A sufficient statistics approach for welfare analysis of oligopolistic third-degree price discrimination*

Takanori Adachi†

Abstract

This paper proposes a sufficient statistics approach to studying the welfare effects of third-degree price discrimination in differentiated oligopoly. Specifically, our sufficient conditions for price discrimination to increase or decrease social welfare simply entail a cross-market comparison of multiplications of such sufficient statistics as *pass-through*, *conduct*, and *profit margin* that are functions of first-order and second-order elasticities of the firm's demand. Notably, these results are derived under a general class of market demand, and can be readily be extended to accommodate heterogeneous firms. These features suggest that our approach has potential for conducting welfare analysis without a full specification of an oligopoly model.

Keywords: Third-Degree Price Discrimination; Oligopoly; Sufficient Statistics.

JEL classification: D43; L11; L13.

*I am grateful to Giacomo Calzolari and Armin Schmutzler (Editors-in-charge) as well as an anonymous referee for insightful comments and suggestions. I also thank Iñaki Aguirre, Jacob Burgdorf, Susumu Cato, Yong Chao, Jacob Gramlich, Makoto Hanazono, Hiroaki Ino, Noritaka Kudoh, Toshifumi Kuroda, Jeanine Miklós-Thal, Ignacio Palacios-Huerta, Dan Sasaki, Susumu Sato, Nicolas Schutz, Shohei Tamura, Tsuyoshi Toshimitsu, Yusuke Zennyo, and conference and seminar participants at numerous places for helpful comments and discussions. An earlier version of the paper was circulated under the title of "Output and Welfare Implications of Oligopolistic Third-Degree Price Discrimination" as a coauthored paper with Michal Fabinger, who contributed to the initial stage of the paper but has unfortunately retreated afterwards. I acknowledge a Grant-in-Aid for Scientific Research (C) (15K03425, 18K01567, 21K01440) from the Japan Society for the Promotion of Science. All remaining errors are my own.

†Graduate School of Management and Graduate School of Economics, Kyoto University, Japan. E-mail: adachi.takanori.8m@kyoto-u.ac.jp

1 Introduction

This paper explores the welfare effects of third-degree price discrimination in oligopoly. Specifically, we consider a fairly general setting, and present sufficient conditions under which oligopolistic third-degree price discrimination increases or decreases Marshallian social welfare (i.e., the sum of the consumer and producer surpluses) when all discriminatory markets are served even in the absence of price discrimination. To do this task, we employ the *sufficient statistics approach* as a unifying methodology: a technique often used in public economics (Chetty 2009; Kleven 2021; Adachi and Fabinger 2022) as well as macroeconomics (Barnichon and Mesters 2022). Our analysis is mainly developed under firm symmetry; however, it can readily be extended to accommodate heterogeneous firms (Online Appendix C). Moreover, our analysis permits a moderate degree of cost differences to exist across separate markets.

Under third-degree price discrimination, consumers are segmented into separate markets and charged different unit prices in accordance with their identifiable characteristics (e.g., age, occupation, location, or time of purchase). In contrast, all consumers are charged the same price if third-degree price discrimination is not practiced (i.e., "uniform pricing"). Without loss of generality, the case of two markets can be considered to understand how price discrimination might change output and welfare in each market. If all firms are symmetric, the prevailing equilibrium price is common in either market whether price discrimination or uniform pricing is implemented. In this situation, if a discriminatory price becomes greater than the uniform price in one market, and the unit price decreases in the other market, the former market is traditionally called a "strong" market (s), and the latter a "weak" market (w) in the literature since Robinson (1933).[1] More formally, this situation is expressed by $p_s^* > \overline{p} > p_w^*$, where p_s^* and p_w^* are the equilibrium prices under price discrimination in the strong and the weak markets, respectively, and $\overline{p}$ is the uniform price.[2] Given such a price change, price discrimination increases output and social welfare in the weak market, but decreases them in the strong market. What are the overall effects of the price change?

In the analysis below, we follow Leontieff (1940), Silberberg (1970), Schmalensee (1981),

[1] To be precise, Robinson (1933, p. 189) originally states "stronger" and "weaker" markets.

[2] In this paper, price discrimination is present when $p_s > p_w$, i.e., when prices between markets are not uniform. As Clerides (2004, p. 402) states, once cost differentials are allowed, "there is no single, widely accepted definition of price discrimination." To understand this, consider symmetric firms and let mc_s and mc_w be the marginal cost at equilibrium output in markets s and w, respectively (they do not necessarily have to be constants for any output levels). Then, two alternative definitions can be considered. One is the *margin* definition: price discrimination occurs when $p_s - mc_s > p_w - mc_w$. The other one is the *markup* definition as per Stigler (1987): price discrimination occurs when $p_s/mc_s > p_w/mc_w$. Our simpler definition is aligned with the former definition, and employed for its tractability and connectivity to the existing literature on third-degree price discrimination with no cost differentials. Moreover, our definition of price discrimination coincides with what Chen and Schwartz (2015) and Chen, Li and Schwartz (2021) call "differential pricing." As long as cost differentials are sufficiently small, these differences will not significantly alter the results because if $mc_s = mc_w$, these three definitions are equivalent.

Holmes (1989), and Aguirre, Cowan, and Vickers (2010) to add the constraint $p_s - p_w = t$, where $t \geq 0$ is interpreted as an artificial constraint on the profit maximization problem for oligopolistic firms under symmetry. Then, the regime change, which is discrete in its nature, is now measured by t and is continuously connected between $t = 0$ as uniform pricing and $t^* \equiv p_s^* - p_w^*$ as price discrimination in equilibrium. This formulation enables us to describe social welfare as a function of t, $W(t)$, and characterize $W'(t)$ in terms of economic concepts based on elasticity terms of market demand. In this way, whether social welfare improves or deteriorates by this global change of the regime can be determined. This methodology shares the central idea of the sufficient statistics approach where welfare consequences of policy changes are derived "in terms of estimable elasticities" (Kleven 2021, p. 516). One benefit of focusing on sufficient statistics "rather than deep primitives" (Chetty 2009, p. 452) in conducting welfare analysis is that one can focus on the deeper *structure* that is "robust across a broad class of underlying models" (Kleven 2021, p. 535) without a particular specification of market demand. If we instead start with a specific class of demand, it remains unclear to what extent the welfare analysis is valid under another class of market demand.[3]

Our sufficient conditions for oligopolistic price discrimination to increase or decrease social welfare are provided by means of a cross-market comparison of the multiplications of two or three of the following economic concepts: (i) *profit margin*, which is the difference between price and marginal cost ($\mu \geq 0$); (ii) *pass-through*, i.e., how the price responds to a small change in marginal cost ($\rho > 0$); and (iii) *conduct*, which measures the degree of market monopolization ($\theta \in [0, 1]$). These three sufficient statistics are determined by the following two first-order and two second-order elasticities: (a) the own price elasticity of the firm's demand (ϵ^{own}), (b) the cross price elasticity of the firm's demand (ϵ^{cross}), (c) the curvature of the firm's demand (α^{own}), and (d) the elasticity of the cross-price effect of the firm's demand (α^{cross}).

Specifically, this paper demonstrates that the product of all three concepts, $\theta\mu\rho$, provides the sufficient condition for the change in *welfare*. As explained in Subsection 3.2, the product of conduct and pass-through evaluated at the discriminatory prices $\theta_m^*\rho_m^*$, $m = s, w$, in Figure 1 (A) is interpreted as *quantity pass-through*, measuring how output in each individual market changes in response to a marginal change in price. To evaluate a marginal change in welfare, profit margin μ_m^* should be considered because it measures the welfare gain or loss that results from a marginal change in quantity under imperfect competition in which the price exceeds

[3] One may criticize that sufficient statistics are only endogenous variables by holding that a sufficient condition is meaningful only when it consists of exogenous parameters. However, in equilibrium, our sufficient conditions are functions of exogenous parameters for the same reason that in equilibrium, endogenous variables are functions of exogenous variables, as demonstrated in Section 4. However, deep parameters themselves do not always allow economic interpretations in a direct manner; for example, in the case of linear demand, the slope coefficient is not directly to related to demand elasticity. In contrast, sufficient statistics such as elasticities almost always have economic interpretations. This *is* the benefit from the sufficient statistics approach because welfare analysis can be conducted based on economic concepts one-level higher that underlie a plausible class of model specification.

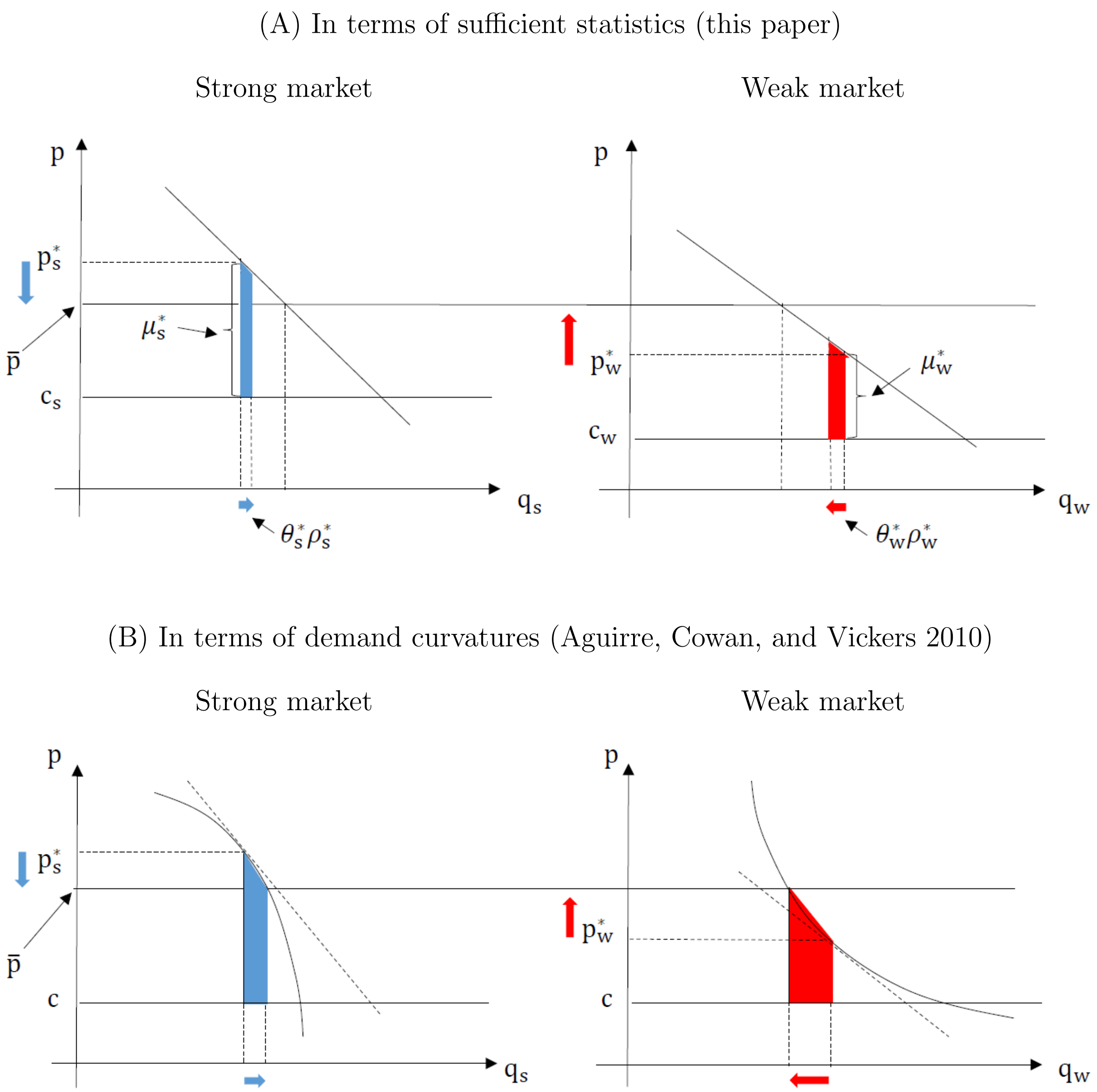

Figure 1: A graphical illustration of welfare changes in strong and weak markets

marginal cost. In this way, the welfare implications can be obtained by means of a cross-market comparison of the quantity change multiplied by the profit margin.

Existing literature on third-degree price discrimination has a centennial tradition, pioneered by Pigou (1920) and Robinson (1933), with their main focus on whether price discrimination increases or decreases social welfare (see Varian (1989); Armstrong (2006, 2008); and Stole (2007) for comprehensive surveys of this literature). Among others, Schmalensee (1981) and Aguirre, Cowan, and Vickers (hereafter, ACV) (2010) study how *demand curvatures* relate to output and welfare effects. Third-degree price discrimination necessarily entails allocative inefficiency because some consumers exist who have the same marginal utility but face different prices simply because they belong to different markets. Thus, for third-degree price discrimination to increase social welfare, it must sufficiently expand aggregate output to offset such misallocation across markets. Schmalensee (1981) shows that an increase in aggregate output is a necessary condition for third-degree price discrimination to increase social welfare—a conclusion that is generalized by Varian (1985) and Schwartz (1990)—and ACV (2010) identify a sufficient condition for price discrimination to raise social welfare: inverse demand in the weak market is more convex than that in the strong market at the discriminatory prices. Figure 1 (B) provides a graphical illustration of ACV's (2010) argument: if uniform pricing is implemented instead, welfare loss in the weak market due to the output reduction that has arisen under price discrimination is sufficiently large (the right panel) as compared to the welfare gain in the strong market (the left panel), provided that the inverse demand in the weak market is sufficiently convex as compared to that in the strong market.

However, these studies are limited to *monopolistic* third-degree discrimination: to date, "there are *virtually no predictions* as to how discrimination impacts welfare" (Hendel and Nevo 2013, p. 2723; emphasis added) when *oligopolistic* competition is considered. For example, Holmes (1989) employs the same technique used by Schmalensee (1981) and ACV (2010) to examine the output effects of third-degree price discrimination in a symmetric oligopoly (see Section 3 for details). However, Holmes (1989) provides no welfare predictions (see also Dastidar 2006).[4] In this paper, we contribute to the literature by providing fairly general conditions regarding whether oligopolistic price discrimination increases or decreases social welfare.[5]

[4] In a similar vein, Armstrong and Vickers (2001) consider a model of symmetric duopoly with product differentiation à la Hotelling (1929), and study the consequences of third-degree price discrimination in the competitive limit around zero transportation costs wherein the equilibrium prices are almost equal to marginal cost. Under this setting, Armstrong and Vickers (2001) show that price discrimination decreases social welfare if the weak market has a lower value of price elasticity of demand (Adachi and Matsushima (2014) also derive a similar result by assuming linear demand in a standard model of symmetrically differentiated duopoly). Our paper aims to fill the gap between monopoly, such as in Schmalensee (1981) and ACV (2010), and Armstrong and Vickers' (2001) competitive limit with respect to welfare implications. I thank Susumu Sato for suggesting this interpretation.

[5] Rhodes and Zhou (2022) incorporate oligopolistic competition into a model of personalized pricing or first-degree price discrimination as the limit case of third-degree price discrimination.

Notably, our analysis does not necessitate the assumption of no cost differentials between discriminatory markets. In almost all theoretical studies on price discrimination, this assumption is made mainly to focus on demand differences. However, in many real-world cases of price discrimination, cost differentials are quite often observed, such as in the typical example of freight charges across regional markets with different transportation and storage costs (Phlips 1983, pp. 5-7). In the narrowest definition of price discrimination, this might not be considered price discrimination because they can be regarded as distinct products. However, airlines can be arguably motivated to offer different types of seats because they seek to exploit heterogeneity among consumers. In light of this observation, this study permits a moderate amount of cost differentials to exist across discriminatory markets. Specifically, our analysis below needs not employ an explicit assumption regarding constant marginal costs in strong and weak market, c_s and c_w, as long as the second-order conditions for profit maximization are satisfied and a sufficiently large discrepancy between c_s and c_w does not change the order of discriminatory prices from the one with no cost differentials.[6] Figure 1 also reflects this generalization: in (A), c_s and c_w are different, whereas in (B), marginal cost, c, is common for both strong and weak markets.

In a closely related study, Chen, Li, and Schwartz (2021) extend Chen and Schwartz' (2015) analysis of monopoly to investigate the welfare effects of cost-based price discrimination ("differential pricing") in oligopoly.[7] In their setting, demand in market $m = 1, 2, ..., M$ with N symmetric firms is given by (using Chen, Li, and Schwartz' (2021) notation) $\lambda_m \cdot \widetilde{D}(\mathbf{p}_m)$, where $\mathbf{p}_m = (p_{m1}, p_{m2}, ..., p_{mN})$ is the price vector in market m and $\lambda_m \in (0, 1)$ is the weight for market m satisfying $\sum_{m=1}^{M} \lambda_m = 1$. As such, market heterogeneity arises only from the supply side—firms' marginal costs are different across markets—because own and cross elasticites are *identical across markets* that result from the common demand component, $\widetilde{D}(\cdot)$. Under this setting, Chen, Li, and Schwartz (2021) are able to identify demand conditions to determine aggregate social welfare—expressed in terms of the weights $(\lambda_1, \lambda_2, ..., \lambda_M)$—is concave or convex as a function of price: because the uniform price lies in between the discriminatory prices, social welfare is higher (corr. lower) under differential pricing if the welfare function is convex (corr. concave).

However, aggregate social welfare is no longer expressed in this simple manner once *demand heterogeneity across markets* is allowed. A typical situation when cost-based price discrimination can be at issue comes from the universal service requirement and fairness concerns (Okada 2014; Geruso 2017; DellaVigna and Gentzkow 2019). In these cases, markets segmented by,

[6] In the context of reduced-fare parking as a form of third-degree price discrimination with cost differentials, Flores and Kalashnikov (2017) characterize a sufficient condition for free parking (drivers receive a price discount in the form of complementary parking while pedestrians do not) to be welfare improving.

[7] See also Galera and Zaratiegui (2006) and Bertoletti (2009) as studies of conditions under which price discrimination increases social welfare when cost differentials between markets are allowed.

e.g., geographical areas would differ in terms of price elasticities of market demand, and if so, Chen, Li, and Schwartz' (2021) methodology is no longer valid. In contrast, our analysis provides welfare implications more directly. As pointed out by Chen and Schwartz (2015, p. 103), our methodology "neither implies nor is implied" by the conditions in the analysis of Chen and Schwartz (2015) for monopoly and Chen, Li, and Schwartz (2021) for oligopoly. In this sense, Chen, Li, and Schwartz' (2021) analysis and mine are not mutually exclusive but are complementary.

Our study is also in line with Mrázová and Neary (2017) who show the usefulness of *demand manifold*—the relationship between demand elasticity and convexity which is not ascribed to a function or a correspondence—in comparative statics by suggesting the linkage between these first- and second-order elasticities and sufficient statistics such as markup and pass-through as shown in an empirical study by De Loecker, Goldberg, Khandelwal, and Pavcnik (2016).[8] Mrázová and Neary (2017) point out that one of the advantages of working with the demand manifold instead of the demand function per se is that it is clearer to understand results from comparative statics and counterfactual experiments because demand elasticity and curvature are more closely related to them than demand primitives themselves.[9] However, Mrázová and Neary (2017) mainly focus on perfect and monopolistic competition: when firm heterogeneity is taken into account, only cost/productivity heterogeneity à la Melitz (2003) is considered. In other words, *neither* ϵ^{cross} *nor* α^{cross} *appears* in Mrázová and Neary's (2017) analysis because product differentiation in a strategic context is not taken into account. Therefore, Mrázová and Neary (2017) are only able to focus on two parameters, ϵ^{own} and α^{own}. While we do not make use of their method directly, we explicitly consider imperfect competition based on product differentiation: further research would be promising to investigate how Mrázová and Neary's (2017) methodology can be more utilized for welfare analysis of imperfectly competitive behavior with the use of sufficient statistics.

Our methodology has the following policy implications. Admittedly, our welfare predictions are not "perfect" in that they are stated only as *sufficient* conditions that justify the current regime: for example, the first part of Proposition 1 below provides one sufficient condition for when price discrimination is justified from a standpoint of social welfare. Hence, one may still miss some other parametric cases of market demand that can also support price discrimination simply because our sufficient condition does not hold. However, our results enable one to conclude that once our sufficient condition holds, a regime change that bans price discrimination *definitely* decreases social welfare. In this sense, our sufficient conditions are "conservative" but

[8]In our context, (i) *profit margin* is determined by the firm-level price elasticity (or the own price elasticity), (ii) *pass-through* is determined mainly by the demand curvature, and (iii) *conduct* is determined by the ratio of the industry-level elasticity to the firm-level elasticity. See the expressions (17) below for the case of price discrimination when market-wise elasticities are defined.

[9]Beggs (2021) derives a necessary and sufficient condition for two demand functions to have the same demand manifold: one is derived from the other by a change in market size and a change in quality.

"secure" in line with the "*in dubio pro reo*" principle behind juridical decisions: it is important to prevent the "innocent" from being mistakenly judged as "guilty".

The remainder of this paper is organized as follows. Section 2 presents our base model of oligopolistic pricing with symmetric firms and constant marginal costs. Then, we derive the sufficient statistics implications of welfare effects of price discrimination in Section 3. Subsequently, Section 4 provide parametric examples of three representative classes of market demand with product differentiation that are often employed in applies studies: linear, CES (constant elasticity of substitution), and multinomial logit with outside option. Section 5 concludes.[10] Implications of aggregate output and consumer surplus are provided in Online Appendix B, and we argue in Online Appendix C that our methodology can readily be extended when firm heterogeneity is introduced.

2 The model of oligopolistic pricing

For ease of exposition, this section follows Holmes (1989) and ACV (2010) to consider the case of two symmetric firms and two separate markets or consumer groups (hereafter, simply called "markets"). It is straightforward to extend the following analysis to the case of more than two symmetric firms and more than two separate markets: in Section 4 below, where we consider parametric examples of market demand, the number of symmetric firms is assumed to be $N \geq 2$.[11] As explained in Introduction, we call one market s (strong), where the equilibrium discriminatory price is higher than the equilibrium uniform price, and the other w (weak), where the opposite is true.

Two firms, A and B, have an identical cost structure in each market. Specifically, each firm has an identical cost function, $c_m(q_{im})$, in market $m = s, w$, where q_{im} is firm i's output ($i = A$, B). For simplicity of exposition, we assume, with a slight abuse of notation, that firms have a constant marginal cost in each market m, $c_m \geq 0$; here, c_s and c_w can be different. However, as mentioned again in Subsection 2.3 below, it is assumed that the strong market either has a higher marginal cost or only slightly lower marginal cost so that its price still increases with price discrimination. In this sense, this paper does not consider the role of cost differences in differential pricing (see also Footnote 2 above).

[10] In this paper, the only policy instrument is an enforcement of uniform pricing. Cowan (2018) studies a model of monopoly to consider a more moderate instrument by which a government regulates the monopolist's profit margins or price-marginal cost ratios across different markets.

[11] See Online Appendix A for the case of a general number of markets. We assume that resale between markets is impossible to prevent consumers in the strong market from being better off buying the good at a lower price in the week market (see Boik (2017) for an empirical analysis of oligopolistic third-degree price discrimination when arbitrage may matter).

2.1 Consumers

In market $m = s, w$, given firms A and B's prices p_{Am} and p_{Bm}, the representative consumer purchases $x_{Am} > 0$ and $x_{Bm} > 0$ to maximize her net utility (i.e., surplus)

$$U_m(\mathbf{x}_m) - p_{Am}x_{Am} - p_{Bm}x_{Bm},$$

where $\mathbf{x}_m = (x_{Am}, x_{Bm})$, U_m is three-times continuously differentiable, $\partial U_m/\partial x_{im} > 0$ and $\partial^2 U_m/\partial x_{im}^2 < 0$ for firm $i = A$, B, and $\partial^2 U_m/(\partial x_{Am}\partial x_{Bm}) < 0$ (i.e., firms A and B produce substitutable products). Here, it is assumed that the representative consumer has a large amount of income so that this maximization problem is valid.

Inverse demands in market m, $p_{im} = P_{im}(x_{im}, x_{-i,m})$, are derived from the representative consumer's utility maximization ($-i = A, B$, $-i \neq i$): $\partial U_m(x_{im}, x_{-i,m})/\partial x_{im} - p_{im} = 0$, which also implicitly defines firm i's *direct demand* in market m, $x_{im} = x_{im}(p_{im}, p_{-i,m})$. We assume that $x_{im}(\cdot)$ is twice continuously differentiable. Because of the assumptions regarding the utility, firm i's demand in market m decreases as its own price increases ($\partial x_{im}/\partial p_{im} < 0$), and it rises as the rival's price increases ($\partial x_{im}/\partial p_{-i,m} > 0$; the firms' products are substitutes).

We also assume that from a viewpoint of consumers, firms are symmetric: $U_m(x', x'') = U_m(x'', x')$ for any $x' > 0$ and $x'' > 0$. Then, the firms' demands in market m are also symmetric: $x_{Am}(p', p'') = x_{Bm}(p', p'')$ for any $p' > 0$ and $p'' > 0$. Because the firms' technologies are also identical, we focus on symmetric Nash equilibrium until we allow firm heterogeneity in Section 5.[12]

We define the demand in symmetric pricing by $q_m(p) \equiv x_{Am}(p, p)$. Note here that

$$q'_m(p) = \underbrace{\frac{\partial x_{Am}}{\partial p_A}(p_A, p)\bigg|_{p_A=p}}_{<0 \text{ (ACV's } q'_m)} + \underbrace{\frac{\partial x_{Am}}{\partial p_B}(p, p_B)\bigg|_{p_B=p}}_{>0 \text{ (strategic)}}. \tag{1}$$

Thus, for $q'_m(p)$ to be negative, we assume that $|\partial x_{Am}(p, p)/\partial p_A| > \partial x_{Am}(p, p)/\partial p_B$. Note also that by symmetry, the following relationship also holds (this corresponds to Holmes' (1989) Equation 4):

$$\underbrace{\frac{\partial x_{Am}}{\partial p_A}(p, p)}_{\text{own}} = \underbrace{q'_m(p)}_{\text{industry}} - \underbrace{\frac{\partial x_{Bm}}{\partial p_A}(p, p)}_{\text{strategic}}.$$

This exchangeability is key in Holmes' (1989) derivation below. Intuitively, each firm, under

[12] Here, $\partial^2 x_{im}(p, p)/\partial p_i^2$ can be positive, zero or negative. Following Dastidar's (2006, p. 234) Assumption 2 (iv), we assume that

$$\frac{\partial^2 x_{im}}{\partial p_i^2}(p, p) + \frac{\partial^2 x_{im}}{\partial p_i \partial p_{-i}}(p, p) \leq 0.$$

symmetry, treats the industry demand $q_m(p)$ as if it is its own demand. Thus, how a firm's pricing behavior affects its own demand as an industry demand has the following two effects: a small decrease in p_A by firm A by deviating from the industry price p (i) not only raises its own demand by $\partial x_{Am}/\partial p_A$ as the *residual* monopolist (*industry effects*), (ii) firm A can now also obtain some of the consumers originally attached to firm B, and this amount is $\partial x_{Bm}/\partial p_A$ (*strategic effects*).

2.1.1 The (first-order) price elasticities of market demand

Under symmetric pricing, we are able to define, following Holmes (1989, p. 245), the *price elasticity of the industry's demand* by

$$\epsilon_m^I(p) \equiv -\frac{pq_m'(p)}{q_m(p)} > 0. \tag{2}$$

As Weyl and Fabinger (2013, p. 542) state, this should not "be confused with the elasticity of the residual demand that any of the firms faces."[13] Similarly, the *own* and the *cross price elasticities of the firm's demand* are defined by

$$\epsilon_m^{own}(p) \equiv -\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p,p) > 0$$

and by

$$\epsilon_m^{cross}(p) \equiv \frac{p}{q_m(p)}\frac{\partial x_{Bm}}{\partial p_A}(p,p) > 0,$$

respectively. Then, Holmes (1989) shows that under symmetric pricing,

$$\epsilon_m^{own}(p) = \epsilon_m^I(p) + \epsilon_m^{cross}(p) \tag{3}$$

holds.[14] This implies that the own-price elasticity must be equal to or greater than the industry's elasticity and greater than the cross-price elasticity (i.e., $\epsilon_m^{own}(p) \geq \epsilon_m^I(p)$ and $\epsilon_m^{own}(p) > \epsilon_m^{cross}(p)$).

[13]Note that ϵ_m^I here is conceptually identical to η in ACV (2010, p. 1603) and ϵ_D in Weyl and Fabinger (2013, p. 542).

[14]In general, when there are $N \geq 2$ symmetric firms as in Section 4 below, this identity still holds if the cross price elasticity is defined by $\epsilon_m^{cross}(p) \equiv (N-1)[p/q_m(p)][\partial x_{Bm}(p,p)/\partial p_A]$.

2.1.2 The second-order price elasticities of market demand

We also consider two second-order elasticities. First, the *curvature of the firm's (direct) demand* in market m is defined by

$$\alpha_m^{own}(p) \equiv -\frac{p}{\partial x_{Am}(p,p)/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_A^2}(p,p),$$

which measures the convexity/concavity of the firm's direct demand, and corresponds to $\alpha_m(p)$ in Aguirre, Cowan and Vickers 2010, p. 1603). Second, we define the *elasticity of the cross-price effect* of the firm's direct demand in market m by

$$\alpha_m^{cross}(p) \quad \equiv \quad -\frac{p}{\partial x_{Am}(p,p)/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p,p),$$

which never appears in monopoly. Here, α_m^{own} and α_m^{cross} are positive (resp. negative) if and only if $\partial^2 x_{Am}/\partial p_A^2$ and $\partial^2 x_{Am}/(\partial p_B \partial p_A)$ are positive (resp. negative), respectively. Note also that the sign of α_m^{own} indicates whether the firm's own part of the demand slope under symmetric pricing given the rival's price p, $\partial x_{Am}(\cdot,p)/\partial p_A$, is *convex* ($\alpha_m^{own}$ is positive) or *concave* (α_m^{own} is negative). On the other hand, α_m^{cross} measures to what extent the rival's price level matters to how many of the firm's customers switch to the rival's product when the firm raises its own price ($\partial x_{Am}/\partial p_A$). Thus, a large α_m^{cross} implies that $\partial x_{Am}/\partial p_A$ is very responsive to a change in p_B, and vice versa.

2.2 Firms

Firm i's profit in market m is written as

$$\pi_{im}(\mathbf{p}_m) = (p_{im} - c_m)x_{im}(\mathbf{p}_m), \tag{4}$$

where $\mathbf{p}_m = (p_{im}, p_{-i,m})$. As in Dastidar's (2006, pp. 235-6) Assumptions 3 and 4, for the existence and the global uniqueness of pricing equilibrium under either uniform pricing or price discrimination, we assume that for each firm $i = A, B$, $\partial^2 \pi_{im}/\partial p_{im}^2 < 0$, $\partial^2 \pi_{im}/(\partial p_{im}\partial p_{-i,m}) > 0$, and

$$-\frac{\partial^2 \pi_{im}/(\partial p_{im}\partial p_{-i,m})}{\partial^2 \pi_{im}/\partial p_{im}^2} < 1$$

(see Dastidar's (2006) Lemmas 1 and 2 for the existence and the uniqueness).

We then define the first-order partial derivative of the profit in market m, evaluated at a symmetric price p, by

$$\partial_p \pi_m(p) \equiv \left.\frac{\partial \pi_{im}(p_{im}, p_{-i,m})}{\partial p_{im}}\right|_{p_{im}=p_{-i,m}=p}$$
$$= q_m(p) + (p - c_m)\frac{\partial x_{Am}}{\partial p_A}(p,p). \quad (5)$$

Under symmetric discriminatory pricing, p_m^* satisfies $\partial_p \pi_m(p_m^*) = 0$ for $m = s, w$, whereas under symmetric uniform pricing, $\overline{p}$ is a (unique) solution of $\partial_p \pi_s(\overline{p}) + \partial_p \pi_w(\overline{p}) = 0$. Throughout this paper, we consider the situation where the weak market is open under uniform pricing (for which $q_w(p_s^*) > 0$ is a sufficient condition).[15]

2.2.1 The second-order derivative of the profit function under symmetry

As a measure of concavity of the market-wise profit function in symmetric equilibrium, we define:

$$\pi_m''(p) \equiv q_m'(p) + \frac{\partial x_{Am}}{\partial p_A}(p,p) + (p - c_m)\frac{d}{dp}\left(\frac{\partial x_{Am}}{\partial p_A}(p,p)\right)$$
$$= \underbrace{\partial_p^2 \pi_m(p)}_{\text{ACV's } \pi_m''} + \underbrace{\frac{\partial x_{Am}}{\partial p_B}(p,p) + (p - c_m)\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p,p)}_{\text{strategic}}, \quad (6)$$

where $\partial_p^2 \pi_m(p)$ is given by

$$\partial_p^2 \pi_m(p) \equiv \left[2 + (p - c_m)\frac{\partial^2 x_{Am}(p,p)/\partial p_A^2}{\partial x_{Am}(p,p)/\partial p_A}\right]\frac{\partial x_{Am}}{\partial p_A}(p,p), \quad (7)$$

which corresponds to ACV's (2010, p. 1603) $\pi_m''(p)$. The second and third terms in Equation (6) arise due to oligopoly. Here, in each m, $\pi_m''(p)$ is assumed to be negative for all $p \geq 0$.[16]

We now argue how π_m'' is expressed in terms of the first- and second-order price elasticities of demand. Note first that Equation (7) implies that

$$\partial_p^2 \pi_m(p) = -\{2 - \underbrace{\frac{p - c_m}{p}}_{=L_m(p)} \underbrace{[-\frac{p}{\frac{\partial x_{Am}}{\partial p_A}(p,p)}\frac{\partial^2 x_{Am}}{\partial p_A^2}(p,p)]}_{=\alpha_m^{own}(p)}\} \underbrace{[-\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p,p)]}_{=\epsilon_m^{own}(p)} \frac{q_m(p)}{p}$$

[15]Note that $q_w(\overline{p}) > q_w(p_s^*)$ because $q_w(\cdot)$ is strictly decreasing and $p_s^* > \overline{p}$. Thus, if $q_w(p_s^*) > 0$, then the weak market is open under uniform pricing, i.e., $q_w(\overline{p}) > 0$. Alternatively, we would be able to show that there exist $\underline{c}_s$ and $\overline{c}_s$, $\underline{c}_s < \overline{c}_s$, such that $p_s^* > p_w^*$ and $q_w(\overline{p}) > 0$ for $c_s \in (\underline{c}_s, \overline{c}_s)$ in a similar spirit of Adachi and Matsushima (2014).

[16]ACV's (2010) Appendix A discusses the concavity of the profit function.

$$= -[2 - L_m(p)\alpha_m^{own}(p)]\epsilon_m^{own}(p)\frac{q_m(p)}{p},$$

where

$$L_m(p) \equiv \frac{p - c_m}{p} \tag{8}$$

is the *markup rate* (i.e., the Lerner index). Then, from Equation (6), it can be verified that $\pi''_m(p)$ is expressed in terms of the four elasticities (ϵ_m^{own}, ϵ_m^{cross}, α_m^{own}, and α_m^{cross}) as well as $q_m(p)$ and p itself:

$$\begin{aligned}
\pi''_m(p) &= -[2 - L_m(p)\alpha_m^{own}(p)]\epsilon_m^{own}(p)\frac{q_m(p)}{p} + [\underbrace{\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_B}(p,p)}_{=\epsilon_m^{cross}(p)}]\frac{q_m(p)}{p} \\
&\quad -\underbrace{\frac{p-c_m}{p}}_{=L_m(p)}[\underbrace{-\frac{p}{\frac{\partial x_{Am}}{\partial p_A}(p,p)}\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}(p,p)}_{=\alpha_m^{cross}(p)}][\underbrace{\frac{p}{q_m(p)}\frac{\partial x_{Am}}{\partial p_A}(p,p)}_{=-\epsilon_m^{own}(p)}]\frac{q_m(p)}{p} \\
&= -\{[2 - (\alpha_m^{own} + \alpha_m^{cross})L_m]\epsilon_m^{own} - \epsilon_m^{cross}\}\frac{q_m}{p}.
\end{aligned} \tag{9}$$

2.2.2 Conduct

Now, we are able to define the *conduct parameter*[17] in market m by $\theta_m(p) \equiv 1 - ADR_m(p)$, where $ADR_m(p)$ is the *aggregate diversion ratio* (Shapiro 1996) in market m, defined by

$$ADR_m(p) \equiv -\frac{\partial x_{Bm}(p,p)/\partial p_A}{\partial x_{Am}(p,p)/\partial p_A} = \frac{\epsilon_m^{cross}(p)}{\epsilon_m^{own}(p)} \geq 0.$$

This concept will be utilized in Section 3. Here, $ADR_m(p)$ measures the intensity of *rivalness*: if $ADR_m(p)$ is close to one, consumers who leave a firm as a response to an increase in its price are mostly switching to its rival's product.[18]

[17] This term originates from the empirical literature where conduct itself is a target of estimation ("parameter") without an exact specification of strategic interaction (see, e.g., Bresnahan 1989; Genesove and Mullin 1998; and Corts 1999). Here, strategic interaction is explicitly modeled (i.e., price competition), and thus the degree of conduct is solely based on product differentiation with no possibility of collusive pricing.

[18] Alternatively, Weyl and Fabinger (2013, p. 531) and Adachi and Fabinger (2022) define the conduct parameter in a market (which, in our interest in price discrimination, can be indexed by m) by $\theta_m \equiv \epsilon_m^I L_m$ (their mc and ϵ_D are replaced by our c_m and ϵ_m^I, respectively) as the Lerner index adjusted by the elasticity of the *industry's* demand. If the first-order condition is given for each market (that is, if full price discrimination is allowed), then $\theta_m(p)$ defined as in Weyl and Fabinger (2013) coincides with $1 - ADR_m(p)$ because $[(p_m - c_m)/p_m]\epsilon_m^{own} = 1$ and thus

$$\begin{aligned}
\epsilon_m^I(p)L_m(p) &= \frac{1}{\epsilon_m^F(p)}\left(-\frac{p}{q_m(p)}\right)q'_m(p) \\
&= -\frac{q_m(p)}{p}\frac{1}{\partial x_{Am}(p,p)/\partial p_A}\left(-\frac{p}{q_m(p)}\right)\left(\frac{\partial x_{Am}}{\partial p_A}(p,p) + \frac{\partial x_{Am}}{\partial p_B}(p,p)\right)
\end{aligned}$$

As Weyl and Fabinger (2013, p. 544) argue, $\theta_m(p)$ captures the degree of *industry-level brand loyalty or stickiness*[19] in market m. To see this, note that the conduct parameter is also expressed by

$$\theta_m(p) = \frac{\epsilon_m^I(p)}{\epsilon_m^{own}(p)}, \tag{10}$$

where $\epsilon_m^{own}(p) \geq \epsilon_m^I(p)$. If $\epsilon_m^{own}(p) \to \infty$ as in the case of the price-taking assumption, $\theta_m(p)$ is zero. On the other hand, if $\epsilon_m^{own}(p)$ is equal to $\epsilon_m^I(p)$, that is, the own elasticity is nothing but the industry's elasticity, then it is monopoly and $\theta_m(p) = 1$.[20] Note here that by using this the Holmes decomposition (Equation 3), we can rewrite Equation (9) as

$$\pi_m''(p) \quad = \quad -\{1 + \theta_m - (\alpha_m^{own} + \alpha_m^{cross})L_m\}\frac{q_m \epsilon_m^{own}}{p}. \tag{11}$$

Note also here that $\theta_m(p)$, which ranges between 0 and 1, better captures the brand stickiness than $L_m(p)$ does: the markup rate, L_m, alone is not appropriate to measure the rivalness within market m because it can be the case that p_m is close to c_m (the markup rate is close to zero) simply because the price elasticity of the industry's demand $\epsilon_m^I(p_m)$ is very large, whereas the brand rivalness is so weak that the cross-price elasticity, ϵ_m^{cross}, remains very small (as a result, in total, ϵ_m^{own} is very large, which is actually the reason for the low markup rate). However, if ϵ_m^{cross} is close to ϵ_m^{own} (i.e., almost of all consumers who leave a firm as a response to its price increase are switching to other rivals' products), then θ_m becomes close to zero *irrespective of the value of the markup rate.*

2.3 Equilibrium

The equilibrium discriminatory price in market $m = s, w$, p_m^*, satisfies the following Lerner formula:

$$\epsilon_m^{own}(p_m^*)L_m(p_m^*) = 1. \tag{12}$$

This shows that the discriminatory price in market m approaches to the marginal cost as the own-price elasticity for the firm, $\epsilon_m^{own}(p_m^*)$, becomes large. Because of Holmes' (1989) elasticity formula explained above, $\epsilon_m^{own}(p_m^*)$ can be large (i) when $\epsilon_m^I(p_m^*)$ is very large even if $\epsilon_m^{cross}(p_m^*)$

$$\begin{aligned} &= \frac{\partial x_{Am}(p,p)/\partial p_A + \partial x_{Bm}(p,p)/\partial p_A}{\partial x_{Am}(p,p)/\partial p_A} \text{ (by symmetry)} \\ &= 1 - ADR_m(p) \equiv \theta_m(p) \end{aligned}$$

is established. It turns out that this alternative definition is more tractable when firm heterogeneity is introduced in Section 5.

[19]Even if firms' products have the same characteristics across different markets (with no product differentiation), brand loyalty may differ across markets, reflecting the differences in market characteristics (as summarized in demand functions).

[20]Because $[(p_m - c_m)/p_m]\epsilon_m^{own} = 1$ and $\epsilon_m^{own} = \epsilon_m^I + \epsilon_m^{cross}$, it is verified that $\theta_m + [(p_m - c_m)/p_m]\epsilon_m^{cross} = 1$. Thus, as long as the products are substitutes ($\epsilon_m^{cross} > 0$), θ_m is less than one.

is close to zero, or (ii) when $\epsilon_m^{cross}(p_m^*)$ is very large even if $\epsilon_m^I(p_m^*)$ is close to zero. Evidently, if there are no cost differentials between markets, which market is strong or weak is solely determined by the difference in the own-price elasticity. As mentioned above, we assume that the marginal cost in the strong market is not sufficiently low to assure that $p_s^* > \overline{p} > p_w^*$ indeed holds.[21,22]

Lastly, let y_m be per-firm (symmetric) *market share* of output in market m, that is, $y_m(p_s, p_w) \equiv q_m(p_m)/[q_s(p_s) + q_w(p_w)]$. Then, the equilibrium uniform price, $\overline{p} \equiv \overline{p}(c_s, c_w)$, satisfies:

$$\sum\nolimits_{m=s,w} \overline{y}_m \epsilon_m^{own}(\overline{p}) L_m(\overline{p}) = 1, \tag{13}$$

where $\overline{y}_m \equiv y_m(\overline{p}(c_s, c_w), \overline{p}(c_s, c_w))$ for $m = s, w$.[23] In this way, the equilibrium level of uniform price is determined by the market-share weighted average of the own price elasticities, whereas the equilibrium level of discriminatory price solely depends on the firm's own price elasticity in that market. In the rest of the paper, the dependence of the equilibrium price is often implicit when there are no confusions. In particular, the superscript star (the upper bar) denotes price discrimination (uniform pricing). For example, we write $(\epsilon_m^I)^* \equiv \epsilon_m^I(p_m^*)$ and $\overline{\epsilon}_m^I \equiv \epsilon_m^I(\overline{p})$ as the industry's elasticities in equilibrium.

3 Welfare analysis

As mentioned in Introduction, we add the constraint $p_s - p_w = t$, where $t \geq 0$, to the firms' profit maximization problem.[24] Then, we express social welfare (as well as aggregate output and consumer surplus) as a function of t in $[0, t^*]$, where $t = 0$ corresponds to uniform pricing,

[21] See Nahata, Ostaszewski, and Sahoo (1990) for an example of all discriminatory prices being lower than the uniform price with a plausible demand structure under monopoly. In the case of oligopoly, Corts (1998) show that best-response asymmetry, in which firms differ in ranking strong and weak markets, is necessary for all discriminatory prices to be lower than the uniform price ("all-out price competition"). As long as symmetric firms are considered, this case never arises.

[22] When price discrimination is allowed, each firm may not price discriminate even if it is allowed to do so because it is still able to set a uniform price (i.e., it is not forced to price discriminate). We assume that $\pi_{im}(\cdot, p_{-i,m}^*)$ is strictly increasing (decreasing) at $p_{im} = \overline{p}$ in market $m = s$ (m) and thus firm i has an incentive to deviate from the equilibrium uniform price if the other firm chooses $p_{-i,s}^*$ and $p_{-i,w}^*$, and that $\pi_{im}(\cdot, p_{-i,m}^*)$ attains the global optimum at $p_{im} = p_{im}^*$.

[23] If there are no cost differentials, i.e., $c_s = c_w$ $(\equiv c)$, then the formula is simpler:

$$\frac{\overline{p} - c}{\overline{p}} = \frac{1}{\sum_{m=s,w} \overline{y}_m \epsilon_m^{own}(\overline{p})}$$

as shown by Holmes (1989, p. 247): the markup rate (common to all markets) is equal to the inverse of the average of own-price elasticities weighted by the output shares.

[24] Alternatively, Vickers (2020) analyzes properties of social welfare and consumer surplus as a scalar argument to make a comparison between price discrimination and uniform pricing in monopoly. Vickers (2020) especially focuses on the case where quantity elasticity or inverse demand curvature is constant for all markets. See also Cowan (2017) for an analysis of the role of price elasticity and demand curvature in determining the effects of monopolistic third-degree price discrimination.

and $t = t^* \equiv p_s^* - p_w^*$ to price discrimination. Note that under this constrained problem of profit maximization, p_w satisfies $\partial_p \pi_s(p_w + t) + \partial_p \pi_w(p_w) = 0$. Thus, we write the solution by $p_w(t)$. Then, we define $p_s(t) \equiv p_w(t) + t$. Applying the implicit function theorem to this equation yields to

$$\begin{cases} p_w'(t) = -\dfrac{1}{1 + \pi_w''/\pi_s''} < 0 \\[2em] p_s'(t) = \dfrac{1}{1 + \pi_s''/\pi_w''} > 0. \end{cases} \tag{14}$$

They show the natural relationship between p_m' and π_m'': as the π_m'' becomes smaller around the equilibrium price, i.e., the profit function in market m becomes flatter at the peak point, the price becomes more responsive in that market. i.e., $|p_m'|$ is larger.

3.1 Preliminaries

We now define the representative consumer's utility in symmetric pricing by $\widetilde{U}_m(q) = U_m(q, q)$. Then, social welfare under symmetric pricing as a function of t is written as

$$\begin{aligned} W(t) &\equiv \widetilde{U}_s(q_s[p_s(t)]) + \widetilde{U}_w(q_w[p_w(t)]) - 2c_s \cdot q_s[p_s(t)] - 2c_w \cdot q_w[p_w(t)] \\ &= (\widetilde{U}_s' - 2c_s) \cdot q_s' \cdot p_s'(t) + (\widetilde{U}_w' - 2c_w) \cdot q_w' \cdot p_w'(t), \end{aligned}$$

which implies (using $\widetilde{U}_m' = \partial U_m/\partial q_A + \partial U_m/\partial q_B = 2(\partial U_m/\partial q_A)$ by symmetry)

$$\begin{aligned} \frac{W'(t)}{2} &= [p_s(t) - c_s] \cdot q_s' \cdot p_s'(t) + [p_w(t) - c_w] \cdot q_w' \cdot p_w'(t) \\ &= \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s'' + \pi_w''}\right)}_{>0} \{z_w[p_w(t)] - z_s[p_s(t)]\}, \end{aligned}$$

where, as in ACV (2010, p. 1605),

$$z_m(p) \equiv \frac{\mu_m(p) q_m'(p)}{\pi_m''(p)}$$

is "the ratio of the marginal effect of a price increase on social welfare to the second derivative of the profit function," and

$$\mu_m(p) \equiv p - c_m \tag{15}$$

is the *profit margin* in market m.[25] In contrast to ACV (2010), our q'_m and π''_m have *strategic effects* as Equations (1) and (6) above show.

Now, if we assume z_m is *increasing* in p (the increasing ratio condition for welfare; IRCW),[26] then as in ACV's (2010) Lemma, it is verified that if there exists $\widehat{t}$ such that $W'(\widehat{t}) = 0$, then $W''(\widehat{t})/2 < 0$. This is because

$$\frac{W''(t)}{2} = \left(-\frac{\pi''_s \pi''_w}{\pi''_s + \pi''_w}\right)(z'_w p'_w - z'_s p'_s) + (z_w - z_s)\frac{d}{dt}\left(-\frac{\pi''_s \pi''_w}{\pi''_s + \pi''_w}\right),$$

and thus $\text{sign}[W''(\widehat{t})/2] = \text{sign}[z'_s p'_s - z'_w p'_w]$ is negative ($\because z'_w p'_w < 0$, $z'_s p'_s > 0$, and $z_w = z_s$ for $t = \widehat{t}$). Hence, $(1/2)W(t)$ is strictly quasi-concave on $[0, t^*]$, and behaves in either manner:

1. If $W'(0) \leq 0$, then $(1/2)W(t)$ is *monotonically decreasing* in r, and as a result $\Delta W/2 \equiv [W(t^*) - W(0)]/2 < 0$; *price discrimination decreases social welfare.*

2. If $W'(0) > 0$, then $(1/2)W(t)$ either

   (a) is *monotonically increasing* (if $W'(t^*) > 0$, this is true), and as a result, $\Delta W/2 > 0$; *price discrimination increases social welfare.*

   (b) *first increases, and then after the reaching the maximum* (where $W'(t) = 0$), *decreases until* $t = t^*$. In this case, *price discrimination may increase or decrease social welfare.*

Below, we focus on the first two cases that provide sufficient conditions for determining the welfare effects of price discrimination. All the three parametric examples in Section 4 below satisfy the IRCW.

[25]Here, $\mu_m(p)q'_m(p)$ can be interpreted as the marginal effect of a price increase on social welfare in market m because:

$$\frac{\mathrm{d}[\overbrace{\frac{1}{2}U_m[q_m(p)] - c_m q_m(p)]}^{\text{per-firm (normalized)}}}{\mathrm{d}p} = \mu_m(p)q'_m(p).$$

[26]Note that

$$z'_m(p) = \frac{[\mu_m(p)q''_m(p) + q'_m(p)]\pi''_m(p) - \mu_m(p)q'_m(p)\pi'''_m(p)}{[\pi''_m(p)]^2}$$

and thus, the IRCW is equivalent to

$$[\mu_m(p)q''_m(p) + q'_m(p)]\pi''_m(p) > \mu_m(p)q'_m(p)\pi'''_m(p).$$

Appendix B of ACV (2010) discusses sufficient conditions for the IRCW (IRC in their abbreviation) to hold in the case of monopoly.

3.2 Sufficient conditions using pass-through

To provide a formal statement that permits the graphical interpretation explained in Introduction, we must define the remaining sufficient statistic—*pass-through* in market m by $\rho_m \equiv \partial p_m / \partial c_m$. It is a function of $t \in [0, t^*]$ of the constrained problem considered above. In particular,

$$\rho_m[p_m(t)] = \begin{cases} \dfrac{\partial x_{Am}/\partial p_A}{\pi_s'' + \pi_w''} & \text{for } t < t^* \\[3em] \dfrac{\partial x_{Am}/\partial p_A}{\pi_m''} \; (\equiv \rho_m^*) & \text{for } t = t^* \end{cases}$$

is obtained by applying the implicit function theorem to $\partial_p \pi_s(p_w + t) + \partial_p \pi_w(p_w) = 0$ for $t < t^*$ and $\partial_p \pi_m(p_m) = 0$ for $t = t^*$ (i.e., under price discrimination).

From Equation (9) it is observed that ρ_m^* as defined above can be expressed as

$$\begin{aligned} \rho_m^* &= \frac{\dfrac{\partial x_{Am}}{\partial p_A}}{\left\{2 - [(\alpha_m^{own})^* + (\alpha_m^{cross})^*](L_m)^* - \dfrac{(\epsilon_m^{cross})^*}{(\epsilon_m^{own})^*}\right\} \dfrac{\partial x_{Am}}{\partial p_A}} \\ &= \frac{1}{2 - \dfrac{(\epsilon_m^{cross})^* + (\alpha_m^{own})^* + (\alpha_m^{cross})^*}{(\epsilon_m^{own})^*}} \end{aligned}$$

because $(L_m)^* = 1/(\epsilon_m^{own})^*$. Note here that in the case of monopoly (i.e., $(\epsilon_m^{cross})^* = 0$ and $(\alpha_m^{cross})^* = 0$),

$$\rho_m^* = \frac{1}{2 - \dfrac{(\alpha_m^{own})^*}{(\epsilon_m^{own})^*}} \tag{16}$$

and $(\alpha_m^{own})^*/(\epsilon_m^{own})^*$ corresponds to ACV's (2010, p. 1603) curvature of the inverse demand, σ_m^*.

Now, using conduct, profit margin, and pass-through, we obtain the following sufficient conditions for price discrimination to increase or decrease social welfare.

Proposition 1. *Given the IRCW, price discrimination increases social welfare if*

$$\mu_s^* \theta_s^* \rho_s^* < \mu_w^* \theta_w^* \rho_w^*$$

holds, and it decreases social welfare if

$$\frac{\overline{\mu}_s \overline{\theta}_s \overline{\rho}_s}{\overline{\pi}_s''} \geq \frac{\overline{\mu}_w \overline{\theta}_w \overline{\rho}_w}{\overline{\pi}_w''}$$

holds, where

$$\bar{\pi}''_m \equiv \pi''_m(\overline{p}) = -\{[2 - (\overline{\alpha}^{own}_m + \bar{\alpha}^{cross}_m)\overline{L}_m]\bar{\epsilon}^{own}_m - \bar{\epsilon}^{cross}_m\}\frac{\overline{q}_m}{\overline{p}},$$

for $m = s, w$.

Proof. See Appendix A. □

In plain words, if either (i) *conduct* (θ), (ii) *profit margin* (μ), or (iii) *pass-through* (ρ) is sufficiently *small* in the *strong* market, then social welfare is likely to be higher under price discrimination. In particular, if these three measures are calculated (or estimated) in each separate market, then it would assist one to judge whether price discrimination is desirable from a society's viewpoint. Specifically, suppose that price discrimination is being conducted. Then, to evaluate it from a viewpoint of social welfare, one only needs the local information: first, θ^*_m, μ^*_m and ρ^*_m for each $m = s, w$, are computed, and if the sufficient condition above is satisfied, then the ongoing price discrimination is justified. In addition, to compute θ^*_m, μ^*_m and ρ^*_m in equilibrium, *information on marginal cost is unnecessary*: once a specific form of demand function, $q_{im} = x_{im}(p_{im}, p_{-i,m})$, is provided (and if the IRCW is satisfied), then the three variables are computed in the following manner:[27]

$$\begin{cases} \theta^*_m = 1 - \dfrac{(\epsilon^{cross}_m)^*}{(\epsilon^{own}_m)^*} \\ \rho^*_m = \dfrac{1}{2 - \dfrac{(\epsilon^{cross}_m)^* + (\alpha^{own}_m)^* + (\alpha^{cross}_m)^*}{(\epsilon^{own}_m)^*}} \\ \mu^*_m = \dfrac{p^*_m}{(\epsilon^{own}_m)^*}. \end{cases} \tag{17}$$

Thus, if the firm's demand for each market m is estimated and the discriminatory price p^*_m is observed, then one can easily compute θ^*_m, μ^*_m, and ρ^*_m, using up to second-order demand elasticities.[28]

To provide an intuitive understanding as explained in Introduction, note that $\theta^*_m\rho^*_m$ is interpreted as *quantity* pass-through in market m under price discrimination if the marginal costs are constant: it is defined as $dq^*_m/d\widetilde{q}$, where $\widetilde{q}$ is an exogenous amount of output with $\pi_{jm}(p_{jm}, p_{-j,m}) = (p_{jm} - c_m)[x_{jm}(p_{jm}, p_{-j,m}) - \widetilde{q}]$, which can be expressed by

$$\begin{aligned} \frac{dq^*_m}{d\widetilde{q}} &= q'_m(p^*_m) \cdot \frac{dp^*_m}{d\widetilde{q}} \\ &= \frac{q'_m}{\partial x_{Am}/\partial p_A} \cdot \frac{\partial x_{Am}}{\partial p_A} \cdot \frac{dp^*_m}{d\widetilde{q}} \end{aligned}$$

[27] An alternative expression for μ^*_m is $\mu^*_m = c_m/[(\epsilon^{own}_m)^* - 1]$ if the cost information is used.

[28] It should be emphasized that the second-order supply property, i.e., the derivative of marginal cost, would be necessary if non-constant marginal cost is allowed, as suggested by Adachi and Fabinger (2022) in the context of general "taxation" (pure taxation and other additional costs from external changes).

$$
\begin{aligned}
&= \left(\frac{q'_m}{\partial x_{Am}/\partial p_A}\right) \cdot \left(\frac{\partial x_{Am}/\partial p_A}{\pi''_m}\right) \\
&= \theta^*_m \cdot \rho^*_m
\end{aligned}
$$

because the first-order condition with $\widetilde{q}$ indicates $dp^*_m/d\widetilde{q} = 1/\pi''_m$.[29] Now, $\mu^*_m \times \theta^*_m \rho^*_m$ approximates the trapezoid generated by a small deviation from (perfect) price discrimination that captures the marginal welfare gain in the strong market and the marginal welfare loss in the weak market (see Figure 1, A). If the latter is larger than the former, such a deviation lowers social welfare, and owing to the IRCW, this argument extends globally so that the regime switch to uniform pricing definitely decreases social welfare.

Note that this comparison is not straightforward when starting at uniform pricing (see the latter part of Proposition 1): why is the adjustment term, $\bar{\pi}''_m$, necessary for the deviation from uniform pricing? This is because pass-through is not defined market-wise unless the pricing regime is "perfect" or "full" price discrimination (i.e., $t = t^*$), where the first-order conditions are given market-wise. Note that if $|\pi''_m|$ is small, then π_m is "flat," and thus the price shift $|\Delta p_m|$ in response to some change would be large (see Expression 14). Hence, the role of π''_w/π''_s is to adjust measurement units for ρ_w/ρ_s. For example, if $|\pi''_w|$ is very small, then ρ_w is "over represented," and thus it should be "penalized" so that the right hand side of the inequality in the proposition becomes small.

Proposition 1 cannot be further simplified even if no cost differentials (i.e., $c_s = c_w$) are additionally assumed. In other words, this expression is already robust to the inclusion of cost differentials. Now, if we further assume that there are no strategic effects (i.e., $\theta_m = 1$), then the condition $\mu^*_w \theta^*_w \rho^*_w \geq \mu^*_s \theta^*_s \rho^*_s$ becomes $(p^*_s - c)/(p^*_w - c) \leq (1/\rho^*_s)/(1/\rho^*_w)$, which coincides with

$$
\frac{p^*_w - c}{2 - \sigma^*_w} \geq \frac{p^*_s - c}{2 - \sigma^*_s}
$$

in Proposition 2 of ACV (2010, p. 1606), where σ^*_m is what they call the curvature of the inverse demand function (under price discrimination), because of $\sigma^*_m = (\alpha^{own}_m)^*/(\epsilon^{own}_m)^*$ and Equation (16). Thus, price discrimination increases social welfare "if the discriminatory prices are not far apart and the inverse demand function in the weak market is locally more convex than that in the strong market" (ACV 2010, p. 1602). As compared to Figure 1 (B), Figure 1 (A) shows the usefulness of the sufficient statistics in welfare evaluation. In Online Appendix B, we extend our arguments to aggregate output and consumer surplus. Online Appendix C also argues that our methodology is readily extended to accommodate heterogeneous firms.

[29] Note that this is the case where $dq^*_m/d\widetilde{q}$ is evaluated at $\widetilde{q} = 0$: Miklós-Thal and Shaffer (2021a) derive a general formula for $\widetilde{q} > 0$, correcting Weyl and Fabinger's (2013) arguments. If marginal costs are non-constant (see Online Appendix D), then $\pi_{im}(p_{im}, p_{-i,m}) = p_{im} \cdot [x_{im}(p_{im}, p_{-i,m}) - \widetilde{q}] - c_m[x_{im}(p_{im}, p_{-i,m}) - \widetilde{q}]$ should be considered, where $c_m(\cdot)$ is the cost function, and thus $\theta^*_m \rho^*_m$ is no longer the quantity pass-through under price discrimination (that is, when $\widetilde{q} = 0$). See Weyl and Fabinger (2013, p. 572) for a precise expression of quantity pass-through with non-constant marginal costs.

3.3 An alternative expression

The next result shows another expression that can be readily verified to be equivalent to Proposition 1.

Corollary 1. *Given the IRCW, price discrimination increases social welfare if*

$$\frac{\mu_s^*\theta_s^*}{1+\theta_s^*-[(\alpha_s^{own})^*+(\alpha_s^{cross})^*]L_s^*}<\frac{\mu_w^*\theta_w^*}{1+\theta_w^*-[(\alpha_w^{own})^*+(\alpha_w^{cross})^*]L_w^*}$$

holds, and it decreases social welfare if

$$\frac{\bar{\mu}_s\bar{\theta}_s}{1+\bar{\theta}_s-(\bar{\alpha}_s^{own}+\bar{\alpha}_s^{cross})\bar{L}_s}\geq\frac{\bar{\mu}_w\bar{\theta}_w}{1+\bar{\theta}_w-(\bar{\alpha}_w^{own}+\bar{\alpha}_w^{cross})\bar{L}_w}$$

holds.

Proof. Using Equations (2), (10), and (11), we can rewrite z_m so that

$$\frac{W'(t)}{2}=\left(-\frac{\pi_s''\pi_w''}{\pi_s''+\pi_w''}\right)\left(\frac{\mu_w\theta_w}{1+\theta_w-(\alpha_w^{own}+\alpha_w^{cross})L_w}-\frac{\mu_s\theta_s}{1+\theta_s-(\alpha_s^{own}+\alpha_s^{cross})L_s}\right)$$

for $t\in[0,t^*]$. □

This corollary indicates that pass-through is, although it facilitates an intuitive interpretation as shown in Proposition 1, not necessary. Instead, the own and cross curvatures are utilized, and the second inequality in Corollary 1 is computationally simpler than the second inequality in Proposition 1 because the second-order derivative for the profit function, π_m'' , is not involved. For this reason, this corollary's result is used for numerical exercises in the next section.

Note that our expression for

$$z_m=\frac{(p_m-c_m)\theta_m}{1+\theta_m-(\alpha_m^{own}+\alpha_m^{cross})L_m}$$

is a generalization of ACV's (2010) Equation (4),

$$z_m=\frac{p_m-c}{2-\alpha_m^{own}L_m}$$

if there are *no strategic effects* (i.e., $\theta_m=1$ and $\alpha_m^{cross}=0$). Additionally, if there are *no cost differentials* (i.e., $c_s=c_w\equiv c$), then the second part of the corollary reduces to ACV's (2010, p. 1605) Proposition 1 ($\overline{\alpha}_s^{own}\geq\overline{\alpha}_w^{own}$ in our notation; in their notation, $\alpha_s(\overline{p})\geq\alpha_w(\overline{p})$) because $L_s(\overline{p})=L_w(\overline{p})$. That is, the firm's "direct demand function in the strong market is at least as convex as that in the weak market at the nondiscriminatory price" (ACV 2010, p. 1602).

4 Parametric examples of market demand

To consider the following three examples of parametric market demand, we consider $N \geq 2$ symmetric firms, assuming that there are still two separate markets (strong and weak): let $\mathbf{x}_m = (x_{1m}, x_{2m}, ..., x_{Nm})$ be the representative consumer's consumption bundle in market $m = s, w$, and $\mathbf{p}_m = (p_{1m}, p_{2m}, ..., p_{Nm})$ be the prices in that market. We focus on (i) linear, (ii) CES (constant elasticity of substitution), and (iii) multinomial logit demands: these demand functions are among the commonly-used demand systems (Quint 2014; Choné and Linnemer 2020), and Online Appendix B verifies that the IRCW holds for these three demands. Note that to save notation, the same β_m is repeatedly used in the following three examples, but with different meanings (similarly, ω_m appears twice: in Subsections 4.1 and 4.3).

Let the set of related parameters be denoted by Θ. If

$$G(\Theta, N) \equiv \frac{\bar{\mu}_s \bar{\theta}_s}{1 + \bar{\theta}_s - (\bar{\alpha}_s^{own} + \bar{\alpha}_s^{cross})\bar{L}_s} - \frac{\bar{\mu}_w \bar{\theta}_w}{1 + \bar{\theta}_w - (\bar{\alpha}_w^{own} + \bar{\alpha}_w^{cross})\bar{L}_w}$$

and

$$H(\Theta, N) \equiv \frac{\mu_s^* \theta_s^*}{1 + \theta_s^* - [(\alpha_s^{own})^* + (\alpha_s^{cross})^*]L_s^*} - \frac{\mu_w^* \theta_w^*}{1 + \theta_w^* - [(\alpha_w^{own})^* + (\alpha_w^{cross})^*]L_w^*}$$

are defined, then, according to Corollary 1, $G(\Theta, N) \geq 0$ implies $\Delta W < 0$ and $H(\Theta, N) < 0$ implies $\Delta W > 0$. Table 1 shows the first- and second-order elasticities as well as the conduct parameter θ_m (from Equation 10) as a function of p: $\overline{p}$ (in the case of uniform pricing) or p_m^* (in the case of price discrimination) is imputed. Interestingly, in the case of CES demand, not only $(\epsilon_m^{own}, \epsilon_m^{cross})$ is constant but so are $(\alpha_m^{own}, \alpha_m^{cross})$ and θ_m. Given any price p, L_m, and μ_m are obtained from Equations (8) and (15), respectively. Then, $G(\Theta, N)$ and $H(\Theta, N)$ are parametrically expressed for each of the demands.

We below focus on cross-market differences in demand in line with the literature on third-degree price discrimination where market differences arise from the demand side. In particular, we focus on one parameter that is closely related to the first-order elasticities, (δ_s, δ_w) for linear demand, (σ_s, σ_w) for CES demand, and (β_s, β_w) for multinomial demand (see below for the definitions).

However, our methodology does not preclude cost differences: in all these examples, we consider both cases of common and different marginal costs across strong and weak markets. To ensure that the strong market is indeed strong when cost differentials are allowed but demand heterogeneity is not allowed, it is sufficient to assume that the marginal cost in the strong market is higher than in that in the weak market, c_w: $c_s > c_w$ (although c_s should not be too much higher than c_w). Hence, we consider c_s that is slightly higher than c_w when cost differentials are allowed. However, this inequality will not be sufficient when demand

(i) Linear	
First-order	
Own: $\epsilon_m^{own}(p)$	$\frac{[1+(N-2)\delta_m]p}{(1-\delta_m)(\omega_m-p)}$
Cross: $\epsilon_m^{cross}(p)$	$\frac{\delta_m p}{(1-\delta_m)(\omega_m-p)}$
Second-order	
Own: α_m^{own}	0
Cross: α_m^{cross}	0
Conduct	
θ_m	$\frac{1+(N-3)\delta_m}{1+(N-2)\delta_m}$
(ii) CES	
First-order	
Own: ϵ_m^{own}	$\frac{1+(N-1)\sigma_m}{N}$
Cross: ϵ_m^{cross}	$\frac{\sigma_m-1}{N}$
Second-order	
Own: α_m^{own}	$\frac{N\sigma_m[N+1+(N-1)\sigma_m]-2(\sigma_m-1)[1+(N-1)\sigma_m]}{N[1+(N-1)\sigma_m]}$
Cross: α_m^{cross}	$\frac{N\sigma_m(\sigma_m-1)-2(\sigma_m-1)[1+(N-1)\sigma_m]}{N[1+(N-1)\sigma_m]}$
Conduct	
θ_m	$\frac{2+(N-2)\sigma_m}{1+(N-1)\sigma_m}$
(iii) Logit	
First-order	
Own: $\epsilon_m^{own}(p)$	$\beta_m p\cdot[1-q_m(p;N)]$
Cross: $\epsilon_m^{cross}(p)$	$\beta_m p\cdot q_m(p;N)$
Second-order	
Own: $\alpha_m^{own}(p)$	$\beta_m p\cdot[1-2q_m(p;N)]$
Cross: $\alpha_m^{cross}(p)$	$\frac{\beta_m p\cdot q_m(p;N)[1-2q_m(p;N)]}{1-q_m(p;N)}$
Conduct	
$\theta_m(p)$	$\frac{1-2q_m(p;N)}{1-q_m(p;N)}$

Table 1: The four elasticities and the conduct 'parameter' under symmetric price p in market m (with N symmetric firms). See the main text for the notations. Note that for the CES demand, the four elasticities and the conduct parameter are constant for any p. For the linear demand, the two second-order elasticites are zero, and the conduct parameter is constant.

differentials are also allowed: we exclude the parameter region where $p_s^* \geq p_w^*$ does not hold in each of the three examples.

4.1 Linear demand

Linear demand is derived from the quadratic utility of the representative consumer in market $m = s, w$ under symmetric product differentiation (Shubik with Levitan 1980):

$$U_m(\mathbf{x}_m) = \omega_m \cdot \sum_{i=1}^{N} x_{im} - \frac{1}{2}\left(\beta_m \sum_{i=1}^{N} x_{im}^2 + 2\gamma_m \sum_{j\neq i} x_{im}x_{jm}\right).$$

This yields linear inverse demand, $P_{im}(x_{im}, \mathbf{x}_{-i,m}) = \omega_m - \beta_m x_{im} - \gamma_m \sum_{j\neq i} x_{jm}$, where $\mathbf{x}_{-i,m} = (x_{jm})_{j=1,2,...,N;\, j\neq i}$, and the corresponding direct demand in market m is

$$\begin{aligned} x_{im}(p_{im}, \mathbf{p}_{-i,m}; \omega_m, \beta_m, \gamma_m) \quad &= \quad \frac{1}{[1+(N-1)\delta_m](1-\delta_m)\beta_m} \\ &\quad \times \left\{ \omega_m(1-\delta_m) - [1+(N-2)\delta_m]p_{im} + \delta_m \sum_{j\neq i} p_{jm} \right\} \end{aligned}$$

for firm i, where $\mathbf{p}_{-i,m} = (p_{jm})_{j=1,2,...,N;\, j\neq i}$, and $\delta_m \equiv \gamma_m/\beta_m \in [0, 1)$ is the *strength of substitutability*: if δ_m is close to one, market m is approximated by perfect competition, whereas if δ_m is equal to zero, each firm behaves as a monopolist.

In symmetric equilibrium with $\mathbf{p}_m = (p, p, ..., p)$, the firm's demand in market m is given by

$$q_m(p) \quad = \quad \frac{\omega_m - p}{[1+(N-1)\delta_m]\beta_m},$$

and thus the own and the cross price elasticities can be obtained as shown in Table 1, which imply that the conduct parameter is given as a constant by

$$\theta_m(p) = \frac{1+(N-3)\delta_m}{1+(N-2)\delta_m} \equiv \widetilde{\theta}_m.$$

Then, the discriminatory price in market m satisfies Equation (12):

$$\begin{aligned} &\underbrace{\frac{p_m^* - c_m}{p_m^*}}_{=L_m(p_m^*)} = \underbrace{\frac{(1-\delta_m)(\omega_m - p_m^*)}{[1+(N-2)\delta_m]p_m^*}}_{=1/\epsilon_m^{own}(p_m^*)} \\ \Leftrightarrow \quad & p_m^* = p_m^*(c_m, \omega_m, \delta_m, N) \equiv \frac{(1-\delta_m)\omega_m + [1+(N-2)\delta_m]c_m}{2+(N-3)\delta_m}, \end{aligned}$$

whereas the equilibrium uniform price, $\overline{p}$, is derived by solving Equation (13):

$$\sum_{m=s,w} \frac{[1+(N-2)\delta_m](\overline{p}-c_m)}{[1+(N-1)\delta_m](1-\delta_m)\beta_m} = \sum_{m=s,w} \frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m},$$

because

$$\begin{aligned} \bar{y}_m \bar{\epsilon}_m^{own} \overline{L}_m &= \frac{\dfrac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}}{\displaystyle\sum_{m=s,w} \frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}} \cdot \frac{[1+(N-2)\delta_m]\overline{p}}{(1-\delta_m)(\omega_m - \overline{p})} \cdot \frac{\overline{p}-c_m}{\overline{p}} \\ &= \frac{\dfrac{[1+(N-2)\delta_m](\overline{p}-c_m)}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}}{\displaystyle\sum_{m=s,w} \frac{\omega_m - \overline{p}}{[1+(N-1)\delta_m]\beta_m}} \end{aligned}$$

for $m = s,\, w$, leading to an explicit solution:

$$\overline{p} = \overline{p}(\boldsymbol{c}, \boldsymbol{\omega}, \boldsymbol{\delta}, \boldsymbol{\beta}, N) \equiv \frac{\displaystyle\sum_{m=s,w} \frac{(1-\delta_m)\omega_m + [1+(N-2)\delta_m]c_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}}{\displaystyle\sum_{m=s,w} \frac{2+(N-3)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}},$$

where $\boldsymbol{c} = (c_s, c_w)$, $\boldsymbol{\omega} = (\omega_s, \omega_w)$, $\boldsymbol{\delta} = (\delta_s, \delta_w)$, and $\boldsymbol{\beta} = (\beta_s, \beta_w)$.

Finally, noting the two curvatures, α_m^{own} and α_m^{cross}, are necessarily zero and the conduct parameter is constant in each market, it is verified that

$$\begin{aligned} G(\boldsymbol{\delta}, \boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{c}, N) &= \frac{\bar{\mu}_s \widetilde{\theta}_s}{1+\bar{\theta}_s} - \frac{\bar{\mu}_w \widetilde{\theta}_w}{1+\bar{\theta}_w} \\ &= \frac{(\overline{p}-c_s)[1+(N-3)\delta_s]}{2+(2N-5)\delta_s} - \frac{(\overline{p}-c_w)[1+(N-3)\delta_w]}{2+(2N-5)\delta_w} \end{aligned}$$

and

$$\begin{aligned} H(\boldsymbol{\delta}, \boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{c}, N) &= \frac{\mu_s^* \widetilde{\theta}_s}{1+\theta_s^*} - \frac{\mu_w^* \widetilde{\theta}_w}{1+\theta_w^*} \\ &= \frac{(1-\delta_s)(\omega_s - c_s)[1+(N-3)\delta_s]}{[2+(N-3)\delta_s][2+(2N-5)\delta_s]} - \frac{(1-\delta_w)(\omega_w - c_w)[[1+(N-3)\delta_w]}{[2+(N-3)\delta_w][2+(2N-5)\delta_w]}. \end{aligned}$$

In Figure 2 with $\omega_s = 1.50$, $\omega_w = 1.00$, and $\beta_s = \beta_w = 1.00$, we consider the two cases of identical marginal costs ($c_s = c_w = 0.20$) on the left panel, and of different marginal costs ($c_s = 0.22$ and $c_w = 0.20$) on the right panel. The top panel assumes $N = 2$, whereas the bottle

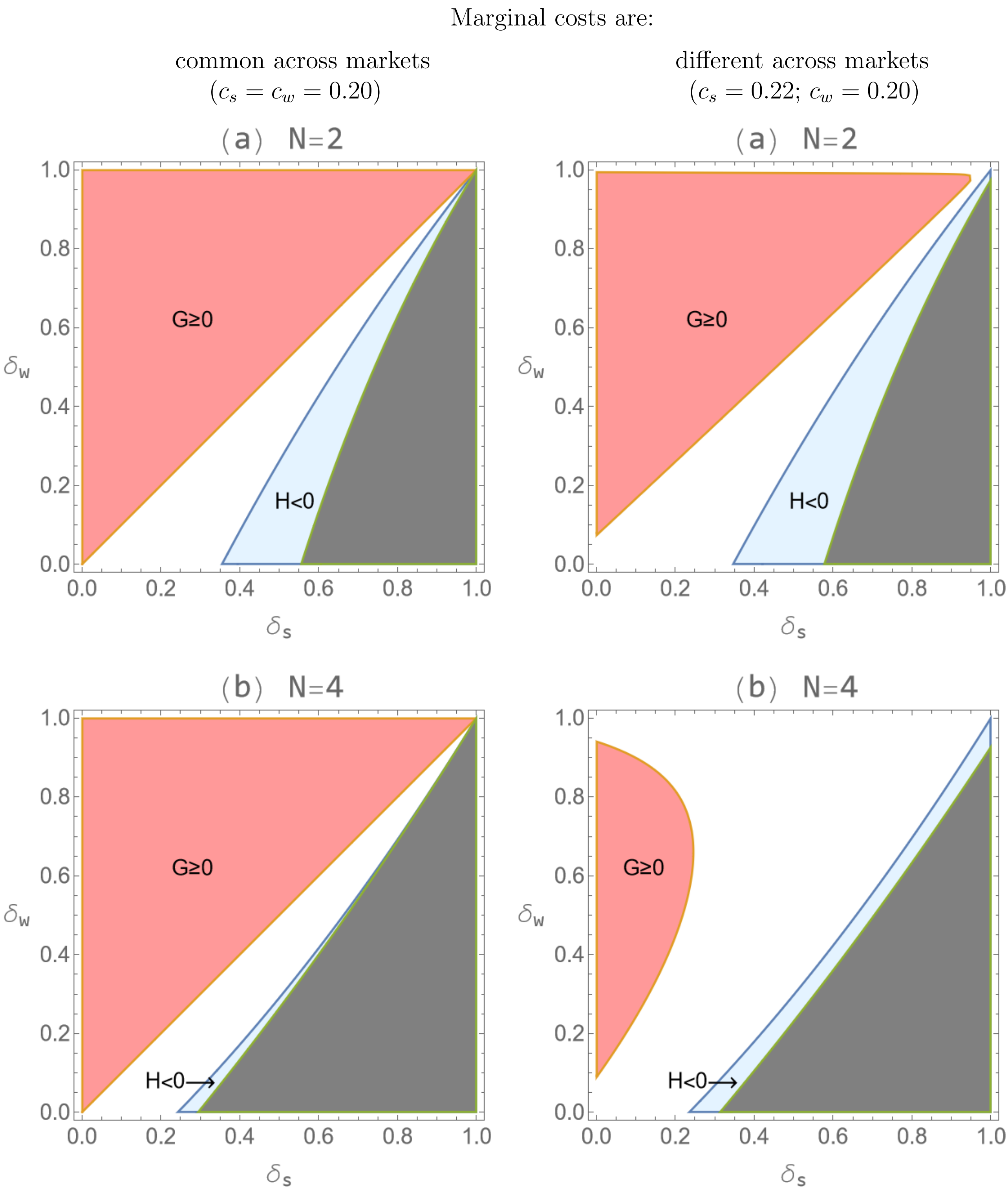

Figure 2: Linear demand with $\omega_s = 1.5$, $\omega_w = 1.0$, and $\beta_s = \beta_w = 1.0$. Note that for p_w^* to be actually lower than p_s^*, δ_s, relative to δ_w, must not be sufficiently large. Specifically, the lower-right shaded region of (δ_s, δ_w) must be excluded. The regions for $H < 0$ and for $G \geq 0$ are colored when $N = 2$ (top) and when $N = 4$ (bottom), depending on whether the two marginal costs are common (left) or different (right). Note also that $H < 0$ is *only a part* of the region where price discrimination improves social welfare and $G \geq 0$ is also *only a part* of the region where it reduces social welfare.

panel has $N = 4$ firms. In each of the four graphs, the dark-shaded are of the (δ_s, δ_w) region is excluded because $p_s^* \geq p_w^*$ does not holds. It is observed that the region for sufficiency for $\Delta W < 0$ when the current regime of uniform pricing is relaxed, i.e, $G(\delta_s, \delta_w; \boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{c}, N) \geq 0$, appears in the north-west side where the degree of substitutability in the weak market is sufficiently large. Here, a marginal increase in welfare gain due to the lower price in the weak market will be relatively low because the intensity of competition in the weak market is already high. Hence, welfare gain in the weak market due to the price reduction is limited and thus price discrimination is likely to reduce social welfare (Adachi and Matsushima (2014) also derive a similar finding from their Figures 4 and 5). This effect from competition is also prominent when the number of firms increases to $N = 4$: the region for $H(\delta_s, \delta_w; \boldsymbol{\omega}, \boldsymbol{\beta}, \boldsymbol{c}, N) < 0$ where sufficiency for $\Delta W > 0$, i.e, social welfare decreases if price discrimination is banned, shrinks.

4.2 CES (constant elasticity of substitution) demand

Suppose that the representative consumer's utility in market m is given by:

$$U_m(\mathbf{x}_m) = \left(\sum_{i=1}^{N} x_{im}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}},$$

where $\sigma_m > 1$ is the *constant elasticity of substitution* across products/firms (Vives 1999, pp. 147-8).[30] Then, the direct demand function for good/firm i is given by

$$x_{im}(p_{im}, \mathbf{p}_{-i,m}; \sigma_m) = \frac{p_{im}^{-\sigma_m}}{\sum_{j=1}^{N} p_{jm}^{1-\sigma_m}},$$

and, in symmetric equilibrium, the firm's demand in market m is

$$q_m(p) = \frac{1}{Np},$$

and thus the own and the cross price elasticities are obtained as constants as indicated in Table 1 and thus the conduct parameter is also given as a constant by

$$\theta_m(p) = \frac{2 + (N-2)\sigma_m}{1 + (N-1)\sigma_m} \equiv \widehat{\theta}_m.$$

[30] Anderson, de Palma, and Thisse (1992, pp. 85-90) discuss how this demand system can be microfounded by discrete choice modeling. The elasticity of substitution between the two goods is *constant*, $1/(1 - \beta_m)$

The own and the cross curvatures are also obtained as constants as shown in Table 1. Hence, let the sum of these two curvatures be denoted by

$$\begin{aligned}
\widehat{\alpha}_m &\equiv \alpha_m^{own}(p) + \alpha_m^{cross}(p) \\
&= \frac{N^2\sigma_m(1+\sigma_m) - 4(\sigma_m - 1)[1+(N-1)\sigma_m]}{N[1+(N-1)\sigma_m]}
\end{aligned}$$

Now, the discriminatory price in market m is obtained explicitly by solving Equation (12):

$$\begin{aligned}
\frac{p_m^* - c_m}{p_m^*} &= \frac{N}{1+(N-1)\sigma_m} \\
\Leftrightarrow \ p_m^* = p_m^*(c_m, \sigma_m, N) &\equiv \frac{1+(N-1)\sigma_m}{(N-1)(\sigma_m - 1)} c_m,
\end{aligned}$$

which indicate constant markup, whereas the equilibrium uniform price satisfies Equation (13):

$$\sum_{m=s,w} \frac{[1+(N-1)\sigma_m](\overline{p} - c_m)}{2N} = \overline{p}$$

because

$$\begin{aligned}
\bar{y}_m \bar{\epsilon}_m^{own} \overline{L}_m &= \frac{q_m(\overline{p})}{q_s(\overline{p}) + q_w(\overline{p})} \cdot \frac{1+(N-1)\sigma_m}{N} \cdot \frac{\overline{p} - c_m}{\overline{p}} \\
&= \frac{[1+(N-1)\sigma_m](\overline{p} - c_m)}{2N\overline{p}},
\end{aligned}$$

which leads to the following explicit solution:

$$\overline{p} = \overline{p}(\boldsymbol{c}, \boldsymbol{\sigma}, N) \equiv \frac{[1+(N-1)\sigma_s]c_s + [1+(N-1)\sigma_w]c_w}{(N-1)(\sigma_s + \sigma_w - 2)}.$$

Then, it is also verified that

$$\begin{aligned}
G(\boldsymbol{\sigma}, \boldsymbol{c}, N) &= \frac{\bar{\mu}_s \widehat{\theta}_s}{1 + \widehat{\theta}_s - \widehat{\alpha}_s \bar{L}_s} - \frac{\bar{\mu}_w \widehat{\theta}_w}{1 + \widehat{\theta}_w - \widehat{\alpha}_w \bar{L}_w} \\
&= \frac{1}{(N-1)\left(\sigma_s + \sigma_w - 2\right)} \\
&\quad \times \left[\frac{[2+(N-2)\sigma_s][-(N-1)\sigma_w\left(c_s - c_w\right) + (2N-1)c_s + c_w]}{[1+(N-1)\sigma_s]\left(\varSigma_s + 1\right)} \right. \\
&\quad \left. - \frac{[2+(N-2)\sigma_w][(N-1)\sigma_s\left(c_s - c_w\right) + (2N-1)c_w + c_s]}{[1+(N-1)\sigma_w]\left(\varSigma_w + 1\right)} \right],
\end{aligned}$$

where

$$\begin{cases} \Sigma_s \equiv \dfrac{\{\sigma_s[(N-2)^2\sigma_s + N(N+4) - 8] + 4\}[(N-1)\sigma_w (c_s - c_w) - (2N-1)c_s - c_w]}{N\left((N-1)\sigma_s + 1\right)\left(c_s\left((N-1)\sigma_s + 1\right) + c_w\left((N-1)\sigma_w + 1\right)\right)} \\ \qquad + \dfrac{2 + (N-2)\sigma_s}{1 + (N-1)\sigma_s} \\ \\ \Sigma_w \equiv -\dfrac{\{\sigma_w[(N-2)^2\sigma_w + N(N+4) - 8] + 4\}[(N-1)\sigma_s (c_s - c_w) + (2N-1)c_w + c_s]}{N\left((N-1)\sigma_w + 1\right)\left(c_s\left((N-1)\sigma_s + 1\right) + c_w\left((N-1)\sigma_w + 1\right)\right)} \\ \qquad + \dfrac{2 + (N-2)\sigma_w}{1 + (N-1)\sigma_w} \end{cases}$$

and

$$\begin{aligned} H(\boldsymbol{\sigma}, \boldsymbol{c}, N) &= \frac{\mu_s^* \widehat{\theta}_s}{1 + \widehat{\theta}_s - \widehat{\alpha}_s L_s^*} - \frac{\mu_w^* \widehat{\theta}_w}{1 + \widehat{\theta}_w - \widehat{\alpha}_w L_w^*} \\ &= \frac{N[2 + (N-2)\sigma_s][1 + (N-1)\sigma_s]c_s}{(\sigma_s - 1)^2\{N - 1 + [1 + N^2(N-2)]\sigma_s\}} \\ &\quad - \frac{N[2 + (N-2)\sigma_w][1 + (N-1)\sigma_w]c_w}{(\sigma_w - 1)^2\{N - 1 + [1 + N^2(N-2)]\sigma_w\}}. \end{aligned}$$

Note that in the case of CES demand, (σ_s, σ_w) is the only pair of demand parameters that determines which market is strong. Specifically, as in Figure 2 above, the degree of substitution in the weak market, σ_w, must be sufficiently high as compared to σ_s for $p_s^* \geq p_w^*$ to hold. Interestingly, Figure 3 shows that if differential costs are not allowed (i.e., the left panel), the region of $H(\sigma_s, \sigma_w; \boldsymbol{c}, N) < 0$ does not appear. Moreover, if $N = 4$, neither the region of $H < 0$ nor $G \geq 0$ appears, indicating that welfare assessment is not possible. However, once differential costs are allowed (i.e., the right panel), the region for $H < 0$ appears: if (σ_s, σ_w) belongs to this region, we can *definitely* conclude that price discrimination increases social welfare. By comparing Figure 3 with Figure 2, we can also find that (δ_s, δ_w) in the linear demand and (σ_s, σ_w) in the CES demand play a similar role.

4.3 Multinomial logit demand with outside option

Lastly, we consider the following share/demand function that each firm i faces in market $m = s, w$:

$$x_{im}(p_{im}, \mathbf{p}_{-i,m}; \omega_m, \beta_m) = \frac{\exp(\omega_m - \beta_m p_{jm})}{1 + \sum_{j=1}^{N} \exp(\omega_m - \beta_m p_{jm})} \in (0, 1),$$

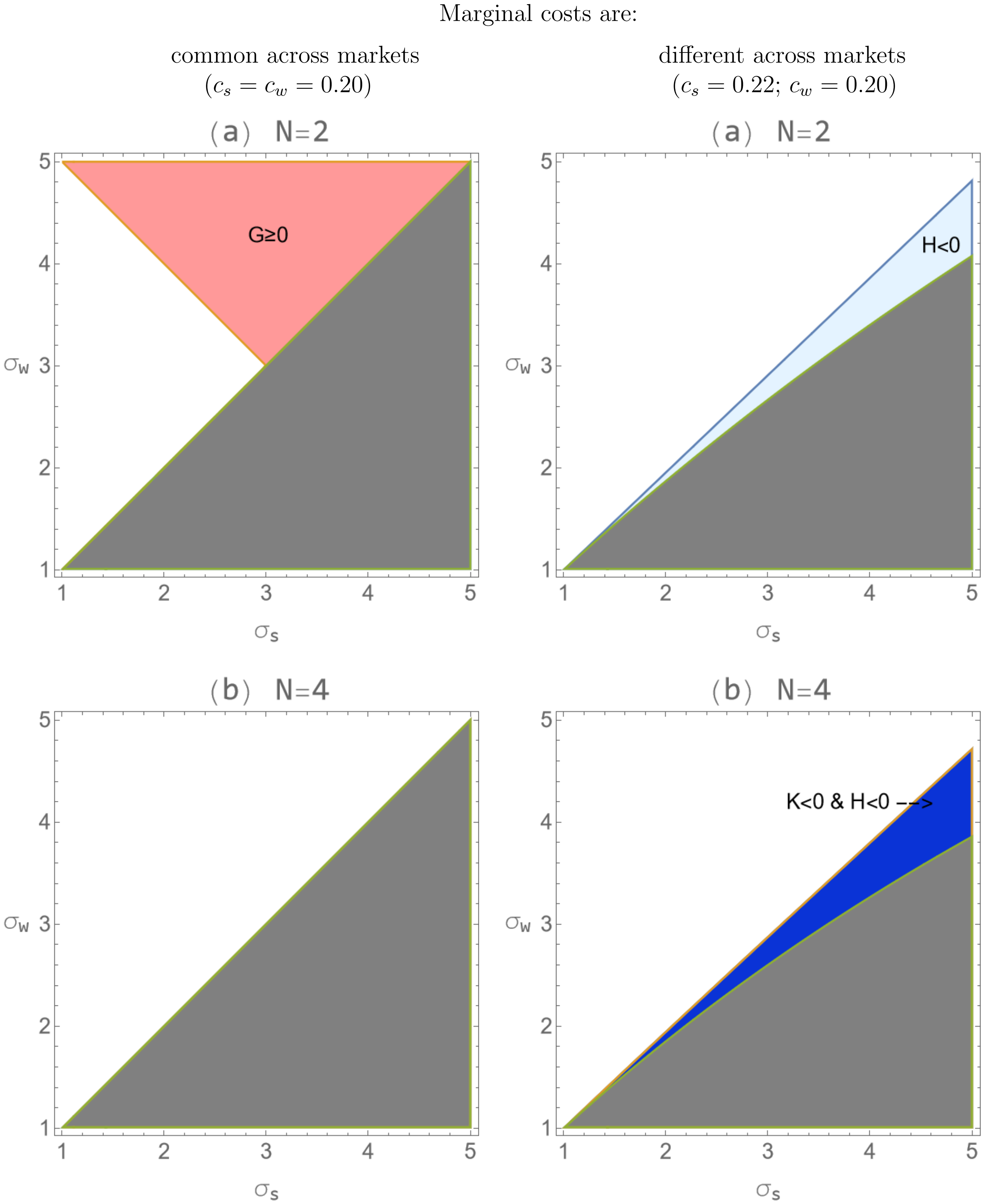

Figure 3: CES demand when $N = 2$ (top) and when $N = 4$ (bottom). As in Figure 2, the dark-shaded region of (σ_s, σ_w) is excluded for p_s^* to be actually higher than p_w^*. The regions for $H < 0$ and for $G \geq 0$ are colored when the two marginal costs are common (left) or different (right). Note also that $H < 0$ is *only a part* of the region where price discrimination improves social welfare and $G \geq 0$ is also *only a part* of the region where it reduces social welfare.

where $\omega_m > 0$ is now the product-specific utility, and $\beta_m > 0$ is the *price responsiveness* of the representative consumer in market m.[31] Then, under symmetric pricing, each firm's market share is given by

$$q_m(p; N) = \frac{\exp(\omega_m - \beta_m p)}{1 + N \cdot \exp(\omega_m - \beta_m p)},$$

where the dependence on N is made explicit for clarity. For any $N \geq 2$, the own and cross price elasticities are $\epsilon_m^{own}(p) = \beta_m p \cdot [1 - q_m(p; N)]$ and $\epsilon_m^{cross}(p) = \beta_m p q_m(p; N)$, respectively. Hence, the conduct parameter is given by

$$\theta_m(p) = \frac{1 - 2q_m(p; N)}{1 - q_m(p; N)}.$$

Similarly, the two curvatures are also obtained as shown in Table 1. the symmetric discriminatory equilibrium price $p_m^* = p_m^*(c_m, \omega_m, \beta_m, N)$ satisfies:

$$\underbrace{p_m^* - c_m}_{=\mu_m^*} - \frac{1}{\beta_m(1 - q_m^*)} = 0$$

and

$$q_m^* \equiv q_m(p_m^*; N) = \frac{\exp(\omega_m - \beta_m p_m^*)}{1 + N \cdot \exp(\omega_m - \beta_m p_m^*)}.$$

Both p_m^* and q_m^* should be jointly solved numerically. Similarly, the equilibrium uniform price $\overline{p} = \overline{p}(\mathbf{c}, \boldsymbol{\omega}, \boldsymbol{\beta}, N)$ satisfies

$$\sum_{m=s,w} q_m(\overline{p}; N) \left\{1 - \beta_m(\overline{p} - c_m)[1 - q_m(\overline{p}; N)]\right\} = 0,$$

which should also be numerically solved.

It is thus shown that

$$G(\boldsymbol{\beta}, \boldsymbol{\omega}, \boldsymbol{c}, N) = \frac{(\overline{p} - c_s)(1 - 2\overline{q}_s)}{2 - 3\overline{q}_s - \beta_s(\overline{p} - c_s)(1 - 2\overline{q}_s)} - \frac{(\overline{p} - c_w)(1 - 2\overline{q}_w)}{2 - 3\overline{q}_w - \beta_w(\overline{p} - c_w)(1 - 2\overline{q}_w)}$$

and

$$H(\boldsymbol{\beta}, \boldsymbol{\omega}, \boldsymbol{c}, N) = \frac{(p_s^* - c_s)(1 - 2q_s^*)}{2 - 3q_s^* - \beta_s(p_s^* - c_s)(1 - 2q_s^*)} - \frac{(p_w^* - c_w)(1 - 2q_w^*)}{2 - 3q_w^* - \beta_w(p_s^* - c_w)(1 - 2q_w^*)}.$$

[31] Anderson, de Palma, and Thisse (1987) argue that the indirect utility of the representative consumer in market m is given by

$$V_m(\mathbf{p}_m) = \frac{\ln\left[\sum_{i=1}^{N} \exp\left(\omega_m - \beta_m p_{im}\right)\right]}{\beta_m}.$$

This demand form can also be microfounded by the random utility model (see, e.g., Anderson, de Palma, and Thisse 1992, Ch. 2).

Figure 4 shows that the same type of argument carries over from the cases of linear demand. These examples exhibit similarity between linear and multinomial demands in terms of predictions based on our analysis using sufficient statistics. Recall that the cost structure is set common for both demands. It is observed that if linear or multinomial logit demand is assumed, intense competition in the weak market due to a lesser degree of product differentiation in that market is likely to justify a ban against price discrimination because this is the case where $G \geq 0$ is likely to hold.

5 Concluding remarks

This paper presents the theoretical implications of oligopolistic third-degree price discrimination with general non-linear demand, allowing cost differentials to exist across separate markets. In this sense, this paper, with the help of the methodology proposed by Weyl and Fabinger (2013), generalizes Aguirre, Cowan, and Vickers' (2010) analysis of monopolistic third-degree price discrimination to complement Chen, Li, and Schwartz' (2015) analysis of oligopolistic differential pricing.

Our theoretical analysis, which accommodates firm heterogeneity, can also be utilized to empirically assess the welfare effects of third-degree price discrimination under oligopoly. In particular, in line with the "sufficient statistics" approach (Chetty 2009; Kleven 2021; and Barnichon and Mesters 2022), our predictions regarding the welfare effects do *not* rely on functional specifications, and are thus considered to be fairly robust, although these sufficient statistics can take different values, depending on functional specifications. However, once the numerical values of sufficient statistics are obtained, there should be no disagreement regarding welfare assessment.

As a promising direction, it would be interesting to apply our methodology to the analysis of the welfare effects of *wholesale/input* third-degree price discrimination (Katz 1987; DeGraba 1990; Yoshida 2000; Inderst and Valletti 2009; Villas-Boas 2009; Arya and Mittenforf 2010; Li 2014; O'Brien 2014; Miklós-Thal and Shaffer 2021b, 2021c; and Gaudin and Lestage 2023).[32] To do so, one would need to properly define the sufficient statistics at each stage of a vertical relationship. Another important issue to consider is the case of *multi-product* oligopolistic firms (Armstrong and Vickers 2018; and Nocke and Schutz 2018). What happens if price discrimination is allowed for some products, whereas uniform pricing is enforced for others? These and other important issues related to third-degree price discrimination await further research.

[32] See Jaffe and Weyl (2013) for such an attempt.

Marginal costs are:

common across markets ($c_s = c_w = 0.20$) | different across markets ($c_s = 0.22$; $c_w = 0.20$)

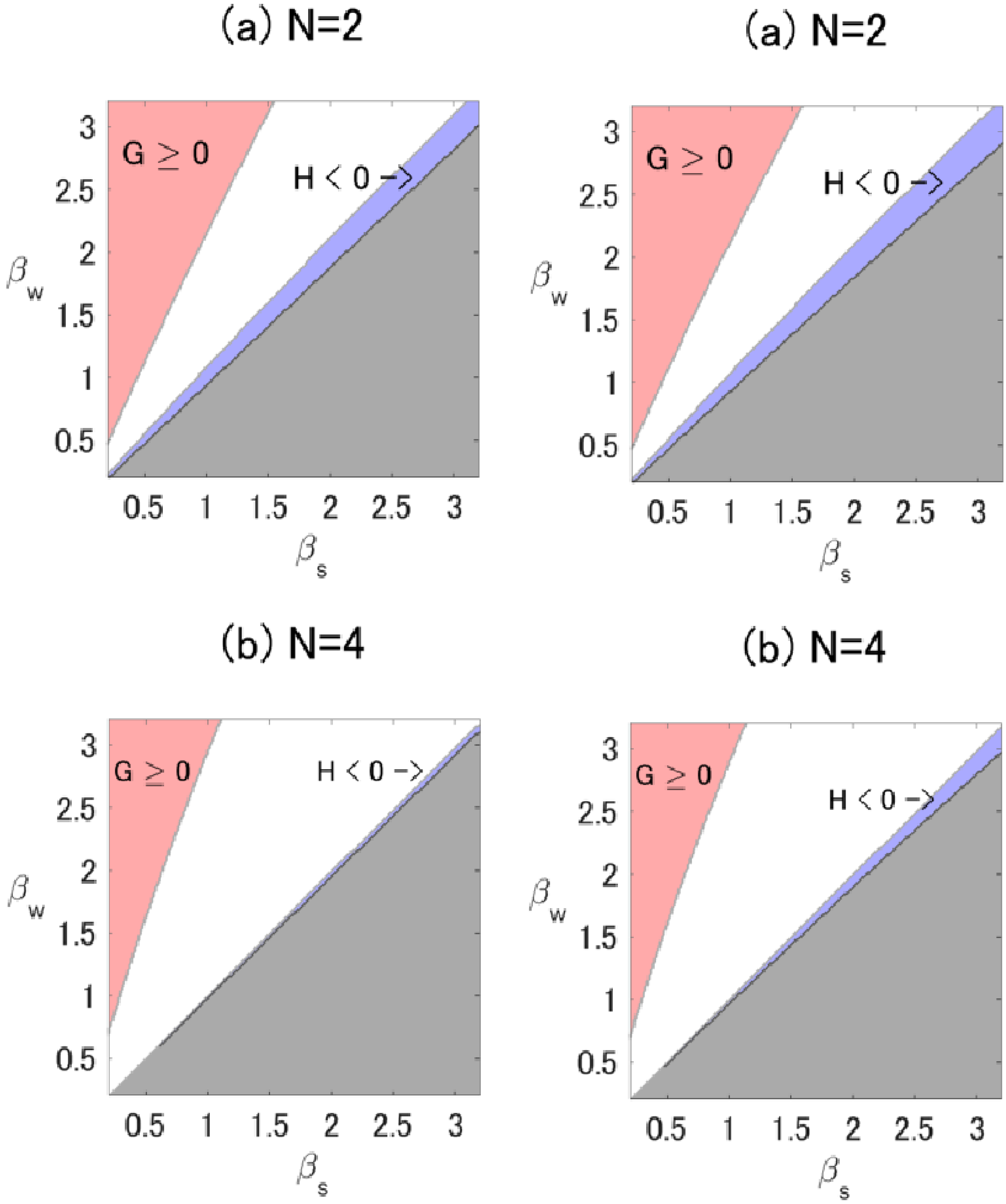

Figure 4: Multinomial logit demand with $\omega_s = 1.5$ and $\omega_w = 1.0$ when $N = 2$ (top) and when $N = 4$ (bottom). As in Figures 2, and 3, the region of (β_s, β_w) where the price coefficient for the strong market β_s relatively large as compared to β_w is excluded (the dark-shaded area). The regions for $H < 0$ and for $G \geq 0$ are colored when the two marginal costs are common (left) or different (right). Note also that $H < 0$ is *only a part* of the region where price discrimination improves social welfare and $G \geq 0$ is also *only a part* of the region where it reduces social welfare.

Appendix A. Proof of Proposition 1

First, for $t = t^*$, it is observed that

$$\begin{aligned} z_m(p_m^*) &= \left(\frac{-q_m(p_m^*)}{\frac{\partial x_{Am}}{\partial p_A}(p_m^*, p_m^*)} \right) \theta_m(p_m^*) \rho_m(p_m^*) \\ &= \mu_m(p_m^*) \theta_m(p_m^*) \rho_m(p_m^*) \end{aligned}$$

and thus

$$\frac{W'(t^*)}{2} = \left(-\frac{\pi_s'' \pi_w''}{\pi_s'' + \pi_w''} \right) (\mu_w^* \theta_w^* \rho_w^* - \mu_s^* \theta_s^* \rho_s^*) > 0$$

if $\mu_w^* \theta_w^* \rho_w^* > \mu_s^* \theta_s^* \rho_s^*$ holds.[33] Given the IRCW, this means that $W(t)$ is strictly increasing in $[0, t^*]$. This completes the proof for the first part.

Next, for $t < t^*$, note that

$$\begin{aligned} z_m(p_m) &= \theta_m \left(\frac{-q_m}{\partial x_{Am}/\partial p_A} \right) \left(\frac{\partial x_{Am}/\partial p_A}{\pi_m''} \right) \\ &= \mu_m \theta_m \underbrace{\left(\frac{\partial x_{Am}/\partial p_A}{\pi_s'' + \pi_w''} \right)}_{=\rho_m} \left(\frac{\pi_s'' + \pi_w''}{\pi_m''} \right). \end{aligned}$$

Thus, it is verified that

$$\begin{aligned} \frac{W'(t)}{2} &= \left(-\frac{\pi_s'' \pi_w''}{\pi_s'' + \pi_w''} \right) \left(\mu_w \theta_w \rho_w \frac{\pi_s'' + \pi_w''}{\pi_w''} - \mu_s \theta_s \rho_s \frac{\pi_s'' + \pi_w''}{\pi_s''} \right) \\ &= \underbrace{(-\pi_s'' \pi_w'')}_{<0} \left(\frac{\mu_w(t) \theta_w(t) \rho_w(t)}{\pi_w''} - \frac{\mu_s(t) \theta_s(t) \rho_s(t)}{\pi_s''} \right), \end{aligned}$$

which implies that given the IRCW, $W(t)$ is strictly decreasing in $[0, t^*]$ if $W'(0) \leq 0$. This completes the proof for the second part.

References

Adachi, T., Fabinger, M., 2022. Pass-through, welfare, and incidence under imperfect competition. *Journal of Public Economics*, 211, 104589.

Adachi, T., Matsushima, N., 2014. The welfare effects of third-degree price discrimination in a differentiated oligopoly. *Economic Inquiry*, 52(3), 1231-1244.

[33] This derivation is partly due to Michal Fabinger.

Aguirre, I., 2016. On the economics of the "Meeting Competition Defense" under the Robinson–Patman Act. *The B.E. Journal of Economic Analysis & Policy*, 16(3), 1213-1238.

Aguirre, I., 2019. Oligopoly price discrimination, competitive pressure and total output. *Economics: The Open-Access, Open-Assessment E-Journal*, 13 (2019-52). 1-16.

Aguirre, I., Cowan, S., Vickers, J., 2010. Monopoly price discrimination and demand curvature. *American Economic Review*, 100(4), 1601-1615.

Anderson, S. P., de Palma, A., Thisse, J.-F., 1987. A representative consumer theory of the logit model. *International Economic Review*, 29(3), 461-466.

Anderson, S. P., de Palma, A., Thisse, J.-F., 1992. *Discrete Choice Theory of Product Differentiation*, The MIT Press.

Armstrong, M., 2006. Recent developments in the economics of price discrimination. In R. Blundell, W. Newey, and T. Persson (eds.), *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, Vol. 2, Cambridge University Press, 97-141.

Armstrong, M., 2008. Price discrimination. In P. Buccirossi (ed.), *Handbook of Antitrust Economics*, The MIT Press, 433-467.

Armstrong, M., Vickers, J., 2001. Competitive price discrimination. *RAND Journal of Economics*, 32(4), 579-605.

Armstrong, M., Vickers, J., 2018. Multiproduct pricing made simple. *Journal of Political Economy*, 126(4), 1444-1471.

Arya, A., Mittenforf, B., 2010. Input price discrimination when buyers operate in multiple markets. *Journal of Industrial Economics*, 58(4), 846-867.

Barnichon, R., Mesters, G., 2022. A sufficient statistics approach for macro policy evaluation. Unpublished manuscript.

Beggs, A., 2021. Demand functions and demand manifolds. *Economic Theory Bulletin*, 9, 195-207.

Bertoletti, P., 2009. On the output criterion for price discrimination. *Economics Bulletin*, 29(4), 2951-2956.

Boik, A., 2017. The empirical effects of competition on third degree price discrimination in the presence of arbitrage. *Canadian Journal of Economics*, 50 (4), 1023-1036.

Bresnahan, T. F., 1989. Empirical studies of industries with market power. In R. Schmalensee and R. D. Willig (eds.), *Handbook of Industrial Organization*, Vol. 2, Elsevier Science Publishers B.V., 1011-1057.

Chen, Y., Li, J., Schwartz, M., 2021. Competitive differential pricing. *RAND Journal of Economics*, 52(1), 100-124.

Chen, Y., Schwartz, M., 2015. Differential pricing when costs differ: A welfare analysis. *RAND Journal of Economics*, 46(2), 442-460.

Chetty, R., 2009. Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods. *Annual Review of Economics*, 1, 451-488.

Choné, P., Linnemer, L., 2020. Linear demand systems for differentiated goods: Overview and user's guide. *International Journal of Industrial Organization*, 73, 102663.

Clerides, S. K. 2004. Price discrimination with differentiated products: Definition and identification. *Economic Inquiry*, 42(3), 402-412.

Corts, K. S., 1998. Third-degree price discrimination in oligopoly: All-out competition and strategic commitment. *RAND Journal of Economics*, 29(2), 306-323.

Corts, K. S., 1999. Conduct parameters and the measurement of market power. *Journal of Econometrics*, 88(2), 227-250.

Cowan, S., 2012. Third-degree price discrimination and consumer surplus. *Journal of Industrial Economics*, 60(2), 333-345.

Cowan, S., 2016. Welfare-increasing third-degree price discrimination. *RAND Journal of Economics*, 47(2), 326-340.

Cowan, S., 2017. A model of third-degree price discrimination with positive welfare effects. Unpublished manuscript.

Cowan, S., 2018. Regulating monopoly price discrimination. *Journal of Regulatory Economics*, 54(1), 1-13.

Dastidar, K. G., 2006. On third-degree price discrimination in oligopoly. *The Manchester School*, 74(2), 231-250.

DellaVigna, S., Gentzkow, M., 2019. Uniform pricing in U.S. retail chains. *Quarterly Journal of Economics*, 134(4), 2011-2084.

Dertwinkel-Kalt, M., Wey, C., 2023. Third-degree price discrimination in oligopoly when markets are covered. *Journal of Industrial Economics*, Forthcoming.

DeGraba, P., 1990. Input market price discrimination and the choice of technology. *American Economic Review*, 80(5), 1246-1253.

De Loecker, J., Goldberg, P. K., Khandelwal, A. K., Pavcnik, N., 2016. Prices, markups and trade reform. *Econometrica*, 84(2), 445-510.

Flores, D., Kalashnikov, V., 2017. Parking discounts: Price discrimination with different marginal costs. *Review of Industrial Organization*, 50(1), 91-103.

Galera, F., Zaratiegui, J. M., 2006. Welfare and output in third-degree price discrimination: A note. *International Journal of Industrial Organization*, 24(3), 605-611.

Gaudin, G., Lestage, R., 2023. Input price discrimination, demand forms, and welfare. *Journal of Industrial Economics*, Forthcoming.

Genesove, D., and Mullin, W. P., 1998. Testing static oligopoly models: Conduct and cost in the sugar industry, 1890-1914. *RAND Journal of Economics*, 29(2), 355-377.

Geruso, M., 2017. Demand heterogeneity in insurance markets: Implications for equity and efficiency. *Quantitative Economics*, 8(3), 929-975.

Hendel, I., Nevo, A., 2013. Intertemporal price discrimination in storable goods markets. *American Economic Review*, 103(7), 2722-2751.

Holmes, T. J., 1989. The effects of third-degree price discrimination in oligopoly. *American Economic Review*, 79(1), 244-250.

Hotelling, H., 1929. Stability in competition. *Economic Journal*, 39(153), 41-57.

Inderst, R., Valletti, T., 2009. Price discrimination in input markets. *RAND Journal of Economics*, 40(1), 1-19.

Jaffe, S., Weyl, E. G., 2013. The first-order approach to merger analysis. *American Economic Journal: Microeconomics*, 5(4), 188-218.

Katz, M. L., 1987. The welfare effects of third-degree price discrimination in intermediate goods markets. *American Economic Review*, 77(1), 154-167.

Kleven, H. J., 2021. Sufficient statistics revisited. *Annual Review of Economics*, 13, 515-538.

Leontief, W., 1940. The theory of limited and unlimited discrimination. *Quarterly Journal of Economics*, 54(3), 490-501.

Li, Y., 2014. A note on third degree price discrimination in intermediate good markets. *Journal of Industrial Economics*, 62(3), 554.

Melitz, M. J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6), 1695-1725.

Miklós-Thal, J., Shaffer, G., 2021a. Pass-through as an economic tool: On exogenous competition, social incidence, and price discrimination. *Journal of Political Economy*, 129(1), 323-335.

Miklós-Thal, J., Shaffer, G., 2021b. Third-degree price discrimination in oligopoly with endogenous input costs. *International Journal of Industrial Organization*, 79, 102713.

Miklós-Thal, J., Shaffer, G., 2021c. Input price discrimination by resale market. *RAND Journal of Economics*, 52(4), 727-757.

Mrázová, M., and Neary, J. P., 2017. Not so demanding: Demand structure and firm behavior. *American Economic Review*, 107(12), 3835-3874.

Nahata, B., Ostaszewski, K., Sahoo, P. K., 1990. Direction of price changes in third-degree price discrimination. *American Economic Review*, 80(5), 1254-1258.

Nocke, V., Schutz, N., 2018. Multiproduct-firm oligopoly: An aggregative games approach. *Econometrica*, 86(2), 523-557.

O'Brien, D. P., 2014. The welfare effects of third-degree price discrimination in intermediate good markets: The case of bargaining. *RAND Journal of Economics*, 45(1), 92-115.

Okada, T., 2014. Third-degree price discrimination with fairness-concerned consumers. *The Manchester School*, 86 (6), 701-715.

Phlips, L., 1983. *The Economics of Price Discrimination*, Cambridge University Press.

Pigou, A. C., 1920. *The Economics of Welfare*, Macmillan.

Quint, D., 2014. Imperfect competition with complements and substitutes. *Journal of Economic Theory*, 152, 266-290.

Rhodes, A., Zhou, J., 2022. Personalized pricing and competition. Unpublished manuscript.

Robinson, J., 1933. *The Economics of Imperfect Competition*, Macmillan.

Schmalensee, R., 1981. Output and welfare implications of monopolistic third-degree price discrimination. *American Economic Review*, 71(1), 242-247.

Schwartz, M., 1990. Third-degree price discrimination and output: Generalizing a welfare result. *American Economic Review*, 80(5), 1259-1262.

Shapiro, C., 1996. Mergers with differentiated products. *Antitrust*, 10(2), 23-30.

Shubik, M., with Levitan, R., 1980. *Market Structure and Behavior*, Harvard University Press.

Silberberg, E., 1970. Output under discriminating monopoly: A revisit. *Southern Economic Journal*, 37(1), 84-87.

Somaini, P., Einav, L., 2013. A model of market power in customer markets. *Journal of Industrial Economics*, 61(4), 938-986.

Stigler, G. J., 1987. *The Theory of Price, Fourth Edition*, Macmillan.

Stole, L. A., 2007. Price discrimination and competition. In M. Armstrong and R. Porter (eds.), *Handbook of Industrial Organization*, Volume 3, Elsevier B.V., 2221-2299.

Varian, H. R., 1985. Price discrimination and social welfare. *American Economic Review*, 75(4), 870-875.

Varian, H. R., 1989. Price discrimination. In R. Schmalensee and R. Willig (eds.), *Handbook of Industrial Organization*, Volume 1, Elsevier B.V., 597-654.

Vickers, J., 2020. Direct welfare analysis of relative price regulation. *Journal of Industrial Economics*, 68(1), 40-51.

Villas-Boas, S. B., 2009. An empirical investigation of the welfare effects of banning wholesale price discrimination. *RAND Journal of Economics*, 40(1), 20-46.

Vives, X., 1999. *Oligopoly Pricing: Old Ideas and New Tools*, The MIT Press.

Weyl, E. G., Fabinger, M., 2013. Pass-through as an economic tool: Principle of Incidence under Imperfect Competition. *Journal of Political Economy*, 121(3), 528-583.

Yoshida, Y., 2000. Third-degree price discrimination in input markets: Output and welfare. *American Economic Review*, 90(1), 240-246.

Online Appendices for "A sufficient statistics approach for welfare analysis of oligopolistic third-degree price discrimination"

Online Appendix A. The case of a general number of markets

Throughout this paper, we assume one strong market and one weak market. More generally, by defining $S \equiv \{m|p_m^* > \overline{p}\}$ and $W \equiv \{m|\overline{p} > p_m^*\}$, we can let $p_s(r)$, $s \in S$, and $p_w(r)$, $w \in S$, consist of the optimal price vector under constraints $|p_m - \overline{p}| \leq r$ for all $m \in S \cup W$, with $r \in [0, \max_m |p_m^* - \overline{p}|]$. Then, for example, social welfare is defined as $W(r; c_s, c_w) \equiv \sum_{s\in S} \widetilde{U}_s(q_s[p_s(r)]) + \sum_{w\in W} \widetilde{U}_w(q_w[p_w(r)]) - 2\sum_{s\in S}(c_s \cdot q_s[p_s(r)]) - 2\sum_{w\in W}(c_w \cdot q_w[p_w(r)])$. Clearly, our two-market analysis does not lose any validity. In this way, an increase in the *weighted* aggregate output, $\sum_{m=s,w}(\overline{p} - c_m)\, q_m'[p_m(r)]p_m'(r) > 0$, can be written as $\mathbb{E}[(\overline{p} - c_m)q_m'p_m'] > 0$ with a general number of markets. For this to hold, $\mathrm{Cov}(\overline{p} - c_m, q_m'p_m') > 0$, that is, on average, the markup under uniform pricing and $q_m'p_m' > 0$ must be positively correlated because $\mathbb{E}[(\overline{p} - c_m)] > 0$ and $\mathbb{E}[q_m'p_m'] > 0$.

Online Appendix B. Aggregate output and consumer surplus

In this appendix, we discuss how we can extend our sufficient statistics approach to analysis of aggregate output as well as consumer surplus.

In a similar way for $W(t)$ defined in Subsection 3.1, aggregate output under pricing is given by

$$Q(t) = Q_s(t) + Q_w(t) = 2\{q_s[p_s(t)] + q_w[p_w(t)]\},$$

whereas consumer surplus is defined by replacing c_m in $W(t)$ by $p_m(t)$ to define

$$CS(t) = U_s(q_s[p_s(t)]) + U_w(q_w[p_w(t)]) - 2p_s(t) \cdot q_s[p_s(t)] - 2p_w(t) \cdot q_w[p_w(t)].$$

Hence, $Q(t)$ and $CS(t)$ are also functions of $t \in [0, t^*]$. The regime change from uniform pricing to price discrimination is captured by a parameter shift from $t = 0$ to $t = t^*$, and vice versa. However, if these functions are globally concave in this range, then the local sign at $t = 0$ or t^* may predict the sign from the regime change. Specifically, consider a representative function, $F(t)$. If the global concavity of $F(t)$ is assured, then $F(t)$ behaves in either manner:

1. If $F'(0) \leq 0$, then $F(t)/2$ is monotonically decreasing in t, and as a result $\Delta F/2 = [F(t^*) - F(0)]/2 < 0$; *price discrimination decreases F*.

2. If $F'(0) > 0$, then $F(t)/2$ either

(a) is monotonically increasing (if $F'(t^*) > 0$, this is true), and as a result, $\Delta F/2 > 0$; *price discrimination increases* F.

(b) first increases, and then after the reaching the maximum (where $F'(t) = 0$), decreases until $t = t^*$. In this case, *price discrimination may increase or decrease* F: it cannot be determined whether $\Delta F/2 < 0$ or $\Delta F/2 > 0$ without further functional and/or parametric restrictions.

Below, we verify whether and how $Q(t)$ and $CS(t)$ are related to this argument.

B1. Output effects

First, we define $h_m(p) \equiv q'_m(p)/\pi''_m(p) > 0$ so that

$$\frac{Q'(t)}{2} = \underbrace{\left(-\frac{\pi''_s \pi''_w}{\pi''_s + \pi''_w}\right)}_{>0} \{h_w[p_w(t)] - h_s[p_s(t)]\}. \tag{18}$$

and assume that this h_m is *increasing* (and call it the increasing ratio condition for quantity; IRCQ). This condition is equivalent to $\varsigma^I_m(p) > \alpha^I_m(p)$, where

$$\varsigma^I_m(p) \equiv -\frac{d\pi''_m}{dp}\frac{p}{\pi''_m} = -\frac{p\pi'''_m}{\pi''_m}$$

is the *industry-level price elasticity of* π''_m and $\alpha^I_m(p) \equiv -pq''_m/q'_m$ is the *industry-level demand curvature.*[34] It is also expressed by $\nu^I_m(p) > 0$, where $\nu^I_m(p) \equiv -ph'_m/h_m$ is the *industry-level price elasticity of* h_m.[35] Now, it is shown that

$$\frac{Q''(t)}{2} = \left(-\frac{q'_s q'_w}{\pi''_s + \pi''_w}\right)[h'_s p'_s - h'_w p'_w] + [h_s - h_w]\frac{d}{dr}\left(-\frac{q'_s q'_w}{\pi''_s + \pi''_w}\right),$$

[34] To see this, note that $h'_m < 0$ is equivalent to $\pi'''_m > (\pi''_m/q'_m)q''_m$, where $\pi''_m/q'_m > 0$, because $h'_m(p) = (\pi'''_m q'_m - \pi''_m q''_m)/(q'_m)^2$, which implies that:

$$h'_m < 0 \Leftrightarrow -\frac{p\pi'''_m}{\pi''_m} > -\frac{pq''_m}{q'_m} \Leftrightarrow \varsigma^I_m > \alpha^I_m.$$

Essentially, the IRCQ states that the profit function, starting from the zero price, increases quickly, attaining the optimal price, and then decreases slowly as p becomes larger and larger beyond the optimum. In this way, the optimal price is reached "close" enough to the zero price, rather than "still climbing up" even far away from it. To see this, if $q''_m > 0$, then it is necessary for π'''_m to be positive. This means that π''_m, which is negative, should be larger (i.e., the negative slope of π''_m should be gentler) as p increases. If $q''_m \leq 0$, then π'''_m should be, whether it is positive or negative, sufficiently large. In either case, as p increases, π_m increases quickly below the optimum, and decreases slowly beyond it.

[35] This is because it is verified that

$$\frac{h'_m}{h_m} = \underbrace{\frac{\pi'''_m q'_m - \pi''_m q''_m}{[q'_m]^2}}_{<0} \cdot \underbrace{\frac{q'_m}{\pi''_m}}_{>0} = \frac{\pi'''_m q'_m - \pi''_m q''_m}{q'_m \pi''_m},$$

so that there exists $\widehat{t}$ such that $Q'(\widehat{t}) = 0$ and

$$\frac{Q''(\widehat{t})}{2} = -\frac{q_s' q_w'}{\pi_s'' + \pi_w''}(h_s' p_s' - h_w' p_w') < 0$$

because $h_s' p_s' < 0$ and $h_w' p_w' > 0$, implying the the global concavity of $Q(t)$ is attained. Based on these results, the following proposition is obtained.

Proposition 2. *Given the IRCQ, price discrimination increases aggregate output if*

$$\theta_s^* \rho_s^* < \theta_w^* \rho_w^*$$

holds, and it decreases aggregate output if

$$\frac{\overline{\theta}_s \overline{\rho}_s}{\overline{\pi}_s''} \geq \frac{\overline{\theta}_w \overline{\rho}_w}{\overline{\pi}_w''}$$

holds.[36]

Proof. First, note that $q_m'/\pi_m'' = \theta_m^* \cdot \rho_m^*$. Then, this implies that from Equality (18) above,

$$\frac{Q'(\bar{t})}{2} = \left(-\frac{\pi_s'' \pi_w''}{\pi_s'' + \pi_w''}\right)(\theta_w^* \rho_w^* - \theta_s^* \rho_s^*).$$

Conversely, for $t < \bar{t}$, it can be verified that:

$$\begin{aligned}
\frac{Q'(t)}{2} &= \left(\frac{q_w'}{\partial x_{A,w}/\partial p_A}\right)\left(\frac{\partial x_{A,w}}{\partial p_A}\right) p_w' + \left(\frac{q_s'}{\partial x_{A,s}/\partial p_A}\right)\left(\frac{\partial x_{A,s}}{\partial p_A}\right) p_s' \\
&= \theta_w \left(\frac{\partial x_{A,w}/\partial p_A}{\pi_s'' + \pi_w''}\right)(\pi_s'' + \pi_w'') p_w' + \theta_s \left(\frac{\partial x_{A,s}/\partial p_A}{\pi_s'' + \pi_w''}\right)(\pi_s'' + \pi_w'') p_s' \\
&= \underbrace{(-\pi_s'' \pi_w'')}_{<0} \left(\frac{\theta_w(t)\rho_w(t)}{\pi_w''} - \frac{\theta_s(t)\rho_s(t)}{\pi_s''}\right),
\end{aligned}$$

which completes the proof. □

Online Appendix E discusses the relationship between Holmes' (1989) expression of the

---

and thus,

$$\nu_m^I = \left(-\frac{p\pi_m'''}{\pi_m''}\right) - \left(-\frac{pq_m''}{q_m'}\right) = \varsigma_m^I - \alpha_m^I,$$

which implies that $h_m' < 0 \Leftrightarrow \nu_m^I > 0$.

[36] As Holmes (1989, fn.2) points out, an increase in aggregate output by price discrimination is necessary for an increase in welfare. In our model, this can be readily observed when marginal costs are identical across markets: the latter inequality in this proposition is implied by the second inequality in Proposition 1.

output effect and ours, and interprets his result in terms of the sufficient statistics.[37]

B2. Consumer surplus

First, note that

$$\begin{aligned}\frac{CS'(t)}{2} &= p_s(r)\cdot {q'}_s \cdot p_s'(t) + p_w(t)\cdot q_w' \cdot p_w'(t) \\ &\quad - p_s'(t)[p_s(t)\cdot q_s' + q_s] - p_w'(t)[p_w(t)\cdot q_w' + q_w] \\ &= -[p_s'(t)q_s + p_w'(t)q_w] \\ &= \underbrace{\left(-\frac{\pi_s''\pi_w''}{\pi_s''+\pi_w''}\right)}_{>0}\{g_s[p_s(t)] - g_w[p_w(t)]\}, \end{aligned} \tag{19}$$

where $g_m(p) \equiv q_m(p)/\pi_m''(p)$. If g_m is assumed to be *decreasing*, then one can use a similar argument. We call this the decreasing marginal consumer loss condition (DMCLC), which is equivalent to $\varsigma_m^I(p) > \epsilon_m^I(p)$.[38] Then, the global concavity of $CS(t)$ is attained. Thus, we can

[37]If h_m is decreasing, as we assume throughout, then z_m is increasing because

$$z_m'(p) = \frac{1 - z_m(p)h_m'(p)}{h_m(p)}$$

so that z_m' is positive if h_m' is negative. That is, the IRCQ is a *sufficient* condition for the IRCW to hold. Thus, our IRCQ is a sophistication of ACV's (2010) monotonicity condition (i.e., their IRC); it is a sophistication that requires "not too convex" demand functions to a stricter degree.

[38]Recall that $\varsigma_m^I(p) \equiv -(d\pi_m''/dp)(p/\pi_m'') = -(p\pi_m'''/\pi_m'')$ was defined as the *industry-level price elasticity of* π_m'' in Appendix B1 above. To see this relationship, note first that $g_m = (q_m'/\pi_m'')(q_m/q_m') = (1/h_m)(q_m/q_m')$. Thus,

$$\begin{aligned} g_m' &= -\frac{h_m'}{[h_m]^2}\times\frac{q_m}{q_m'} + \frac{1}{h_m}(1-\sigma_m^I) \\ &= \frac{1}{h_m}[-\underbrace{\left(-\frac{ph_m'}{h_m}\right)}_{=\nu_m^I}\underbrace{\left(-\frac{q_m}{pq_m'}\right)}_{=\frac{1}{\epsilon_m^I}} + (1-\sigma_m^I)], \end{aligned}$$

where $\sigma_m^I(p) \equiv q_m q_m''/(q_m')^2$ corresponds to what ACV (2010, p. 1603) call the *curvature of the the inverse demand.*, and it is also written as:

$$\sigma_m^I = \underbrace{\left(\frac{-pq_m''}{q_m'}\right)}_{=\alpha_m^I}\underbrace{\left(-\frac{q_m}{pq_m'}\right)}_{=\frac{1}{\epsilon_m^I}} = \frac{\alpha_m^I}{\epsilon_m^I}.$$

Hence,

$$g_m' = \frac{1}{h_m}\left[1 - \frac{\nu_m^I + \alpha_m^I}{\epsilon_m^I}\right],$$

which implies that $g_m' < 0 \Leftrightarrow \nu_m^I(p) + \alpha_m^I(p) > \epsilon_m^I(p) \Leftrightarrow \varsigma_m^I(p) > \epsilon_m^I(p)$. it is also verified that $g_m' < 0$ is equivalent to $\pi_m''' > (q_m'\pi_m'')/q_m$, where the right hand side is positive, because $g_m' = (q_m'\pi_m'' - q_m\pi_m''')/(\pi_m'')^2$. Now, recall that the IRCQ is equivalent to $\pi_m''' > q_m''\pi_m''/q_m'$: if $q_m'\pi_m''/q_m > q_m''\pi_m''/q_m' \Leftrightarrow q_m''(p) < [q_m'(p)]^2/q_m(p)$, that is $q_m(p)$ is not "too convex," then the DMCLC is a *sufficient* condition for the IRCQ to hold. Thus, determine the sign of $CS'(0)$: it follows that

$$\text{sign}[CS'(0)] = \text{sign}[\frac{q_s(\overline{p})}{\pi''_s(\overline{p})} - \frac{q_w(\overline{p})}{\pi''_w(\overline{p})}],$$

and thus, the following proposition is obtained.

Proposition 3. *Given the DMCLC, price discrimination decreases consumer surplus if the output in the weak market at the uniform price* $\overline{p}$ *is sufficiently large, i.e.,*

$$\frac{\overline{q}_s}{\overline{\pi}''_s} < \frac{\overline{q}_w}{\overline{\pi}''_w}$$

holds.

Then, using profit margin and pass-through, we can rewrite Equality (19) as

$$\frac{CS'(t)}{2} = \underbrace{(-\pi''_s\pi''_w)}_{<0}\left(\frac{\mu_w(t)\rho_w(t)}{\pi''_w} - \frac{\mu_s(t)\rho_s(t)}{\pi''_s}\right)$$

for $t < t^*$, and

$$\frac{CS'(t^*)}{2} = \underbrace{\left(-\frac{\pi''_s\pi''_w}{\pi''_s + \pi''_w}\right)}_{>0}(\mu^*_w\rho^*_w - \mu^*_s\rho^*_s)$$

for $t = t^*$ which immediately leads to the following proposition.

Proposition 4. *Given the DMCLC, price discrimination increases consumer surplus if*

$$\mu^*_s\rho^*_s < \mu^*_w\rho^*_w$$

holds, and it decreases consumer surplus if

$$\frac{\overline{\mu}_s\overline{\rho}_s}{\overline{\pi}''_s} \geq \frac{\overline{\mu}_w\overline{\rho}_w}{\overline{\pi}''_w}$$

holds.

B3. "DMCLC ⇒ IRCQ ⇒ IRCW"

As discussed in Footnote 38, the DMCLC implies the IRCQ (see Figure OA1), and the IRCQ implies the IRCW. A simpler way to establish "DMCLC ⇒ IRCQ ⇒ IRCW" is to recall first

under this "not too convex" assumption, the relationship, "DMCLC ⇒ IRCQ ⇒ IRCW," holds if $\sigma^I_m(p) < 1$ is additionally imposed.

$$\boxed{\begin{array}{c}\text{DMCLC: } g'_m < 0 \\ \Updownarrow \\ \varsigma^I_m > \epsilon^I_m\end{array}}$$

$$\Downarrow \qquad\qquad (\text{If } \sigma^I_m < 1 \Leftrightarrow \epsilon^I_m > \alpha^I_m \text{ is imposed})$$

$$\boxed{\begin{array}{c}\varsigma^I_m > \alpha^I_m \\ \Updownarrow \\ \text{IRCQ: } h'_m < 0\end{array}}$$

Figure OA1: DMCLC implies IRCQ

that

$$g'_m \quad = \quad -\frac{h'_m}{[h_m]^2} \times \frac{q_m}{q'_m} + \frac{1}{h_m}(1 - \sigma^I_m),$$

where $1 - \sigma^I_m > 0$ is assumed, that is, q_m is not " too convex." Hence, $g'_m < 0$ if and only if

$$h'_m < \underbrace{\left(\frac{q'_m h_m}{q_m}\right)}_{<0} \cdot \underbrace{\left(1 - \frac{q_m q''_m}{[q'_m]^2}\right)}_{>0},$$

indicating that "if $g'_m < 0$ (DMCLC), then $h'_m < 0$ (IRCQ)." However, the converse is not true because $h'_m < 0$ is not sufficient to ensure that $g'_m < 0$. Next, recall that $z_m(p) = (p - c_m)/h_m(p)$, which implies that

$$z'_m = \frac{h_m - (p - c_m)h'_m}{[h_m]^2}.$$

Thus, if $h'_m < 0$, then $h_m > 0 > (p - c_m)h'_m$, indicating that "if $h'_m < 0$ (IRCQ), then $z'_m > 0$ (IRCW)." However, the converse is not true because $z'_m > 0$ is not sufficient to ensure that $h'_m < 0$.

Below, we discuss whether the DMCLC holds in each of the three parametric examples in Section 4.

B4. Whether the DMCLC holds in the parametric examples

Let $\mathbf{p}_{-i} = (p, p, ..., p)$ be a tuple of $N - 1$ symmetric prices, p. Our results are summarized in Table 2.

B4.1. Linear demand First, it is observed that

$$\pi''_m(p) \quad = \quad q'_m(p) + \frac{\partial x_{im}}{\partial p_i}(p, \mathbf{p}_{-i}) + (p - c_m)\frac{d}{dp}\left(\frac{\partial x_{im}}{\partial p_i}(p, \mathbf{p}_{-i})\right)$$

	Linear	CES	Logit
DMCLC	No	Yes	?
IRCQ	No	Yes	Yes
IRCW	Yes	Yes	Yes

Table 2: Whether each condition is satisfied in the three examples of market demand

$$= \quad -\frac{2+(N-3)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m} < 0$$

because

$$q'_m(p) = -\frac{1}{[1+(N-1)\delta_m]\beta_m}$$

and

$$\frac{\partial x_{im}}{\partial p_i}(p, \mathbf{p}_{-i}) = -\frac{1+(N-2)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m},$$

both of which are constant. Then, it is verified that

$$\begin{aligned} g_m(p) &= \frac{q_m(p)}{\pi''_m(p)} \\ &= \frac{\left(\dfrac{\omega_m - p}{[1+(N-1)\delta_m]\beta_m}\right)}{\left(-\dfrac{2+(N-3)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}\right)} \\ &= -\frac{(\omega_m - p)(1-\delta_m)}{2+(N-3)\delta_m} \end{aligned}$$

and thus $g'_m(p) = (1-\delta_m)/[2+(N-3)\delta_m] > 0$; the *DMCLC never holds.* Moreover, the *IRCQ does not hold, either*, because

$$\begin{aligned} h_m(p) &= \frac{1}{q'_m(p)/\pi''_m(p)} \\ &= \frac{\left(-\dfrac{2+(N-3)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}\right)}{\left(-\dfrac{1}{[1+(N-1)\delta_m]\beta_m}\right)} \\ &= \frac{2+(N-3)\delta_m}{1-\delta_m}, \end{aligned}$$

implying that $h'_m(p) = 0$.

However, the *IRCW does hold.* This is because

$$z_m(p) = \frac{(p-c_m)q'_m(p)}{\pi''_m(p)}$$

$$= \frac{(p-c_m)\left(-\frac{1}{[1+(N-1)\delta_m]\beta_m}\right)}{\left(-\frac{2+(N-3)\delta_m}{[1+(N-1)\delta_m](1-\delta_m)\beta_m}\right)}$$
$$= (p-c_m)\left(\frac{1-\delta_m}{2+(N-3)\delta_m}\right),$$

which implies that $z_m'(p) = (1-\delta_m)/[2+(N-3)\delta_m] > 0$.

B4.2. CES (constant elasticity of substitution) demand First, it is verified that

$$\pi_m''(p) = -\frac{1}{Np^2} - \frac{1+(N-1)\sigma_m}{N^2p^2} + (p-c_m)\frac{2[1+(N-1)\sigma_m]}{N^2p^3}$$
$$= -\frac{1}{N^2p^2}\left\{N + [1+(N-1)\sigma_m]\left[1-\frac{2(p-c_m)}{p}\right]\right\}$$

because

$$q_m'(p) = -\frac{1}{Np^2}$$

and

$$\frac{\partial x_{im}}{\partial p_i}(p, \mathbf{p}_{-i}) = -\frac{1+(N-1)\sigma_m}{N^2p^2}.$$

Then, it is observed that

$$g_m(p) = \frac{q_m(p)}{\pi_m''(p)}$$
$$= -\frac{Np^2}{Np - [1+(N-1)\sigma_m](p-2c_m)}$$

and thus

$$g_m'(p) = \frac{N\{(N-1)(\sigma_m-1)p - 4[1+(N-1)\sigma_m]c_m\}}{\{(N-1)(\sigma_m-1)p - 2[1+(N-1)\sigma_m]c_m\}^2}p,$$

which implies that g_m is decreasing (i.e., the *DMCLC holds*) for

$$p < \frac{4[1+(N-1)\sigma_m]c_m}{(N-1)(\sigma_m-1)} = 4p_m^*.$$

Although g_w is not globally decreasing, it is decreasing in the price range for p_w^*, $[p_w^*, \overline{p}]$ because $\overline{p} \leq 4p_s^*$ holds.[39] Then, g_s is also globally decreasing in the price range, $[\overline{p}, p_w^*]$. Therefore, the

[39] To see this, note first that $\overline{p} \leq 4p_s^*$ is equivalent to $p_s^* \leq [(4\sigma_s + 3\sigma_w - 7)/(\sigma_s - 1)]p_w^*$. Therefore, since $p_s^* \geq p_w^*$, it must hold that $(4\sigma_s + 3\sigma_w - 7)/(\sigma_s - 1) \geq 1$. This is equivalent to $2\sigma_s + \sigma_w \geq 3$, which holds by assumption ($\sigma_m > 1$ for $m = s, w$).

IRCQ, and hence, the *IRCW* always hold in the relevant price range.

Now, we define K by $K(\boldsymbol{\sigma}, \boldsymbol{c}, N) \equiv \mu_s^* \rho_s^* - \mu_w^* \rho_w^*$. Then, from Proposition 4, $K < 0$ implies $\Delta CS > 0$. Now, it is verified that

$$\begin{aligned} K(\boldsymbol{\sigma}, \boldsymbol{c}, N) &= \frac{Nc_s}{(N-1)(\sigma_s - 1)} \cdot \frac{[1+(N-1)\sigma_s]^2}{(\sigma_s - 1)\{[(N-1)N-1]\sigma_s + 1\}} \\ &\quad - \frac{Nc_w}{(N-1)(\sigma_w - 1)} \cdot \frac{[1+(N-1)\sigma_w]^2}{(\sigma_w - 1)\{[(N-1)N-1]\sigma_w + 1\}} \end{aligned}$$

because

$$\begin{aligned} (\epsilon_m^{cross})^* + (\alpha_m^{own})^* + (\alpha_m^{cross})^* &= \frac{\sigma_m - 1}{N} + \frac{N^2\sigma_m(1+\sigma_m) - 4(\sigma_m - 1)[1+(N-1)\sigma_m]}{N[1+(N-1)\sigma_m]} \\ &= \frac{\{[3+(N-3)N]\sigma_m + N(N+3) - 6\}\sigma_m + 3}{N[1+(N-1)\sigma_m]} \end{aligned}$$

and hence

$$\begin{aligned} \rho_m^* &= \frac{1}{2 - \dfrac{(\epsilon_m^{cross})^* + (\alpha_m^{own})^* + (\alpha_m^{cross})^*}{(\epsilon_m^{own})^*}} \\ &= \frac{[1+(N-1)\sigma_m]^2}{(\sigma_m - 1)\{[(N-1)N-1]\sigma_m + 1\}}. \end{aligned}$$

Figure 5 illustrates when $K < 0$ holds. As in Figure 3, there is an interesting contrast between the left and right panels. If the marginal costs are common across markets, the region of $K < 0$ disappears once the constraint $p_s^* \geq p_w^*$ is imposed. However, if the marginal cost in the strong market becomes 10% higher, the area of $K < 0$ appears for $N = 2$ and $N = 4$ where σ_s is sufficiently large as compared σ_w.

B4.3. Multinomial logit demand with outside option Unfortunately, it cannot be shown that the DMCLC holds for the logit demand, though $\sigma_m^I(p) < 1$ is satisfied.[40] To see this, first, recall (from Footnote 38) that $g_m' < 0$ if and only if $\varsigma_m^I > \epsilon_m^I$ ($\Leftrightarrow -(p\pi_m'''/\pi_m'') >$

[40] Recall that the logit demand under symmetric pricing is given by:

$$q_m(p) = \frac{\exp(\omega_m - \beta_m p)}{1 + N\exp(\omega_m - \beta_m p)},$$

which implies that $q_m'(p) = -\beta_m q_m(p)[1 - N \cdot q_m(p)]$ and $q_m''(p) = -\beta_m q_m'(p)[1 - 2N \cdot q_m(p)]$. Hence,

$$q_m''(p) < \frac{[q_m'(p)]^2}{q_m(p)} \Leftrightarrow -\beta_m[1 - 2N \cdot q_m(p)] > \frac{q_m'(p)}{q_m(p)} \Leftrightarrow 1 - 2N \cdot q_m(p) < 1 - N \cdot q_m(p),$$

which must be true.

Marginal costs are:

common across markets ($c_s = c_w = 0.20$) | different across markets ($c_s = 0.22$; $c_w = 0.20$)

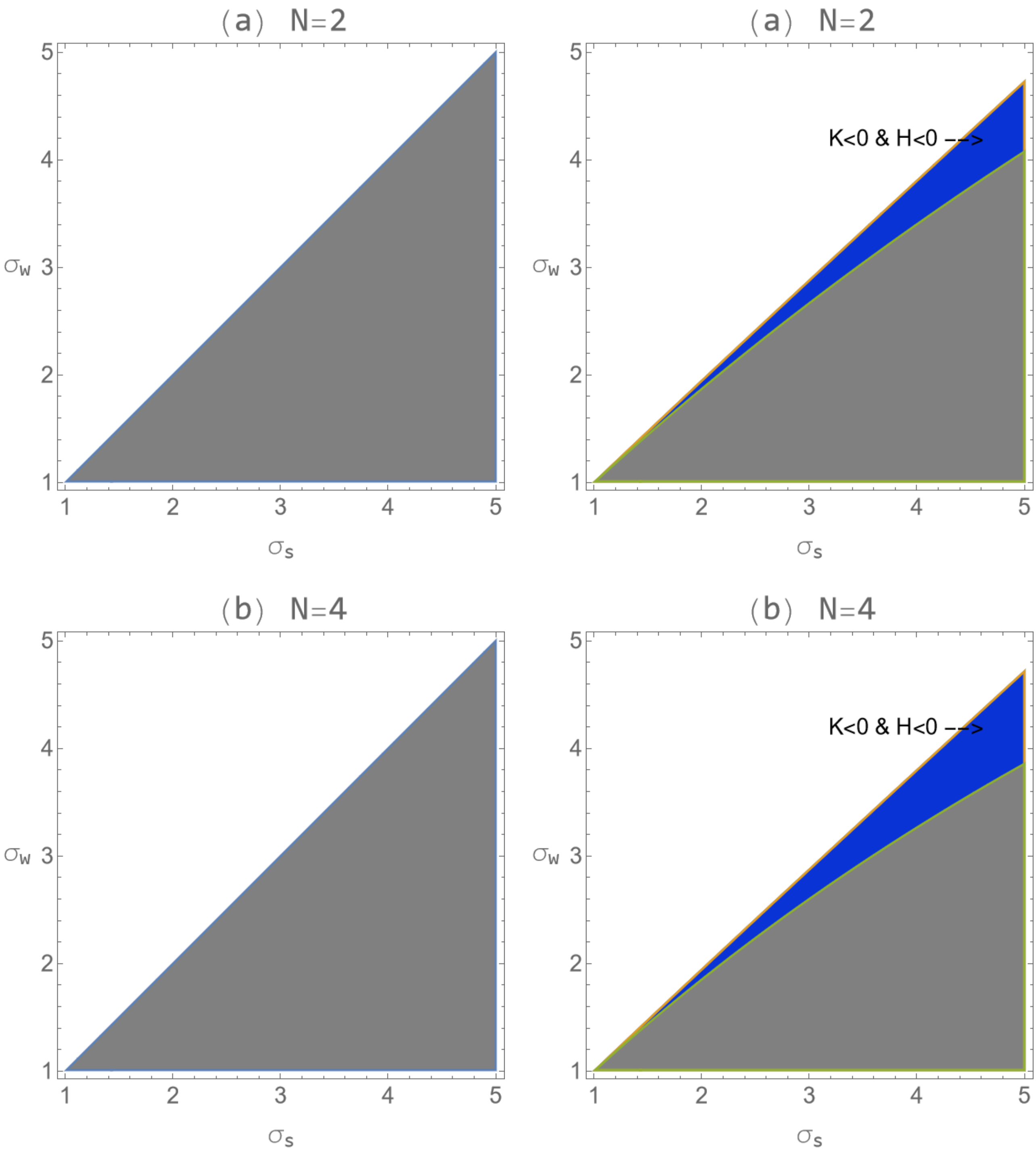

Figure 5: CES demand when $N = 2$ (top) and when $N = 4$ (bottom). In the left panel with common marginal costs, the region of $K < 0$ does not appear because it is fully covered by the dark-shaded area of (σ_s, σ_w) where $p_s^* \geq p_w^*$ does not hold. In the right panel with different marginal costs across markets, the region for $H < 0$ and $K < 0$ is colored.

$-(pq'_m/q_m))$, that is,[41]

$$\pi'''_m(p) \quad > \quad -\beta_m \cdot [1 - Nq_m(p)]\pi''_m(p), \tag{20}$$

where $\pi''_m(p)$ and $\pi'''_m(p)$ are given as:

$$\begin{aligned}\pi''_m(p) &= q'_m(p) + \frac{\partial x_{im}}{\partial p_i}(p, \mathbf{p}_{-i}) + (p - c_m)\frac{d}{dp}\left(\frac{\partial x_{im}}{\partial p_i}(p, \mathbf{p}_{-i})\right) \\ &= -\beta_m q_m \cdot (1 - Nq_m) - \beta_m q_m \cdot (1 - q_m) - \beta_m (p - c_m) q'_m (1 - 2q_m) \\ &= -\beta_m q_m \cdot [2 - (N+1)q_m - \beta_m (p - c_m)(1 - 2q_m)(1 - Nq_m)]\end{aligned}$$

and thus

$$\begin{aligned}\pi'''_m(p) &= -\beta_m (1 - Nq_m)\pi''_m - \beta_m q_m \cdot [-(N+1)q'_m - \beta_m (1 - 2q_m)(1 - Nq_m) \\ &\qquad + \beta_m (p - c_m)(N + 2 - 4Nq_m)q'_m]\,.\end{aligned}$$

This implies that Inequality (20) is rewritten as

$$\beta_m \underbrace{[2 + N(1 - 4q_m)]}_{>0}(p - c_m) > \underbrace{3 + N - \frac{1}{q_m}}_{\gtrless 0},$$

which cannot be verified to be true.

However, the logit demand *does satisfy the IRCQ* (and hence the IRCW). This is because h'_m is negative if and only if $\varsigma^I_m > \alpha^I_m$ ($\Leftrightarrow -\frac{p\pi'''_m}{\pi''_m} > -\frac{pq''_m}{q'_m}$), that is

$$\begin{aligned}&\pi'''_m(p) > -\beta_m \cdot [1 - 2N \cdot q_m(p)]\pi''_m(p) \\ \Leftrightarrow\ & 2\beta_m N \cdot \underbrace{(\frac{1}{N} - q_m)}_{>0}(p - c_m) > -\underbrace{(\frac{1}{q_m} - 2)}_{>0} - \frac{N-1}{N \cdot \underbrace{(\frac{1}{N} - q_m)}_{>0}},\end{aligned}$$

which holds for any $N \geq 2$ as long as $p \geq c_m$.

Online Appendix C. Firm heterogeneity

In this section, we argue that the main thrusts under firm symmetry also hold when heterogeneous firms are introduced. Without loss of generality, we keep considering one strong market and one weak market. We also assume Corts' (1998, p. 315) *best-response symmetry*: all firms agree on which market is strong and which market is weak. The case of best response *asymmetry*

[41] Here, the dependence of $q_m(p)$ on N is suppressed for notational simplicity.

is studied by Corts (1998) (see also Footnote 22 above).[42]

The number of firms is N (≥ 2),[43] and each firm $i = 1, 2, ..., N$ has the constraint, $p_{is} - p_{iw} \leq t_i$. Then, as above, firm i's price in the weak market under all of these constraints is written as $p_{iw}(\mathbf{t})$ as a function of $\mathbf{t} = (t_1, t_2, ..., t_N)^{\mathrm{T}}$, where T denotes transposing. Accordingly, firm i's price in the strong market is written as $p_{is}(\mathbf{t}) = p_{iw}(\mathbf{t}) + t_i$. Therefore, the firms' price pair in market $m = w, s$ is written as $\mathbf{p}_m(\mathbf{t}) = (p_{1m}(\mathbf{t}), p_{2m}(\mathbf{t}), ..., p_{Nm}(\mathbf{t}))^{\mathrm{T}}$. Then, social welfare is defined as a function of $\mathbf{t}$:

$$W(\mathbf{t}) \equiv U_s(\mathbf{x}_s[\mathbf{p}_s(\mathbf{t})]) + U_w(\mathbf{x}_w[\mathbf{p}_w(\mathbf{t})]) - \mathbf{c}_s^{\mathrm{T}} \cdot \mathbf{x}_s[\mathbf{p}_s(\mathbf{t})] - \mathbf{c}_w^{\mathrm{T}} \cdot \mathbf{x}_w[\mathbf{p}_w(\mathbf{t})],$$

where $\mathbf{x}_m[\mathbf{p}_m(\mathbf{t})] = (x_{1m}[\mathbf{p}_m(\mathbf{t})], x_{2m}[\mathbf{p}_m(\mathbf{t})], ..., x_{Nm}[\mathbf{p}_m(\mathbf{t})])^{\mathrm{T}}$ and $\mathbf{c}_m = (c_{1m}, c_{2m}, ..., c_{Nm})^{\mathrm{T}}$.[44]

Now, let $t_i^* \equiv p_{is}^* - p_{iw}^*$ for each j so that $\mathbf{t}^* \equiv (t_1^*, t_2^*, ..., t_N^*)^{\mathrm{T}}$. Then, each firm's constraint is written as $0 \leq t_i = \lambda t_i^* \leq t_i^*$, with $\lambda \in [0, 1]$. Using this, we re-define the functions of $\mathbf{t}$ as functions of *one-dimensional* variable, λ. In particular, the social welfare is written as:

$$W(\lambda) = U_s(\mathbf{x}_s[\mathbf{p}_s(\lambda)]) + U_w(\mathbf{x}_w[\mathbf{p}_w(\lambda)]) - \mathbf{c}_s^{\mathrm{T}} \cdot \mathbf{x}_s[\mathbf{p}_s(\lambda)] - \mathbf{c}_w^{\mathrm{T}} \cdot \mathbf{x}_w[\mathbf{p}_w(\lambda)],$$

where $\mathbf{p}_m$ can also be interpreted as a function of λ. Hence, the equilibrium uniform price is written as $\overline{\mathbf{p}} \equiv \mathbf{p}_s(0) = \mathbf{p}_w(0)$, whereas the equilibrium discriminatory prices are $\mathbf{p}_s^* \equiv \mathbf{p}_s(1)$ and $\mathbf{p}_w^* \equiv \mathbf{p}_w(1)$.[45]

We then use $\boldsymbol{\partial}_{\mathbf{x}} U_m = \mathbf{p}_m$ from the representative consumer's utility maximization problem

[42]Cort's (1998) Proposition 6 states a stronger result: price symmetry can be attained even with best-response asymmetry. This result would further justify that introducing firm heterogeneity with a small amount should not invalidate our analysis assuming firm heterogeneity.

[43]Here, all N firms are assumed to be present in both markets. This aspect of symmetry might be relaxed: the difference in the intensity of competition across the markets can also depend on the difference in the number of active firms across them. For example, Aguirre (2019) shows that price discrimination increases aggregate output under linear demand either with Cournot competition or product differentiation if the number of firms in the strong market is larger than that in the weak market. This counters the well-know result that in monopoly price discrimination never changes aggregate output under linear demand (see, e.g., Robinson 1933; Schmalensee 1981; Varian 1989). A similar finding is also obtained by Miklós-Thal and Shaffer (2021c) in the context of intermediate price discrimination. We thank Iñaki Aguirre for pointing this out to us.

[44]By allowing cost differences across firms and markets, Dertwinkel-Kalt and Wey (2023) study oligopolistic third-degree price discrimination under the demand system proposed by Somaini and Einav (2013), in which demand in each separate market is covered by all firms and thus no consumers are opting out. Under this demand system, Dertwinkel-Kalt and Wey (2023) show that each firm's profit margin (i.e., the Lerner index) under uniform pricing is expressed as the weighted harmonic mean of its market-specific Lerner indices under price discrimination. This result indicates that the profit margin is strictly lower the weighted arithmetic mean of the market-specific margins. In this sense, the market power measured by the Lerner concept is always lower under uniform pricing, and consumer surplus is strictly greater than under price discrimination. Note, however, that a change in social welfare is *not* an issue under this demand system because each firm's output remains the same for both regimes.

[45]Online Appendix F discusses why the exogenous quantity approach by Weyl and Fabinger (2013) and Miklós-Thal and Shaffer (2021a) is not applicable once firm heterogeneity is allowed.

in each market m, where

$$\boldsymbol{\partial}_{\mathbf{x}} U_m \equiv \left(\frac{\partial U_m}{\partial x_{1m}}, \frac{\partial U_m}{\partial x_{2m}}, ..., \frac{\partial U_m}{\partial x_{Nm}} \right),$$

to derive

$$\underbrace{W'(\lambda)}_{1\times 1} = \sum_{m=s,w} [\underbrace{\boldsymbol{\mu}_m(\mathbf{p}_m)^{\mathrm{T}}}_{1\times N} \cdot (\underbrace{\boldsymbol{\partial}_{\mathbf{p}_m} \mathbf{x}_m}_{N\times N} \cdot \underbrace{\mathbf{p}'_m}_{N\times 1})],$$

where $\boldsymbol{\mu}_m(\mathbf{p}_m) \equiv \mathbf{p}_m - \mathbf{c}_m$ is the profit margin vector,

$$\boldsymbol{\partial}_{\mathbf{p}_m} \mathbf{x}_m \equiv \left(\underbrace{\begin{pmatrix} \frac{\partial x_{1m}}{\partial p_{1m}} \\ \vdots \\ \frac{\partial x_{Nm}}{\partial p_{1m}} \end{pmatrix}}_{\equiv \boldsymbol{\partial}_{p_{1m}} \mathbf{x}_m} \cdots \underbrace{\begin{pmatrix} \frac{\partial x_{1m}}{\partial p_{Nm}} \\ \vdots \\ \frac{\partial x_{Nm}}{\partial p_{Nm}} \end{pmatrix}}_{\equiv \boldsymbol{\partial}_{p_{Nm}} \mathbf{x}_m} \right)$$

is the Jacobian for market demands, and $\mathbf{p}'_{\mathrm{m}} \equiv (p'_{1m}(\lambda), p'_{2m}(\lambda), ..., p'_{Nm}(\lambda))^{\mathrm{T}}$.

Firm j's profit function in market $m = s, w$ is given by Equation (4), where $\mathbf{p}_m$ now consists of N firms' prices as above, and

$$\partial_{p_{jm}} \pi_{jm}(\mathbf{p}_m) \equiv x_{jm}(\mathbf{p}_m) + (p_{jm} - c_{jm}) \frac{\partial x_{jm}}{\partial p_{jm}}(\mathbf{p}_m)$$

is defined. Now, we apply the implicit function theorem to $\mathbf{f}(\mathbf{p_w}, \lambda) = \mathbf{0}$, where

$$\mathbf{f}(\underbrace{\mathbf{p_w}}_{N\times 1}, \underbrace{\lambda}_{1\times 1}) \equiv \begin{pmatrix} \partial_{p_{1s}} \pi_{1s}(\mathbf{p}_w + \lambda \mathbf{t}^*) + \partial_{p_{1w}} \pi_{1w}(\mathbf{p}_w) \\ \vdots \\ \partial_{p_{is}} \pi_{is}(\mathbf{p}_w + \lambda \mathbf{t}^*) + \partial_{p_{iw}} \pi_{iw}(\mathbf{p}_w) \\ \vdots \\ \partial_{p_{Ns}} \pi_{Ns}(\mathbf{p}_w + \lambda \mathbf{t}^*) + \partial_{p_{Nw}} \pi_{Nw}(\mathbf{p}_w) \end{pmatrix},$$

is a collection of all firms' first-order conditions for profit maximization under regime λ, to obtain $\mathbf{p}'_w(\lambda) = -[\mathbf{D}_{\mathbf{p}_w} \mathbf{f}]^{-1} [\mathbf{D}_\lambda \mathbf{f}]$, where

$$\mathbf{D}_{\mathbf{p}_w} \mathbf{f} \equiv \underbrace{\begin{pmatrix} \frac{\partial^2 \pi_{1s}}{\partial p_{1s}^2} + \frac{\partial^2 \pi_{1w}}{\partial p_{1w}^2} & \cdots & \frac{\partial^2 \pi_{1s}}{\partial p_{Ns} \partial p_{1s}} + \frac{\partial^2 \pi_{1w}}{\partial p_{Nw} \partial p_{1w}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi_{Ns}}{\partial p_{1s} \partial p_{Ns}} + \frac{\partial^2 \pi_{Nw}}{\partial p_{1w} \partial p_{Nw}} & \cdots & \frac{\partial^2 \pi_{Ns}}{\partial p_{Ns}^2} + \frac{\partial^2 \pi_{Nw}}{\partial p_{Nw}^2} \end{pmatrix}}_{\equiv \mathbf{K}}$$

$$= \underbrace{\begin{pmatrix} \frac{\partial^2 \pi_{1s}}{\partial p_{1s}^2} & \cdots & \frac{\partial^2 \pi_{1s}}{\partial p_{Ns} \partial p_{1s}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi_{Ns}}{\partial p_{1s} \partial p_{Ns}} & \cdots & \frac{\partial^2 \pi_{Ns}}{\partial p_{Ns}^2} \end{pmatrix}}_{\equiv \mathbf{H}_s} + \underbrace{\begin{pmatrix} \frac{\partial^2 \pi_{1w}}{\partial p_{1w}^2} & \cdots & \frac{\partial^2 \pi_{1w}}{\partial p_{Nw} \partial p_{1w}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \pi_{Nw}}{\partial p_{1w} \partial p_{Nw}} & \cdots & \frac{\partial^2 \pi_{Nw}}{\partial p_{Nw}^2} \end{pmatrix}}_{\equiv \mathbf{H}_w}$$

and $\mathbf{D}_\lambda \mathbf{f} = \mathbf{H}_s \mathbf{t}^*$ (with no confusion, we use $\mathbf{H}$ and $\mathbf{K}$ here which are different from H and K in Section 4 and Appendix B).

Here, the elasticity matrix and the curvature matrix can be defined by

$$\begin{aligned} \boldsymbol{\epsilon}_m &= \begin{pmatrix} \epsilon_{11,m} & \epsilon_{21,m} & \cdots & \epsilon_{N1,m} \\ \epsilon_{12,m} & \epsilon_{22,m} & & \vdots \\ \vdots & & \ddots & \vdots \\ \epsilon_{1N,m} & \epsilon_{2N,m} & \cdots & \epsilon_{NN,m} \end{pmatrix} \\ &\equiv \begin{pmatrix} -\frac{p_{1m}}{x_{1m}} \frac{\partial x_{1m}}{\partial p_{1m}} & \frac{p_{1m}}{x_{1m}} \frac{\partial x_{2m}}{\partial p_{1m}} & \cdots & \frac{p_{1m}}{x_{Nm}} \frac{\partial x_{Nm}}{\partial p_{1m}} \\ \frac{p_{2m}}{x_{1m}} \frac{\partial x_{1m}}{\partial p_{2m}} & -\frac{p_{2m}}{x_{2m}} \frac{\partial x_{2m}}{\partial p_{2m}} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{p_{Nm}}{x_{1m}} \frac{\partial x_{1m}}{\partial p_{Nm}} & \cdots & \cdots & -\frac{p_{Nm}}{x_{Nm}} \frac{\partial x_{Nm}}{\partial p_{Nm}} \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} \boldsymbol{\alpha}_m &= \begin{pmatrix} \alpha_{11,m} & \alpha_{21,m} & \cdots & \alpha_{N1,m} \\ \alpha_{12,m} & \alpha_{22,m} & & \vdots \\ \vdots & & \ddots & \vdots \\ \alpha_{1N,m} & \cdots & \cdots & \alpha_{NN,m} \end{pmatrix} \\ &\equiv \begin{pmatrix} -\frac{p_{1m}}{\partial x_{1m}/\partial p_{1m}} \frac{\partial^2 x_{1m}}{\partial p_{1m}^2} & -\frac{p_{2m}}{\partial x_{2m}/\partial p_{2m}} \frac{\partial^2 x_{2m}}{\partial p_{2m} \partial p_{1m}} & \cdots & -\frac{p_{Nm}}{\partial x_{Nm}/\partial p_{Nm}} \frac{\partial^2 x_{Nm}}{\partial p_{Nm} \partial p_{1m}} \\ -\frac{p_{1m}}{\partial x_{1m}/\partial p_{1m}} \frac{\partial^2 x_{1m}}{\partial p_{1m} \partial p_{2m}} & -\frac{p_{2m}}{\partial x_{2m}/\partial p_{2m}} \frac{\partial^2 x_{2m}}{\partial p_{2m}^2} & & \vdots \\ \vdots & & \ddots & \vdots \\ -\frac{p_{1m}}{\partial x_{1m}/\partial p_{1m}} \frac{\partial^2 x_{1m}}{\partial p_{1m} \partial p_{Nm}} & \cdots & \cdots & -\frac{p_{Nm}}{\partial x_{Nm}/\partial p_{Nm}} \frac{\partial^2 x_{Nm}}{\partial p_{Nm}^2} \end{pmatrix}, \end{aligned}$$

respectively. Note also that for $i = 1, 2, ..., N$,

$$\frac{\partial^2 \pi_{im}(\mathbf{p}_m)}{\partial p_{im}^2} = 2 \frac{\partial x_{im}}{\partial p_{im}}(\mathbf{p}_m) + (p_{im} - c_{im}) \frac{\partial^2 x_{im}}{\partial p_{im}^2}(\mathbf{p}_m)$$

$$
\begin{aligned}
&= -\frac{x_{im}}{p_{im}}\left(-\frac{p_{im}}{x_{im}}\frac{\partial x_{im}}{\partial p_{im}}\right)\left[2-\frac{p_{im}-c_{im}}{p_{im}}\left(-\frac{p_{im}}{\partial x_{im}/\partial p_{im}}\frac{\partial^2 x_{im}}{\partial p_{im}^2}\right)\right] \\
&= -\left[2-L_{im}(p_{im})\alpha_{ii,m}\right]\epsilon_{ii,m}\frac{x_{im}}{p_{im}}
\end{aligned}
$$

and for $i, j = 1, 2, ..., N$, $i \neq j$,

$$
\begin{aligned}
\frac{\partial^2 \pi_{im}(\mathbf{p}_m)}{\partial p_{im}\partial p_{jm}} &= \frac{\partial x_{im}}{\partial p_{jm}}(\mathbf{p}_m) + (p_{im}-c_{im})\frac{\partial^2 x_{im}}{\partial p_{im}\partial p_{jm}}(\mathbf{p}_m) \\
&= \frac{x_{im}}{p_{jm}}\left(\frac{p_{jm}}{x_{im}}\frac{\partial x_{im}}{\partial p_{jm}}\right) \\
&\quad \times\left[1+\left(\frac{p_{jm}}{p_{im}}\right)\left(\frac{p_{im}-c_{im}}{p_{im}}\right)\left(\frac{-(p_{im}/x_{im})(\partial x_{im}/\partial p_{im})}{(p_{jm}/x_{im})(\partial x_{im}/\partial p_{jm})}\right)\left(-\frac{p_{im}}{\partial x_{im}/\partial p_{im}}\frac{\partial^2 x_{im}}{\partial p_{im}\partial p_{jm}}\right)\right] \\
&= \left[1+\left(\frac{p_{jm}}{p_{im}}\right)L_{im}(p_{im})\frac{\epsilon_{ii,m}}{\epsilon_{ij,m}}\alpha_{ij,m}\right]\epsilon_{ij,m}\frac{x_{im}}{p_{jm}},
\end{aligned}
$$

which implies that $\mathbf{H}_s$ and $\mathbf{H}_w$ are expressed in terms of the sufficient statistics.

Now, we further proceed to obtain:

$$
\begin{cases}
\mathbf{p}'_w(\lambda) = [-\underbrace{\mathbf{K}^{-1}}_{N\times N}\underbrace{\mathbf{H}_s}_{N\times N}]\underbrace{\mathbf{t}^*}_{N\times 1} \\
\mathbf{p}'_s(\lambda) = -\mathbf{K}^{-1}\mathbf{H}_s\mathbf{t}^* + \mathbf{t}^* = \underbrace{[\mathbf{I}-\mathbf{K}^{-1}\mathbf{H}_s]}_{N\times N}\underbrace{\mathbf{t}^*}_{N\times 1},
\end{cases}
$$

so that

$$
\begin{aligned}
W'(\lambda) &= [\boldsymbol{\mu}_s^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_s}\mathbf{x}_s][\mathbf{I}-\mathbf{K}^{-1}\mathbf{H}_s]\mathbf{t}^* - [\boldsymbol{\mu}_w^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_w}\mathbf{x}_w][\mathbf{K}^{-1}\mathbf{H}_s]\mathbf{t}^* \\
&= \{[\boldsymbol{\mu}_s^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_s}\mathbf{x}_s]\mathbf{K}^{-1}[\mathbf{K}-\mathbf{H}_s] - [\boldsymbol{\mu}_w^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_w}\mathbf{x}_w][\mathbf{K}^{-1}\mathbf{H}_s]\}\mathbf{t}^* \\
&= \big\{\left([\boldsymbol{\mu}_s^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_s}\mathbf{x}_s]\mathbf{H}_s^{-1}\right)\left(\mathbf{H}_s\mathbf{K}^{-1}[\mathbf{K}-\mathbf{H}_s]\right) \\
&\qquad - \left([\boldsymbol{\mu}_w^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_w}\mathbf{x}_w]\mathbf{H}_w^{-1}\right)\left(\mathbf{H}_w\mathbf{K}^{-1}\mathbf{H}_s\right)\big\}\,\mathbf{t}^*.
\end{aligned}
$$

Subsequently, we define

$$
\mathbf{Z}_m(\mathbf{p}) \equiv \underbrace{[\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}}\boldsymbol{\partial}_{\mathbf{p}_m}\mathbf{x}_m(\mathbf{p})]}_{1\times N}\underbrace{\mathbf{H}_m^{-1}(\mathbf{p})}_{N\times N}
$$

to proceed:

$$
\begin{aligned}
W'(\lambda) &= \{\underbrace{\mathbf{Z}_w}_{1\times N} - \underbrace{\mathbf{Z}_s}_{1\times N}\cdot\underbrace{(\mathbf{K}-\mathbf{H}_s)\mathbf{H}_w^{-1}}_{N\times N}\}\underbrace{(-\mathbf{H}_w\mathbf{K}^{-1}\mathbf{H}_s)\mathbf{t}^*}_{N\times 1} \\
&= (\mathbf{Z}_w - \mathbf{Z}_s)(\boldsymbol{\Gamma}\mathbf{t}^*),
\end{aligned}
$$

where $\boldsymbol{\Gamma} \equiv -\mathbf{H}_w\mathbf{K}^{-1}\mathbf{H}_s \gg \mathbf{0}$ is assumed. We also assume that multi-dimensional version of the IRCW: for each market m and each firm i, Z_{im} is increasing in p_k, $k = 1, 2, ..., N$.

We then define the multi-dimensional version of the conduct parameter (Weyl and Fabinger 2013, p. 552; see Footnote 18) by:

$$\boldsymbol{\theta}_m(\mathbf{p})^{\mathrm{T}} \equiv \begin{pmatrix} \dfrac{\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}}\boldsymbol{\partial}_{p_{1m}}\mathbf{x}_m(\mathbf{p})}{-x_{1m}(\mathbf{p})} & \cdots & \dfrac{\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}}\boldsymbol{\partial}_{p_{im}}\mathbf{x}_m(\mathbf{p})}{-x_{im}(\mathbf{p})} & \cdots & \dfrac{\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}}\boldsymbol{\partial}_{p_{Nm}}\mathbf{x}_m(\mathbf{p})}{-x_{Nm}(\mathbf{p})} \end{pmatrix}$$

as well as the pass-through matrix by:

$$\boldsymbol{\rho}_m(\lambda) = \begin{cases} \begin{pmatrix} \dfrac{\partial x_{1m}}{\partial p_{1m}}(\mathbf{p}) & 0 & \cdots & 0 \\ 0 & \dfrac{\partial x_{2m}}{\partial p_{2m}}(\mathbf{p}) & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \dfrac{\partial x_{Nm}}{\partial p_{Nm}}(\mathbf{p}) \end{pmatrix} \mathbf{K}^{-1}(\mathbf{p}) & \text{for } \lambda < 1 \\ \\ \begin{pmatrix} \dfrac{\partial x_{1m}}{\partial p_{1m}}(\mathbf{p}_m^*) & 0 & \cdots & 0 \\ 0 & \dfrac{\partial x_{2m}}{\partial p_{2m}}(\mathbf{p}_m^*) & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \dfrac{\partial x_{Nm}}{\partial p_{Nm}}(\mathbf{p}_m^*) \end{pmatrix} \mathbf{H}_m^{-1}(\mathbf{p}_m^*) & \text{for } \lambda = 1 \end{cases}$$

as in the case of firm symmetry (recall the definition in Subsubsection 3.2).[46]

Hence, the three sufficient statistics under price discrimination are given by $\boldsymbol{\theta}_m^* \equiv \boldsymbol{\theta}_m(\mathbf{p}_m^*)$, $\boldsymbol{\mu}_m^* \equiv \boldsymbol{\mu}_m(\mathbf{p}_m^*)$, and $\boldsymbol{\rho}_m^* \equiv \boldsymbol{\rho}_m(1)$. Similarly, those under uniform pricing are $\overline{\boldsymbol{\theta}}_m \equiv \boldsymbol{\theta}_m(\overline{\mathbf{p}})$ $\overline{\boldsymbol{\mu}}_m \equiv \boldsymbol{\mu}_m(\overline{\mathbf{p}})$, and $\overline{\boldsymbol{\rho}}_m \equiv \boldsymbol{\rho}_m(0)$.

We are now able to generalizes Proposition 1 in Section 3 for the case of firm heterogeneity.

Proposition 5. *Given the IRCW, if* $[[\boldsymbol{\theta}_w^*]^{\mathrm{T}} \circ [\boldsymbol{\mu}_w^*]^{\mathrm{T}}]\boldsymbol{\rho}_w^* > [[\boldsymbol{\theta}_s^*]^{\mathrm{T}} \circ [\boldsymbol{\mu}_s^*]^{\mathrm{T}}]\boldsymbol{\rho}_s^*$ *holds, where* $\circ$ *indicates element-by-element multiplication, then price discrimination increases social welfare. Conversely, if* $[[\overline{\boldsymbol{\theta}}_w]^{\mathrm{T}} \circ [\overline{\boldsymbol{\mu}}_w]^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w < [[\overline{\boldsymbol{\theta}}_s]^{\mathrm{T}} \circ [\overline{\boldsymbol{\mu}}_s]^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s\overline{\boldsymbol{\Delta}}$, *where* $\overline{\boldsymbol{\Delta}} \equiv \overline{\mathbf{K}}\,\overline{\mathbf{H}}_s^{-1}\overline{\mathbf{H}}_w\overline{\mathbf{K}}^{-1}$ *is defined for adjustment, where* $\overline{\mathbf{K}} \equiv \mathbf{K}(\overline{\mathbf{p}})$ *and* $\overline{\mathbf{H}}_m \equiv \mathbf{H}_m(\overline{\mathbf{p}})$, $m = s, w$, *holds, then price discrimination decreases social welfare.*

[46] Given this definition, $\boldsymbol{\theta}_m$ is rewritten as

$$\boldsymbol{\theta}_m^{\mathrm{T}} = \begin{pmatrix} \sum_{k=1}^N L_{km}\epsilon_{k1,m}(-x_{km}/x_{1m}) & \cdots & \sum_{k=1}^N L_{km}\epsilon_{ki,m}(-x_{km}/x_{im}) & \cdots & \sum_{k=1}^N L_{km}\epsilon_{kN,m}(-x_{km}/x_{Nm}) \end{pmatrix},$$

which implies that it can be expressed in terms of the sufficient statistics. Note also that $\boldsymbol{\rho}_m$ can be expressed in terms of the sufficient statistics as well because $\partial x_{im}/\partial p_{im} = -(x_{im}/p_{im})\epsilon_{ii,m}$ holds for $i = 1, 2, ..., N$.

Proof. Using the definition of $\boldsymbol{\theta}_m(\mathbf{p})$, we can rewrite:

$$\mathbf{Z}_m[\mathbf{p}_m(\lambda)] = [\underbrace{[\boldsymbol{\theta}_m[\mathbf{p}_m(\lambda)]]^{\mathrm{T}}}_{1\times N} \circ \underbrace{(-x_{1m}[\mathbf{p}_m(\lambda)], ..., -x_{jm}[\mathbf{p}_m(\lambda)], ..., -x_{Nm}[\mathbf{p}_m(\lambda)])}_{1\times N}]\underbrace{\mathbf{H}_m^{-1}[\mathbf{p}_m(\lambda)]}_{N\times N}$$

$$= [[\boldsymbol{\theta}_m[\mathbf{p}_m(\lambda)]]^{\mathrm{T}} \circ [\boldsymbol{\mu}_m[\mathbf{p}_m(\lambda)]]^{\mathrm{T}}] \begin{pmatrix} \frac{\partial x_{1m}}{\partial p_{1m}}[\mathbf{p}_m(\lambda)] & 0 & \cdots & 0 \\ 0 & \frac{\partial x_{2m}}{\partial p_{2m}}[\mathbf{p}_m(\lambda)] & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial x_{Nm}}{\partial p_{Nm}}[\mathbf{p}_m(\lambda)] \end{pmatrix} \mathbf{H}_m^{-1}[\mathbf{p}_m(\lambda)],$$

where

$$\boldsymbol{\mu}_m(\mathbf{p})^{\mathrm{T}} = \begin{pmatrix} \frac{-x_{1m}(\mathbf{p})}{\partial x_{1m}(\mathbf{p})/\partial p_{1m}} & \cdots & \frac{-x_{im}(\mathbf{p})}{\partial x_{im}(\mathbf{p})/\partial p_{im}} & \cdots & \frac{-x_{Nm}(\mathbf{p})}{\partial x_{Nm}(\mathbf{p})/\partial p_{Nm}} \end{pmatrix}$$

is used.

Then, for the first part of the proposition, it is immediate to see that

$$\mathbf{Z}_m(\mathbf{p}_m^*) = [[\boldsymbol{\theta}_m(\mathbf{p}_m^*)]^{\mathrm{T}} \circ [\boldsymbol{\mu}_m(\mathbf{p}_m^*)]^{\mathrm{T}}]\boldsymbol{\rho}_m(\mathbf{p}_m^*),$$

which is interpreted as a result of applying the implicit function theorem to $\mathbf{g}(\mathbf{p}_m, \mathbf{c}_m) = \mathbf{0}$, where

$$\mathbf{g}(\underbrace{\mathbf{p}_m}_{N\times 1}, \underbrace{\mathbf{c}_m}_{N\times 1}) = \begin{pmatrix} \partial_{p_{1m}}\pi_{1m}(\mathbf{p}_m; c_{1m}) \\ \vdots \\ \partial_{p_{im}}\pi_{im}(\mathbf{p}_m; c_{im}) \\ \vdots \\ \partial_{p_{Nm}}\pi_{Nm}(\mathbf{p}_m; c_{Nm}) \end{pmatrix}$$

so that $\boldsymbol{\rho}_m(\mathbf{p}_m^*) = -[\mathbf{D}_{\mathbf{p}_m}\mathbf{g}]^{-1}[\mathbf{D}_{\mathbf{c}_m}\mathbf{g}]$. Now, note that $W'(1) > 0$ if the inequality in this proposition holds. Thus, given the IRCW, $W(\lambda)$ is strictly increasing in $[0, 1]$, meaning that social welfare is higher under price discrimination than under uniform pricing. This complete the proof for the first part of the proposition.

For the second part, we proceed:

$$\mathbf{Z}_m(\lambda) = [[\boldsymbol{\theta}_m^{\mathrm{T}}(\lambda) \circ \boldsymbol{\mu}_m^{\mathrm{T}}(\lambda)]\boldsymbol{\rho}_m(\lambda)[\mathbf{K}(\lambda)\mathbf{H}_m^{-1}(\lambda)],$$

for $\lambda < 1$, and thus

$$W'(\lambda) = \left\{[\boldsymbol{\theta}_w^{\mathrm{T}} \circ \boldsymbol{\mu}_w^{\mathrm{T}}]\boldsymbol{\rho}_w[\mathbf{K}\mathbf{H}_w^{-1}\mathbf{K}^{-1}] - [\boldsymbol{\theta}_s^{\mathrm{T}} \circ \boldsymbol{\mu}_s^{\mathrm{T}}]\boldsymbol{\rho}_s[\mathbf{K}\mathbf{H}_s^{-1}\mathbf{K}^{-1}]\right\} \underbrace{(\mathbf{K}\boldsymbol{\Gamma}\mathbf{t}^*)}_{<<0}.$$

Then, it is verified that

$$\begin{aligned} W'(0) < 0 &\Leftrightarrow [\overline{\boldsymbol{\theta}}_w^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_w^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w[\overline{\mathbf{K}}\,\overline{\mathbf{H}}_w^{-1}\overline{\mathbf{K}}^{-1}] > [\overline{\boldsymbol{\theta}}_s^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_s^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s[\overline{\mathbf{K}}\,\overline{\mathbf{H}}_s^{-1}\overline{\mathbf{K}}^{-1}] \\ &\Leftrightarrow [\overline{\boldsymbol{\theta}}_w^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_w^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w < [\overline{\boldsymbol{\theta}}_s^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_s^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s[\overline{\mathbf{K}}\,\overline{\mathbf{H}}_s^{-1}\overline{\mathbf{K}}^{-1}][\overline{\mathbf{K}}\,\overline{\mathbf{H}}_w^{-1}\overline{\mathbf{K}}^{-1}]^{-1} \\ &\Leftrightarrow [\overline{\boldsymbol{\theta}}_w^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_w^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w < [\overline{\boldsymbol{\theta}}_s^{\mathrm{T}} \circ \overline{\boldsymbol{\mu}}_s^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s[\overline{\mathbf{K}}\,\overline{\mathbf{H}}_s^{-1}\overline{\mathbf{H}}_w\overline{\mathbf{K}}^{-1}], \end{aligned}$$

which completes the proof. □

Note that for each i, $Z_{im}^* = \sum_{k=1}^{N} \theta_{km}^* \mu_{km}^* \rho_{ikm}^*$, which is interpreted as the *weighted sum* of firm i's own pass-through (ρ_{iim}^*) and the collection of its cross pass-through (ρ_{ikm}^*, $k \neq i$). For aggregate output and consumer surplus, we can readily generalize our previous results to the case of firm heterogeneity in a similar manner by noting that

$$Q(\lambda) = \sum_{i=1}^{N} x_{is}[\mathbf{p}_s(\lambda)] + \sum_{i=1}^{N} x_{iw}[\mathbf{p}_w(\lambda)]$$

and

$$CS(\lambda) = U_s(\mathbf{x}_s[\mathbf{p}_s(\lambda)]) + U_w(\mathbf{x}_w[\mathbf{p}_w(\lambda)]) - [\mathbf{p}_s(\lambda)]^{\mathrm{T}} \cdot \mathbf{x}_s[\mathbf{p}_s(\lambda)] - [\mathbf{p}_w(\lambda)]^{\mathrm{T}} \cdot \mathbf{x}_w[\mathbf{p}_w(\lambda)],$$

respectively. Table 3 summarizes our results for heterogeneous firms given relevant constraints that are similar to the DMCLC and the IRCQ.

Online Appendix D. Non-constant marginal costs

Notice that our results so far do not crucially depend on the assumption of constant marginal costs. The only caveat is the definition of pass-through: to properly define pass-through in accommodation with non-constant marginal costs, we introduce a small amount of unit tax $t_m > 0$ in market m: the firm's first-order derivative of the profit with respect to its own price (Equation 3 in the main text) is now replaced by

$$\partial_p \pi_m(p) = q_m(p) + (p - t_m - mc_m[q_m(p)]) \frac{\partial x_{Am}}{\partial p_A}(p, p),$$

(a) Social Welfare
If $[[\boldsymbol{\theta}_w^*]^{\mathrm{T}} \circ [\boldsymbol{\mu}_w^*]^{\mathrm{T}}]\boldsymbol{\rho}_w^* > [[\boldsymbol{\theta}_s^*]^{\mathrm{T}} \circ [\boldsymbol{\mu}_s^*]^{\mathrm{T}}]\boldsymbol{\rho}_s^*$, then $W^* > \overline{W}$.
If $[[\overline{\boldsymbol{\theta}}_w]^{\mathrm{T}} \circ [\overline{\boldsymbol{\mu}}_w]^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_w < [[\overline{\boldsymbol{\theta}}_s]^{\mathrm{T}} \circ [\overline{\boldsymbol{\mu}}_s]^{\mathrm{T}}]\overline{\boldsymbol{\rho}}_s\overline{\boldsymbol{\Delta}}$, then $W^* < \overline{W}$.
(b) Aggregate Output
If $[\boldsymbol{\theta}_w^*]^{\mathrm{T}}\boldsymbol{\rho}_w^* > [\boldsymbol{\theta}_s^*]^{\mathrm{T}}\boldsymbol{\rho}_s^*$, then $Q^* > \overline{Q}$.
If $[\overline{\boldsymbol{\theta}}_w]^{\mathrm{T}}\overline{\boldsymbol{\rho}}_w < [\overline{\boldsymbol{\theta}}_s]^{\mathrm{T}}\overline{\boldsymbol{\rho}}_s\overline{\boldsymbol{\Delta}}$, then $Q^* < \overline{Q}$.
(c) Consumer Surplus
If $[\boldsymbol{\mu}_w^*]^{\mathrm{T}}\boldsymbol{\rho}_w^* > [\boldsymbol{\mu}_s^*]^{\mathrm{T}}\boldsymbol{\rho}_s^*$, then $CS^* > \overline{CS}$.
If $[\overline{\boldsymbol{\mu}}_w]^{\mathrm{T}}\overline{\boldsymbol{\rho}}_w < [\overline{\boldsymbol{\mu}}_s]^{\mathrm{T}}\overline{\boldsymbol{\rho}}_s\overline{\boldsymbol{\Delta}}$, then $CS^* < \overline{CS}$.

Table 3: Summary of the Sufficient Conditions (with N heterogeneous firms, $\boldsymbol{\theta}_m$, $\boldsymbol{\mu}_m$, and $\boldsymbol{\rho}_m$ are the conduct vector ($N \times 1$), the profit margin vector ($N \times 1$), and the pass-through matrix ($N \times N$), respectively, in market $m = s, w$; asterisks and upper bars indicate price discrimination and uniform pricing, respectively; and $\overline{\boldsymbol{\Delta}}$ is a term for adjustment defined in the text).

where $mc_m = c_m'[q_m(p)]$ is the marginal cost at $q_m(p)$. Then, pass-through is defined by $\rho_m \equiv \frac{\partial p_m}{\partial t_m}$, and no other changes should be made to derive the results above. In fact, the usefulness of pass-through is that it can easily be accommodated with non-constant marginal costs (Weyl and Fabinger 2013, and Adachi and Fabinger 2022). An additional caveat is that $\theta_m^* \rho_m^*$ is no longer interpreted as quantity pass-through under price discrimination (Weyl and Fabinger 2013, p. 572): one needs to take into account the "elasticity of the marginal cost" to approximate the trapezoids of the welfare gain and loss by a deviation from (full) price discrimination.

Online Appendix E. Reinterpretation of Holmes' (1989) result on the output effects in terms of sufficient statistics

The following lemma holds when cost differentials are permitted.

Lemma. $Q'(t) > 0$ *if and only if (suppressing the dependence on* $p_m(t)$*)*

$$\underbrace{L_w \cdot \frac{\alpha_w^{own} + \alpha_w^{cross}}{\theta_w} - L_s \cdot \frac{\alpha_s^{own} + \alpha_s^{cross}}{\theta_s}}_{\textit{adjusted concavity}} + \underbrace{\frac{1}{\theta_s} - \frac{1}{\theta_w}}_{\textit{elasticity ratio}} > 0, \tag{21}$$

where, following Holmes (1989), we call the first and the second terms in the left hand side of *inequality the* adjusted-concavity part, *and the third and the fourth terms the* elasticity-ratio part.

Proof. It is immediate to see that $Q'(t)/2$ is also given by

$$\begin{aligned}\frac{Q'(t)}{2} &= q'_w \cdot p'_w + q'_s \cdot p'_s \\ &= -\frac{\pi''_s q'_w}{\pi''_s + \pi''_w} + \frac{\pi''_w q'_s}{\pi''_s + \pi''_w} \\ &= \underbrace{\left(-\frac{q'_s q'_w}{\pi''_s + \pi''_w}\right)}_{>0} \left(\frac{\pi''_s}{q'_s} - \frac{\pi''_w}{q'_w}\right),\end{aligned}$$

where (see Equation 9)

$$\begin{aligned}\frac{\pi''_m(p)}{q'_m(p)} &= \{2 - L_m(p)[\alpha_m^{own}(p) + \alpha_m^{cross}(p)]\} \underbrace{\frac{\epsilon_m^{own}(p)}{\epsilon_m^I(p)}}_{=\frac{1}{\theta_m(p)}} - \underbrace{\frac{\epsilon_m^{cross}(p)}{\epsilon_m^I(p)}}_{=\frac{1-\theta_m(p)}{\theta_m(p)}} \\ &= \frac{2 - L_m \cdot (\alpha_m^{own} + \alpha_m^{cross}) - (1 - \theta_m)}{\theta_m},\end{aligned}$$

Hence,

$$\frac{Q'(t)}{2} = \underbrace{\left(-\frac{q'_s q'_w}{\pi''_s + \pi''_w}\right)}_{>0} \left[\left(L_w \cdot \frac{\alpha_w^{own} + \alpha_w^{cross}}{\theta_w} - \frac{1}{\theta_w}\right) - \left(L_s \cdot \frac{\alpha_s^{own} + \alpha_s^{cross}}{\theta_s} - \frac{1}{\theta_s}\right)\right],$$

which completes the proof. This is also interpreted as a generalization of ACV's (2010, p. 1608) Equation (6). □

Consider the adjusted-concavity part. A larger α_w^{own} and/or α_w^{cross} make a positive $Q'(t)$ more likely. A larger α_w^{own} means that the firm's own part of the demand in the weak market ($\partial x_{A,w}/\partial p_A$) is more convex ("the output expansion effect"). Similarly, a larger α_w^{cross} means that how many of the firm's customers switch to the rival's product as a response to the firm's price increase is not so much affected by the current price level ("the countervailing effect"). In this sense, the strategic concerns in the firm's pricing are small. Thus, both a larger α_w^{own} and a larger α_w^{cross} indicate that the weak market is competitive. Even if $\partial x_{A,w}/\partial p_A$ is not so convex, a larger α_w^{cross} can substitute it. Here, the intensity of market competition, $1/\theta_w$, magnifies both effects, resulting in α_w^{own}/θ_w and $\alpha_w^{cross}/\theta_w$. A similar argument also holds for α_s^{own} and α_s^{cross}.

Additionally, we are able to show that Holmes' (1989) expression for $Q'(t)$ (expression (9) in Holmes 1989, p. 247) is equivalent to the left hand side of Inequality (21) above. First, Holmes (1989, p. 247), who assumes no cost differentials ($c \equiv c_s = c_w$) as in most of the papers on third-degree price discrimination, derives a necessary and sufficient condition for $Q'(t) > 0$ under symmetric oligopoly. It is (using our notation) written as:

$$\underbrace{\frac{p_s - c}{q_s'(p_s)} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s, p_s)}{\partial p_A}\right) - \frac{p_w - c}{q_w'(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w, p_w)}{\partial p_A}\right)}_{\text{adjusted-concavity condition (Robinson 1933)}}$$
$$+ \underbrace{\frac{\epsilon_s^{cross}(p_s)}{\epsilon_s^I(p_s)} - \frac{\epsilon_w^{cross}(p_w)}{\epsilon_w^I(p_w)}}_{\text{elasticity-ratio condition (Holmes 1989)}} > 0.$$

Recall that

$$\frac{1}{\theta_s} - \frac{1}{\theta_w} = \frac{\epsilon_s^{cross}}{\epsilon_s^I} - \frac{\epsilon_w^{cross}}{\epsilon_w^I}.$$

Then, the first and the second terms in the left hand side of Holmes' (1989) inequality is rewritten as:

$$\begin{aligned} & \frac{p_s - c}{q_s'(p_s)} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s, p_s)}{\partial p_A}\right) - \frac{p_w - c}{q_w'(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w, p_w)}{\partial p_A}\right) \\ = \; & L_w(p_w) \cdot \left[\left(-\frac{p_w}{q_w'(p_w)}\right)\frac{d}{dp_w}\left(\frac{\partial x_{A,w}(p_w, p_w)}{\partial p_A}\right)\right] \\ & -L_s(p_s) \cdot \left[\left(-\frac{p_s}{q_s'(p_s)}\right)\frac{d}{dp_s}\left(\frac{\partial x_{A,s}(p_s, p_s)}{\partial p_A}\right)\right]. \end{aligned}$$

Now, it is also observed that

$$\begin{aligned} \frac{\alpha_m^{own} + \alpha_m^{cross}}{\theta_m} &= \frac{\partial x_{Am}/\partial p_A}{q_m'}\left(-\frac{p_m}{\partial x_{Am}/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_A^2} - \frac{p_m}{\partial x_{Am}/\partial p_A}\frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}\right) \\ &= -\frac{p_m}{q_m'}\left(\frac{\partial^2 x_{Am}}{\partial p_A^2} + \frac{\partial^2 x_{Am}}{\partial p_B \partial p_A}\right). \\ &= \left(-\frac{p_m}{q_m'(p_m)}\right)\frac{d}{dp_m}\left(\frac{\partial x_{Am}(p_m, p_m)}{\partial p_A}\right). \end{aligned}$$

Therefore, Inequality (21) above is another expression for Holmes' (1989, p. 247) Inequality (9) because

$$\frac{p_s - c}{q_s'} \cdot \frac{d}{dp_s}\left(\frac{\partial x_{A,s}}{\partial p_A}\right) - \frac{p_w - c}{q_w'(p_w)} \cdot \frac{d}{dp_w}\left(\frac{\partial x_{A,w}}{\partial p_A}\right) = L_w \cdot \frac{\alpha_w^{own} + \alpha_w^{cross}}{\theta_w} - L_s \cdot \frac{\alpha_s^{own} + \alpha_s^{cross}}{\theta_s}.$$

Online Appendix F. Why the exogenous quantity method does not work under firm heterogeneity

Weyl and Fabinger (2013) and Miklós-Thal and Shaffer (2021a) use what they the exogenous quantity method to study the welfare effects of price discrimination (see Footnote 29 in the main text, but refer to their original papers for more details). However, this method is not readily extendible to the case of firm heterogeneity. To see this, note that their welfare arguments rely on the *deadweight loss* (DWL) in market $m \in \{s, w\}$ under $\mathbf{x}_m = (x_{1m}, x_{2m}, ..., x_{Nm})$ by

$$\begin{aligned} DWL_m(\mathbf{x}_m) &= U_m(\mathbf{x}_m^{FB}) - U_m(\mathbf{x}_m) - \sum_{i=1}^{N} c_{im} \cdot (x_{im}^{FB} - x_{im}) \\ &= \int_{x_{1m}}^{x_{1m}^{FB}} \cdots \int_{x_{Nm}}^{x_{Nm}^{FB}} \frac{\partial^N U(s_1, s_2, ..., s_N)}{\partial s_1 \cdots \partial s_N} ds_1 \cdots ds_N \\ &\qquad - \sum_{i=1}^{N} c_{im} \cdot (x_{im}^{FB} - x_{im}), \end{aligned} \tag{22}$$

where $\mathbf{x}_m^{FB} = (x_{1m}^{FB}, x_{2m}^{FB}, ..., x_{Nm}^{FB})$ is the first-best output pair that satisfies:

$$\begin{cases} \dfrac{\partial U_m}{\partial x_{1m}}(\mathbf{x}_m^{FB}) = c_{1m} \\ \cdots \\ \dfrac{\partial U_m}{\partial x_{Nm}}(\mathbf{x}_m^{FB}) = c_{Nm}. \end{cases}$$

Under firm symmetry, the DWL expression (22) is simplified to

$$\begin{aligned} DWL_m(x) &= \int_{x}^{x^{FB}} \left[\frac{\partial U}{\partial s}(s) - c\right] ds \\ &= \int_{x}^{x^{FB}} [p(s) - c]\, ds, \end{aligned} \tag{23}$$

where $p(q)$ is the inverse demand, as Weyl and Fabinger (2013, p. 537) and Miklós-Thal and Shaffer (2021a, p. 325) define. This expression has a nice feature in that $p(s) - c$ is the markup value. However, under firm heterogeneity, the DWL expression (22) cannot be as simple as (23): this is the reason why the exogenous quantity method cannot be utilized as it draws on this expression.