis formulation, $w(x,t)$ is expressed as

$$w(x,t)=\sum_{n=1}^{\infty}\varphi_n(x)q_n(t) \tag{2}$$

in which $\varphi_n(x)$ is the n-th mode shape, and $q_n(t)$ is the n-th mode vibration.

Hence, Equation (1) is derived into the following equations considering the damping term.

$$\ddot{q}_n(t)+2h\omega_n\dot{q}_n(t)+\omega_n{}^2q_n(t)=-\frac{2}{n\pi}\{\ddot{z}_1(t)+(-1)^{n-1}\ddot{z}_2(t)\} \tag{3}$$

$$\varphi_n(x)=\sin\frac{n\pi x}{L} \tag{4}$$

in which h is the damping factor and ω_n is the n-th natural circular frequency defined as following.

$$\omega_n=(n\pi)^2\sqrt{\frac{\mathrm{EI}}{\mathrm{mL}^4}} \tag{5}$$

In Equations (3) and (4), a constant h value is assumed for all modes.

From Equation (3), the odd modes are affected by the sum of the input accelerations, and the even modes are affected by the difference of the input accelerations. Therefore, only odd modes appear in the responses subjected to the identical input motions into both supports.

3. Multi-Input Response Spectra

Using the previous equations, the three-dimensional multi-input response spectra is defined in Figure 2. The x-axis denotes the location of the simple beam normalized by length L. The y-axis denotes the first natural period of the simple beam and the z-axis denotes the maximum response during earthquake excitation. Three kinds of spectra can be defined like the ordinary response

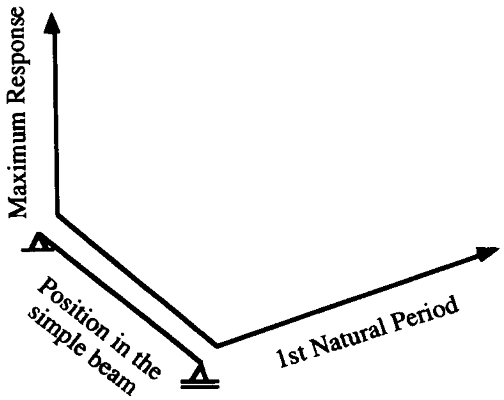


Fig. 2 Definition of each axis for multi-input response spectrum

spectra: the absolute acceleration response spectrum, the relative velocity response spectrum and the relative displacement response spectrum.

For the relative velocity and displacement response spectra, relative values are defined as the distance from the straight line which connects two supports. And for the rigid beam which has a natural period of 0 seconds, the absolute acceleration response spectrum is defined as the maximum value of Equation (1) divided by the mass per unit length: m.

This method is valid for the transverse vibrations as well as the vertical vibrations. However, longitudinal components of earthquake records require another equation of motions which describes coupled vibration with longitudinal direction and vertical direction.

4. Examples of Multi-Input Response Spectra

4.1 Array Observation System for Multi-Input Response Spectra

As numerical examples of the multi-input response spectra, two different types of earthquake records were used. They were recorded by the array observation system installed by the authors near the construction site of Akashi Kaikyo bridge in Japan[8]. Akashi Kaikyo bridge will be the world's longest suspension bridge, which is expected to be completed in 2000. It will have a center span of 1990 m long, and the first natural period for bending of the girder will be about 16 seconds. Three sets of servo velocity meters, whose reliable frequency ranges are 0.025 to 70 Hz, have been set near both ends of this bridge to record long-period ground motions. One is on Awaji-shima Island, and the others are on mainland Honshu (Akashi City and Kobe City). The locations of the observation stations are shown in Figure 3. Each site has a time code generator which enables the exact time to be clocked to an accuracy of ±0.1 sec.

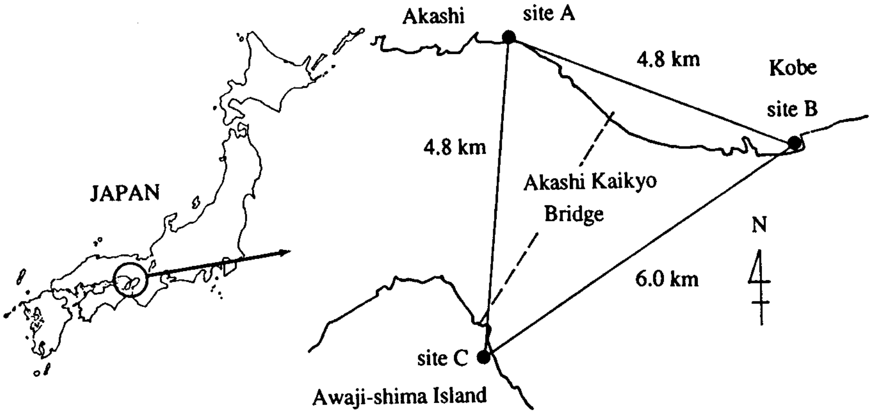


Fig. 3 Location of array observation system and Akashi Kaikyo bridge

4.2 The 1991 Myanmar Earthquake

First, the records of the 1991 Myanmar earthquake occurred on the 6th of January in 1991 in Myanmar (about 4000 km away from the observation sites) with magnitude of 7.2. Figure 4 shows two time histories of NS components for the Myanmar earthquake, at site A (Akashi City) and site B (Kobe City), together according to their time signals. Site A and site B are about 4.8 km apart from each other.

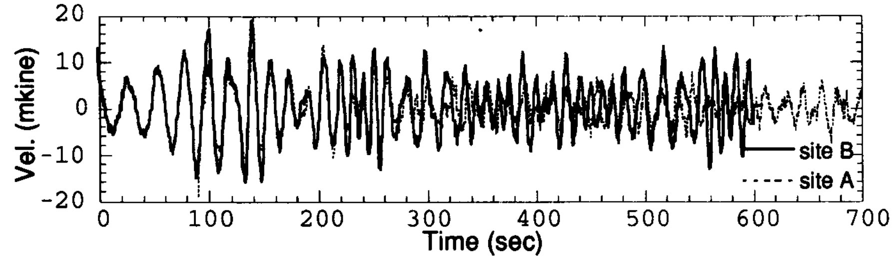


Fig. 4 Velocity time histories for NS components of Akashi (site A) and Kobe (site B) records of the 1991 Myanmar earthquake

Although the maximum velocity was only about 40 mkine, it lasted more than 10 minutes. The dominant period was from 10 to 30 seconds, which includes the first natural period of Akashi Kaikyo bridge. Figure 5 shows the cross correlation function of these 2 records normalized to have the maximum value of 1.0 given in radial, transverse and vertical directions. They have a sinusoidal wave form due to a high correlation between the 2 records. The phase lag of these records was calculated to have the maximum cross correlation results in 1.0 second.

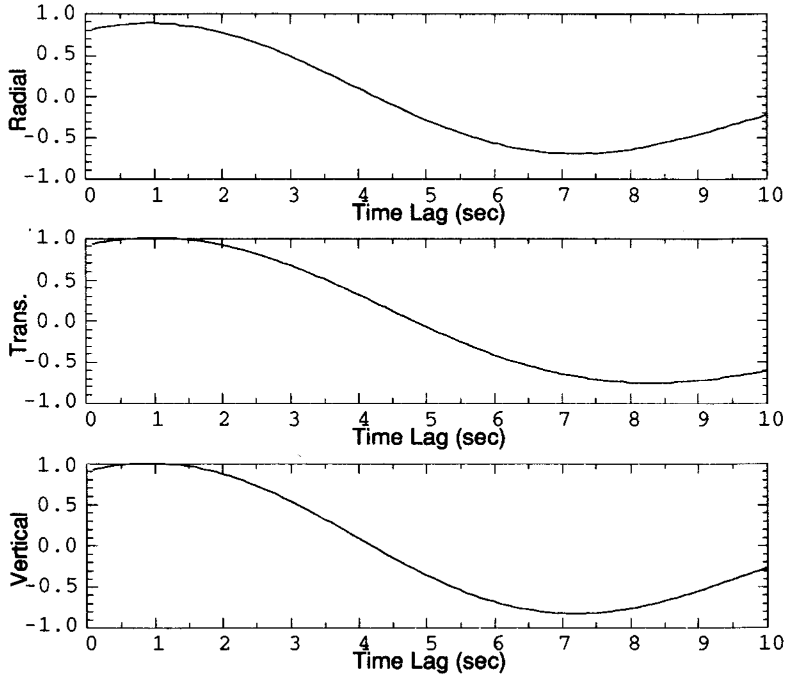


Fig. 5 Normalized cross correlation function between Akashi (site A) and Kobe (site B) records of the 1991 Myanmar earthquake given in radial, transverse and vertical direction, respectively

Figure 6 shows the multi-input response spectra for the identical input motions, and Figure 7 for the different input motions; Figure 6 was calculated using NS component of Kobe (site B) record of the 1991 Myanmar earthquake as input motions into both supports, and Figure 7 was calculated using NS components of Akashi (site A) and Kobe (site B) records as separate input motions into each support. Figures 6(a) and 7(a) show the absolute acceleration response,

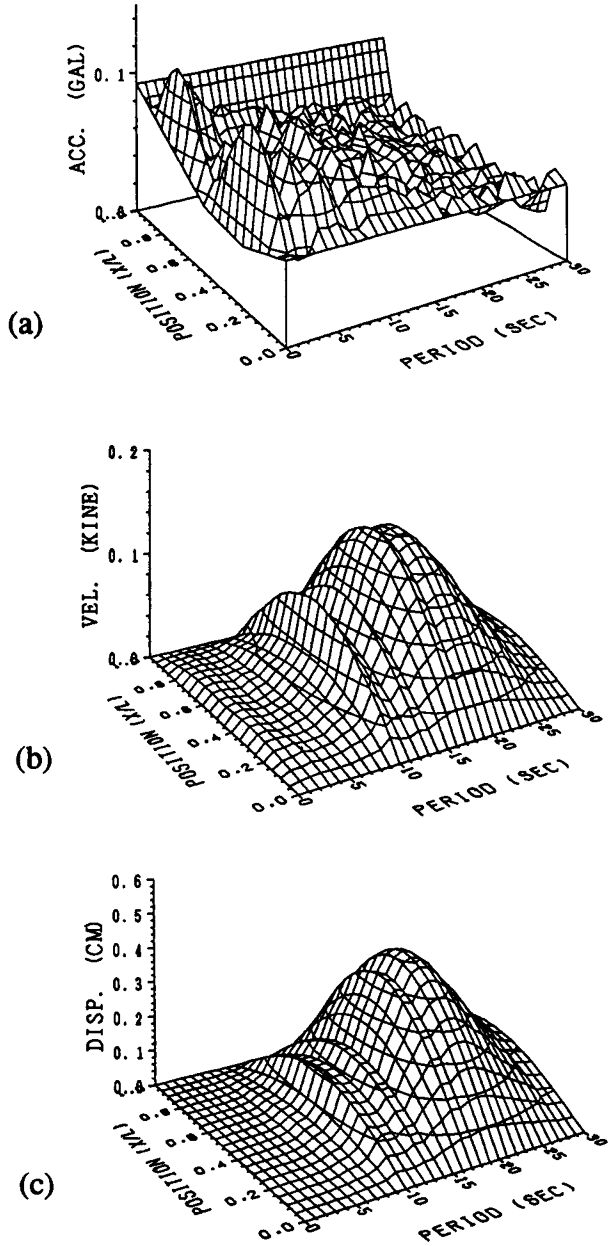


Fig. 6 Multi-input response spectra using NS component of Kobe (site B) record for the 1991 Myanmar earthquake as input motions into both supports (a) Absolute acceleration response (b) Relative velocity response (c) Relative displacement response

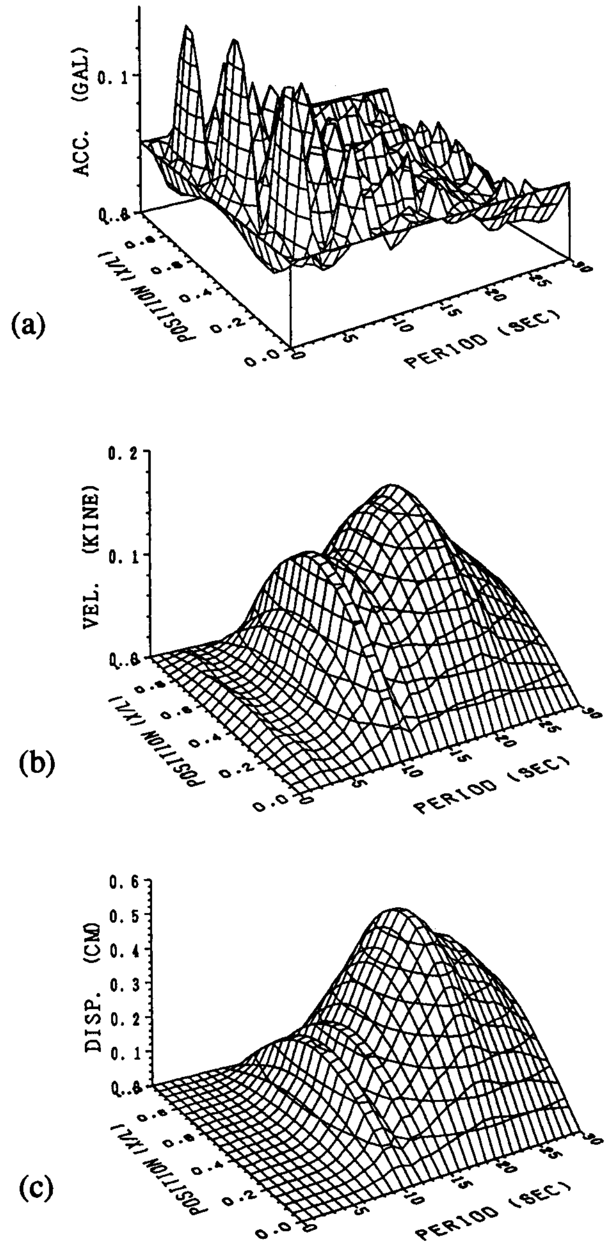
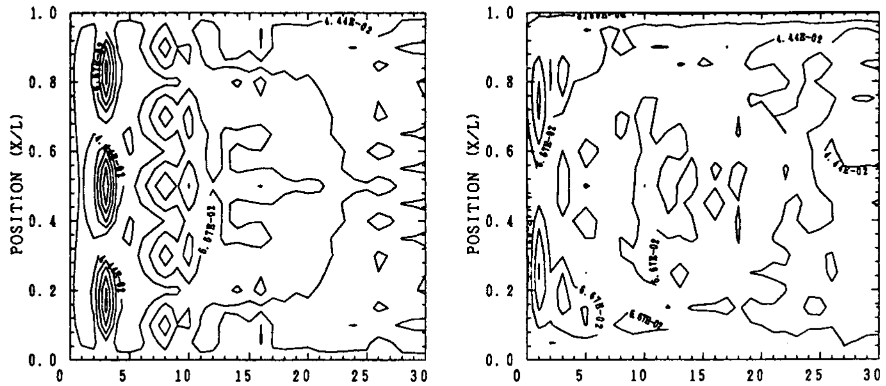


Fig. 7 Multi-input response spectra using NS components of Akashi (site A) and Kobe (site B) records for the 1991 Myanmar earthquake as input motion into each support (a) Absolute acceleration response (b) Relative velocity response (c) Relative displacement response

Figures 6(b) and 7(b) show the relative velocity response, and Figures 6(c) and 7(c) show the relative displacement response. The damping factor of Figures 6 and 7 is set to $h=2\%$, and the y-axes are set to between 0 and 30 seconds which includes the first natural period of Akashi Kaikyo bridge. And the first 10 modes were involved in the mode superposition, as the participation factor of the 10th mode becomes 1/10 of that of the 1st mode.

The relative velocity and displacement response for the different input motions of Figures 7(b) and 7(c) showed smaller values compared to the identical input motions of Figures 6(b) and 6(c), especially for the long-period beam which has a first natural period of 25 to 30 seconds.

For the absolute acceleration response, the dominant vibration mode changed when the different input motions were applied compared to the identical input motions case. Figure 8 shows contour maps of the three-dimensional absolute acceleration response spectra of Figures 6(a) and 7(a) for precise discussion. For the identical input motions shown in Figures 6(a) and 8(a), the 3rd mode dominates for the beam with the first natural period of 4 seconds. Moreover, the maximum response of this beam is also the maximum value compared to the beams with the other natural periods. On the contrary, for the different input motions shown in Figures 7(a) and 8(b), the response of the 4 second beam is small and that of the 1 second beam takes the maximum value by the 2nd vibration mode. This even



(a) (b)

Fig. 8 Contour map of multi-input response spectra for absolute acceleration (a) For the identical input motions using NS component of Kobe (site B) record for the 1991 Myanmar earthquake (b) For the different input motions using NS components of Akashi (site A) and Kobe (site B) records for the 1991 Myanmar earthquake vibration mode is characteristic for the different input motions. Also, there is no clear dominant vibration mode for the beams whose natural period is longer than 5 seconds.

In summing up, this method using three-dimensional response spectra can clearly verify the dominant vibration mode and the critical location of the maximum response. The center of the span does not always show the maximum response.

In calculating this multi-input response spectra for 1 damping factor and 30 natural periods of simple beams, 60 seconds were needed as the CPU time using the supercomputer Fujitsu FACOM VP-2600 of the Kyoto University Data Processing Center. It is desirable that a more efficient algorithm should be considered.

4.3 The 1991 Nara-Ken Nanbu Earthquake

Instead of the strong and distant earthquake which was mentioned in the previous section, a small and near earthquake was used in this section. The same array observation system at Akashi Kaikyo bridge was able to catch many records of small near field earthquakes. One of them was the Nara-Ken Nanbu earthquake which occurred on June 16th of 1991. The magnitude of this earthquake was only 4.3, however, as it occurred as near as 80 km away from the array sites, all 3 stations could get records of this earthquake simultaneously.

Figure 9 shows 2 time histories of EW components for the Nara-Ken Nanbu earthquake at site A (Akashi City) and site C (Awaji-shima Island) together according to their time signals. Site C is located about 6.0 km away from site A. The maximum velocity ranged from 10 to 20 mkine and the dominant periods were less than 1 second. As the local site effect dominates small earthquakes, the

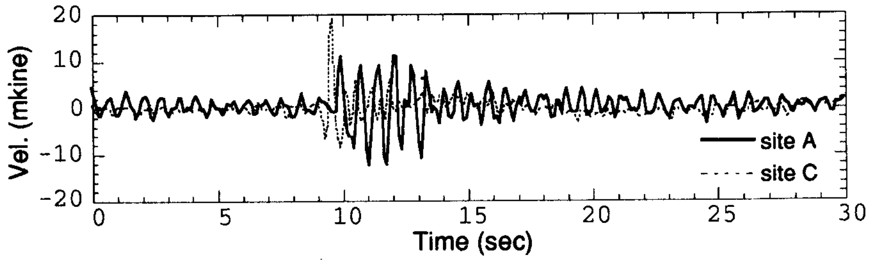


Fig. 9 Velocity time histories for EW components of Akashi (site A) and Awaji-shima (site C) records for the 1991 Nara-Ken Nanbu earthquake

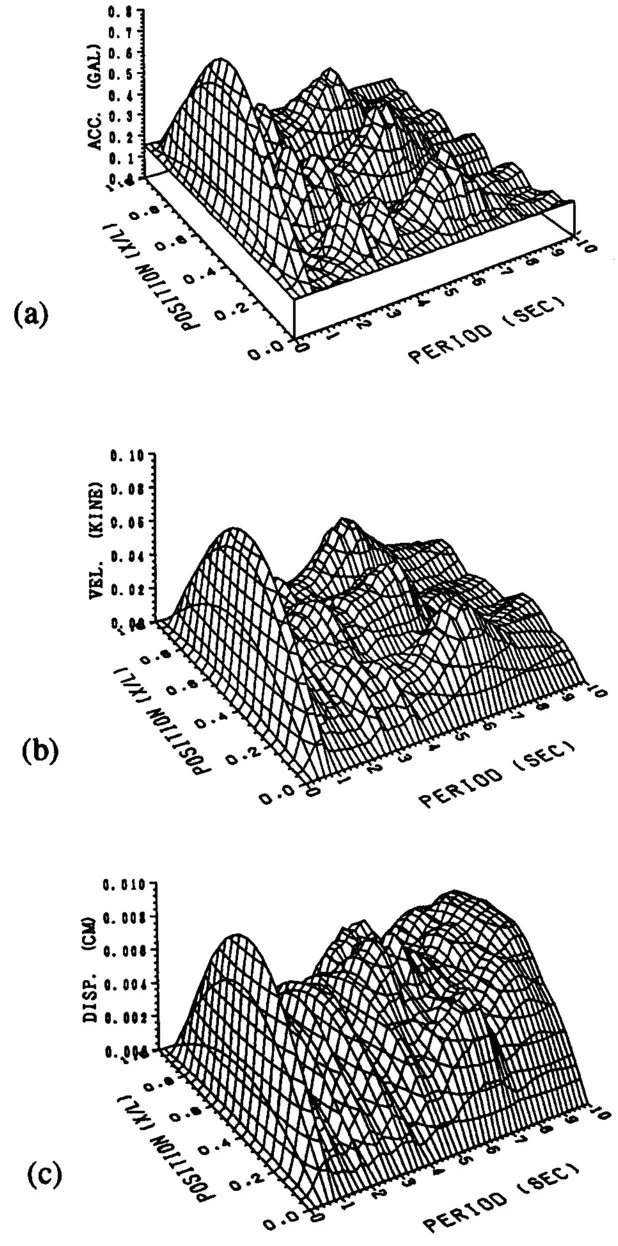


Fig. 10 Multi-input response spectra using EW component of Akashi (site A) record for the 1991 Nara-Ken Nanbu earthquake as input motions into both supports (a) Absolute acceleration response (b) Relative velocity response (c) Relative displacement response

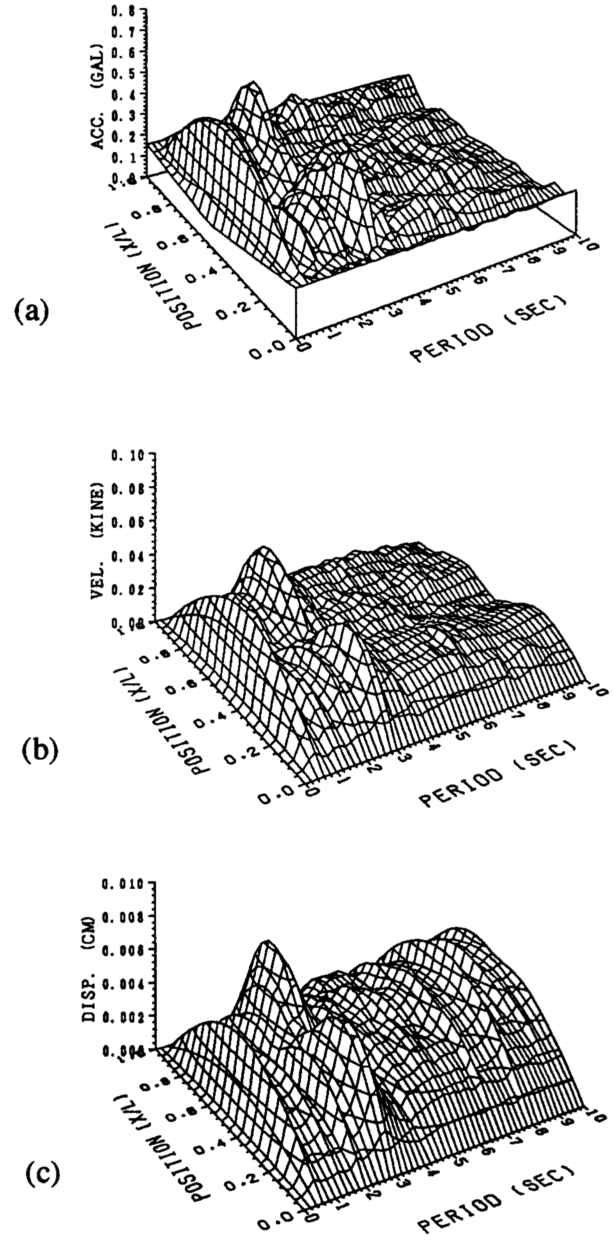


Fig. 11 Multi-input response spectra using EW components of Akashi (site A) and Awaji-shima (site C) records for the 1991 Nara-Ken Nanbu earthquake as input motion into each support into each support (a) Absolute acceleration response (b) Relative velocity response (c) Relative displacement response

correlation between the 2 records was not high compared with the records of the Myanmar earthquake in the previous section.

Figure 10 shows the multi-input response spectra for the identical input motions, and Figure 11 for the different input motions; Figure 10 was calculated using the EW components of the Akashi (site A) record of the 1991 Nara-Ken Nanbu earthquake as input motions into both supports, and Figure 11 was calculated using the EW components of Akashi (site A) and Awaji-shima (site C) records as input motions to each support. Figures 10(a) and 11(a) show the absolute acceleration responses, Figures 10(b) and 11(b) show the relative velocity responses and Figures 10(c) and 11(c) show the relative displacement responses. The damping factor of Figures 10 and 11 is set to $h=2\%$, and the y-axes are set to between 0 and 10 seconds to see the shorter periods more precisely than in the previous section.

Figure 10 shows the large values at 0.6 seconds for the 1st mode and 6 seconds for the 3rd mode. On the other hand, those values in Figure 11 are smaller, and the 2nd mode at 3.0 seconds becomes dominant. The identical input motions in Figure 10 take larger values than the different input motions in Figure 11, from a global point of view. Sometimes, however, as with the beam of 3.0 seconds, the different input motions make the response almost double compared with the identical input motions. The position where the maximum response occurs may also change in these cases. Furthermore, the absolute acceleration response for the different input motions in Figure 11(a) shows there is no clear dominant vibration mode for the beams whose natural period is longer than 5 seconds, like Figure 7(a) of the Myanmar earthquake.

Generally, the low correlation between the 2 records of the Nara-Ken Nanbu earthquake results in a significant effect on the response under multiple-support excitations.

5. Comparison with the Ordinary Response Spectra Method

In this section, the ordinary response spectra method using the root mean square values was examined to check efficiency of the proposed multi-input response spectra. The maximum response was estimated using the NS components of the records at Akashi (site A) and Kobe (site B) for the 1991 Myanmar earthquake as discussed in the previous section.

First, the root mean square values were calculated for each mode of the beam using the ordinary response spectra of each record shown in Figure 12. Then, the approximated maximum response was derived by the sum of the absolute

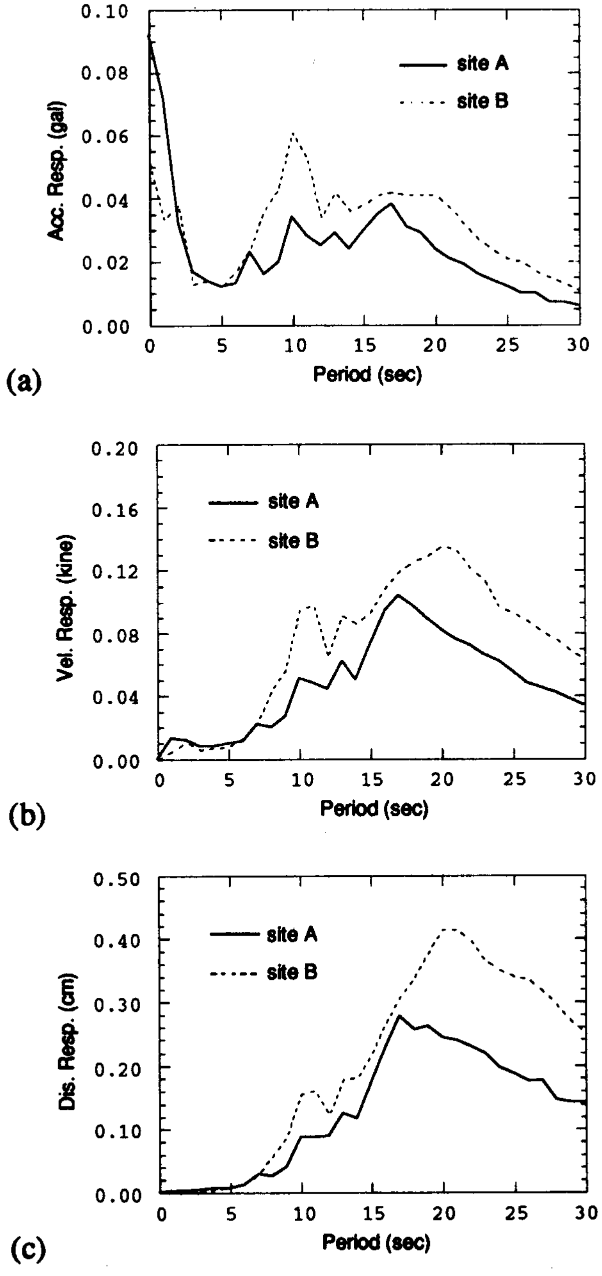


Fig. 12 Response spectra for NS components of Akashi (site A) and Kobe (site B) records for the 1991 Myanmar earthquake (a) Absolute acceleration response (b) Relative velocity response (c) Rleative displacement response

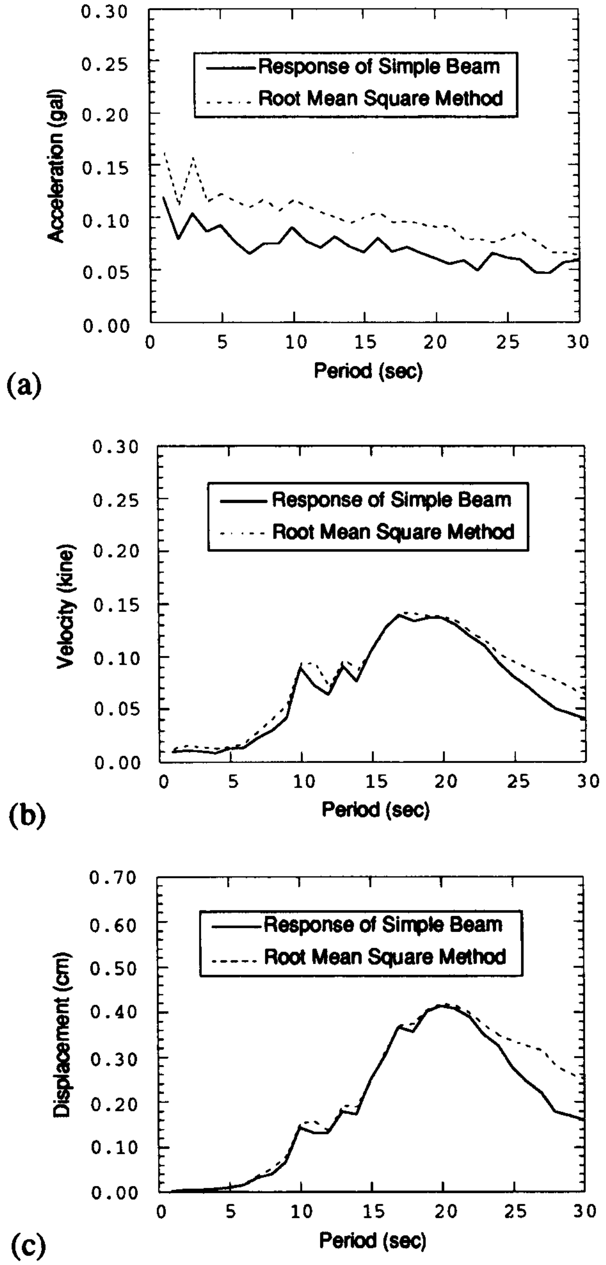


Fig. 13 Comparison of the maximum response between simulation and RMS method for NS components for the 1991 Myanmar earthquake (a) Absolute acceleration response (b) Relative velocity response (c) Relative displacement response

values for each input record. These approximated values were plotted in Figure 13 with the real maximum values calculated from the multi-input response spectra of Figure 7. The bold lines in Figure 13 represent the values from the multi-input response spectra, and the dotted lines are the approximated values from the ordinary response spectra method.

The maximum relative velocity response of Figure 13(b) and the maximum relative displacement response of Figure 13(c) were evaluated sufficiently by the ordinary method for the beams of shorter natural periods of less than 25 seconds; which means that the approximate method could be applicable for ordinary structures. For the beams with quite long natural period of longer than 25 seconds, the approximate method gives 1.5 times higher estimation. The maximum absolute acceleration responses of Figure 13(a) were evaluated as about 1.5 times larger by the ordinary method for the whole range of the natural period. It means a larger resisting force might be needed in the design of the beam with multi-input excitations by the approximate method.

Sakurai, et al.7) pointed out that the ordinary response spectra method had been overestimated when the two records showed a low correlation between them. This research had been done for structures with a natural period of shorter than 0.5 seconds. Though the records used in this analysis showed a high correlation, as mentioned in Figure 5, the long natural periods of the beams of over 20 seconds made the input motions relatively uncorrelated.

6. Conclusions

In this paper, new three-dimensional response spectra for multiple-support input motions were proposed and the efficiency of the spectra was shown by the numerical examples using the array observed earthquake records. The main conclusions obtained were as follows:

(1) A multi-input response spectrum was proposed and was drawn in a three-dimensional figure using the response of simple beams under multiple-support excitation with various natural periods. Numerical examples showed that the three-dimensional response spectra could clearly verify the dominant vibration mode and the critical location of the maximum response.

(2) Some multi-input response spectra were calculated for the earthquake records observed by a newly-installed array at the Akashi Kaikyo bridge construction site. For the different input motions to each support of the simple beam, the multi-input response spectra showed different dominant modes and different critical locations compared with the identical input motions.

(3) The maximum response of the beam was estimated by the ordinary response spectra method as well as by the proposed multi-input response spectra. For the absolute acceleration response, the estimated values from the ordinary root mean square method showed about 50% higher values than those of the multi-input response spectra for the whole range of the natural periods. For the relative velocity and relative displacement response, the estimation by the ordinary method was almost the same as that by the new method for the beams with natural periods of less than 25 seconds; and about 50% higher values were estimated for the beams with natural periods of more than 25 seconds.

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