Lifetime Analysis of Repairable Systems by Daubechies Wavelets

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1 Introduction

Lifetime analysis is quite beneficial to understand the failure mechanism of engineering systems by means of the theoretical statistics and can be applied to design their maintenance planning [2, 3]. In general, there are two types of systems in the lifetime analysis; repairable systems and non-repairable systems. Especially, the repairable system reliability modeling plays a central role in reliability engineering practices [1], because non-repairable systems are maintained by the so-called perfect repair, which is equivalent to replace the failed units/components by new ones. That is to say, the perfect repair offers a new unit which is as good as new when the operating unit fails. On the other hand, the minimal repair only restores the failed unit to its functioning condition just prior to the failure, where the minimal repair means the age (in the sense of effectiveness) of the unit will not be changed after a repair activity. Since the seminal contribution by Barlow and Hunter [2], a great number of researchers have discussed the minimal repair and its mathematical features. Nakagawa and Kowada [5] used the failure rate to define minimal repair, and gave an insight to deal with the minimal repair process mathematically. Aven [6] considered the optimal replacement policies under a minimal repair assumption. Brown and Proschan [7] introduced the imperfect repair by combining the minimal repair with the perfect repair. Block et al. [8] extended the notation of imperfect repair by introducing the time dependency. Kijima [9] introduced the notion of general repair and succeeded to represent a wider class of repair activities. Although the general repair is a well-defined stochastic model to describe the repair activities, its analytical treatment is rather troublesome. In other words, due to the analytical difficulty of the general repair model, it has not been frequently used in the industry. Instead, since the minimal repair approximately represents repair effects under a plausible assumption, i.e., it only restores the failed unit to the latest functioning condition just prior to the failure, the research for the minimal repair has been still conducted in some literatures [10–12].

In this paper we concern the statistical inference problem of the minimal repair process, which denotes the cumulative number of failures/minimal repairs. It is well known that the minimal repair process is given by a non-stationary Poisson process. Hence, the statistical inference problem of the minimal repair process is equivalent to one for the Poisson process. For the representative parametric models, the power law model by

Duane [13] and Cox and Lewis model [15] have been often used in the lifetime analysis. Lee and Lee [16] and Crow [17] gave the fundamental results for the power law model; point estimation based on the maximum likelihood method and the confidence intervals, respectively. Muralidharan et al. [18, 19] derived the predictive distribution of the power law model and the conditional interval estimate for the ratio of intensity parameters, respectively. Saito et al. [20, 21] applied parametric and non-parametric bootstrapping techniques to estimate the optimal preventive replacement times with minimal repair, respectively.

In this paper, we consider the lifetime analysis of repairable systems via the Daubechies wavelets [22]. Kuhl and Bhairgond [23] proposed a Daubechies wavelet estimator for the intensity function of a non-stationary Poisson process. Their estimator is based on an approximation with a naive estimate of the intensity function, and provides a nice estimation performance as a non-parametric estimator, though they did not clarify the derivation procedure. We apply the wavelet-based approach to the inference problem of the minimal repair process. Especially, we propose an exact approach with a naive estimator and a non-parametric maximum likelihood estimation in estimating non-stationary Poisson processes [24]. Finally, throughout a simulation experiment, we compare our Daubechies wavelet approaches with the classical maximum likelihood estimation for two representative parametric models.

2 Repairable Systems

Minimal repair and replacement are frequently used as practical maintenance activities for real engineering systems, where the minimal repair is a maintenance activity to repair the failed component, so that its function is recovered without changing its age, while a replacement restores the entire component into the new condition, so that it behaves as a new component. Suppose at the moment that there are two components, where the first (second) component has the absolutely continuous lifetime distribution $F(t)$ $(G(t))$ with $F(0) = 0$ and $F(\infty) = 1$ $(G(0) = 0$ and $G(\infty) = 1)$. When the first component fails and the failed component is replaced by the second component, the distribution function of the time to failure of the second of two components is given by the Stieltjes convolution:

$$F * G(t) = \int_0^t F(t-u)dG(u), \tag{1}$$

where the replacement component is assumed to be new on installation. On the other hand, when one replaces the failed component by one with equal age, the survivor function of the first and second components is given by the relevation transform (see Krakowski [25]):

$$F \circ G(t) = \overline{F}(t) + \int_0^t \frac{\overline{G}(t)}{\overline{G}(u)} dF(u), \tag{2}$$

where in general $\overline{\psi}(\cdot) = \psi(\cdot)$. Barlow and Hunter [2] and Barlow and Proschan [3] summarized the notion of overhaul with minimal repair for any intervening failures, where the failure rate of component remains undisturbed by any repair of failures.

If the replacement is made with identical components, say $F(t) = G(t)$ in Eq.(1), and the time to replace an failed component by a new one is negligible, then the cumulative number of failures/replacements by time t is given by the renewal counting process $N(t) = \max\{n : \sum_{i=1}^{n} T_i \leq t\}$, where $F(t) = P\{T_i \leq t\}$ $(i = 1, 2, \ldots)$. On the other hand, if the minimal repair is made at each failure and the time to repair an failed component is negligible, then the cumulative number of failures/repairs by time t is, from $F(t) = G(t)$ in Eq.(2), given by the non-stationary Poisson process (see Baxter [26]):

$$P(N(t) = n) = \frac{\Lambda(t;\boldsymbol{\theta})^n}{n!} e^{-\Lambda(t;\boldsymbol{\theta})}, \tag{3}$$

where

$$\Lambda(t;\boldsymbol{\theta}) = \mathrm{E}[N(t)] = -\log \overline{F}(t;\boldsymbol{\theta}) = \int_0^t \lambda(u;\boldsymbol{\theta}) du \tag{4}$$

is the mean value function with the parameter vector $\boldsymbol{\theta}$, say, $F(t) = F(t;\boldsymbol{\theta})$, and $\lambda(t;\boldsymbol{\theta}) = d\Lambda(t;\boldsymbol{\theta})/dt$ is called the intensity function.

The commonly used technique to estimate the parameter $\boldsymbol{\theta}$ in $\Lambda(t;\boldsymbol{\theta})$ and $\lambda(t;\boldsymbol{\theta})$ is the maximum likelihood estimation. Suppose that n failure/repair times $\boldsymbol{t} = (t_1, t_2, \ldots, t_n)$ with right truncation at T $(\geq t_n)$ are observed for each component. Then, the log likelihood function is given by

$$LLF(\boldsymbol{\theta};\boldsymbol{t}) = \sum_{i=1}^{n} \lambda(t_i;\boldsymbol{\theta}) - \Lambda(T;\boldsymbol{\theta}). \tag{5}$$

By maximizing $LLF(\boldsymbol{\theta};\boldsymbol{t})$ with respect to $\boldsymbol{\theta}$, we get the maximum likelihood estimate $\hat{\boldsymbol{\theta}}$.

In the lifetime analysis, it is common to assume the power law model [13] and Cox and Lewis model [15]. In the power-law model, the intensity function is given by $\lambda(t;\boldsymbol{\theta}) = abt^{b-1}$, $\boldsymbol{\theta} \in (a, b)$. It is known as a flexible model with increasing intensity (b>1) and decreasing intensity (0<b<1) functions. When $b = 1$, it is reduced to a homogeneous Poisson process with constant intensity. In Cox and Lewis model, the intensity function is given by $\lambda(t;\boldsymbol{\theta}) = -\exp(a + bt)$, $\boldsymbol{\theta} \in (a, b)$. Because the maximum likelihood method provides an unbiased estimate in many cases, if we have an enough number of data and can know the real intensity form, it is possible to infer the failure process with minimal repair accurately. Of course, since knowing the real intensity function in advance is difficult, the parametric method above has some limitations.

3 Wavelet-based Approach

3.1 Daubechies Wavelets

Daubechies [22] proposed a set of continuous and conpactly supported wavelets, which are very popular in wavelet analysis field. The Daubechies wavelets are not defined in closed form, where the Daubechies scaling function and wavelet function are defined in the following forms:

$$\phi(t) = \sum_{i=0}^{n} h_i \phi(2t - i), \tag{6}$$

$$\psi(t) = \sum_{i=0}^{n} (-1)^i h_{n-i} \psi(2t - i). \tag{7}$$

In Eqs.(6) and (7), the filter coefficients h_i are given in [22], and n is the support width and determines the smoothness of the functions $\phi(t)$ and $\psi(t)$. The starting values $\{\phi(t),\ t = 1, 2, \ldots, n-1\}$ can be obtained by solving the recursive formula:

$$\begin{cases} \sum_{t=0}^{n} \phi(t) = 1, \\ \phi(0) = 0, \\ \phi(n) = 0, \\ \phi(t) = \sum_{i=0}^{n} h_i \phi(2t - i), \quad t = 1, 2, \ldots, n-1. \end{cases}$$

The other values of $\phi(t)$ with $t \in [0, n]$ and $t \neq 1, 2, \ldots, n-1$ are calculated in Eq. (6).

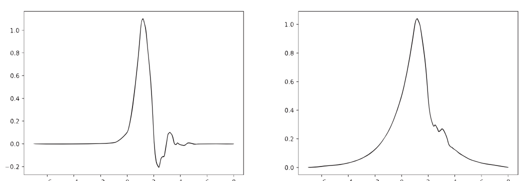
3.2 Wavelet Estimator

Since the Daubechies scaling function $\phi(t)$ can take negative values, one needs a positive basis function for approximating a positive $\lambda(t)$. Walter and Shen [27] developed a positive basis function for estimating probability density functions.

Let $\phi(t)$ be the Daubechies scaling function having the compact support. The positive basis function by Walter and Shen [27] is given by

$$P_r(t) = \sum_{j \in \mathbb{Z}} r^{\|j\|} \phi(t - j) \tag{8}$$

with parameter r satisfying $a \leq r < 1$, where $\mathbb{Z}$ is a set of all integers. The parameter r controls the minimum value of $P_r(t)$, so that the minimum value of $P_r(t)$ is greater than or equal to 0 when $r = a$ (>0). Figure 1 illustrates the positive basis functions with $r = 0.1$ and $r = 0.5$ when $\phi(t)$ with $n = 7$. In this case, we can see that the minimum value of $P_r(t)$ is less than 0 when $r = 0.1$.



(a) $n = 7$ and $r = 0.1$. (b) $n = 7$ and $r = 0.5$.

Figure 1: Positive basis functions.

By using the positive basis function $P_r(t)$, a positive reproducing kernel, $k_r(t, s) \in V_0$, is given by

$$k_r(t, s) = \left(\frac{1-r}{1+r}\right)^2 \sum_{a=-\infty}^{\infty} P_r(t - a) P_r(s - a). \tag{9}$$

Let $k(t, s)$ denote a reproducing kernel satisfying

$$\int_{-\infty}^{\infty} k(t, s) \lambda(s) ds = \lambda(t), \tag{10}$$

where $\lambda(t)$ is an arbitrary continuous function. For $\lambda(t) \in L_2(\mathbb{R})$, an approximation of the function $\lambda_0(t) \in V_0$ is constructed as

$$\lambda_0(t) \approx \int_{-\infty}^{\infty} k_r(t,s)\lambda(s)ds. \tag{11}$$

In general, the approximation of an arbitrary function $\lambda(t) \in V_m$ is given by

$$\lambda_m(t) = \int_{-\infty}^{\infty} k_{r,m}(t,s)\lambda(s)ds, \tag{12}$$

where $k_{r,m}(t,s)$ in Eq.(12) is the positive reproducing kernel in V_m and is given by

$$k_{r,m}(t,s) = 2^m k_r(2^m t, 2^m s). \tag{13}$$

Kuhl and Bhairgond [23] proposed a wavelet estimator based on Eq. (12) as follows:

$$\hat{\lambda}_{r,m}(t) = 2^m \left(\frac{1-r}{1+r}\right)^2 \sum_{a=-k}^{k} \left\{ \sum_{i=1}^{n} P_r(2^m t_i - a) \right\} P_r(2^m t - a), \tag{14}$$

where t_i $(i = 1, 2, \ldots, n)$ are the failure/repair times, the parameter a is determined in the range in which the positive basis function covers the entire failure/repair times, and the resolution level m is determined based on the detail of the approximation. Unfortunately, Kuhl and Bhairgond [23] did not clarify the derivation procedure of their wavelet estimator in Eq. (14). Here, we derive the same result to complete the discussion and improve Kuhl and Bhairgond's estimator [23].

Let $\lambda_{naive}(t)$ denote the well-known naive estimator of non-stationary Poisson intensity function:

$$\hat{\lambda}_{naive}(t) = \sum_{i=1}^{n} \frac{1}{t_i - t_{i-1}} I_i(t), \tag{15}$$

where

$$I_i(t) = \begin{cases} 1, & t_{i-1} < t \le t_i \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Substituting the naive estimator $\lambda(t)$ in Eq. (15). into Eq.(12), we can obtain the naive wavelet estimator (NWE):

$$\begin{aligned} \hat{\lambda}_{NWE}(t) &= \int_{-\infty}^{\infty} k_{r,m}(t,s)\hat{\lambda}_{naive}(s)ds \\ &= \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} k_{r,m}(t,s) \frac{1}{t_i - t_{i-1}} ds \\ &= 2^m \left(\frac{1-r}{1+r}\right)^2 \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \sum_{a=-\infty}^{\infty} P_r(2^m t - a) \\ &\times Pr(2^m s - a) \frac{1}{t_i - t_{i-1}} ds \\ &= 2^m \left(\frac{1-r}{1+r}\right)^2 \sum_{a=-\infty}^{\infty} \Big\{ \sum_{i=1}^{n} \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \\ &\times Pr(2^m s - a) ds \Big\} P_r(2^m t - a). \end{aligned} \tag{17}$$

Hence it is seen that the exact form of NWE contains the integrals. If each integral is approximated by the elementary rectangular approximation method, it is given by

$$\begin{aligned}\hat{\lambda}_{RNWE}(t) = 2^m \left(\frac{1-r}{1+r}\right)^2 \sum_{a=-\infty}^{\infty} \left\{ \sum_{i=1}^{n} \frac{1}{t_i - t_{i-1}} \int_{t_{i-1}}^{t_i} \right. \\ \times Pr(2^m t_i - a) ds \} \, P_r(2^m t - a) \\ = 2^m \left(\frac{1-r}{1+r}\right)^2 \sum_{a=-\infty}^{\infty} \left\{ \sum_{i=1}^{n} \frac{1}{t_i - t_{i-1}} (t_i - t_{i-1}) \right. \\ \times Pr(2^m t_i - a) \} \, P_r(2^m t - a) \\ = 2^m \left(\frac{1-r}{1+r}\right)^2 \sum_{a=-\infty}^{\infty} \left\{ \sum_{i=1}^{N} P_r(2^m t_i - a) \right\} \\ \times P_r(2^m t - a). \qquad (18)\end{aligned}$$

We called the above estimator the rectangular approximation naive wavelet estimator (RNWE), which is equivalent to Kuhl and Bhairgond's estimator [23]. Strictly speaking, RNWE is an approximation of NWE from the computational point of view. In order to compute the NWE, $\hat{\lambda}_{NWE}(t)$, more accurately, we need to apply any numerical integration algorithm for Eq.(12).

As we have already shown in the above, both NWE and RNWE are based on the naive estimator in Eq.(15) of the non-stationary Poisson process. However, it is also well known that the naive estimator is the most intuitive estimator but causes the overfitting problem. In other words, the native estimator lacks the generalization ability as a statistical estimator. In this paper, as the alternative baseline estimators of the non-stationary Poisson process, we employ the non-parametric maximum likelihood estimator (NPMLE) and kernel estimator. Boswell [24] proposed the following NPMLE of the intensity function by maximizing the upper bound of the empirical log likelihood function:

$$\hat{\lambda}_{NPMLE}(t) = \begin{cases} 0, & 0 \le t < t_1, \\ r_k(t), & t_k \le t < t_{k+1}, \\ r_{n-1}(t), & t = t_n, \\ \infty, & t_n < t, \end{cases} \qquad (19)$$

where

$$r_k(t) = \min_{v \ge k+1} \max_{u \le k} \left[\frac{v-u}{J(u,v)} \right],$$

$$J(u,v) = \sum_{i=u+1}^{v} \left\{ (n-i+1)(t_i - t_{i-1}) \right\}.$$

Then we define the non-parametric maximum likelihood wavelet estimator (NPMLWE) by

$$\hat{\lambda}_{NPMLWE}(t) = \int_{-\infty}^{\infty} k_{r,m}(t,s) \hat{\lambda}_{npmle}(s) ds. \qquad (20)$$

Dohi [14] proposed the kernel estimation for intensity function of NHPP in software reliability model. The kernel estimation of intensity function is given by

$$\hat{\lambda}_{kernel}(x) = \frac{1}{h}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right), \tag{21}$$

where the function $K(\cdot)$ is the kernel function and h (<0) is the bandwidth. Barghout, Littlewood, and Abdel-Ghaly [4] and Wang, Wang, and Liang [28] assumed the Gaussian kernel function. In this article we use the following four kernel functions:

$$K_1(x) = (1 - |x|)I_{[-1,1]}(x), \tag{22}$$

$$K_2(x) = \frac{3}{4}(1 - x^2)^2 I_{[-1,1]}(x), \tag{23}$$

$$K_3(x) = \frac{15}{16}(1 - x^2)^2 I_{[-1,1]}(x), \tag{24}$$

$$K_4(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \tag{25}$$

$$K_5(x) = \frac{70}{81}(1 - |x|^3)^3 I_{[-1,1]}(x), \tag{26}$$

where

$$I_{[-1,1]}(x) = \begin{cases} 1 & \text{for } x \in [-1,1] \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

Then we define the kernel wavelet estimator (KWE) by

$$\hat{\lambda}_{KWE}(t) = \int_{-\infty}^{\infty} k_{r,m}(t,s)\hat{\lambda}_{kernel}(s)ds. \tag{28}$$

4 A Simulation Experiment

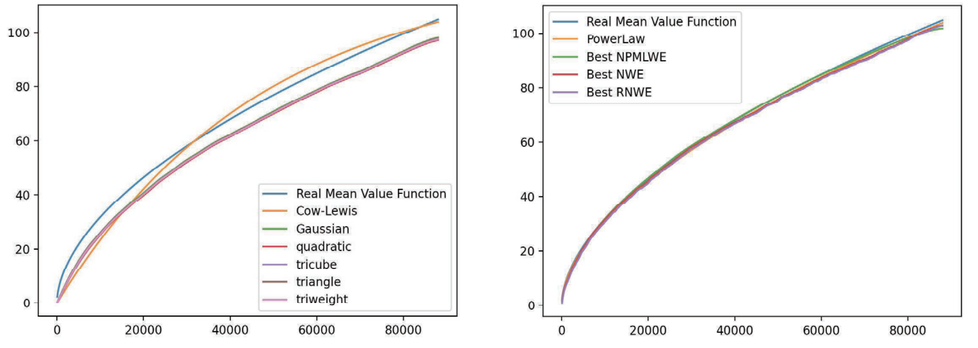
4.1 Experimental Set Up

In this section, we carry out the Monte Calro simulation and estimate the intensity function characterizing the minimal repair process with the common parametric methods and the non-parametric wavelet methods. Our concern here is to compare the Daubechies wavelet approaches with the maximum likelihood methods for two representative minimal repair processes in terms of the goodness-of-fit. For the parametric models, we assume the power law model [13] and Cox and Lewis model [15] and apply the maximum likelihood method to estimate the model parameters. For non-parametric methods, we apply three Daubechies wavelets; NWE, RNWE, NPMLWE and KWE to estimate the intensity function of non-stationary Poisson process. In the Monte Carlo simulation, it is assumed that the intensity function of a real non-stationary Poisson process is given by the power law model with $\lambda_{real}(t;\boldsymbol{\theta}) = abt^{b-1}$ for the time interval $(0,T]$, where $a = 0.20$, $b = 0.55$ and $T = 88,000$. We apply the standard linear congruential generator [29] to generate the pseudo random numbers as failure/repair times $\boldsymbol{t} = (t_1, t_2, \ldots, t_n)$. Finally we generate 30 failure/repair time data sets and estimate the number of minimal repairs when 30 independent and identical components are running in parallel.

As a goodness-of-fit performance measure, we apply the mean absolute error (MAE):

$$\text{MAE} = \frac{\sum_{i=1}^{n} |\lambda_{real}(t_i;\boldsymbol{\theta}) - \hat{\lambda}(t_i)|}{n} \tag{29}$$

for each component, where t_i is the i-th sample point, n is the number of samples for each component, $\hat{\lambda}(t)$ is an estimate of the intensity function. Based on the assumption that 30 independent and identical components run in parallel, we observe 30 failure/minimal repair time sequence with the same truncation point T. Formally, let $\boldsymbol{T} = (\boldsymbol{t}_1, \boldsymbol{t}_2, \ldots, \boldsymbol{t}_{30})$ be the failure/minimal repair time sequence. For parametric approach with the power law model and Cox-Lewis model, the maximum likelihood estimation is carried out by maximizing $LLF(\boldsymbol{\theta};\boldsymbol{T}) = \sum_{i=1}^{30} LLF(\boldsymbol{\theta};\boldsymbol{t}_i)$. For the wavelet approach with NWE, RNWE and NPMLWE, we estimate each intensity function for $\boldsymbol{t}_i$ $(i = 1, 2, \ldots, 30)$ and take the average them to obtain the final estimates.



(a) (b)

Figure 2: Estimation of the cumulative number of failuress/minimal repairs.

In the Daubechies wavelets, the parameter j in the positive basis function $P_r(t)$ can take all values in $\mathbb{Z}$. Xiao and Dohi [30] gave a useful range for j as $j \in [-7, 7]$ in their experiment on software fault data analysis. Since the function $P_r(t)$ decays to zero quickly, we adjusted as $j \in [-7, 8]$ in our case. The range of the parameter s in $\hat{\lambda}_{r,m}(t)$ is determined so as to satisfy that the positive basis function covers the entire failure/minimal repair times. Xiao and Dohi [30] also took the range of s as

$$s \in [-(\text{Integer part of } 2^m t_n + 7),\ \text{Integer part of } 2^m t_n + 7\ ], \tag{30}$$

where t_n is the n-th failure/repair time.

4.2 Discussion

In the simulation experiment, we compare three wavelet shrinkage estimators; NWE, RNWE and NPMLWE in terms of goodness-of-fit, where we turn up the parameters as $r = 0.3,\ 0.5,\ 0.7$, $m = 5,\ 6,\ 7,\ 8,\ 9,\ 10$ and the support width of wavelet is given by 7. Table

Table 1: Sensitivity of model parameters m and r in the wavelet shrinkage methods.

m\r	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
	NWE			NPMLWE			RNWE			KWE with Gaussian		
2	12.09	17.83	30.86	8.94	15.11	29.02	10.02	14.96	26.3	10.91	16.46	29.88
3	8.22	13	26	5.49	10.43	24.18	6.77	10.3	20.9	8.24	12.44	25.49
4	5.84	9.78	22.84	3.36	7.48	21.15	4.7	7.29	17.44	6.87	10.19	22.95
5	4.23	7.66	20.83	1.89	5.53	19.18	3.39	5.38	15.2	6.27	9.11	21.63
6	3.12	6.29	19.43	0.92	4.31	17.95	2.53	4.17	13.8	6.04	8.67	21.06
7	2.25	5.28	18.56	0.54	3.44	17.12	1.91	3.36	12.91	5.96	8.52	20.86
8	1.62	4.59	17.92	0.65	2.84	16.55	1.5	2.78	12.28	5.93	8.47	20.8
9	1.28	4.18	17.52	0.8	2.43	16.14	1.73	2.49	11.86	5.93	8.46	20.79
10	0.96	3.93	17.3	0.94	2.23	15.94	2.48	2.59	11.67	5.93	8.46	20.81
	KWE with Quadratic			KWE with tricube			KWE with Triangle			KWE with Triweight		
2	11.21	16.63	29.95	11.02	16.51	29.89	11.06	16.57	29.93	11.08	16.56	29.92
3	8.7	12.74	25.65	8.45	12.54	25.53	8.45	12.6	25.59	8.5	12.61	25.58
4	7.48	10.66	23.23	7.19	10.41	23.06	7.11	10.4	23.09	7.21	10.45	23.11
5	6.99	9.72	22.05	6.67	9.43	21.84	6.53	9.34	21.8	6.67	9.45	21.86
6	6.82	9.38	21.59	6.5	9.08	21.35	6.31	8.93	21.26	6.48	9.06	21.35
7	6.77	9.28	21.44	6.46	8.97	21.2	6.24	8.78	21.07	6.43	8.95	21.19
8	6.76	9.25	21.41	6.44	8.95	21.17	6.21	8.73	21.01	6.41	8.92	21.15
9	6.76	9.25	21.42	6.44	8.95	21.18	6.2	8.72	21	6.41	8.92	21.15
10	6.76	9.26	21.44	6.44	8.96	21.2	6.2	8.72	21.02	6.41	8.92	21.18

1 presents the MAE of the three wavelet estimators with different turning parameters. It can be seen that MAE decreases, as m increases and r decreases, respectively, in almost all wavelet estimators, except NPMLWE and RNWE. Especially, when $r = 0.3$ and $m = 10$, KWEs and NWE could show the best goodness-of-fit performance. When $r = 0.3$ and $m = 7$, NPMLE shows the best goodness-of-fit performance. When $r = 0.3$ and $m = 8$, RNWE shows the best goodness-of-fit performance. Figure 2 shows the behavior of the cumulative number of failures/minimal repairs on the operation time, where the real mean value function for two parametric models are estimated based on the maximum likelihood method, and all wavelet shrinkage estimates with best goodness-of-fit performance are plotted. It is observed that almost all estimates, except Cox and Lewis model and KWEs, are close to the real mean value function. The result of Cox and Lewis model shows that missing the model selection causes the worse estimation of the cumulative number of failures/repairs. Table 2 compares all the MAE results. It is seen that KWEs provided the worst result in terms of the minimization of MAE. If we can know the real failure/repair model completely, the resulting MAE in the power law model was 1.21. On the other hand, it is worth noting that NPMLWE gave smaller MAEs than the maximum likelihood method with the real model. Hence, for the purpose on goodness-of-fit by minimizing MAE, it can be concluded that NPMLWE outperformed the others. However, this does not always imply that our wavelet methods are superior to the parametric approach. Because NPMLWE might result the over fitting estimates. In our simulation experiment, the failure/repair time data employed in the analysis are large enough, so that the MAE for the power law model might be small enough. Hence, the superiority of the wavelet approach should be further investigated by checking the

asymptotic behavior of estimates and the predictive performance for unknown future trend.

Table 2: Comparison of MAEs in simulation experiment.

Model Name	MAE	Model Name	MAE
Power Law	0.93	Best KWE with Gaussian	5.93
Cow-Lewis	3.31	Best KWE with quadratic	6.76
Best NWE	0.96	Best KWE with tricube	6.44
Best NPMLWE	0.54	Best KWE with triangle	6.2
Best RNWE	1.5	Best KWE with triweight	6.41

5 Conslusion

In this paper, we have developed novel non-parametric estimation methods based on the Daubechies wavelets for failure/repair time data arising in the lifetime analysis. More specifically, we have revisited Kuhl and Bhairgond estimator [23], and given the exact solution based on the naive estimator of the intensity function for a non-stationary Poisson process and alternative method based on the non-parametric maximum likelihood method by Boswell [24]. Throughout a simulation experiment, we have compared our Daubechies wavelet approaches with the classical maximum likelihood estimation for two parametric models. It has been shown that two wavelet-based estimation methods; Kuhl and Bhairgond estimator [23] and the non-parametric maximum likelihood wavelet estimator, gave the smaller mean absolute errors than the parametric models. In the future, we will propose another non-parametric wavelet-based method with the kernel function.

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