Cognitive Neuroscientific Research for Developing Diagram Use Instruction for Effective Mathematical Word Problem Solving

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The research conducted for this dissertation examined the requirements for acquiring knowledge and skills that enable students to use diagrams effectively for solving mathematical word problems. The findings have useful implications for not only diagram use instruction, but also mathematics instruction, and the cultivation of effective strategy use in students. This dissertation consists of nine chapters, including five that describe studies that have been carried out. Chapter 1 presents the general introduction and the structure of the dissertation, Chapter 2 the scope of the dissertation, Chapter 3 an overview of the principles and theories behind the dissertation, Chapters 4 to 8 the details of the five studies, and Chapter 9 a general discussion and the conclusions that can be drawn.

Cultivating the ability to use multiple representations is essential in 21st-century education (e.g., National Research Council, 2012). Students should be able to utilize not only verbal representations (e.g., written and spoken words) but also visual representations, such as diagrams, to think about and understand information, to solve problems, and to communicate. Despite the generally accepted importance, teachers rarely provide instructions about using diagrams to their students. Most schools teach mathematical word problem solving to students so that they can apply what they have learned in mathematics to real life. Previous studies have repeatedly reported evidence that diagram use is an effective strategy for such problem solving (e.g., Hembree, 1992), while also revealing that some serious problems stand in the way of students acquiring the ability to use diagrams for such purposes (e.g., Uesaka et al., 2007). This dissertation firstly clarifies those.

The first problem is a general lack of spontaneity in using diagrams (production deficit; e.g., Dufour-Janvier et al., 1990; Uesaka et al., 2007; van Garderen et al., 2013). Mathematical word problem solving requires basic cognitive abilities such as reading, mathematical, and visual skills (Boonen et al., 2014; Daroczy et al., 2015; Oostermeijer et al., 2014). Diagrams are generally considered advantageous in facilitating such cognitive processes. However, students tend not to use diagrams. This lack of spontaneity is a serious problem because it means that many students cannot benefit from applying the strategy of diagram use to their problem solving.

The second problem is that many students fail to choose the appropriate kind of diagram to use in their problem solving (i.e., they lack of conditional knowledge in diagram use). This has been reported in previous studies about mathematical word problem solving (e.g., Corter & Zahner, 2007; Hegarty & Kozhevnikov, 1999; Van Garderen et al., 2013). One main reason is that the rules that govern conditional knowledge about the appropriate kinds of diagrams to use for particular problem types are not specified anywhere. Researchers have examined various cognitive models for solving mathematical word problems (e.g., Kintsch & Greeno, 1985) and the cognitive properties of some diagrams in relation to certain kinds of problems (e.g., Novick & Hurley, 2001). However, they have not clarified the correspondence between kinds of diagrams and types of mathematical word problems that, for example, junior high school students should learn how to solve. As a result, teachers rarely teach conditional knowledge explicitly in classroom settings.

The third problem is that many students fail to derive correct answers in problem solving even when they use the kind of diagram that is appropriate for the problem (Uesaka & Manalo, 2006; Uesaka et al., 2007, 2010). Even if the teacher manages to get students to construct appropriate diagrams for mathematical word problems they are attempting to solve (e.g., by giving them the opportunity to consult with classmates or providing a prompt to use the diagram), many still fail to correctly solve the problems. The likely cause of such failure is that students cannot correctly undertake the procedures for using the diagram and, subsequently, to draw the necessary inferences from it to derive the answer required. This suggests that sufficient knowledge and skills in construction, use, and drawing of inferences from diagrams need to be cultivated in students if they are to benefit from using diagrams.

Outlines of the five studies undertaken for this dissertation are provided below.

Study 1 was a preliminary investigation into how correspondences between problem types and kinds of diagrams are represented in textbooks. One government-approved textbook series for elementary school level in Japan was examined for the types of mathematical word problems that the books deal with, and the kinds of diagrams presented with those problems. The analyses revealed significant differences in association between kinds of diagrams and types of problems. More concrete diagrams were included with problems involving change, combination, variation, and visualization of quantities; while number lines were more often used with comparison and variation problems. Tables and graphs corresponded to problems requiring organization of quantities, and more concrete diagrams and graphs to problems involving quantity visualization.

Study 2 investigated the effects of problem-appropriate diagram use instruction using an adaptation of the multiple baseline design to be able to distinguish the effects of each kind of diagram instruction on student performance on the different problem types as well as their perceptions of the mental effort required (cognitive load). The instruction for using line diagrams, tables, and graphs was provided to 67 junior high school students in a staggered manner, and the effects on their performance in solving three different types of problems was examined. The results showed that, following the appropriate diagram use instruction for each problem type, construction and use of problem-appropriate diagrams increased in test problems and persisted over time. More importantly, the instruction led to increases in problem solving performance (correct answer rates) and to decreases in perceived cognitive load.

Study 3 investigated the problem-appropriate diagram knowledge necessary to facilitate effective solving of particular kinds of mathematical word problems. We provided forty participants (15.0 ± 3.1 years) with problem-appropriate diagram knowledge (use of tables or graphs) and then mathematical word problems. In the pre-test, they seldom used diagrams. After intervention, increases in diagram use and correct answer rates occurred only in the problems that matched the problem-appropriate diagram knowledge provided. In addition, the results of EEG frequency power comparing the matched conditions detected differences corresponding to the behavioral results in both table and graph knowledge. More specifically, in the table matched condition, theta power increased in the left prefrontal area during construction and in the parietal area during use. In contrast, in the graph matched condition, theta power in the left parietal area increased during construction but showed no particular changes during use. The results clarified the need to teach students about using particular kinds of diagrams for different types of problems.

Study 4 was conducted to better understand the development of table use competence following the provision of task-appropriate instruction, focusing on both behavioral and neurophysiological evidence (i.e., brain activity, using functional near-infrared spectroscopy or fNIRS). Sixteen students (mean age 15.7 ± 2.9 years) were asked to solve mathematical word problems for which the use of tables (which is one kind of diagram) was deemed effective. Data collection progressed in three phases: (1) Pre-test without the demand for diagram use, (2) Pre-test with demand to use a table, and (3) Post-test (after participants received instruction on table use for problem solving). Although table use increased in Phase 2, it was only in Phase 3 that such use led to increases in correct answers. In Phase 3, fNIRS measurements also indicated an increase in blood flow to the frontal area

(DLPFC, VLPFC) of the brain usually associated with working memory activity. These results demonstrate important neurophysiological changes resulting from task-appropriate instruction that promotes effective strategy use and improves learning performance.

Study 5 also aimed to better understand the development of diagram use competence following the provision of task-appropriate instruction, following almost the same procedure as that used in Study 4. While Study 4 focused on tables, Study 5 focused on graphs. We asked twenty students (mean age 14.4 ± 2.5 years) to solve mathematical word problems for which the use of graphs (one kind of diagram) was deemed effective. Data collection progressed in three phases: (1) Pre-test without the demand for diagram use, (2) Pre-test with the demand to use a graph, and (3) Post-test (after the instruction on graph use). Although graph use increased in Phase 2, it was only in Phase 3 that such use led to increases in correct answers. In Phase 3, fNIRS measurements also indicated increased blood flow to the brain's frontal area. Similar to Study 4, these results demonstrate important neurophysiological changes resulting from task-appropriate instruction that promotes effective strategy use and improves learning performance. Thus, the results on graphs replicated those on tables.

Chapter 9, General Discussion, summarizes and integrates the findings from the five studies, and then discusses their theoretical, research, and practical significance. In Section 1, these four main findings are explained and discussed.

(1) Japanese mathematics textbooks suggest scaffolding diagrams (from concrete to abstract diagrams). However, for solving more complex word problems, textbooks propose only visualization but no explicit instruction for the specific appropriate diagrams. Hence, there is a clear gap in what they provide in relation to what needs to be learned (Study 1).
(2) Students' acquisition of problem-appropriate diagram knowledge decreases cognitive load, increasing spontaneous and appropriate use of

diagrams and enhancing success when solving the corresponding mathematical word problems (Study 2 and 3).

(3) Comparing behavioral (diagram use and correct answer rates) and neuroscientific (EEG frequency power) evidence corresponding to the acquisition of problem-appropriate diagram skills, we detected particular differences in both table and graph knowledge (Study 3).

(4) Instruction that includes procedural and conditional knowledge about problem-appropriate diagrams enhances the correct use of diagrams and success in mathematical word problem solving. Furthermore, it activates regions of the frontal lobe responsible for working memory activity when in the process of solving the problems (Study 4 and 5).

Section 2 describes the following three points as this dissertation's theoretical, research, and practical contributions/significance.

(1) Theoretical contributions/significance

This research provides useful insights into the use of self-constructed diagrams in problem solving. Our findings show the importance of paying attention to the cultivation of procedural and conditional knowledge, and of providing explicit instruction in problem-appropriate diagram use. We were able to confirm that such instruction leads to a reduction in cognitive load, which is necessary not only for promoting spontaneous diagram use, but also because it frees up cognitive resources for use in the actual problem solving. Furthermore, we were able to identify brain areas that are linked to cognitive processes involved in diagram construction and use for mathematical word problem solving.

(2) Research contributions/significance

From a study design perspective, we were able to demonstrate the value of using the multiple baseline design for demonstrating specific instructional effects on students' problem solving performance. The tasks and procedures we were able to successfully use to identifying links between instruction, problem solving performance, and brain activity are further research contributions.

(3) Practical contributions/significance

We believe that the instructional methods we have developed in this research can genuinely be useful for promoting effective diagram use in mathematical word problem solving. Teachers should be able to use them in their classroom instruction, and such use should enable more students to more successfully solve mathematical word problems.

Section 3 describes the following three points as limitations of this dissertation.

(1) The diagrams examined were only line diagrams, tables, and graphs.

(2) The sample sizes were small.

(3) The instruction sessions were mainly provided by the researcher, not real school teachers. This means we need to verify in real school settings the usefulness of those instructional methods.

Section 4 explains important directions for future research, including a need to further examine the cognitive properties that determine the problem-appropriateness of different kinds of diagrams, for problem solving purposes, but also for other purposes such as for learning, thinking, and communication.