Bayesian model updating of a simply-supported truss bridge based on dynamic responses

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ABSTRACT

This study intends to investigate the application of model updating based on forced vibration data to a simply-supported truss bridge. A fast Bayesian FFT method was used to perform the modal identification obtained from field tests, and the Transitional Markov Chain Monte Carlo (TMCMC) algorithm is employed to generate samples. Although updating as many parameters as possible is the ideal model update process, it is not practical to identify all the parameters because of limitation of the experimental data. The bridge was thus divided into several clusters, and the values of the updated parameters of the members in the same cluster are assumed to be equal. Two model updating schemes were discussed as an example to investigate the effect of parameter selection, such as how to model the spring at each support, in model updating process. It was observed that although models with more parameters tend to fit better, the updated result often showed a different trend from the engineering prediction.

Keywords: Bayesian model updating, Transitional Markov Chain Monte Carlo, Field vibration test, Damage detection, Simply-supported truss bridge

1. INTRODUCTION

In structural engineering, finite element (FE) models are widely used for structural analysis. Comparing to field experiments, FE analysis can save time and costs. However, due to the limited available information and simplification in modeling, uncertainties, such as material properties, geometric properties, boundary conditions and load conditions, invariably exist in the system. Model updating methods would calibrate these uncertain parameters in the FE model based on the measurement data, so called a data-driven model calibration.

One type of model updating method is based on Bayesian theory, which tries to find a probability distribution function (PDF) of the model parameters [1-11]. Au and Beck [3] and Beck and Au [4] applied the Bayesian-based method to reliability analysis. Beck and Yuen [5] and Muto and Beck [6] sought the most probable model from several model classes. Goller and Schueller [7] and Goller et al. [8] investigated uncertainties in the Bayesian model updating and weighting factors of each mode in the objective function. Yuen et al. [9] and Lam et al. [10,11] extended the applicability and efficiency of the Bayesian-based method.

Although the Bayesian model updating method can provide a posterior distribution, the complexity of its PDF makes it difficult to generate samples directly from the posterior distribution. Therefore, an efficient sampler is necessary. A lot of sampling methods have been proposed, especially Metropolis–Hastings (MH) algorithm [12,13], a special case of Markov chain Monte Carlo (MCMC) [14,15]. Beck and Au [16] proposed Adaptive MH algorithm (AMH), using a sequence of intermediate distributions to converge on target distribution. Ching and Chen [17] used the importance sampling to replace the kernel density estimate (KDE) in AMH, called the transitional Markov chain Monte-Carlo (TMCMC) method. Goller et al. [18] proposed the parallelized MCMC.

The degree of freedom of the engineering structure is much higher than the laboratory model, and a typical engineering structure may consist of hundreds to thousands of members. Therefore, it is not practical for a member-level or element-level model updating of the actual engineering structure. Grouping multiple members into a cluster and assuming the members in the same cluster will share the same values of uncertain parameters can reduce the number of uncertain parameters to be updated if a suitable parameter selection scheme is available. Especially for the FE model update for damage detection, a proper design of the model update scheme will directly link to the accuracy of the damage detection by mans of the FE model update. Moreover, with the growing interest in digital twins for civil infrastructures, the study of the applicability of FE model updating to various structural states is an extremely important topic. However, few studies investigate how to group structural members in to a cluster considering engineering significance of the updated model even in terms of damage detection, which is a driving force of this study.

This study intends to investigate the effect of updating structural parameters selection on the model updating of an actual simply-supported truss bridge even including the bridge with damage based on the Bayesian model updating method. The fast Bayesian FFT [19,20] was used to identify the structural dynamic properties, and the TMCMC algorithm was used to generate samples. Two different model updating parameter schemes focusing the spring at each support were examined and discussed about the engineering significance of the updated results.

2. METHODOLOGY

2.1. Bayesian model updating method

The formulation of the posterior PDF of uncertain parameter vector θ ($\theta \in \mathbf{R}^{N_\theta}$), under the condition system response D is given as follows.

$$P(\theta|D,M)=\frac{P(D|\theta,M)P(\theta|M)}{P(D|M)} \tag{1}$$

where M is the assumed probabilistic model class for the structure; $P(D|M)$ is the evidence of model class M, and $P(D|M)=\int P(D|\theta,M)P(\theta|M)d\theta$; $P(\theta|M)$ is the prior PDF; $P(D|\theta,M)$ is the likelihood function, which donates the conditional probability of D given θ; N_θ is the number of uncertain parameters.

Even if the experimental data available is insufficient to constraint all updated parameters, the Bayesian model updating methods also can provide a posterior distribution of the uncertain parameters.

Assuming the variances is same for each natural frequencies and mode shapes, the likelihood function becomes:

$$P(D|\theta,M)=ce^{-J(\theta)/2\sigma^2} \tag{2}$$

where $J(\theta)=\sum_{i=1}^{N_m}\left((1-\langle\varphi_i(\theta),\hat{\varphi}_i\rangle^2)+\left(1-f_i(\theta)/\hat{f}_i\right)^2\right)$; $f_i(\theta)$ and $\varphi_i(\theta)$ are the ith natural frequency and normalized mode shape vector under the given uncertain parameter vector θ obtained from the FE model while $\hat{f}_i$ and $\hat{\varphi}_i$ are the measured values from the experiments; N_m is the total number of modes; c_l is the normalized constant which lead $\int P(D|\theta,M)dD=1$.

2.2. Transitional Markov chain Monte Carlo

In high-dimensional space, the MH algorithm also cannot keep a higher efficiency. Therefore, many researchers have proposed improved MCMC samplers. In this study, the TMCMC is used generate samples efficiently. The TMCMC is based on the AMH method. The advantage of TMCMC is that it does not need kernel density estimation (KDE) which is difficult to calculate in high-dimension space. The essence of TMCMC is to use a series of asymptotic intermediate distributions, $(P(\theta|D)^{(j)}(j=1,2,3\ldots))$, to approach the final distribution. The importance sampling method is used to transfer between the intermediate distributions, $(P_j(\theta|D), j=0,1\ldots m,\ \theta\in\mathbf{R}^{N_\theta})$. With the value of j increasing, the $P(\theta|D)^{(j)}$ becomes more closed to the target distribution $P(\theta|D)$. MCMC approach is to solve the problem that the number of distinct samples reduces due to the re-sampling progress.

For the asymptotic intermediate distribution $P_j(\theta|D,M)$, the values of their variances $(\sigma_j, j=1\ldots m)$ are different, and $P_j(\theta|D,M)$ is shown as:

$$P_j(D|\theta,M)=c_je^{-J(\theta)/2\sigma_j^2} \qquad (\sigma_1>\ldots>\sigma_m=\sigma, j=1,2\ldots m) \tag{3}$$

Especially, assuming $P_0(D|\theta,M)=c_0$ follows uniform distribution. The adjacent intermediate distributions are connected by importance sampling, and the weighting $w_j(\theta)$ is shown as:

$$w_j(\theta)=\frac{P_{j+1}(\theta|D,M)}{P_j(\theta|D,M)} \tag{4}$$

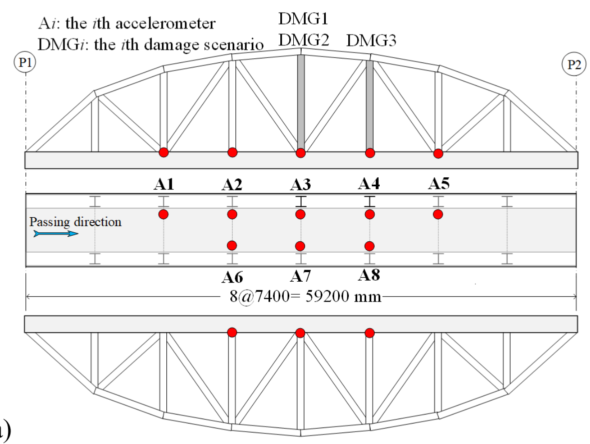
Then, with the normalized value weighting $w_{j,k}^n$, the sample sequence of $\theta_{j+1,k}$, which follows the distribution of $P_{j+1}(\theta|D,M)$, is generated from $\theta_{j,k}$.

$$\theta_{j+1,k}=\theta_{j,k}, \text{ with the probability } w_{j,k}^n=\frac{w_j(\theta_{j,k})}{\sum_{k=1}^{N_s}w_j(\theta_{j,k})} \tag{5}$$

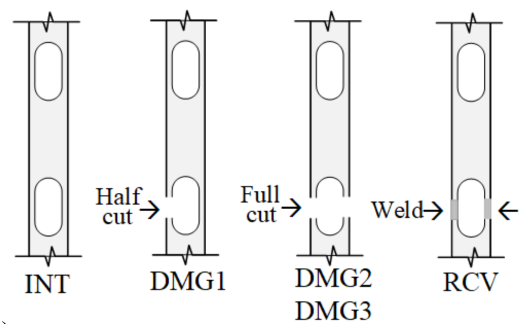
3. TARGET BRIDGE AND VIBRATION TEST

This section would briefly introduce the target bridge, field tests, and modal identification. More details can be found in reference [21-23].

Figure 1. Mode shapes of the building.



(a)



(b)

Figure 2. (a) Sensor layout and (b) damage scenario

3.1. Target bridge and FE model

The target structure was a simply-supported steel truss bridge, as shown in Fig 1. The length and width of the main span are 59.2m and 3.6m. It was built in 1959 and removed in 2012. The ambient and vehicle-induced vibration experiments were conducted before the bridge removal. The bridge was closed prior to the experiment.

The FE model in ABAQUS is created based on shell elements for concrete slabs and beam and truss elements for steel members. The boundary conditions were treated as perfect roller and pin. However, the boundary conditions are a significant source of uncertainties in the model, and three types of springs were considered and added to the supports at P1 and P2.

3.2. Field test and Damage scenario

Eight uniaxial accelerometers were installed on the deck of the bridge to measure the vibration data as shown in Fig. 2 (a). Five damage scenarios (INT, DMG1, DMG2, RCV, and DMG3) were considered consecutively in this bridge as shown in Fig. 2(b). As a damage, two tension members were severed as shown in Fgi.2(b). A 21 kN vehicle was used to excite the bridge with an average speed of about 20 km/h under each damage scenario.

3.3. Modal identification

Prior to the model updating process, modal identification based on experimental data is necessary. This section would take the INT state as an example to illustrate the modal identification process.

The PSD and singular value spectrum of acceleration data measured by all eight sensors under the INT state are shown in Fig. 3. Vibration modes of six peaks indicate well-excited modes, and are taken as the candidate modes to be updated. The natural frequencies of these six modes were close to 3 Hz, 5 Hz, 7 Hz, 9.5 Hz, 10.5 Hz, and 13Hz, respectively. To reduce random errors, the same experimental procedure was repeated about 10 times. The last mode, which is close to 13Hz, is not stable because it cannot be found in most runs, therefore, this mode is not considered. Their histograms with the normal distribution fit are shown in Fig. 4 with those from damage cases (DMG1, DMG2, RCV, and DMG3). The MPV value and coefficient of variation of the frequency are summarized in Table 1. The corresponding mode shapes of INT state are presented in Fig. 5(a). It includes first bending mode, first torsional mode, second bending mode, second torsional mode, and third bending mode from the top to the bottom. Those mode shapes for the DMG2 and DMG3 states are shown in Fig. 5 (b) and Fig. (c) respectively. It is noted that modes shapes under DMG3 states differed from those under INT and DMG2 states.

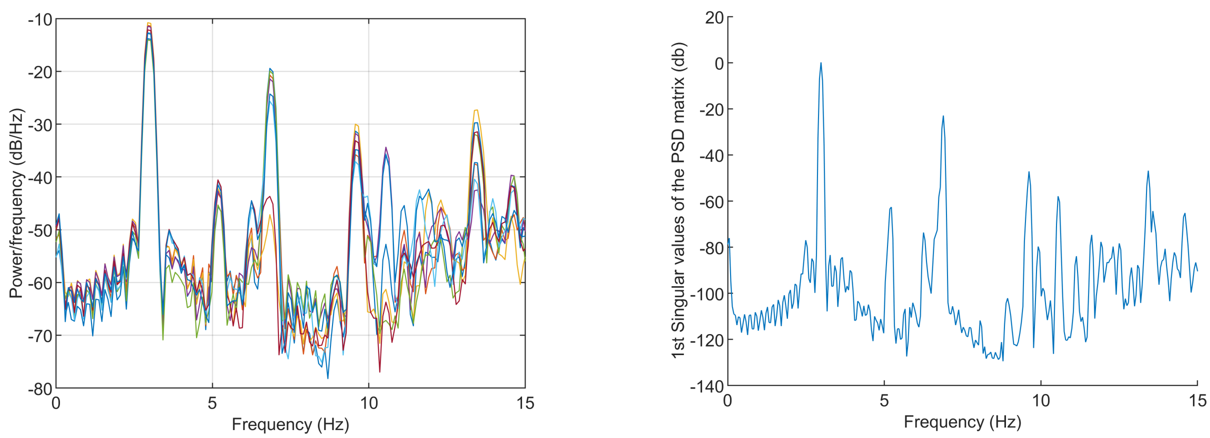


Figure 3. (a) PSD and (b) singular value spectrum estimate for all channels.

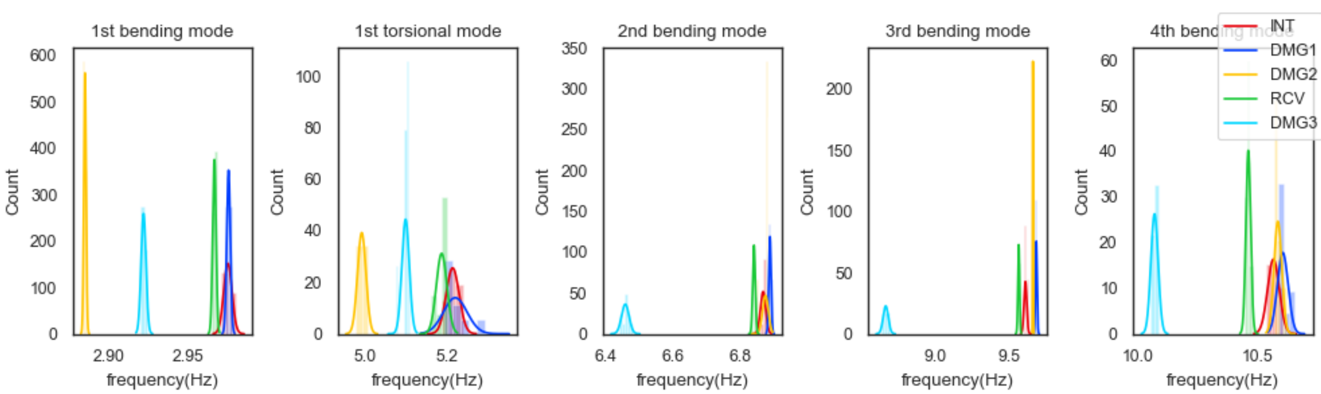
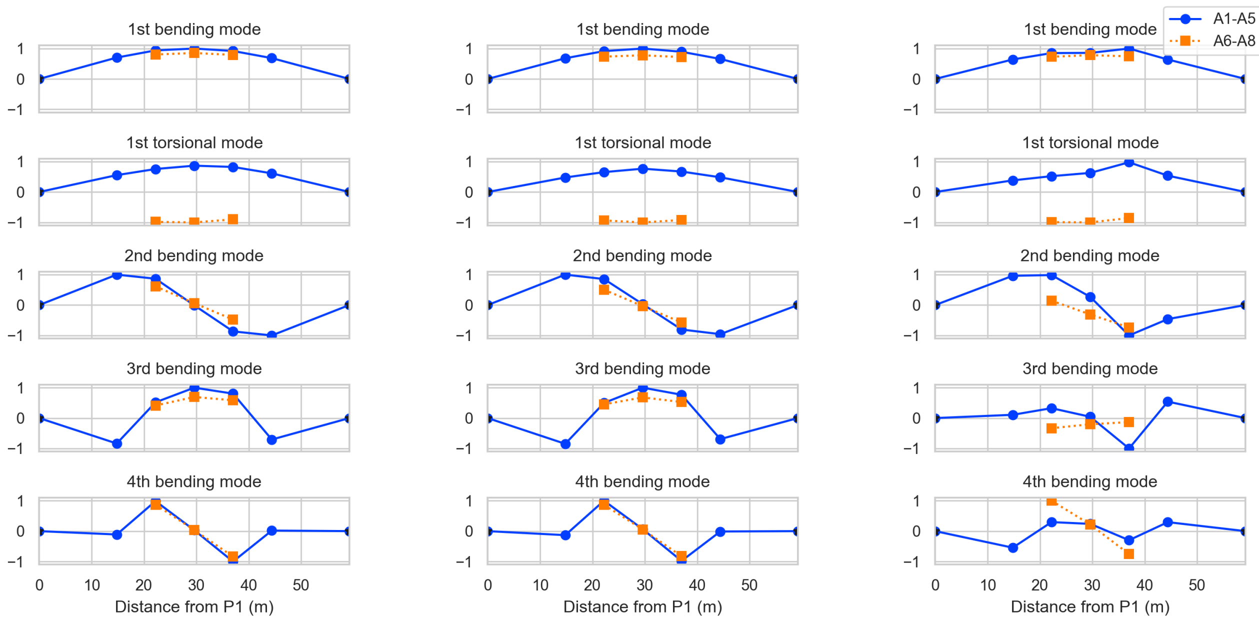


Figure 4. Histograms with the normal distribution fit of the identified frequencies

Table 1. Identified natural frequencies (MPV: most probable value; CV: coefficient of variance)

Mode	1st bending frequency		1st torsional frequency		2nd bending frequency		3rd bending frequency		4th bending frequency	
	MPV (Hz)	CV (%)	MPV (Hz)	CV (%)	MPV (Hz)	CV (%)	MPV (Hz)	CV (%)	MPV (Hz)	CV (%)
INT	2.98	0.0932	5.21	0.3181	6.87	0.0088	9.61	0.1087	10.57	0.2472
DMG1	2.98	0.0397	5.22	0.5803	6.89	0.0508	9.69	0.0571	10.61	0.2229
DMG2	2.89	0.0259	4.99	0.2232	6.88	0.1308	9.67	0.0195	10.59	0.1623
RCV	2.97	0.0378	5.19	0.2603	6.84	0.0571	9.57	0.0603	10.46	0.1015
DMG3	2.92	0.0555	5.10	0.1870	6.46	0.1800	9.66	0. 3200	10.07	0.1686



(a) INT state *(b) DMG2 state* *(c) DMG3 state*

Figure 5: Identified mode shape

4. MODEL UPDATE

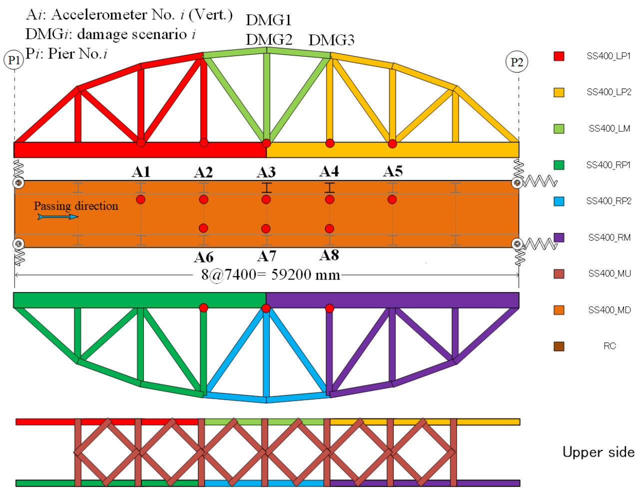
4.1. Model updating scheme

The FE model consists of more than one hundred members and contains approximately three thousand elements. As a result of the limited experimental information, no matter member-level or element-level the model updating is impossible. To reduce the number of the updating parameters, several substructure blocks, each of which consists of multiple members, were imported into the system. In the same block, the material properties of members are set to be the same value. Indeed, the types of defined uncertain parameters have a significant impact on the Bayesian FE model updating process.

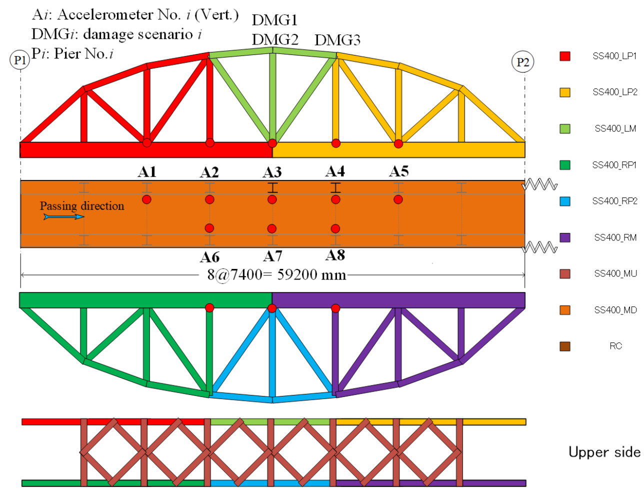
To discuss the effect of different model updating schemes, two types of schemes were investigated, as presented in Fig. 6 (a) and Fig. 6(b), in which Case 1 model indicates the FE model considering two horizontal springs, four vertical springs, and four rotation springs at supports while Case 2 model indicates the FE model considering springs in the longitudinal direction at P1. The main part of bridge model was divided into eight blocks, and three types of model parameters, such as spring constants, steel stiffnesses and concrete stiffnesses, are considered in both two cases. The parameters starting with 'SS400' and 'spring' indicate the stiffness of steel members and spring constants, respectively. 'RC' means the stiffness of the reinforced concrete. Three types of springs, two horizontal springs, four vertical springs, and four rotation springs, are taken as candidate parameters to present the uncertainties in the boundary conditions. The difference between Case 1 and Case 2 is the number of considered springs. In Case 1, all three types of springs were taken into account; in Case 2, only the two horizontal springs were considered, constants of vertical springs $=\infty$, and constants of rotation springs = 0.

4.2. Case 1 model: FE model considering two horizontal springs, four vertical springs, and four rotation springs at supports

Table 2 presents the MPV of the updated natural frequencies and MACs for Case 1. From INT state to DMG2 state, a full cut was applied to the tension member on the sensor A3. Therefore, the decreasing of SS400_LM was the expected phenomenon. However, as shown in Figure 7 the updated distributions SS400_LM is increased while the updated spring constants are decreased.

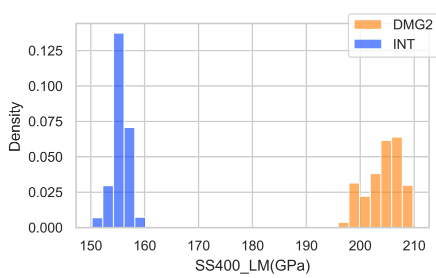


(a) Case 1 model: FE model considering two horizontal springs, four vertical springs, and four rotation springs at supports

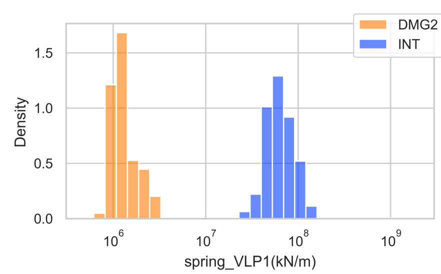


(b) Case 2 model: FE model considering springs in the longitudinal direction at P1

Figure 6. Model updating scheme



(a) SS400_LM



(b) Spring_VLP1

Figure 7. Updated parameters of Case 1 model: FE model considering two horizontal springs, four vertical springs, and four rotation springs at supports.

Table 2. Updated natural frequencies of Case 1 model: FE model considering two horizontal springs, four vertical springs, and four rotation springs at supports.

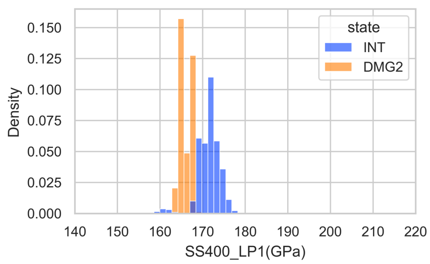
Mode	1st bending mode		1st torsional mode		2nd bending mode		3rd bending mode		4th bending mode	
	MPV (Hz)	MAC	MPV (Hz)	MAC	MPV (Hz)	MAC	MPV (Hz)	MAC	MPV (Hz)	MAC
INT	3.04	0.9967	4.69	0.9968	6.70	0.9947	9.59	0.9982	10.54	0.9978
DMG2	2.99	0.9986	4.84	0.9848	6.99	0.9912	9.78	0.9907	10.85	0.9909

MPV: most probable value of the frequency; MAC: modal assurance criteria

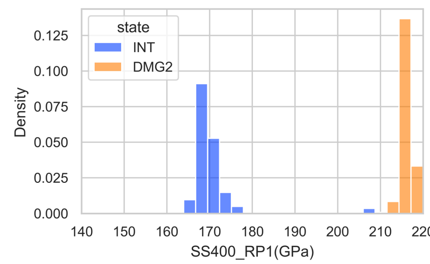
Table 3. Updated natural frequencies of Case 2 model: FE model considering springs in the longitudinal direction at P1.

Mode	1st bending mode		1st torsional mode		2nd bending mode		3rd bending mode		4th bending mode	
	MPV (Hz)	MAC	MPV (Hz)	MAC	MPV (Hz)	MAC	MPV (Hz)	MAC	MPV (Hz)	MAC
INT	3.0928	0.9997	5.0579	0.9939	6.6838	0.9808	10.117	0.9961	10.631	0.9668
DMG2	3.0857	0.9992	4.7919	0.9852	6.5582	0.9921	9.8276	0.9907	10.281	0.9986

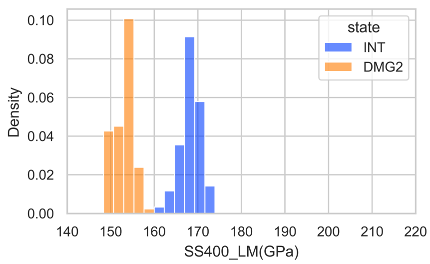
MPV: most probable value of the frequency; MAC: modal assurance criteria



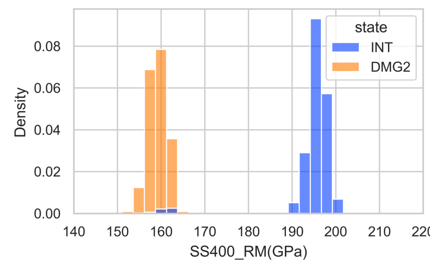
(a) SS400_LP1



(b) SS400_RP1



(c) SS400_LM



(d) SS400_RM

Figure 8. Updated parameters of Case 2 model: FE model considering springs in the longitudinal direction at P1.

4.3. Case 2 model: FE model considering springs in the longitudinal direction at P1

MPV of the updated natural frequencies and MACs for Case 2 are summarized in Table 3. Fig. 8 shows the distributions of some updated parameters. A decrease of SS400_LM can be observed as expected. Due to the DMG2 damage, SS400_LP1 and SS400_LP2 were almost unchanged, but SS400_RM was decreased despite an increase of SS400_RP1 and SS400_RP2.

4.4. Discussion

It was not reasonable to consider many springs at supports like Case 1 model. For Case 2 model, after removing vertical and torsional springs, the updated result as expected was observed even though the updated stiffness for some members in the opposite sides of the damage was increased. Different amounts of sensor information of the members on two sides might affect the model updating results. It indicates the importance of the parameter selection based on the number of sensors, the location of sensors, the type of sensor, and the design of the bridge model for FE model update considering engineering goals.

5. CONCLUSIONS

This study indicates that the deployment of sensors, how many sensors are installed, or where is the location of the sensors, affects the selection of parameters. How to choose the updated parameters is a significant problem in model updating especially for damage detection. On the one hand, the limited data cannot constrain too many parameters, which leads to an unreasonable solution; on the other hand, decreasing the number of parameters means less identified information from the updated model.

Model updating problem is essentially an optimization problem, and the criterion of the optimal model is the objective function. The model update process finds the mathematically optimal solution based on the objective function. However, the purpose of model updating is not to obtain a model that can fit the objective function well but to find parameter values that can correctly reflect the real situation of the bridge. In other words, the goal of the FE model update is to identify the parameter distributions with physical meanings. The real challenge in the application of the model updating to damage detection is narrowing the gap between the engineering meaning and mathematical optimal model.

Existing studies on the FE model update have not fully investigated the feasibility of the FE model update for damaged structures and damage detection. Therefore, considering the FE model update for damage simulation and damage detection, further comprehensive investigations, such as proper parameter selection, proper deployment of sensors, are needed.

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