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Fuzzy Approach to Conflict Analysis

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Abstract

Based on the concept of fuzzy sets and fuzzy relations, in this paper, a new approach is presented for modeling and analyzing conflicts. In analyzing conflicts, it is fundamental to evaluate feasible outcomes according to the preference of each player. A fuzzy preference matrix is first defined to evaluate preference relations between outcomes for each player. Several actions and reactions of players are investigated, and a new method of stability analysis is then proposed to derive the grades of membership of stability, instability and equilibrium. The new approach determines a different set of equilibria, depending on the fuzzy environment and the threshold.

1. Introduction

We have many complex situations, modeled as conflicts, where various objectives, interests and groups oppose each other. To model and solve conflict situations, various analysis methods have been proposed [1–5]. Conflict Analysis [1–3] is one method of analyzing conflicts, where a preference vector, an ordering of outcomes according to the preference of each player is fundamental.

In order to analyze the preferences, evaluations of outcomes according to various factors, not only quantitative but also intangible, are needed. The concept of fuzzy sets and fuzzy relations [6–9] will give an important mathematical base to analyze such problems. In this paper, a new approach for analyzing conflicts is presented, based on the concept of fuzzy relations and Conflict Analysis.

Following the description of basic factors in conflict analysis, we define a fuzzy preference matrix to investigate the fuzzy preference relations between outcomes for each player. This matrix plays a fundamental role in this paper. The actions and reactions of players are then classified, and the grades of membership for the actions and reactions are evaluated for stability analysis of outcomes. Finally, an equilibrium of conflict in a fuzzy environment is newly defined, and its grade of membership is derived. As shown by simulation analyses, the present

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method determines different sets of equilibria, depending on the fuzzy environment and the threshold. The present approach will thus provide flexible support and information for conflict analysis.

2. Modeling conflict in a fuzzy environment

Following the description of basic factors in Conflict Analysis [1,2], we define a fuzzy preference matrix in order to evaluate a unilateral change of strategy and to derive a fuzzy UI matrix for further stability analysis.

2.1 Basic factors in conflict analysis

A conflict is modeled in terms of players, options, strategies, outcomes and preference relations between outcomes. Each player has options, and a possible selection of options by a player is called a strategy. The situation where each player chooses a strategy is called an outcome. The selection of an option by a player is indicated by a 1, and a 0 indicates that the option is not taken. An outcome is thus indicated by a column of ones and zeros. The column is converted to decimal form. Infeasible outcomes are removed from further consideration. The set of all feasible outcomes, where the outcomes are ordered from most preferred on the left to least preferred on the right, is called the preference vector of a player. When a player can unilaterally change his situation from an outcome to a more preferred outcome by changing his strategy, the more preferred outcome is called a unilateral improvement (UI) from the original outcome for the player.

The stability analysis of the conflict is carried out by determining the stability of each outcome for each player. An outcome is stable for a player if it it not reasonable for him to change his strategy to attain any other outcome. Each outcome for a given player is classified into the following three types: An outcome is rational if the player has no UI from the outcome. An outcome is unstable if he has at least one UI which can improve his situation to a preferable outcome, no matter what sequential actions the other player takes. An outcome is stable if, due to the other players' sequential sanctioning, he has no UI which can improve his situation to a preferable outcome. When an outcome is unstable for two or more players, stability by simultaneity is to be assessed. After examining the stability of each outcome for each player, the stability of outcomes across players is analyzed. An outcome is an equilibrium if it is stable for all players.

2.2 Fuzzy preference matrix

Players have different preferences within the total set of feasible outcomes. In order to analyze these preferences, the outcomes are to be evaluated by various factors not only in a quantitative but also in an intangible way. The concept of fuzzy sets and fuzzy relations provides a mathematical base to analyze such problems. In this paper, we investigate the preferences in a frame of fuzzy relations instead of preference vectors.

Let the total set of feasible outcomes be $0=\{o_1, o_2, \cdots, o_n\}$. Then, a fuzzy preference relation between the outcomes for player A is represented by an nxn **fuzzy preference matrix** P_A as

$$P_A=[(p_A)_{ij}], \qquad i,j=1,2,\cdots,n, \tag{2.1}$$

where the element $(p_A)_{ij}$ is given by

$$(p_A)_{ij}=\mu_A(o_i, o_j), \qquad 0\leqq\mu_A\leqq 1, \qquad i,j=1,2,\cdots,n. \tag{2.2}$$

The element in (2.2) represents the grade of which outcome o_j is preferred by player A to outcome o_i.

It is assumed throughout the present paper that the fuzzy preference relation in the product space 0×0 satisfies the following laws.

1. Irreflexive Law

$$\mu_A(o_i, o_i)=0 \qquad \text{for } {}^{\forall}o_i \in 0,$$

2. Asymmetric Law

$$\mu_A(o_i, o_j)>0 \text{ then } \mu_A(o_j, o_i)=0 \text{ for } {}^{\forall}o_i, o_j \in 0 \ (i\neq j),$$

3. Transitive Law

$$\mu_A(o_i, o_k)\geqq \bigvee_j \{\mu_A(o_i, o_j)\wedge\mu_A(o_j, o_k)\}$$

$$\text{for } {}^{\forall}o_i, o_j, o_k \in 0, \ (i\neq j\neq k).$$

2.3 Fuzzy UI matrix

A player can change an outcome to attain a new outcome by changing his strategy, assuming that the other players do not change their strategies. The new outcome is called a **unilateral change (UC)** from the original outcome for the player. We construct an nxn binary UC matrix which indicates the existence of UCs from each outcome for player A as

$$UC_A=[(uc_A)_{ij}], \qquad i,j=1,2,\cdots,n, \tag{2.3}$$

where the element $(uc_A)_{ij}$ is given by

$$(uc_A)_{ij}=\begin{cases}1 & \text{if A can unilaterally change } o_i \text{ to } o_j,\\ 0 & \text{if A cannot unilaterally change } o_i \text{ to } o_j.\end{cases} \tag{2.4}$$

When a player can unilaterally improve an outcome to a new outcome by changing his strategy, the new outcome is called a **unilateral improvement (UI)** from the original outcome for the player. The UI is the UC which is more preferred to the original outcome for the player. We define an nxn fuzzy UI matrix for player A as

$$UI_A=[(ui_A)_{ij}], \qquad i,j=1,2,\cdots,n, \tag{2.5}$$

where the element $(ui_A)_{ij}$ represents the grade of which the outcome o_j is preferred by player A to the outcome o_i. Then, the element $(ui_A)_{ij}$ is represented by

$$(ui_A)_{ij}=(uc_A)_{ij}\wedge(p_A)_{ij}, \tag{2.6}$$

$$=\min[(uc_A)_{ij}, (p_A)_{ij}]. \tag{2.7}$$

The two fuzzy matrices P_A and UI_A in this chapter can be derived on the basis of preference information. These matrices for player A and also for other players form the basis for a fuzzy approach in the present paper.

3. Fuzzy stability analysis

By defining several grades of membership for the actions of players, we develop a new method for conflict analysis in a fuzzy environment.

Each outcome will be analyzed for stability for each player and finally across the players. An outcome is stable for a player if it is not reasonable for him to change the outcome by changing his strategy. When an outcome is stable for all players in the conflict, the outcome is a solution to the conflict.

In the following, we discuss the stability of outcome q for player A in a conflict between two players (A and B), where the players are assumed to act out the following behaviors:

1. Each player is myopic: Each player can change his strategy to his UIs only once.
2. The other player will react to the original player by changing his UI to the least preferred outcome for the original player.
3. The original player will choose the most preferred UI among his UIs, considering the other player's reactions.

3.1 Grade of rationality

When an outcome q does not have any UIs for a player, the player can not unilaterally improve the outcome. The outcome in this case is thus stable and is referred to as being **rational** for the player. The grade of rationality of outcome q for player A is represented by

$$(r_A)_q = \min_{p_i \varepsilon PP_A} \overline{(UI_A)_{qp_i}} \tag{3.1}$$

$$= 1 - \max_{p_i \varepsilon PP_A} (UI_A)_{qp_i}, \tag{3.2}$$

where PP_A is the set of UIs of player A from outcome q and is given by

$$PP_A = \{p_i \mid (UI_A)_{qp_i} > 0\}. \tag{3.3}$$

When player A has a UI from outcome q to outcome p_i, player A attains outcome p_i which is preferable to the original outcome q, if player B does not have any UIs from outcome p_i and cannot react to A's UI. It is then reasonable for player A to take his UI. The outcome q in this case is **unstable** for player A. The grade of instability of outcome q is represented by

$$(u_A^1)_{qp_i} = (UI_A)_{qp_i} \wedge (r_B)_{p_i}. \tag{3.4}$$

3.2 Grade of reaction

Consider a case in which player B can react to player A's UI from outcome q to outcome p_i by taking B's UI from outcome p_i to outcome y_{ij}. If the resulting outcome y_{ij} is preferred to the original outcome q by player A, it is reasonable for player A to change the original outcome q to outcome p_i. This case is called a **nonsanction.** On the other hand, if the resulting outcome y_{ij} is less preferred by player A to the original outcome q, it is not reasonable for player A to change the original outcome q to outcome p_i. This case is called a **sanction** by player B against player A's UI. In order to evaluate the grade of these cases with respect to the initial change of player A, we divide the set of UIs of player B into two subsets as follows:

$$Y_i=\{y_{ij}\,|\,(UI_B)_{p_i y_{ij}}>0\}=YP_i+YM_i, \tag{3.5}$$

$$YP_i=\{y_{ij}\,|\,(P_A)_{q y_{ij}}>0\}, \tag{3.6}$$

$$YM_i=\{y_{ij}\,|\,(P_A)_{q y_{ij}}=0\}, \tag{3.7}$$

where Y_i is the set of player B's UIs from player A's UI. YP_i is the set of player B's UIs, preferred by player A to the original outcome q, and YM_i is the set of player B's UIs, less preferred by player A to the original outcome q.

Grade of nonsanction

A nonsanction case occurs when player B's sequential reaction from outcome p_i to outcome y_{ij} against player A's UI from outcome q to outcome p_i results in a more preferred outcome for player A. In this case, it is reasonable for player A to take his UI, and outcome q is thus unstable for player A. When player B has several UIs, which make no sanctions against player A's UI, player B will take one, which is least preferred by player A. Thus, the grade of nonsanction for player A's UI is represented by

$$(u_A^2)_{qp_i}=\bigwedge_{y_{ij}\in YP_i}[(UI_A)_{qp_i}\wedge(UI_B)_{p_i y_{ij}}\wedge(P_A)_{q y_{ij}}]. \tag{3.8}$$

Grade of sanction

A sanction case occurs when player B's sequential reaction from outcome p_i to outcome y_{ij} against player A's UI from outcome q to outcome p_i results in a less preferred outcome for player A. In this case, it is not reasonable for player A to take his UI, and outcome q is then stable for player A. When player B has several UIs, which make sanctions against player A's UI, player B will take one which is least preferred by player A. Thus, the grade of sanction for player A's UI is represented by

$$(s_A^2)_{qp_i} = \bigvee_{y_{ij} \varepsilon YM_i} [(UI_A)_{qp_i} \wedge (UI_B)_{p_i y_{ij}} \wedge (P_A)_{y_{ij}q}], \tag{3.9}$$

where $(P_A)_{y_{ij}q}$ represents the grade of which outcome y_{ij} is not preferred by player A to the original outcome q.

Based on these reactions by player B and the assumptions about the behaviors of players, we analyze the grades of **stability and instability** of outcome q for player A. When player A has plural UIs from the original outcome q, he will select one, which minimizes the grade of stability of outcome q. And if there exist several UIs which give the same minimum grade, he will take one which maximizes the grade of instability of outcome q. In relation to player B's reactions, the grade of stability and instability of outcome q for player A is then represented by

$$(s_A)_q = \bigwedge_{p_i \varepsilon PP_A} (s_A^2)_{qp_i}, \tag{3.10}$$

$$(u_A)_q = \bigvee_{p_i \varepsilon PP_{A'}} [(u_A^1)_{qp_i} \vee (u_A^2)_{qp_i}], \tag{3.11}$$

where $PP_{A'}$ is a partial set of PP_A, whose elements give $(s_A)_q$ in (3.10).

3.3 Grade of stability by simultaneity

When both players simultaneously change an outcome to respective UIs, a new outcome, different from respective UIs, is obtained. If the new outcome is less preferred by a player than the original outcome, then it is a sanction against his change, and the original outcome is stable for the player. This case will be referred to as a **stability by simultaneity**. Here, we evaluate the grade of this case. Let a_i be a UI of player A from an outcome q, and let b_j be a UI of player B from the same outcome q. $(a_i - q)$ is the value of the net change made by player A on outcome q, and $(b_j - q)$ is the corresponding value made by player B. The resulting outcome c [1, 2] is then represented by

$$c = q + (a_i - q) + (b_j - q) \tag{3.12}$$

$$=(a_i+b_j)-q. \tag{3.13}$$

When the resulting outcome c is less preferred by player A to the original outcome q, outcome q is stable by simultaneity for player A. When players A and B have plural UIs from outcome q, the grade of stability by simultaneity of outcome q for player A is given by

$$(ss_A)_q=\bigwedge_{a_i}\Big[\bigvee_{b_j}[(u_A)_q\wedge(u_B)_q\wedge(UI_A)_{qa_i}\wedge(UI_B)_{qb_j}\wedge(P_A)_{cq}]\Big]. \tag{3.14}$$

3.4 Grade of equilibrium

The foregoing investigations on actions of players now enable us to analyze the **total stability and instability** of each outcome for each player. An outcome is stable for a player if it is rational, sanctioned, or stable by simultaneity. An outcome is unstable if at least one UI from it is not sanctioned.

The total grade of stability and instability of outcome q for player A is thus represented as follows:

$$(S_A)_q=(r_A)_q\vee(s_A)_q\vee(ss_A)_q, \tag{3.15}$$

$$(U_A)_q=(u_A)_q. \tag{3.16}$$

After the stability analysis for each outcome for each player is made separately, an overall stability analysis across the players is needed to derive solutions to the conflict. When an outcome is stable for all players, the outcome is a solution to the conflict. Then, when the total grade of stability of outcome q is greater than its total grade of instability for all players, as

$$(S_A)_q>(U_A)_q, \tag{3.17}$$

$$(S_B)_q>(U_B)_q, \tag{3.18}$$

the outcome q is referred to as an **equilibrium** in the present fuzzy approach. The grade of equilibrium of outcome q, which satisfies (3.17) and (3.18), is given by

$$(E_A)_q=(S_A)_q\wedge(S_B)_q. \tag{3.19}$$

The grade of stability and instability of outcome q and also its grade of equilibrium, defined in this chapter, are fundamental for the present conflict analysis. A threshold drives away the fuzziness, and a definite set of equilibria can be determined, depending on its value.

4. Simulation analyses

We evaluate the characteristics of the present fuzzy approach by simulation analyses for a two player (A and B) conflict. It is assumed that each player has two options respectively and the set of feasible outcomes in decimal form is given by

$$0=\{0,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15\}.$$

UC matrices (UC_A and UC_B) for players A and B are given as Table 1.

We assume two cases (Case 1 and Case 2) for fuzzy preference matrices (P_A and P_B), as shown in Table 2 and Table 3 respectively. Both cases have the same ordering between outcomes in the preference of players. That is, the preference vectors of the players are given as

Player A: 8 12 10 14 9 13 11 0 15

Player B: 0 14 15 11 13 10 12 9 8

for both cases. However, the grades in fuzzy preference matrix in Case 1 are

Table 1 UC Matrices

Player A

	0	8	9	10	11	12	13	14	15
0	0	1	0	0	0	1	0	0	0
8	1	0	0	0	0	1	0	0	0
9	0	1	0	0	0	1	0	0	0
10	0	0	0	0	0	0	1	0	0
11	0	0	0	0	0	0	0	1	0
12	0	0	0	0	0	0	0	0	1
13	1	1	0	0	0	0	0	0	0
14	0	0	1	0	0	0	0	0	0
15	0	0	0	1	0	0	0	0	0

Player B

	0	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0
8	0	0	1	1	1	0	0	0	0
9	0	1	0	1	1	0	0	0	0
10	0	1	1	0	1	0	0	0	0
11	0	1	1	1	0	0	0	0	0
12	0	0	0	0	0	0	1	1	1
13	0	0	0	0	0	1	0	1	1
14	0	0	0	0	0	1	1	0	1
15	0	0	0	0	0	1	1	1	0

Table 2 Fuzzy Preference Matrices for Case 1

Player A

	0	8	9	10	11	12	13	14	15
0	0.0	0.4	0.1	0.2	0.1	0.3	0.1	0.1	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.2	0.0	0.1	0.0	0.2	0.0	0.1	0.0
10	0.0	0.2	0.0	0.0	0.0	0.1	0.0	0.0	0.0
11	0.0	0.4	0.1	0.2	0.0	0.3	0.1	0.1	0.0
12	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.3	0.1	0.2	0.0	0.2	0.0	0.1	0.0
14	0.0	0.2	0.0	0.1	0.0	0.1	0.0	0.0	0.0
15	0.1	0.5	0.1	0.3	0.1	0.3	0.1	0.2	0.0

Player B

	0	8	9	10	11	12	13	14	15
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.5	0.0	0.1	0.1	0.2	0.1	0.1	0.3	0.3
9	0.3	0.0	0.0	0.1	0.1	0.1	0.1	0.3	0.2
10	0.3	0.0	0.0	0.0	0.1	0.0	0.1	0.2	0.2
11	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.1
12	0.4	0.0	0.0	0.1	0.1	0.0	0.1	0.3	0.2
13	0.2	0.0	0.0	0.0	0.1	0.0	0.0	0.2	0.1
14	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0

rather small, compared with the grades in Case 2. This means that each player has rather nearly equal preference for all outcomes in Case 1.

Table 4 and Table 5 represent simulation results for Case 1 and Case 2 respectively. The grade of rationality in Case 1 is greater than its grade in Case 2 for almost all outcomes for both players. This characteristic is also shown in the grade of equilibrium for all outcomes. These are due to the fact that each player has a rather nearly equal preference for all outcomes in Case 1 unlike in Case 2. A definite set of equilibria is determined by introducing a threshold. In Case 1, all outcomes form equilibria for a case of threshold 0.6, and for a case of threshold 1.0, the only outcome, 11, forms an equilibrium. Thus, depending on the fuzzy environment and threshold, a different set of equilibria is obtained. Simulation analyses indicate that the introduction of fuzzy preference relations between outcomes produces a different set of equilibria, even if the ordering of outcomes in the preference of players is the same. The present fuzzy approach is thus flexible, and provides various information for analyses of real

Table 3 Fuzzy Preference Matrices for Case 2

Player A

	0	8	9	10	11	12	13	14	15
0	0.0	1.0	0.7	0.9	0.6	0.9	0.7	0.8	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.9	0.0	0.7	0.0	0.8	0.0	0.6	0.0
10	0.0	0.8	0.0	0.0	0.0	0.5	0.0	0.0	0.0
11	0.0	1.0	0.6	0.9	0.0	0.9	0.6	0.7	0.0
12	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	1.0	0.5	0.8	0.0	0.8	0.0	0.7	0.0
14	0.0	0.9	0.0	0.6	0.0	0.7	0.0	0.0	0.0
15	0.5	1.0	0.8	0.7	0.6	1.0	0.8	0.9	0.0

Player B

	0	8	9	10	11	12	13	14	15
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	1.0	0.0	0.4	0.5	0.8	0.5	0.7	0.9	0.8
9	1.0	0.0	0.0	0.5	0.7	0.5	0.7	0.8	0.7
10	0.9	0.0	0.0	0.0	0.6	0.0	0.5	0.7	0.6
11	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.4
12	0.9	0.0	0.0	0.4	0.6	0.0	0.5	0.8	0.6
13	0.8	0.0	0.0	0.0	0.5	0.0	0.0	0.7	0.6
14	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.0

Table 4 Simulation Results for Case 1

Outcomes	Player A			Player B			Grade of Equilibrium
	r_A	S_A	U_A	r_B	S_B	U_B	
0	0.6	0.6	0.4	1.0	1.0	0.0	0.6
8	1.0	1.0	0.0	0.8	0.8	0.2	0.8
9	1.0	1.0	0.0	0.9	0.9	0.1	0.9
10	1.0	1.0	0.0	0.9	0.9	0.1	0.9
11	1.0	1.0	0.0	1.0	1.0	0.0	1.0
12	0.8	0.8	0.2	0.7	0.7	0.3	0.7
13	0.9	0.9	0.1	0.8	0.8	0.2	0.8
14	0.9	0.9	0.1	1.0	1.0	0.0	0.9
15	0.9	0.9	0.1	0.8	0.8	0.2	0.8

Table 5 Simulation Results for Case 2

Outcomes	Player A			Player B			Grade of Equilibrium
	r_A	S_A	U_A	r_B	S_B	U_B	
0	0.0	0.0	0.4	1.0	1.0	0.0	0.0
8	1.0	1.0	0.0	0.2	0.2	0.8	0.0
9	1.0	1.0	0.0	0.3	0.3	0.7	0.0
10	1.0	1.0	0.0	0.4	0.4	0.6	0.0
11	1.0	1.0	0.0	1.0	1.0	0.0	1.0
12	0.3	0.7	0.2	0.2	0.2	0.6	0.0
13	0.5	0.5	0.5	0.3	0.3	0.5	0.0
14	0.4	0.6	0.4	1.0	1.0	0.0	0.6
15	0.4	0.4	0.6	0.5	0.5	0.4	0.0

conflict situations.

5. Conclusion

In this paper, we proposed a new method for analyzing conflicts. The method is based on fuzzy preference relations. A stability analysis is performed on the grades of stability, instability and equilibrium, as newly defined in this paper. The definite structure of the conflict can be obtained by introducing a threshold, which would represent a grade of our understanding of the situation. If the result of this analysis is different from the understanding, we can repeat this analysis by changing the fuzzy preference matrices and/or the threshold. This repetition complements the result and provides really effective support for conflict analysis.

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